Application of Local Lyapunov Exponents to Maneuver Design and Navigation in the Three-Body Problem

Rodney L. Anderson[i], Martin W. Lo[ii], George Born[i]

Abstract

Dynamical systems theory has recently been employed for several missions to design trajectories within the three-body problem. This research applied a stability technique, the calculation of local Lyapunov exponents, to such trajectories. Local Lyapunov exponents give an indication of the effects that perturbations or maneuvers will have on trajectories over a specified time. Results from this technique present a possible explanation for the effectiveness of maneuvers at certain points on unstable orbits, and appear to have the potential to aid maneuver design. It also has applications to navigation and the planning of tracking for spacecraft missions.

Extended Abstract

New methods have recently been developed using dynamical systems theory in an effort to aid in the design of trajectories within the three-body problem. These techniques have been applied to trajectory design for several different missions. These missions include the Genesis mission, which has successfully traveled along the initial portion of its trajectory[1,2]. Scheeres et al. have also applied dynamical systems techniques to navigation and maneuver design for such missions[3,4]. For a spacecraft traveling on a Lissajous orbit around one of the Lagrange points, it is desirable to search for points in state space where perturbations might have the greatest effect on the trajectory. Determination of these points would indicate where it would be most desirable to reduce uncertainties in the spacecraft's state. It would also reveal the points where a maneuver could be most effective. A possible method for achieving this objective would be to apply stability techniques to the three-body problem.

One stability technique for quantifying the effect of perturbations at different points on a trajectory is the calculation of local Lyapunov exponents. Local Lyapunov exponents are based on Lyapunov exponents, which are used to measure the convergence or divergence of nearby trajectories in a dynamical system[5]. Whereas Lyapunov exponents characterize the dynamical system as a whole and do not depend on any specific orbit, local Lyapunov exponents provide an indication of the effect a perturbation would have on a particular trajectory over finite times. Abarbanel et al. have examined the calculation of local Lyapunov exponents for several dynamical systems[6]. In this

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paper, the calculation of local Lyapunov exponents has been implemented in both the circular restricted three-body problem and in JPL's libration point mission design tool (LTool), which includes the JPL ephemerides for the desired bodies. The local exponents have been calculated for Lissajous orbits as well as along their stable and unstable manifolds.

The results from LTool for a quasi-periodic orbit around the L1 libration point in the Earth-Moon system are shown in Figure 1. In this plot, the trajectory is shown in the rotating coordinate frame, and the magnitude of the local Lyapunov exponents, which were calculated over two-day time periods, is represented by the color. The magnitude of the local Lyapunov exponents in this case indicates the effect a perturbation to a trajectory would be expected to have over two days. The trajectory in this case was obtained from AUTO2000[7] scripts that were used to obtain initial conditions for a differential corrector in LTool[8].

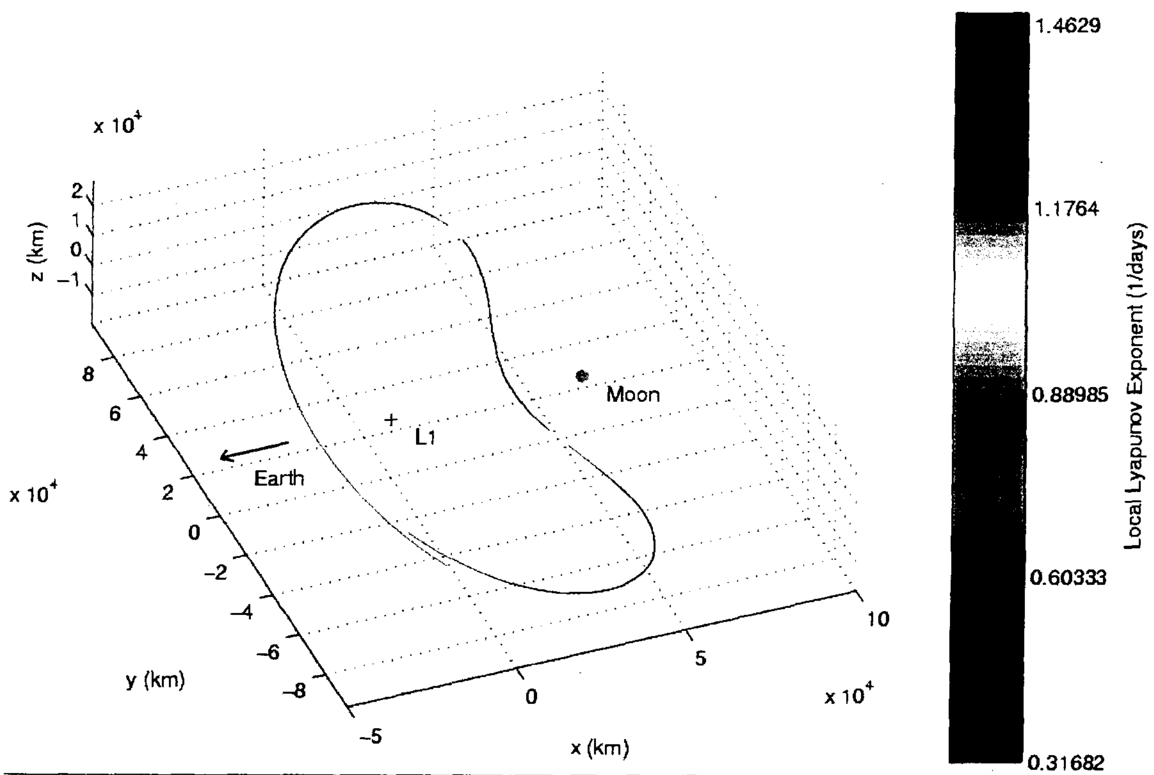

Figure 1 Local Lyapunov exponents on a quasi-periodic orbit in the Earth-Moon system

The local Lyapunov exponents in this plot generally increase on the portions of the trajectory that are closest to the Earth and Moon. In this case, the most obvious increase is on the portion of the trajectory near the Moon. It has been observed that maneuvers on quasi-periodic orbits appear to be most effective when they take place near the line connecting the primaries[9]. The increase in the local Lyapunov exponents near these areas does indeed indicate that perturbations or ΔVs would have a greater effect and may provide an explanation for why this occurs. This could help select the locations for statistical maneuvers in order to reduce the magnitude or number of these small burns and extend the life of the mission. Currently, the local Lyapunov exponents are being compared to actual perturbations and ΔVs to verify some of these observations.

As mentioned previously, the calculation of local Lyapunov exponents should provide a method for determining the possible effect of a perturbation to the state. These perturbations could include deviations from a nominal trajectory caused by uncertainties in the state. In reality, the magnitude of the actual uncertainties would be related to the number and timing of the spacecraft observations. This technique would therefore provide a method for scheduling the best times and locations for taking observations of the spacecraft to reduce unnecessary observations and ensure that the desired uncertainties are obtained. In general, the local Lyapunov exponents are related to how the dynamics will affect knowledge of the position in the future given that the uncertainties at some initial epoch are known. If nearby trajectories diverge, it will be more important to know the state well. This method is being compared with conventional navigation approaches in order to determine the benefits it may have for unstable orbits.

Overall, these methods may contribute to both maneuver design and navigation. They may be able to aid in determining the effect dynamics will have on uncertainties as well as in planning observations. They might also aid in maneuver design and perhaps eventually allow the automation of maneuver planning on unstable orbits.

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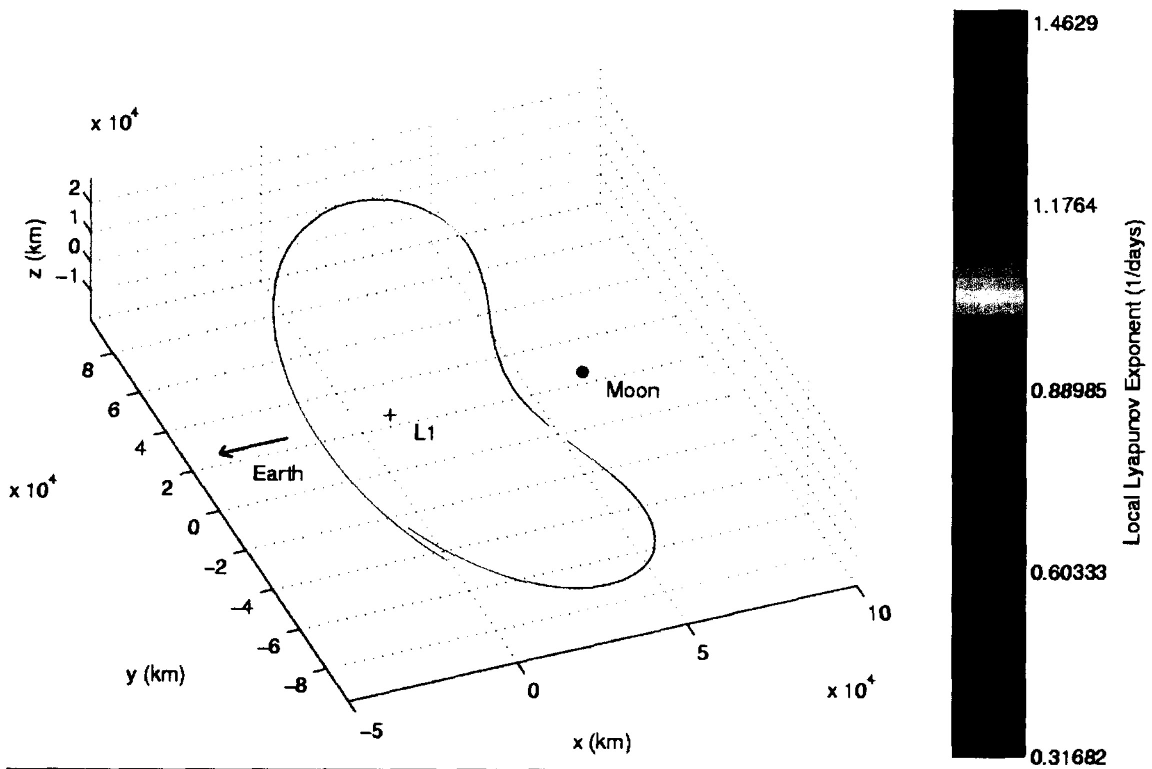

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