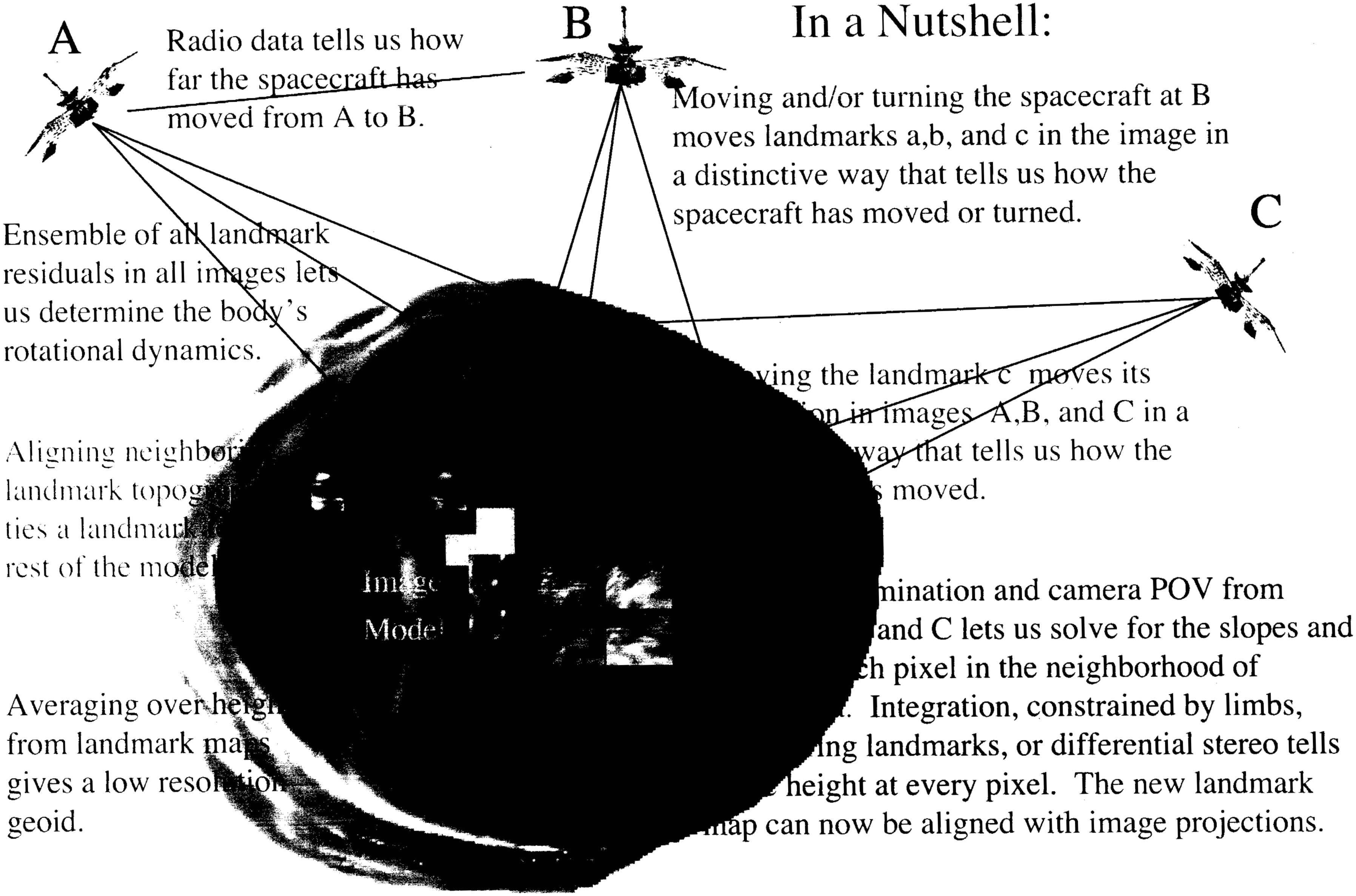
In a Nutshell:

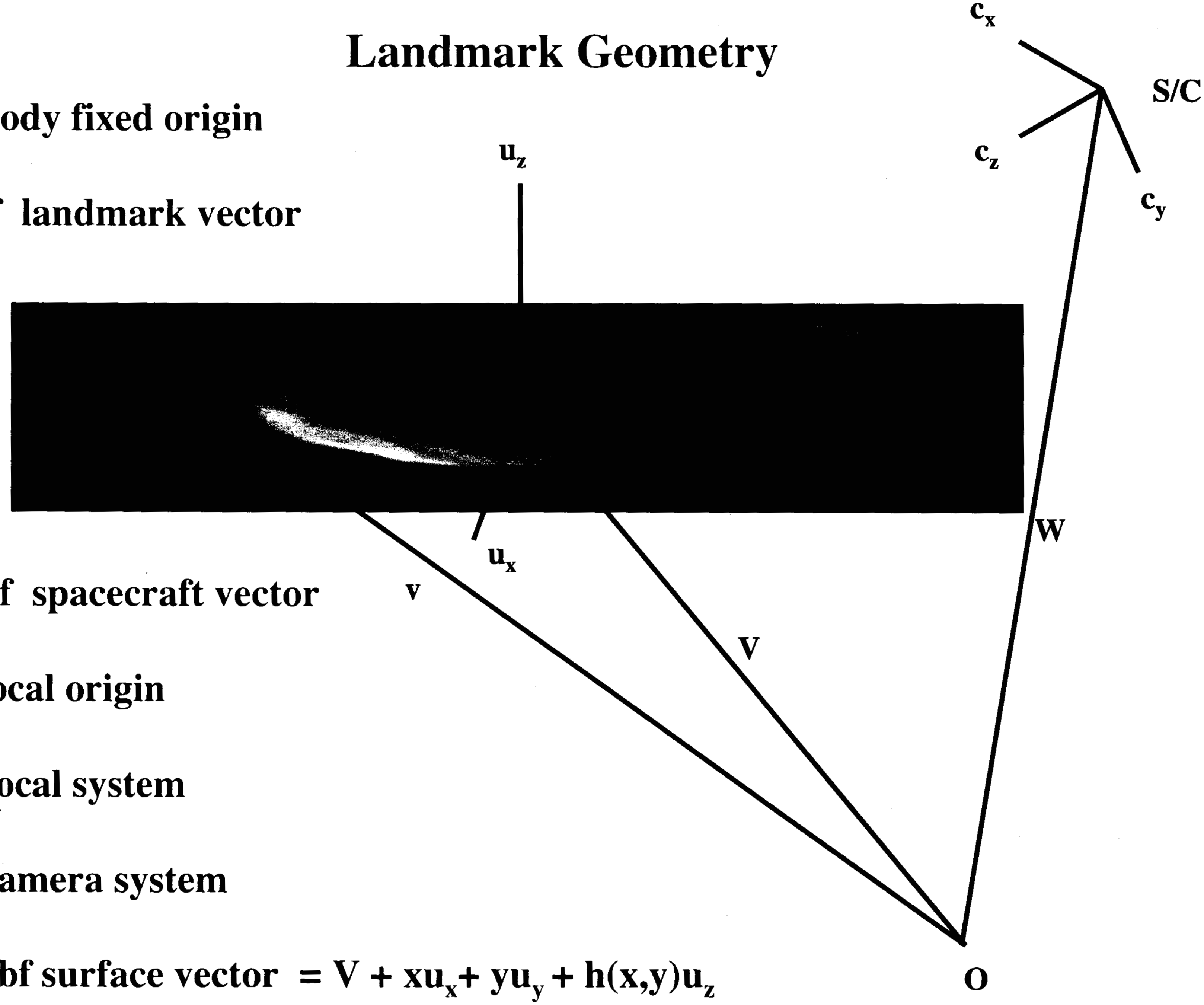


Integrating over landmark slopes, projected onto the geoid and constrained by a sparse set of geoid vectors, produces a high resolution global topography model.

Landmark Geometry

O = body fixed origin

V = bf landmark vector



W = bf spacecraft vector

o = local origin

u_k = local system

c_k = camera system

v = bf surface vector $= V + xu_x + yu_y + h(x,y)u_z$

Landmark Illumination

Landmark model illuminated in local frame according to

$I(x,y) = I_0(1+t_3(x,y))F(\cos i,\cos e)+ \Phi$

$\cos i = (s_1t_1+s_2t_2+s_3)/\sqrt{(1+t_1^2+t_2^2)}$, $\cos e = (e_1t_1+e_2t_2+e_3)/\sqrt{(1+t_1^2+t_2^2)}$

$t_1 = -\partial h/\partial x$, $t_2 = -\partial h/\partial y$, $1+t_3$ = relative albedo, Φ = background,

I_0 =normalization, s_k= local sun vector, e_k= local camera vector

The function F(cosi,cose)=cosi+2cosi/(cosi+cose) does a good job of reproducing imaging data.

The re-illuminated landmark model can be correlated with …

Landmark Image Projection

Landmark point (x,y,h) maps to focal plane location (X,Y) with

$$X = f((V-W)\bullet c_1+M_{11}x+M_{12}y+M_{13}h)/((V-W)\bullet c_3+M_{31}x+M_{32}y+M_{33}h)$$

$$Y = f((V-W)\bullet c_2+M_{21}x+M_{22}y+M_{23}h)/((V-W)\bullet c_3+M_{31}x+M_{32}y+M_{33}h)$$

where f=focal length and $M_{ij}=c_i\bullet u_j$

... data extracted from the corresponding images to locate the landmark center accurately in each image.

It is these locations that enable the stereographic determination of the landmark vectors, spacecraft location and orientation, and the object's rotational dynamics.

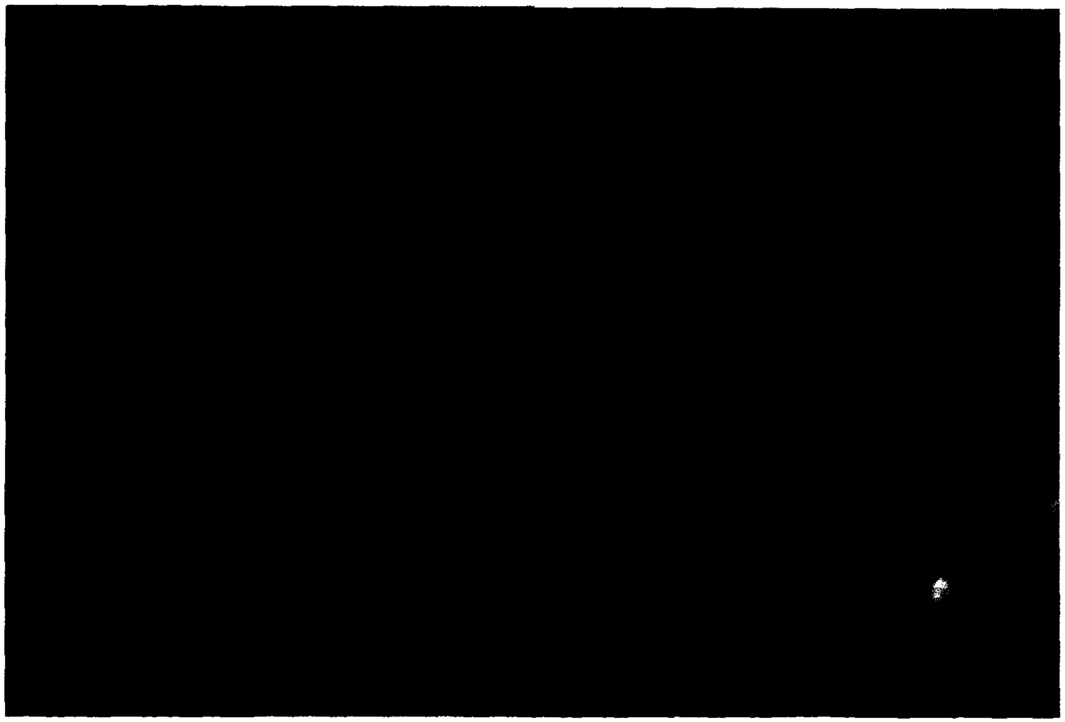
Mutual Landmark Registration

A landmark model is registered to the shape model or to another overlapping landmark map by correlating gradients

Limb Projection

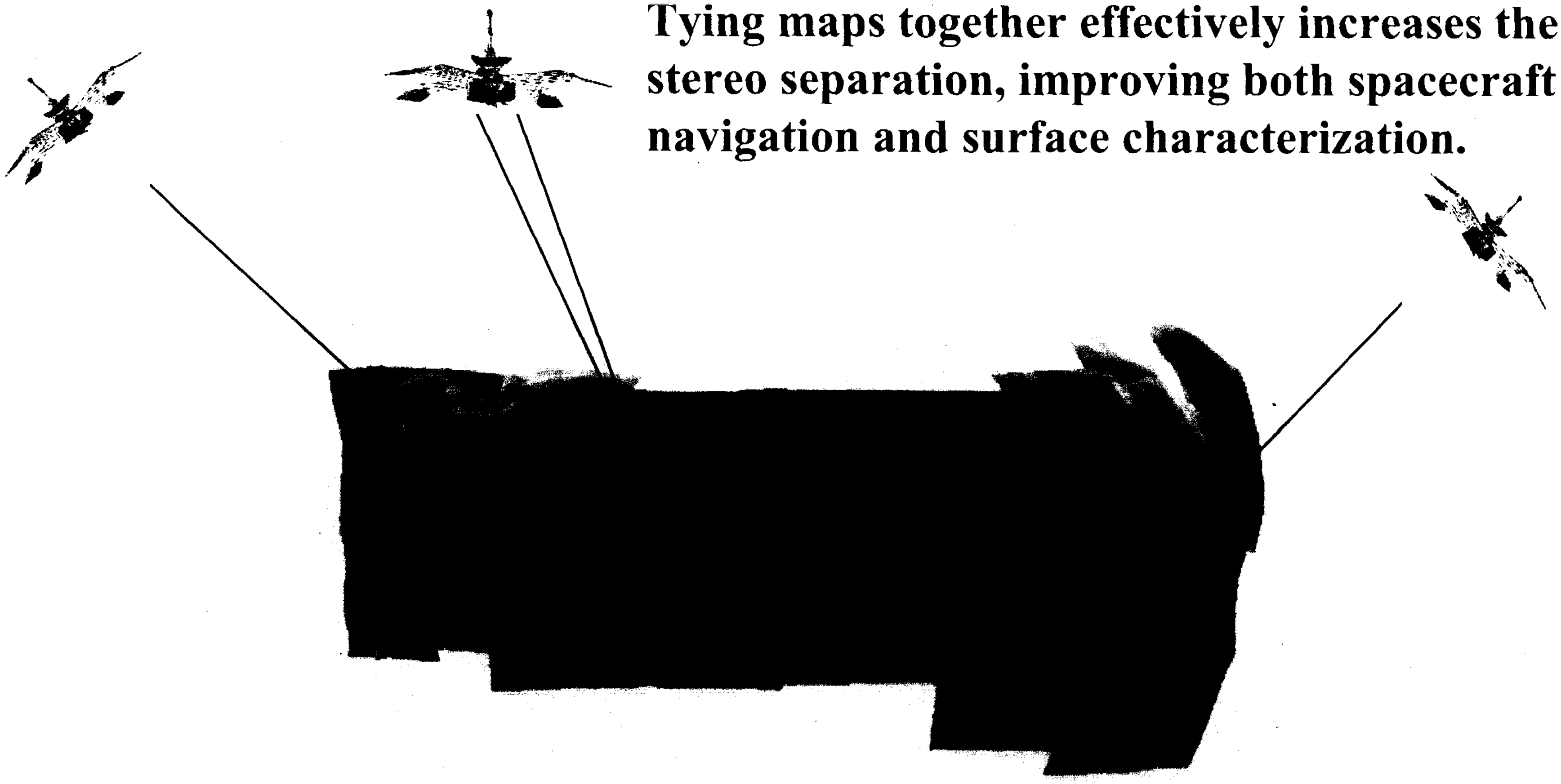
Landmark model is projected into image space and limb residuals are determined

Phobos landmark map (60 m) | **Viking Orbiter image 315A11 (cropped)**



Enhanced Stereography

Tying maps together effectively increases the stereo separation, improving both spacecraft navigation and surface characterization.



Formal uncertainties for a typical landmark:

simple stereo (28 images):	311m	simple stereo + limbs (35 images):	179m
enhanced stereo:	118 m	enhanced stereo + limbs:	95 m
Post fit residuals (all lmk):	47 m	rms residuals from shape (all lmk):	15 m

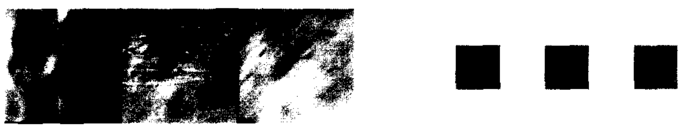
Stereophotoclinometry

Many images with many sun and viewing angles are used to construct the landmark templates. The slopes $-t_1$ and $-t_2$ and the relative albedos $1+t_3$ are determined from the following minimization procedure (see Landmark Illumination slide):

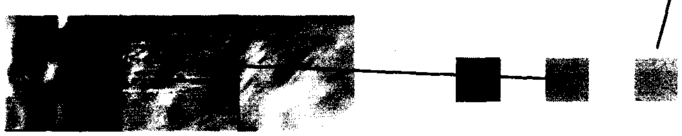
At each pixel (x,y) of the map, minimize

$$\sum_k (E_k(x,y) - I_k(x,y,t) - \delta t \bullet \nabla_t I_k(x,y,t))^2$$

E_k = Data from kth image at (x,y)



I_k = Prediction for kth image data at (x,y)



Since illumination and/or viewing angle are different, predicted pixel brightness will change differently for each image as slope and or albedo changes. We try to get predicted brightness as close to extracted brightness as possible.

Since pixels will not line up exactly, maps will be fuzzy at first. As vectors, heights and camera pointing become better, maps will sharpen up.

Only relative photometry is used. The normalization factor I_0 and background Φ are solved for based on the large scale topographic variations known from stereo, shape, limb, or overlapping map data. Essentially, this provides an interpolation algorithm for topography down to the pixel scale.

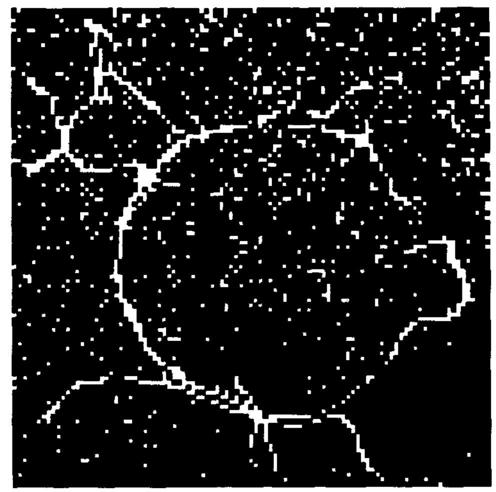
Height Integration

The height at each location (x,y) is determined from the neighboring heights, and a possible constraining height h_c from shape model, differential stereography, limb or overlapping map data, according to:

$$
\begin{aligned}
h(x,y) = [&w_c h_c(x,y) \\
&+h(x+s,y)+s(t_1(x,y)+t_1(x+s,y))/2 \\
&+h(x-s,y)-s(t_1(x,y)+t_1(x-s,y))/2 \\
&+h(x,y+s)+s(t_2(x,y)+t_2(x,y+s))/2 \\
&+h(x,y-s)-s(t_2(x,y)+t_2(x,y-s))/2]/(w_c+4)
\end{aligned}
$$

where s is the map pixel spacing and w_c is a small constraining weight.

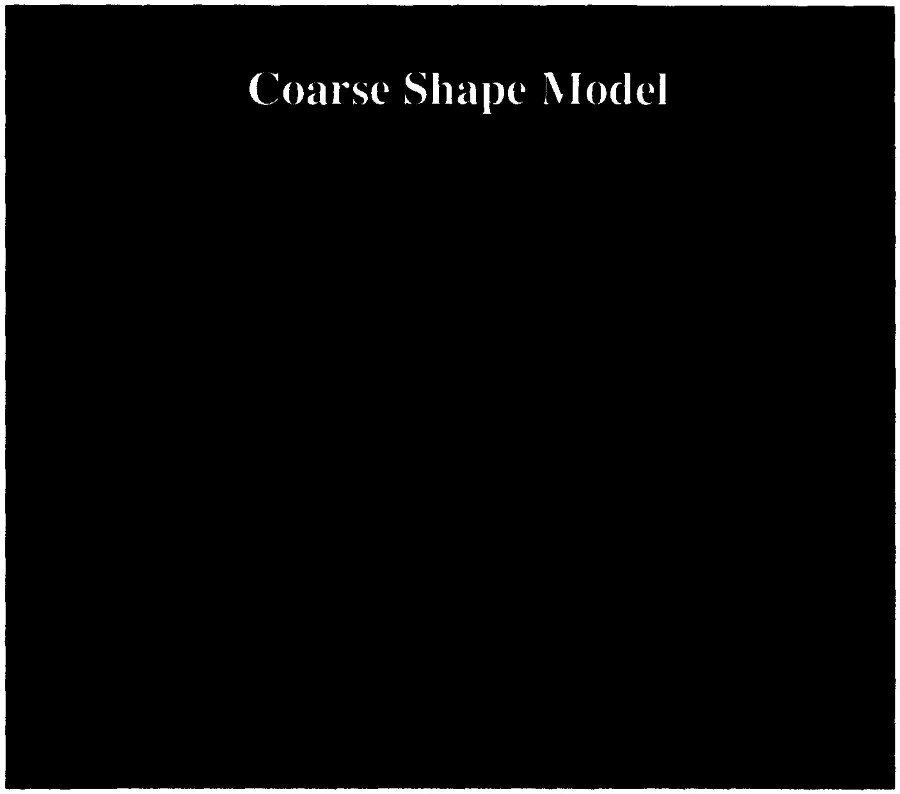
This equation is applied repeatedly to map points chosen at random until a converged solution is reached. If any height does not exist, its term is not included in the average.



Limb and overlapping map points used in a slope to height integration.

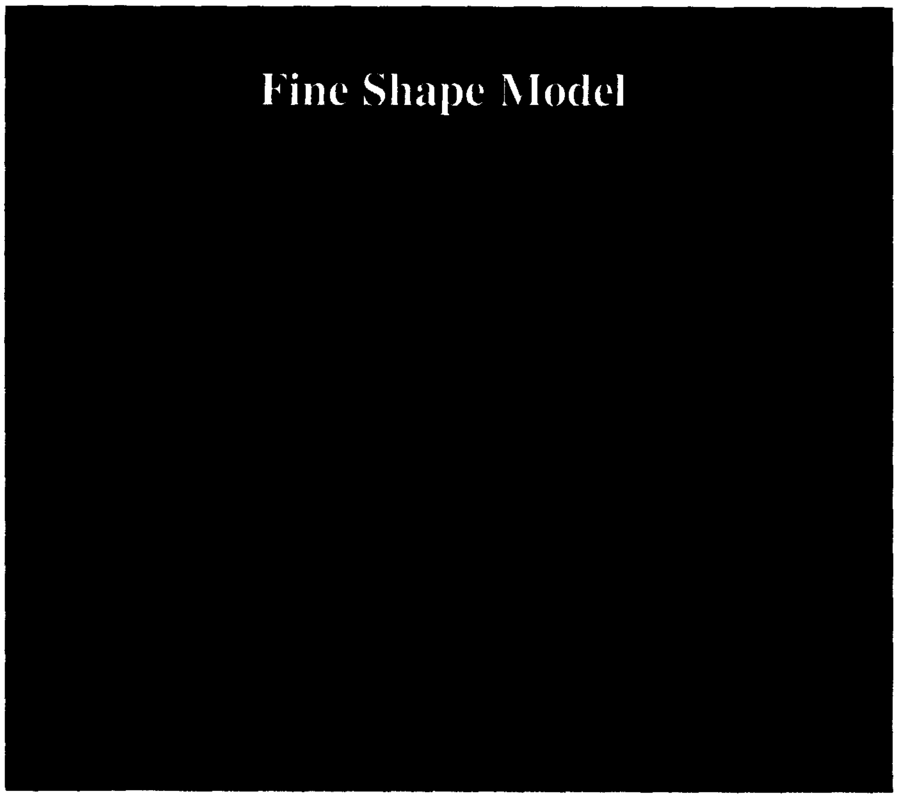
Coarse Shape Model (Geoid)

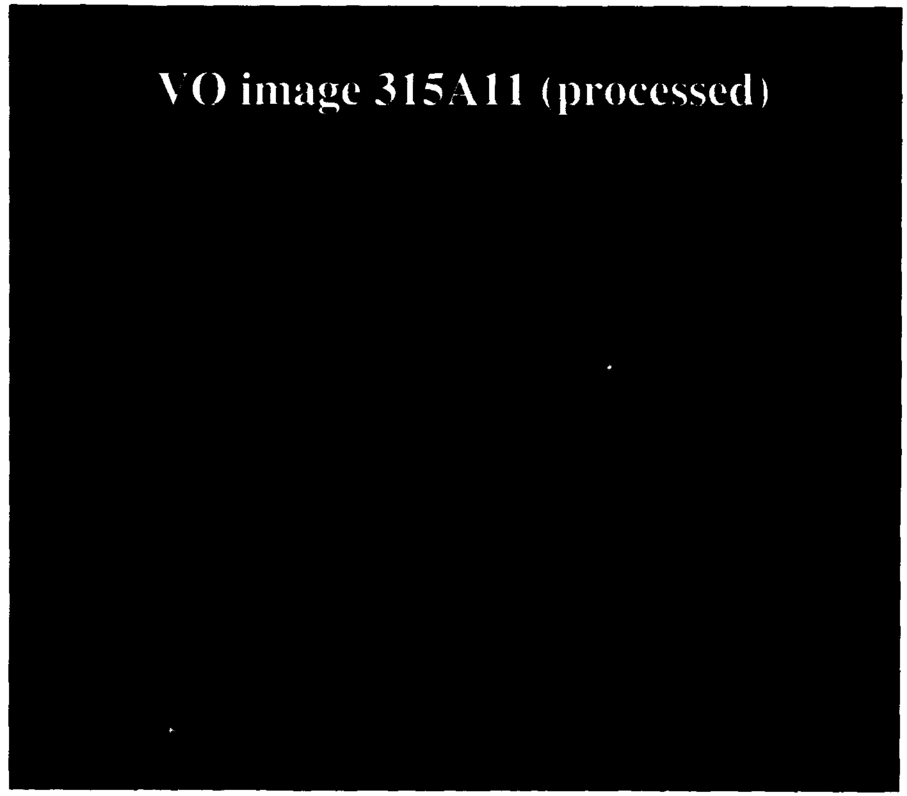
After aligning the images to a set of landmarks, the body is tiled with a set of larger templates called maps. A coarse shape model is constructed from an average of these maps. For Phobos, 248 overlapping maps were used, each containing about 10,000 vectors.

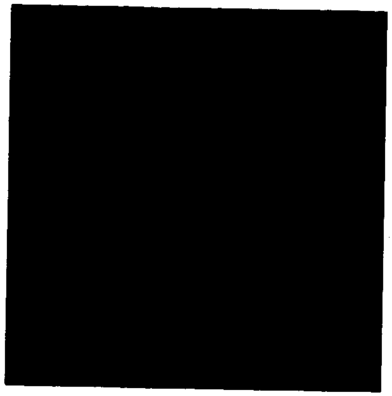


Fine Shape Model

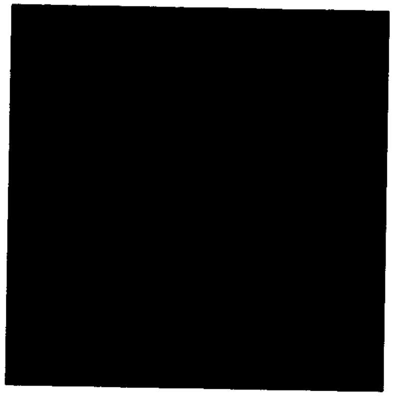
Maps gradients are averaged at a dense set of interpolated surface points. The topography is determined by integrating these gradients over the whole body, similar to the slope-to-height integration for a single map. A small number of seed vectors, 8000 for the Phobos model, constrain the integration.



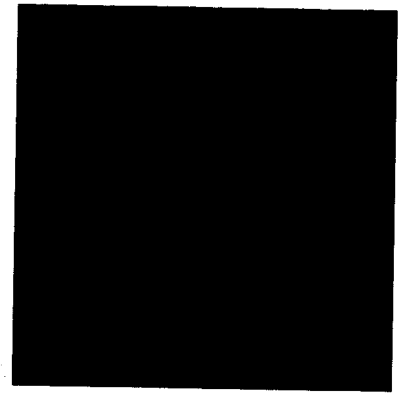




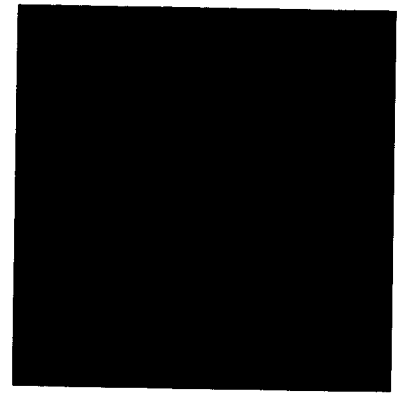
0°



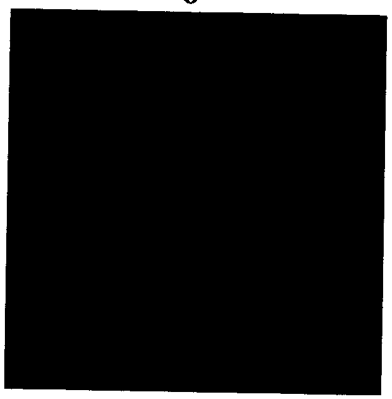
45° E



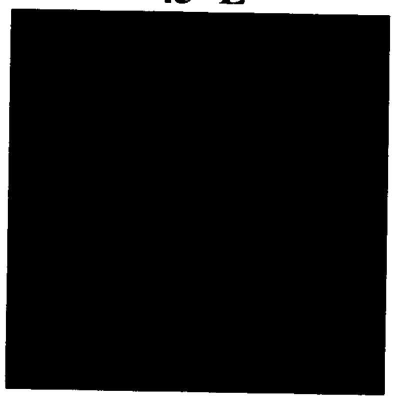
90° E



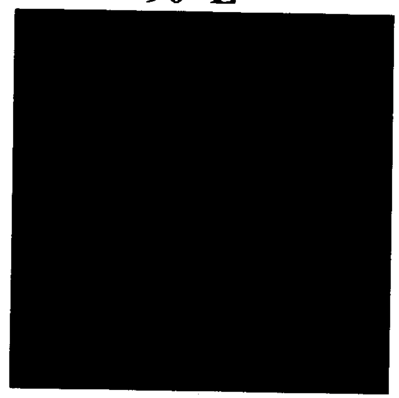
135° E



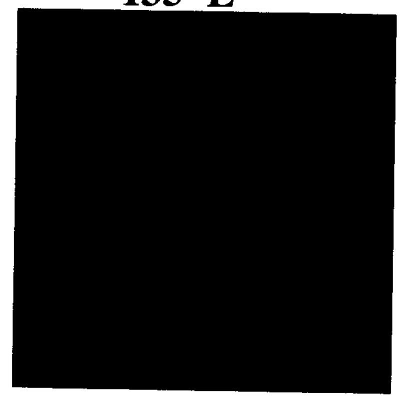
180° E



225° E



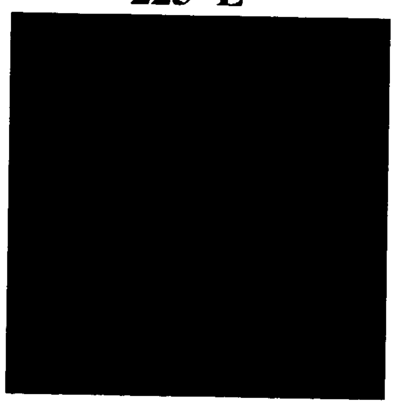
270° E



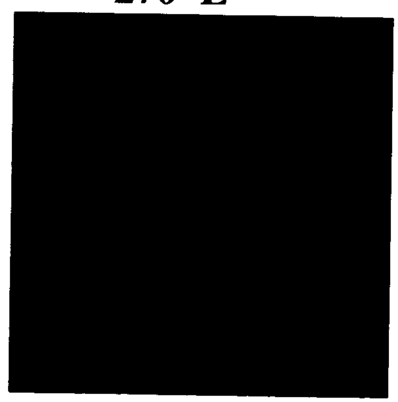
315° E

Phobos Results

1.57 million vector shape and topography model



90° N



90° S

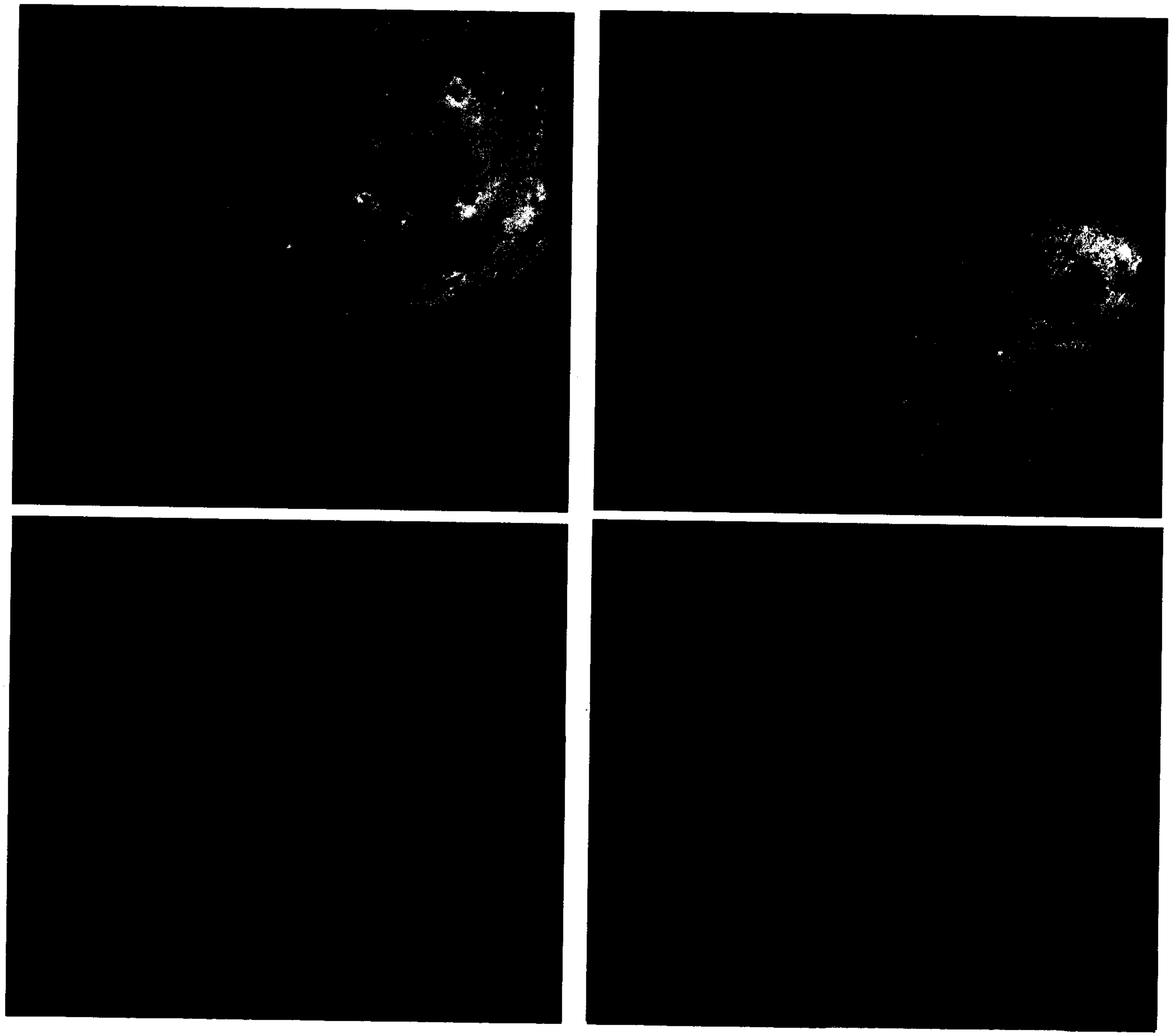
Volume = 5748 km^3

I_{xx}/M = 43.89 km^2

I_{yy}/M = 51.22 km^2

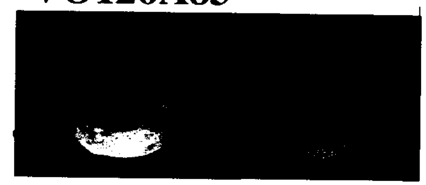
I_{zz}/M = 60.02 km^2

Phobos Model vs. Imaging Data



VO436A43

VO126A83



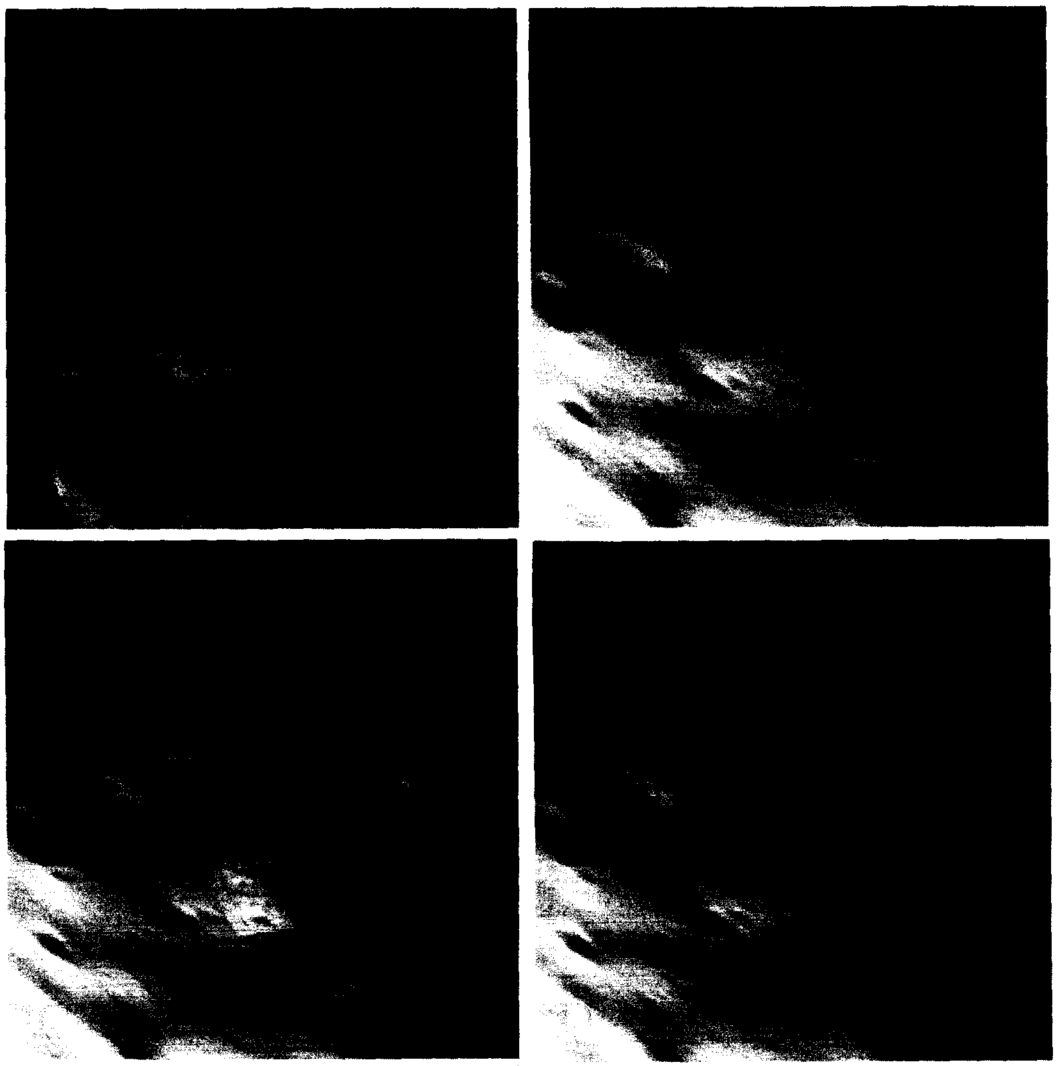
VO149B22

VO203A32

VO405A05

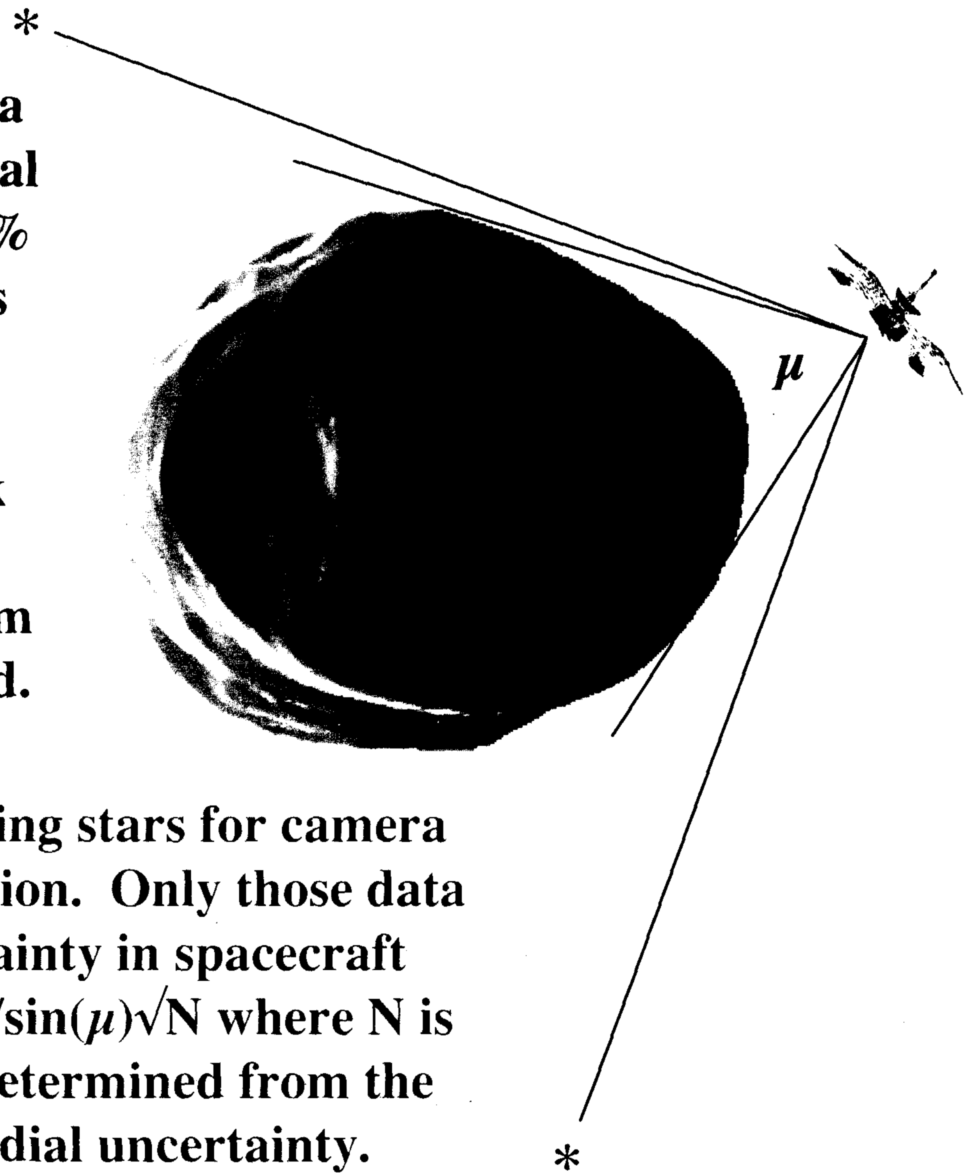
VO458A05

Resolution Considerations



Although the base resolution of the shape model is about 25 meters, most maps used in its construction range from 60 to 80 meters in resolution. A few 25 m maps were used as well. As higher resolution images become available, they can easily be included in the data set. A shows a 12 m resolution image, a portion of VO249A03. B shows the current shape model. C shows the addition of one 30 m context map and 9 overlapping 12 m maps. D shows the resulting integrated shape model. Eventual inclusion of higher resolution maps will significantly refine the final shape model.

Navigation with the Shape Model



- The shape model plays the role of a very extended landmark map. Radial uncertainties are typically about 10% of scale, and tangential uncertainties perhaps 50%.

- Navigation by matching landmark maps to extracted imaging data is a very fast integral part of the program and may eventually be done onboard.

- A low-data alternative involves using stars for camera pointing and limb points for navigation. Only those data need to be downloaded. The uncertainty in spacecraft location should be on the order of $s/\sin(\mu)\sqrt{N}$ where N is the number of limb points and s is determined from the image pixel scale and the model's radial uncertainty.