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Sensitivity analysis of radiative transfer for atmospheric remote sensing in thermal IR: Atmospheric weighting functions and surface partials

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Radiances of terrestrial planets observed across the spectrum contain information of both the planetary atmosphere and the planetary surfaces. In this presentation, we apply the adjoint sensitivity analysis of radiative transfer in thermal IR to the general case of the analytic evaluation of the weighting functions of atmospheric parameters together with the partial derivatives for the surface parameters. Applications to remote sensing of atmospheres of Mars and Venus are discussed.

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1. Introduction

Atmospheric weighting functions and surface partials *are* the sensitivities of observed radiances (observables) to the atmospheric profiles and surface parameters respectively:

$$\delta_X R = \int K^{(X)}(z)\, \delta X(z)\, dz$$

$$\delta_{X_s} R = \frac{\partial R}{\partial X_s}\, \delta X_s$$

Sensitivity analysis yields a general approach to find them. Adjoint sensitivity analysis yields an effective approach to find them.

Pioneering work on using this approach in the atmospheric remote sensing was done by Marchuk [1] who imported from the nuclear transport physics the general expression for the variation of oberables δR through variations of the operator L and right-hand term δS of the forward RT problem $LI = S$.

Alternative approach, using the direct linearization of the observables obtained via the discrete ordinate method was recently implemented by Spurr et al. [2]. The linearization approach involves direct (and, technically, very complicated) linearization of the solution of the forward RT problem itself.

The adjoint approach, since late 80's, is actively develope-d by Box and coworkers (see the recent review in [3]). Recent developments include applications to polarized radiation [4].

This author pursues the adjoint approach since early 90's for applications in the solar spectral region [5], [6], [7], and, more recently, in thermal IR spectral region [8], [9]. Here, we describe a general way to obtaining of atmospheric weighting function-s and surface partials via intermediate sensitivities to radiative parameters directly involved in radiative transfer.

2. Forward RT problem and observables

RT equation and boundary conditions:

$$u\frac{dI}{dz} + \alpha(z)I(z,u) - \frac{1}{2}\alpha(z)\int_{-1}^{1} p(z;u,u')I(z,u')\,du' = \alpha(z)(1-\omega_0(z))B(z),$$

$$I(0,u) = 0, \quad u > 0,$$

$$I(z_0,u) - 2A\int_{0}^{1} I(z_0,u')u'\,du' = \varepsilon B_s. \quad u < 0.$$

I – intensity of radiation;
$z = H_g \ln p$ – geopotential depth measured from TOA downward;
$\theta = \cos^{-1} u$ – nadir angle of propagation of radiation;
α – total extinction coefficient;
p – total phase function of atmospheric scattering normalized to the single scattering albedo ω_0;
B – intensity of blackbody radiation.
(Monochromatic frequency ν is implied)

Observables: Monochromatic monodirectional radiances propagating in direction with zenith angle $\cos^{-1}\mu$, observed at TOA:

$$R(\mu) = I(0,-\mu) = \int_{0}^{z_0} dz \int_{-1}^{1} du\, W(z,u;\mu)\, I(z,u)$$

where

$$W(z,u;\mu) = \delta(z)\cdot\delta(u+\mu)$$

is a corresponding *observables's weighting function.*

3. Radiative parameters

Radiative parameters: All atmospheric and surface parameters directly entering the RT equation and boundary conditions:

Atmospheric parameters:

Total extinction coefficient α;
Total phase function of atmospheric scattering p;
Total single scattering albedo ω_0 (derivative of phase function p);
Atmospheric blackbody radiance (Planck function) B.

Surface parameters:

Surface albedo A;
Surface emissivity ε (related to surface albedo $A = 1 - \varepsilon$);
Surface blackbody radiance (Planck function) B_s;
Surface geopotential depth z_0.

For a composite atmosphere consisting of several components (absorbing gases and scattering/absorbing aerosols):

$$\alpha = \sum_k \alpha_k$$

$$p = \sum_k \frac{\alpha_k}{\alpha} p_k$$

or, better yet,

$$\omega_\ell = \sum_k \frac{\alpha_k}{\alpha} (\omega_\ell)_k$$

In particular,

$$\omega_0 = \sum_k \frac{\alpha_k}{\alpha} (\omega_0)_k$$

Same kind of representation by components is applicable to the surface parameters A and ε (end member analysis).

4. Physical parameters

Physical parameters: All parameters of the physical model of the atmosphere/surface system which, via radiative parameters, impact the atmospheric radiation.

Atmospheric parameters:

Atmospheric temperature;
Mixing ratios of atmospheric gases;
Extinction coefficients of atmospheric aerosols;
Refractivities and size distributions of atmospheric aerosols,
(Etc., etc., etc.)

Surface parameters:

Surface temperature;
Surface pressure;
All surface parameters impacting albedo A, and emissivity ε.

Ground rule: Any physical parameters may be considered, *provided* we know quantitatively the dependence of the radiative parameters on them. Cross-dependence for atmospheric parameters:

Dependence: on of	Atmospheric temperature T	Other atm. parameters $\{X_j\}$	Aerosol parameters $\{x_i\}$
B	$+$	$-$	$-$
α	$+$	$+$	$+?$
$\{\omega_\ell\}$	$+?$	$-?$	$+$

5. Radiative and physical parameters

Variate radiative parameters; corresponding δR:

$$\delta_B R = \int_{-\infty}^{z_0} K^{(B)} \delta B \, dz$$

$$\delta_\alpha R = \int_{-\infty}^{z_0} K^{(\alpha)} \delta\alpha \, dz, \quad \delta_{\omega_\ell} R = \int_{-\infty}^{z_0} K_\ell^{(\omega)} \delta\omega_\ell \, dz$$

Variations of radiative parameters due to physical parameters:

$$\delta B = \frac{\partial B}{\partial T} \delta T$$

$$\delta\alpha = \sum_{j=1}^{N} \frac{\partial \alpha}{\partial X_j} \delta X_j + \frac{\partial \alpha}{\partial T} \delta T, \quad \delta\omega_\ell = \sum_{j=1}^{N} \frac{\partial \omega_\ell}{\partial x_i} \delta x_i$$

Variations of physical parameters:

$$\delta_T R = \int_{-\infty}^{z_0} \left(K^{(B)} \frac{\partial B}{\partial T} + K^{(\alpha)} \frac{\partial \alpha}{\partial T} \right) \delta T \, dz$$

$$\delta_{X_j} R = \int_{-\infty}^{z_0} K^{(\alpha)} \frac{\partial \alpha}{\partial X_j} \delta X_j \, dz, \quad \delta_{x_i} R = \int_{-\infty}^{z_0} \left(\sum_\ell K_\ell^{(\omega)} \frac{\partial \omega_\ell}{\partial x_i} \right) \delta x_i \, dz$$

Atmospheric weighting functions:

$$K^{(T)} = K^{(B)} \frac{\partial B}{\partial T} + K^{(\alpha)} \frac{\partial \alpha}{\partial T}$$

$$K^{(X_j)} = K^{(\alpha)} \frac{\partial \alpha}{\partial X_j}, \quad K^{x_i} = \sum_\ell K_\ell^{(\omega)} \frac{\partial \omega_\ell}{\partial x_i}$$

Surface partials: Surface temperature:

$$\frac{\partial R}{\partial T_s} = \frac{\partial R}{\partial B_s} \cdot \frac{\partial B_s}{\partial T_s}$$

6. Adjoint approach to the sensitivity analysis

The forward RT problem is represented as a single linear operator equation

$$L\,I = S.$$

The observables R are represented as an inner product

$$R = (W, I).$$

We need to find the variation of observables due to variation of any model parameters contained in L and/or S resulting in δI

$$\delta R = (W, \delta I).$$

To do that we variate the forward problem

$$\delta L I + L \delta I = \delta S,$$

and get the equation for δI

$$L\delta I = \delta S - \delta L I$$

Further on, we left-multiply by an arbitrary (so far) function I^*

$$(I^*, L\delta I) = (I^*, \delta S - \delta L I)$$

and use the definition of an *adjoint* operator L^*

$$(I^*, L\delta I) = (L^* I^*, \delta I)$$

to equate

$$(L^* I^*, \delta I) = (I^*, \delta S - \delta L I)$$

Demanding that the function I^* be the solution of the *adjoint* problem

$$L^*\, I^* = W,$$

we get the necesary expression for the variation δR

$$\delta R = (I^*, \delta S - \delta L I)$$

7. Sensitivities to the radiative parameters

Considering the variations of δL and δS due to various radiative parameters we get sensitivities to all radiative parameters.

Atmospheric weighting functions

$$K^{(B)}(z,\mu) = \int_{-1}^{1} du\, I^*(z,u;\mu)\cdot \alpha(z)(1-\omega_0(z))$$

$$K^{(\alpha)}(z,\mu) = \int_{-1}^{1} du\, I^*(z,u;\mu)\left[(1-\omega_0(z))\,B(z) - I(z,u) + \frac{1}{2}\alpha(z)\int_{-1}^{1} p(z;u,u')I(z,u')\,du'\right]$$

$$K_\ell^{(\omega)}(z,\mu) = \int_{-1}^{1} du\, I^*(z,u;\mu)\,P_\ell(u)\cdot\left[\frac{1}{2}\alpha(z)\int_{-1}^{1}\Big(I(z,u)-B(z)\Big)\,P_\ell(u)\,du\right]$$

Surface partials

$$\frac{\partial R(\mu)}{\partial B_s} = \int_{-1}^{0} du\,(-u)\,I^*(z_0,u;\mu)$$

$$\frac{\partial R(\mu)}{\partial \varepsilon} = -\frac{\partial R}{\partial A} = \int_{-1}^{0} du\,(-u)\,I^*(z_0,u;\mu)\cdot\left[B_s - 2\int_{-\infty}^{1} I(z_0,u)\,u\,du\right]$$

$$\frac{\partial R}{\partial z_0} = \alpha(z_0)\,K^{(\alpha)}(z,\mu)$$

Case of blackbody atmosphere ($\omega_0(z) \to 0$)

Forward problem:

$$u\frac{dI}{dz} + \alpha(z)I(z,u) = \alpha(z)B(z),$$
$$I(0,u) = 0, \quad u > 0,$$
$$I(z_0,u) = \varepsilon B_s. \quad u < 0.$$

and its solution:

$$I(z,u) = \int_{-\infty}^{z} t(z,z';u)\, B(z)\, \alpha(z')\frac{dz'}{u}, \quad u > 0$$
$$I(z,u) = \int_{z}^{z_0} t(z',z;u)\, B(z)\, \alpha(z')\frac{dz'}{u} + t(z_0,z;u)\, \varepsilon B_s, \quad u < 0$$

Adjoint problem:

$$-u\frac{dI^*}{dz} + \alpha(z)I^*(z,u) = 0,$$
$$I^*(0,u) = \frac{1}{\mu}\, \delta(z)\, \delta(u+\mu), \quad u < 0,$$
$$I^*(z_0,u) = 0. \quad u > 0.$$

and its solution:

$$I^*(z,u) = 0, \quad u > 0$$
$$I^*(z,u) = \frac{1}{\mu}\, \delta(u+\mu)\, t(z,\mu) \quad u < 0$$

Resulting atmospheric weighing functions and surface partials:

$$K^{(B)}(z,\mu) = \frac{1}{\mu}\, \alpha(z)\, t(z,\mu) = \left(-\frac{\partial t}{\partial z}\right)$$
$$K^{(\alpha)}(x,\mu) = -\frac{1}{\mu}\, \left[\int_{z}^{z_0} t(z',\mu)\, dB(z) + t(z,\mu)\Big(\varepsilon B_s - B(z_0)\Big)\right]$$
$$\frac{\partial R}{\partial B_s} = t(z_0)\, \varepsilon, \quad \frac{\partial R}{\partial \varepsilon} = t(z_0)\, B_s, \quad \frac{\partial R}{\partial z_0} = \alpha(z_0)\, K^{(\alpha)}(z,\mu)$$

8. Discussion: Retrievability and ability to retrieve

Retrievability (of parameters): In this context, the amenability of given physical parameter to be retrieved.

(Our) ability to retrieve: In this context, an extent to which any given parameter can be independently retrieved, given the available information content of data.

This presentation argues that all physical parameters are retrievable.

Ability to retrieve a given parameter *is not* a subject of this study. Availability of explicit sensitivities to given parameters is, nevertheless, instrumental in the analysis of ability to retrieve parameters of interest from given data set.

9. Applications to Mars and Venus

Mars

The atmosphere is (most of time) semitransparent, except in the 15 μ band of CO_2. Both the atmosphere and surface are accessible by remote sensing. (Btw, MGS/TES was originally conceived as a geological, not atmospheric instrument.)

With known atmospheric scattering, retrieval of atmospheric and surface parameters is straightforward, (assuming, Lambertian model of surface is adequate. If atmospheric scattering is treated as unknown, the need on angular dependence of observed radiances becomes mandatory.

If the surface cannot be treated as Lambertian, its bidirectional reflectance/emissivity function needs to be expressed through a limited number of surface physical parameters before any responsible retrievals become possible.

Venus

The atmosphere is opaque everywhere, over the thermal IR spectral range and over the globe. As for Mars, with known atmospheric scattering, retrieval of atmospheric and surface parameters is straightforward. As for Mars, if atmospheric scattering is treated as unknown, the need on angular dependence of observed radiances becomes mandatory.

Extensive variation of temperature through the atmosphere, from ca. 750 K at the surface to $\leq$ 250 K at the visible cloud tops, combined with near-conservative atmospheric scattering in the near IR, provides a unique opportunity of atmospheric sounding by proper thermal radiation, as was demonstrated in mid-90's using data from the Hubble WFC-2 instrument.

10. Conclusion

Atmospheric weighting functions and surface partial derivatives are, by their physical meaning, sensitivities of observed data (observables) to the atmospheric and surface parameters.

Sensitivities of observables to radiative parameters (directly involved in radiative transfer) represent a "gate" to sensitivities of observables to all physical parameters.

Adjoint sensitivity analysis provides an effective way to evaluate these sensitivities.

Present study refers to the case of remote sensing of plane parallel atmosphere in thermal IR, with nadir-viewing geometry.

The same approach can be applied to:

– Thermal (IR and microwave) spectral region, with polarization;

– Shortwave (solar) spectral region, with and w/o polarization;

– Limb-viewing geometry;

– Various combinations of the above.

It can be expected that this area will see accelerating development in the near future.

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