Monte Carlo Calculation of the Cumulative Probability of TID Levels Based on the Variability of the Radiation Environment at Europa

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I. Introduction

The Europa Clipper mission will repeatedly pass in and out of the Jovian radiation belts as it explores the icy moon. In order to evaluate the effectiveness of any safety margins for total ionizing dose (TID) ratings on Europa Clipper spacecraft components, the variability of the expected radiation environment must be characterized. This study outlines a Monte Carlo calculation of the cumulative probability that the TID will deviate from the nominal value of 150 krad(Si) based on the lognormal distributions of electron fluxes observed in the Jupiter environment by the Galileo Energetic Particle Detector (EPD) instrument. Our model shows that parts hardened to 300 krad(Si) for a radiation design factor (RDF) of 2 are expected to withstand the measured variations of the Jupiter radiation environment with margin. This result can be scaled to apply to all parts and materials with different TID levels for the Europa Clipper mission.

II. Radiation Environment

Galileo Energetic Particle Detector (EPD) measurements show that the distribution of electron fluxes in the Jupiter environment are well-approximated by a lognormal distribution, as shown in Figure 1 from reference [Jun, 2005]. This same lognormal distribution is applied to the dose arising from the electron flux.

Total ionizing dose (TID) of Europa Clipper spacecraft components will be dominated by the trapped electrons encountered in the 8-10 Jupiter radii (R_j) region—effectively the perijove of each orbit. Figure 1 plots the average counts per second 8-10 R_j region in 3 EPD channels from the Galileo tour (DC3 for >11 MeV electrons, B1 for 1.5 – 10.5 MeV electrons, and B0 for 3.2 – 10.1 MeV protons). A cursory inspection of Fig. 1 reveals the lack of obvious orbit-to-orbit trends in the Galileo EPD data. If the environment fluctuations on the scale of orbital periods are effectively random, the environmental variability can be modeled as a random draw from the lognormal distribution corresponding to the radiation environment. If there is a significant trend in the environment such as some high or low flux persisting long enough to include several orbits or an overall trend, a model of the radiation environment would have to include that behavior. Trends in the environment that do not persist over more than one orbit period will not affect TID since the narrow 8-10 R_j zone will be the only significant contributor.

In the absence of clear periodicity over the observed timescale, the first-order assumption is to assume a linear trend in time. The calculation of the correlation coefficient r for DC3 over the elapsed time does indicate a small 0.13 correlation. Given the sample size, this level of correlation has ~63% of occurring randomly [Taylor, 1997]. We therefore cannot rule out some very small orbit-to-orbit correlation in the variability of the environment. This question is treated in detail in section III *B*.

Figure 1. Average counts per second observed in DC3, B1, and B0 channels of the Galileo EPD corresponding to 8-10 Rj for each orbit.

III. Monte Carlo Calculation

A. Simulation of Statistically Independent Orbits

We begin with the assumption that each orbit can be approximated as a random draw from a lognormal distribution representing the radiation environment. We developed an Excel tool that calculates the empirical cumulative distribution of TID from 4000 Monte Carlo trials where each random number corresponds to the dose from a single orbit. Each of the 4000 trials is the sum of 60 lognormal random numbers—representing the cumulative TID from 60 simulated orbits for the Europa Clipper tour.

Each lognormal random number is calculated using Excel's built-in LOGNORM.INV function. Consider the lognormal cumulative distribution

$$D(x) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\ln[x]-\mu}{\sigma\sqrt{2}}\right)\right] \quad (1)$$

which returns the cumulative probability of occurrence D (bounded between 0 and 1) given input variable x, the lognormal mean and standard deviation of $\ln(x)$ (μ and σ respectively), where erf is the error function. LOGNORM.INV returns the x value corresponding to the inputs D, μ, and σ in Eq. (1). A random number between 0 and 1 is input for D using the RAND() Excel function, resulting in lognormally distributed random numbers, x. The particular lognormal

Figure 2. The empirical cumulative distribution of 4000 sums (blue) and 16,000 sums (green) of 60 lognormal random numbers each. The standard deviation of the lognormal distribution is based on Galileo measurements of the Jupiter radiation environment relevant to the Europa Clipper mission.

distribution these numbers come from depends on the choice of μ and σ.

The resulting distribution of 4000 sums is centered about the average, exp $(\mu + \frac{\sigma^2}{2})$. The average TID for 60 orbits, together with the standard deviation, are adjustable inputs to the Excel worksheet. In this example, values for these inputs are a nominal RDF=1 TID of 150 krad(Si) for the average, and 1.7 for the standard deviation based on the lognormal distribution of 11 MeV integral flux measurements at 9.5 R_j [Jun, 2005].

The key result of this calculation is the empirical cumulative distribution of the 4000 trials as shown in Fig. 2. Given the large number of trials, the quantiles can be taken to represent the cumulative probability of occurrence. To test for convergence, a 16,000 tour Monte Carlo calculation was also performed. The 5th, 50th, and 95th percentiles agree from one distribution to the other to within less than 1 krad(Si). While 16,000 trials were too computationally taxing to use repeatedly for all the analyses in the study, it served to show that the method is consistent with the 4000-trial case as shown in Fig. 2.

The 16,000 tour calculation shows that given an RDF=1 values of 150 krad(Si), the scenario for a tour of 60 orbits receiving 198 krad(Si) corresponds to the 99.99375 percentile on the green curve in Fig. 2. While this is not a rigorous statement of probability, it shows that the based on our model, and RDF of 2 provides ample margin compared to the variability of the environment.

B. Sensitivity to Inter-Orbit Correlations

Let us consider relaxing the assumption that each orbit is independent. In other words, suppose the variability of the high-radiation environment near Europa follows some kind of trend that persists long enough to influence one or more subsequent flybys. If this were the case, modeling TID with random draws from lognormal distribution may underestimate TID compared to the case of dosing from the higher end of the lognormal distribution many times. Note that any persistence in the environmental variability over the course of one orbit or less will not invalidate the use of random draws in the simulation since essentially all of the dose comes from only a narrow region of roughly 8-10 R_j.

Figure 3. Correlation sensitivity study for the empirical cumulative distribution of 4000 sums of 60 lognormal random numbers. Except for the case of 0% correlation (blue curve), not all of the random numbers in each sum are intendent. The black curve is the distributions of TID for 100% correlation case. The shaded region corresponds to intermediate percentages of correlated orbits. The example case for the radiation environment persisting over 10 orbits—modeled with 10 of the 60 random numbers reused by design—is the dashed line.

The limiting worst-case is that the random draw for the first orbit will determine the entire mission, *i.e.*, that high (or low) radiation compared to the average will persist in the Jovian environment thought the entire mission. This approach equates the statistics of the entire mission with the statistics for a single orbit. This is the black curve in Fig. 3. Intermediate levels of correlation were simulated by forcing a percentage of the 60 orbits in each of the 4000 trials to use the same random number. According to this calculation, if even 10 of the 60 orbits were to be perfectly correlated and the rest independent, then there would still be less than 0.025% likelihood of exceeding an RDF of 2 at 300 krad(Si). At 20 correlated orbits the probability goes to 0.2% and ~1% at 30 correlated orbits, compared ~6% likelihood at full correlation. This illustrative example does not consider the effect of more physically likely behavior like oscillations over several orbits or some kind of weak correlation—it assumes only mixtures of fully correlated and fully independent orbits to gauge the sensitivity of the results to the environment persisting over multiple orbits. This shows that if a few orbits are correlated—including an unusually severe environment persisting over several orbits—it will not significantly stretch the distribution toward infringing on RDF=2. Nevertheless, correlation over many orbits, or indeed all of them, would obviously be cause for concern.

The correlation observed in the Galileo EPD data is smaller than negligible levels of correlation in this sensitivity calculation, therefore we conclude that modeling each orbit's TID as independent from every other orbit is a valid approximation.

C. Mission-Tailored Calculation

For each of the 4000 simulated tours calculated in Section A., 60 orbits were simulated using random draws from the same lognormal distribution based on the RDF=1 of 150 krad(Si) for the entire tour divided equally between each of the 60 orbits.

The standard deviation for the lognormal distribution was 1.7, based on the measured 11 MeV integral flux at 9.5 Rj [Jun, 2005].

From orbit to orbit, the perijove of the Europa Clipper spacecraft will vary somewhat as the tour progresses. Candidate tour geometries specify each perijove together with the expected TID per orbit. This information was used to create a more representative simulation of the environmental variability specifically for an example tour by drawing random numbers from the individual lognormal distribution corresponding to each flyby. The end-of-mission TID—together with each individual TID/orbit—can calculated using the GIRE3 radiation model [Divine and Garrett, 1983; Garrett et al., 2005] and then scaled to expected RDF=1 TID at any given spacecraft component. For the purposes of this calculation each orbit TID was scaled such that the TID for the whole tour would be 150 krad(Si). The mean of each lognormal distribution was adjusted accordingly. Fig. 7 of [Jun, 2005] shows the standard 10^(Standard Deviation of $\log_{10}$(flux)) in bins of R_j width. It has been observed that in [Jun, 2005] natural logarithms were not used but note that ln[10^ Standard Deviation of ($\log_{10}$(flux))] = ln[e^ Standard Deviation of (ln(flux))] = Standard Deviation of (ln(flux)). As noted above, the TID is dominated by the environment closest to Jupiter [Jun, 2005]. Therefore, the standard deviation was selected according to the bin corresponding to perijove for each flyby. To be conservative, the highest standard deviation was used in each case. Up to about 13 R_j, the standard deviation is approximately the same for the energy channels in Fig. 7 of [Jun, 2005]. In any case, the flybys with larger perijove values and higher standard deviations also had the lowest average TID per flyby.

It was noted that a ~0.19 average correlation was observed for the new tour-specific simulation (compared to nearly zero for the same calculation in Section A.). This new small correlation was due to the fact that the expected TID per orbit changed with time due to the geometry of the new tour. This does not represent an orbit-to-orbit correlation in the environment itself. Nevertheless, a small correlation on the order of what was observed in the Galileo EPD data had serendipitously been introduced to the simulation. Fig. 4 shows that the effect of modeling a realistic Europa Clipper tour is small, especially compared to the unphysical case of full orbit-to-orbit correlation.

IV. Conclusions

A Monte Carlo simulation consisting of 4000 sums of 60 lognormal random numbers each was used to estimate orbit-to-orbit for the Europa Clipper mission. The lognormal distribution assumption and its standard deviation were derived from measurements by the Galileo EPD instrument. It is clear from the empirical cumulative distribution plot that it is extremely unlikely that the inherent variability in the radiation environment will cause the TID to exceed or even approach a design factor of RDF=2, *i.e.*, 300 krad(Si). There are small variations on the order of a few krad(Si) per percentile each time a new random seed is generated but this variation is negligible when considering an RDF of 2.

Figure 4. The red curve—plotted together with the curves in Fig. 2—was calculated with a bespoke lognormal distribution for each flyby in a candidate Europa Clipper tour.

The very small orbit-to-orbit correlation observed in the Galileo EPD data is shown to be not significant enough to invalidate modeling each orbit with independent random draws from a lognormal distribution representing the radiation environment near Europa. Using the scaled TID per flyby and standard deviation corresponding to the perijove at each flyby, an updated simulation of 4000 Europa Clipper tour scenarios was used to calculate the probability distribution representing the variability of the mission TID. This method can be generalized as a mission design tool to compare the variability of the radiation environment for different candidate tours of the Jovian environment. Even with correlation in the updated simulation exceeding the correlation in the Galileo EPD data, the chance of the TID exceeding 198 krad(Si) if the RDF=1 is 150 krad(Si) is < 0.1%. The original simulation that did not consider small changes R_j at perijove estimated < 0.1% chance of the TID exceeding 188 krad(Si). We acknowledge the possibility of known unknowns such as systematic errors in the EPD or presently unobserved phenomena in the Jovian radiation environment than cannot be addressed here.

Acknowledgment

The research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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