

Jet Propulsion Laboratory
California Institute of Technology

Low-Thrust Transfer Design Based on Collocation Techniques: Applications in the Restricted Three-Body Problem

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Outline

- I. Motivation
- II. Background
- III. Sample Applications
 - I. Trajectory Stacking Technique
 - II. Orbit Chaining Technique
 - III. Deep Space Gateway Transfer
- IV. Concluding Remarks

Outline

I. Motivation

II. Background

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I. Trajectory Stacking Technique

II. Orbit Chaining Technique

III. Deep Space Gateway Transfer

IV. Concluding Remarks

Motivation

- NASA and private companies seek to establish an enduring human presence in cis-lunar space, and this vision is aided by:
 - Efficient low-thrust propulsion.
 - Stable orbits that can be maintained at low-cost for long time spans.
- Thus, low-thrust transfers between stable periodic orbits will also be advantageous.

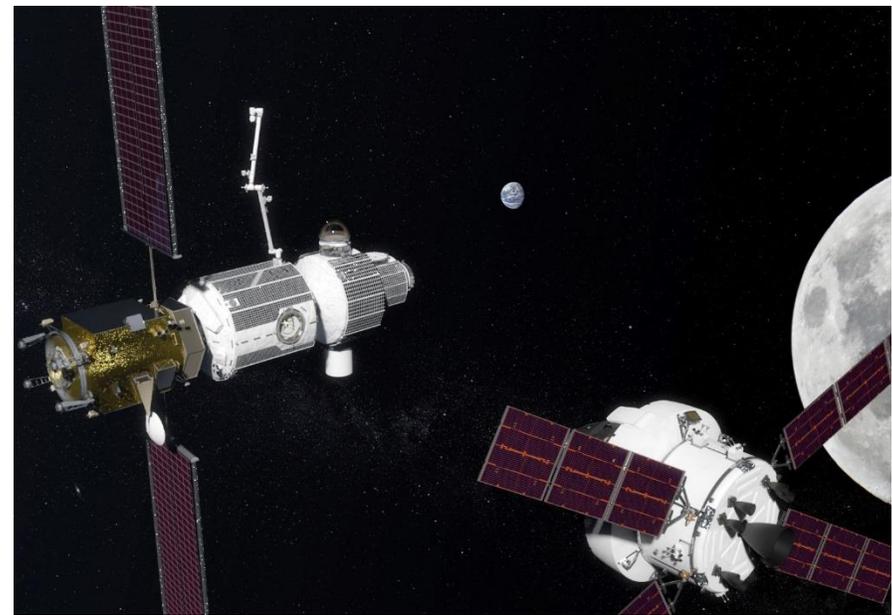


Image Courtesy of nasa.gov

Deep Space Gateway Concept

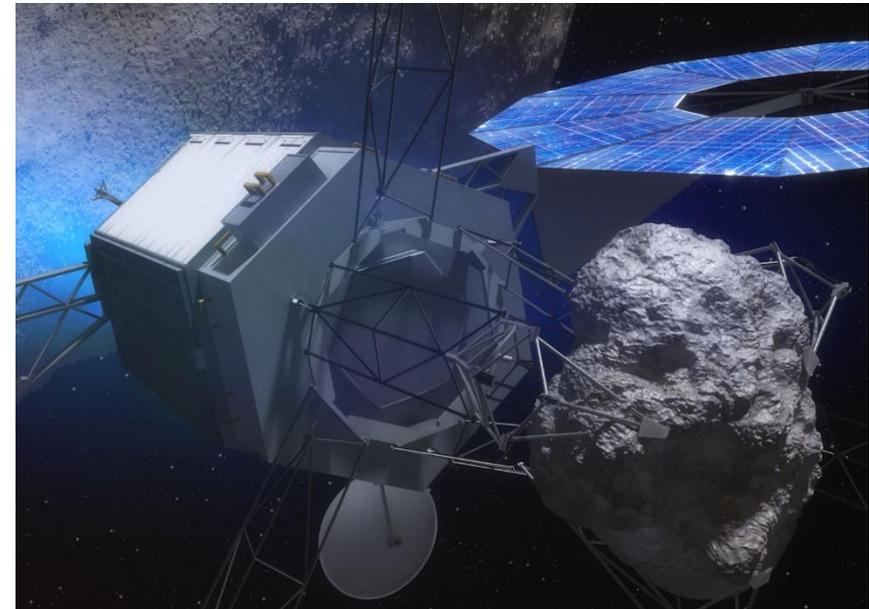
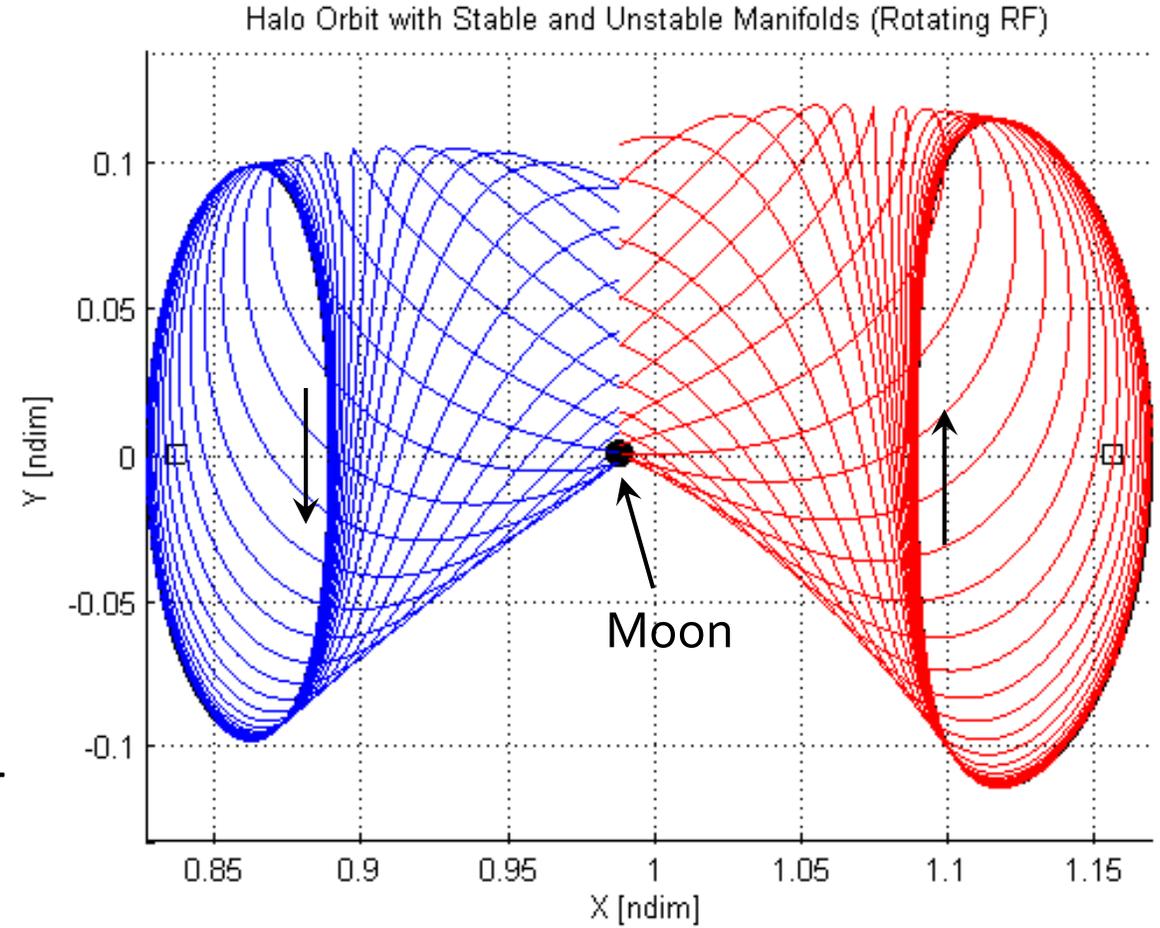


Image Courtesy of nasa.gov

Asteroid Robotic Redirect Mission Concept

Motivation

- Invariant manifold structures offer efficient paths into and out of periodic orbits.
- Therefore, manifolds may assist low-thrust transfer design.
- However, stable and near-stable periodic orbits do not possess manifolds that can assist with trajectory design.
- Thus, additional techniques for low-thrust trajectory design are required



Motivation

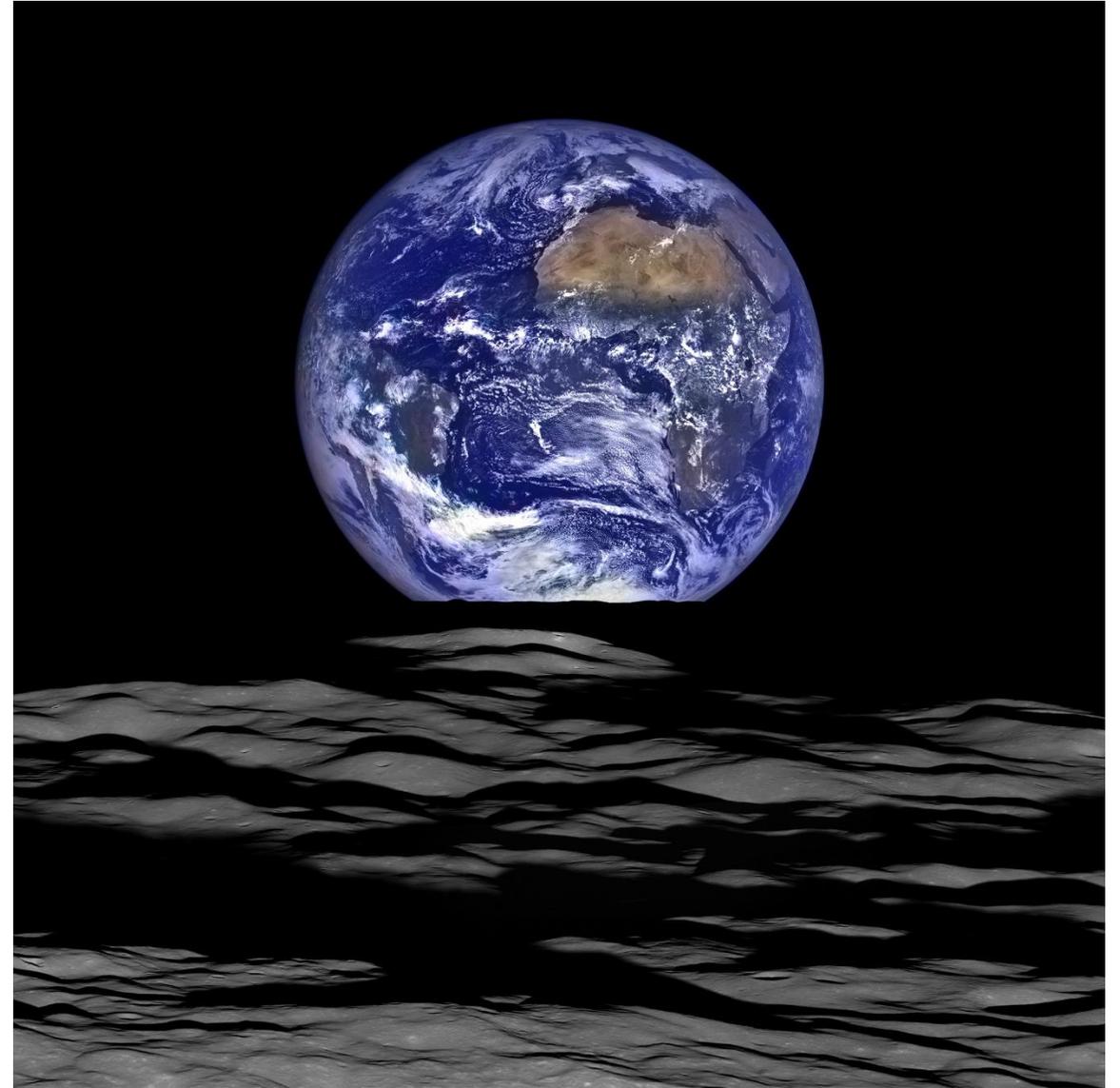
- Collocation is a method for numerically integrating ordinary differential equations.
- This method has been successfully employed to compute low-thrust trajectories when little intuition is available for constructing an initial guess.
 - Grebow, Ozimek, and Howell, 2010
 - Herman, 2015
 - Parrish, et al., 2016



Motivation

Goal: Develop additional techniques for conducting low-thrust trajectory design that leverage collocation to compute optimal trajectories when little intuition is available for the construction of an initial guess.

Sample Application: Low-thrust transfer design between stable and near-stable periodic orbits in the Earth-Moon system.



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I. Trajectory Stacking Technique

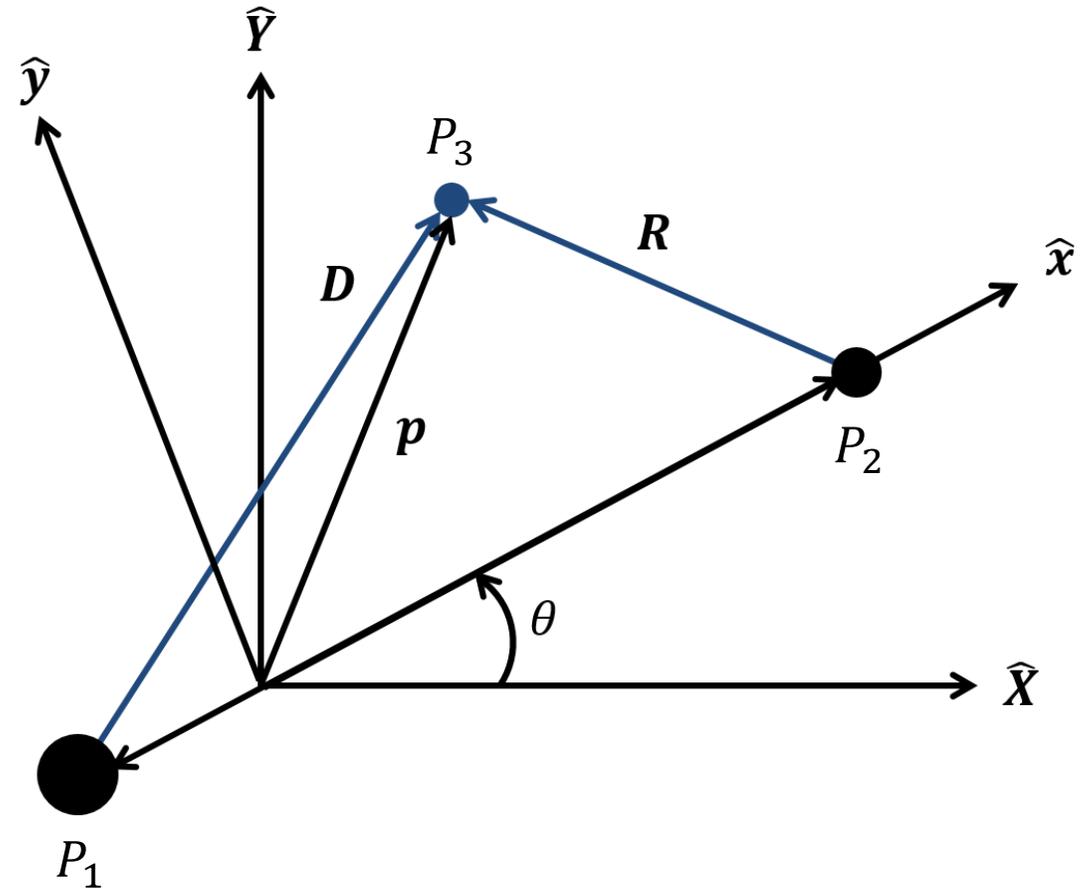
II. Orbit Chaining Technique

III. Deep Space Gateway Transfer

IV. Concluding Remarks

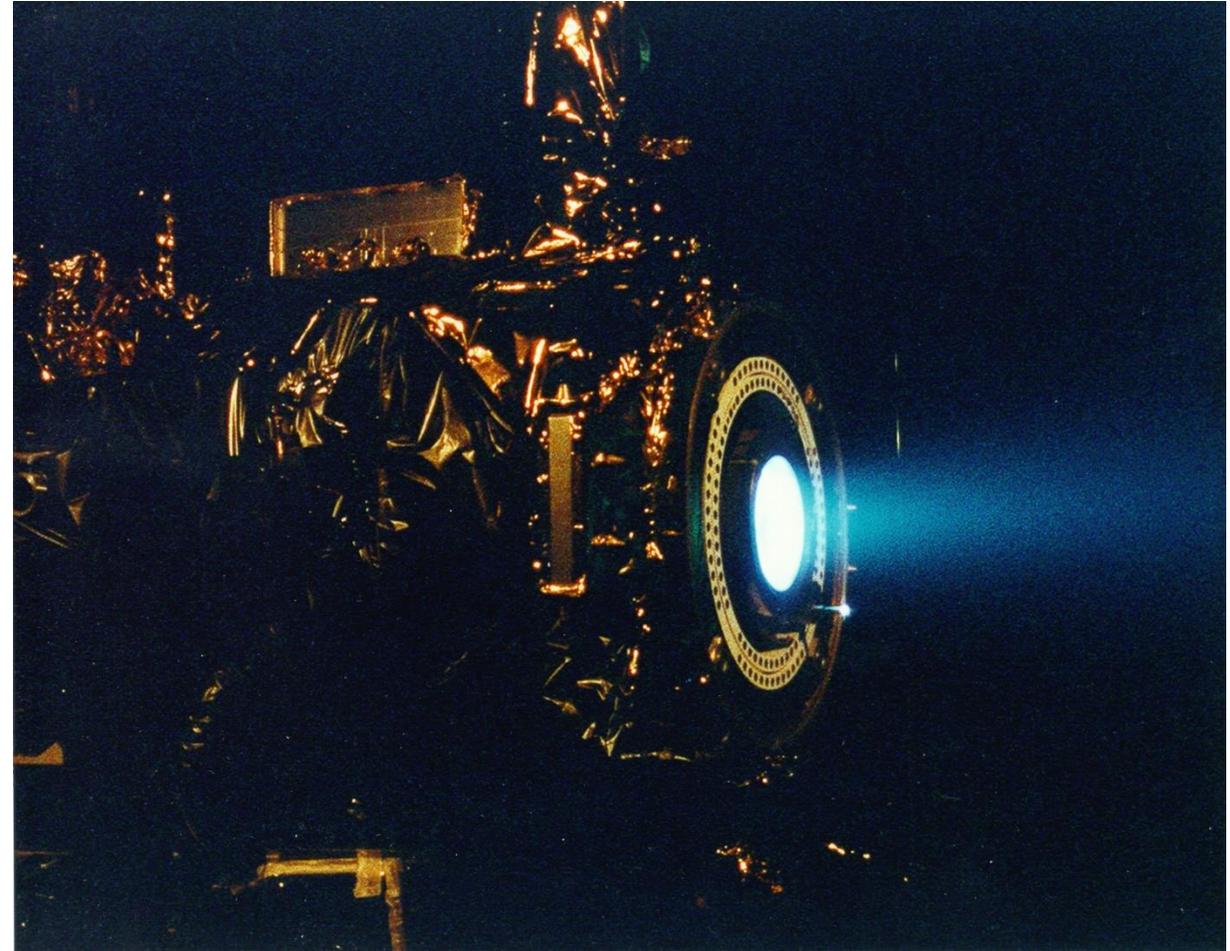
Background

- The circular restricted three-body problem (CR3BP) is employed for initial trajectory design.
 - Gravitational interaction of primaries may be leveraged to compute low-energy trajectories.
 - Avoids complexity of ephemeris model:
 - Time dependence
 - Additional gravitational perturbations



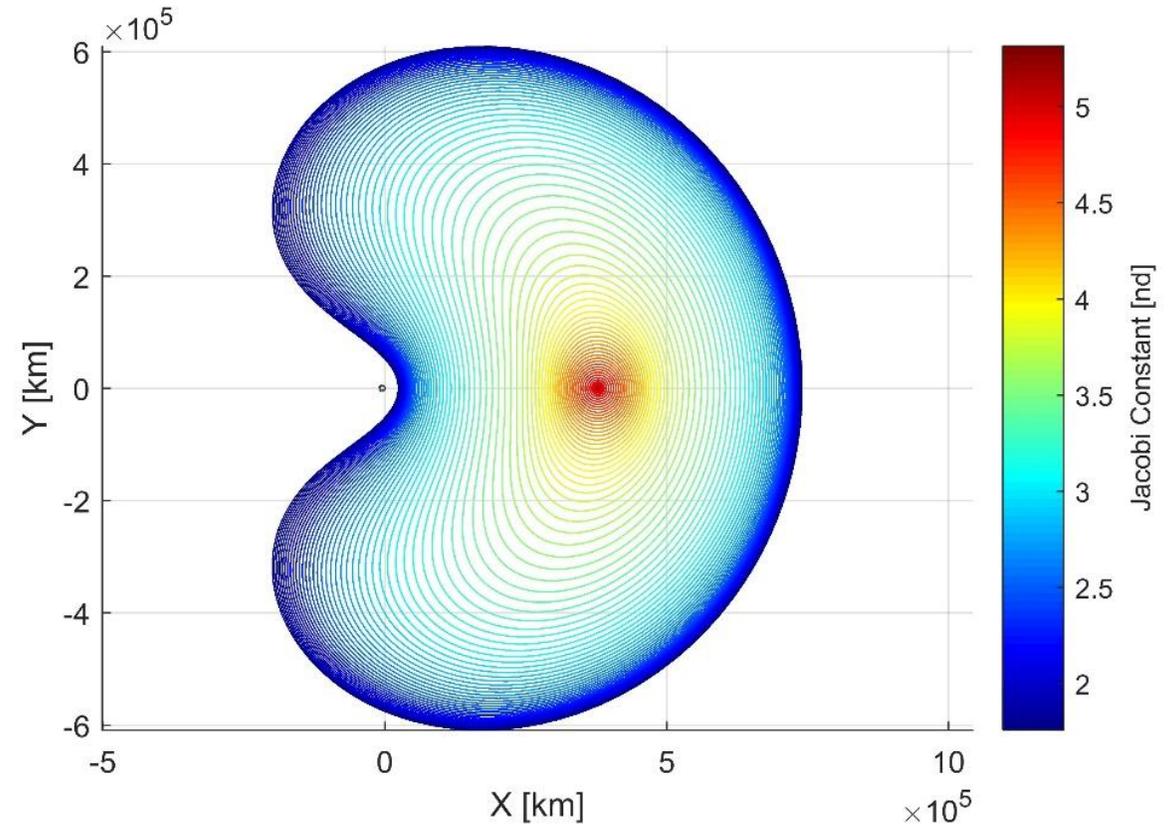
Background

- A low-thrust engine representative of current technological capabilities is employed for all transfers
 - $m_0 = 500 \text{ kg}$
 - $I_{sp} = 2000 \text{ sec}$
 - $T_{max} = 100 \text{ mN}$
- These trajectory design techniques are applicable to a wide-range of engine parameters



Background

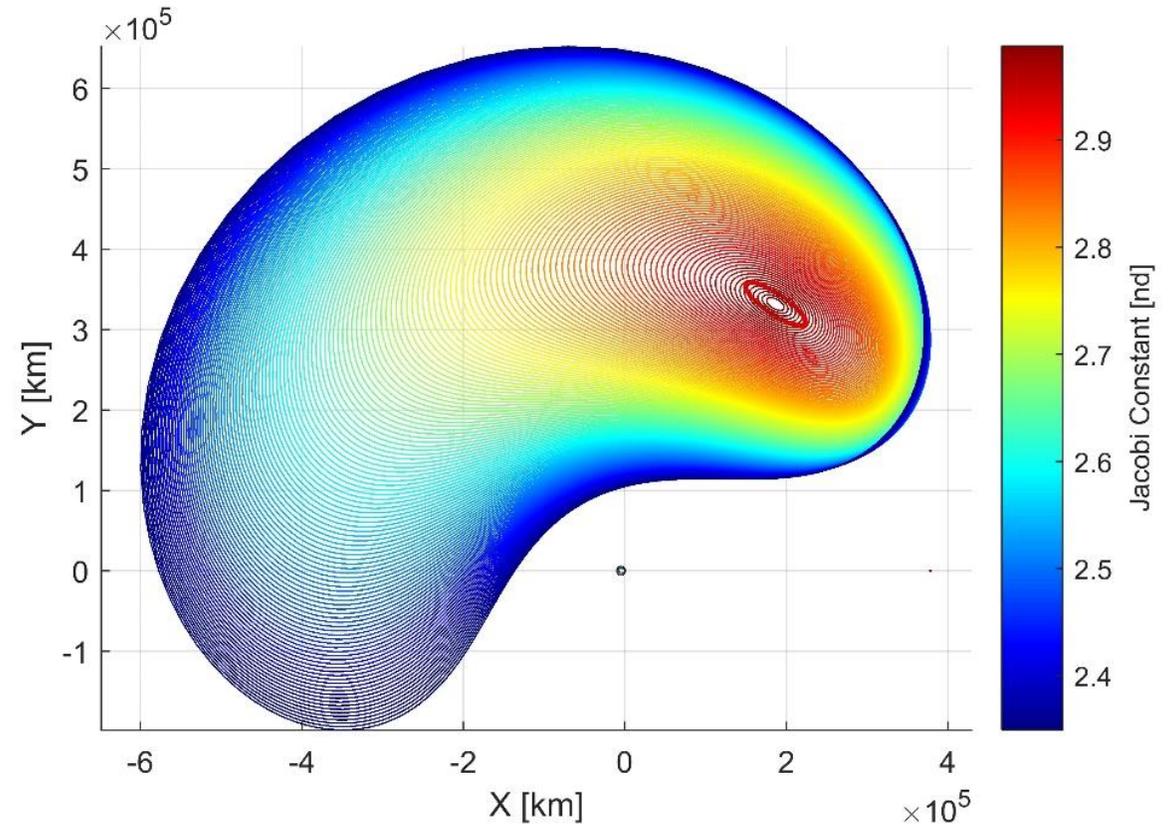
- A variety of stable or near-stable periodic orbits are available in the Circular Restricted Three-Body Problem (CR3BP)
 - Distant Retrograde Orbits (DRO)



Lunar DRO Family

Background

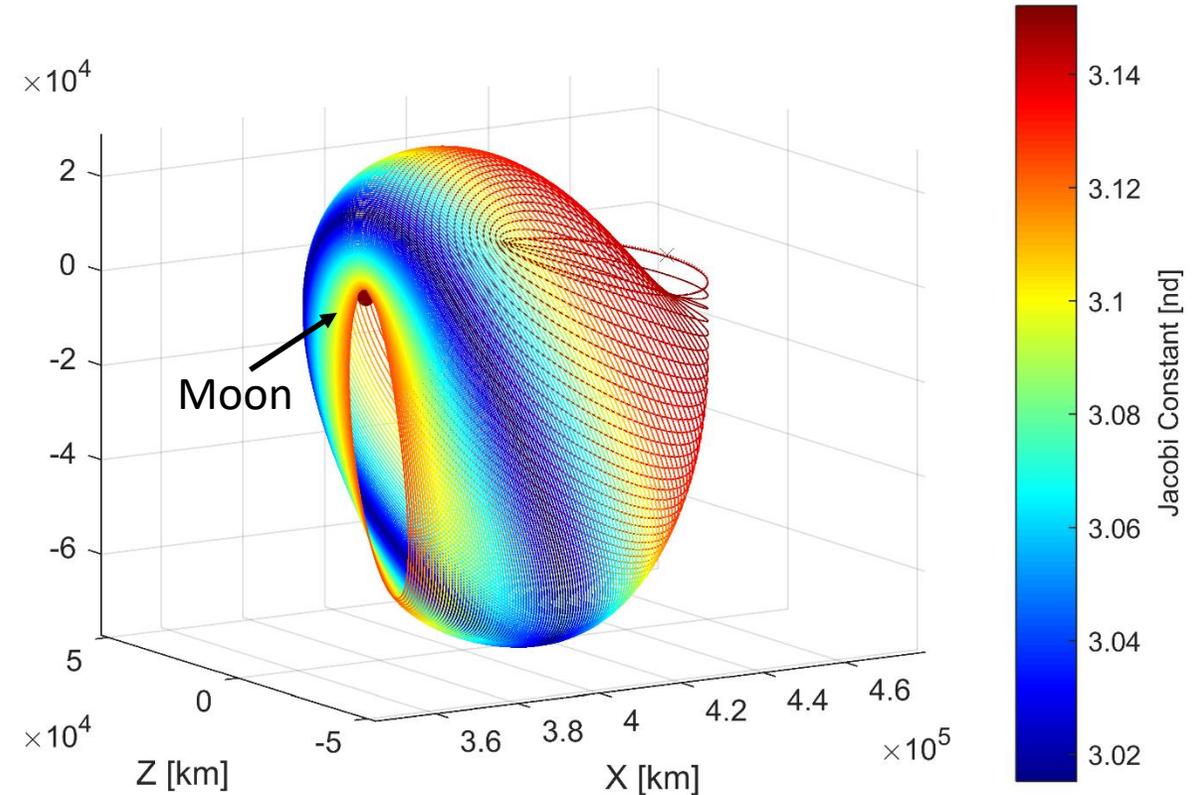
- A variety of stable or near-stable periodic orbits are available in the Circular Restricted Three-Body Problem (CR3BP)
 - Distant Retrograde Orbits (DRO)
 - $L_{4/5}$ Short Period Orbits (SPO)



L_4 SPO Family

Background

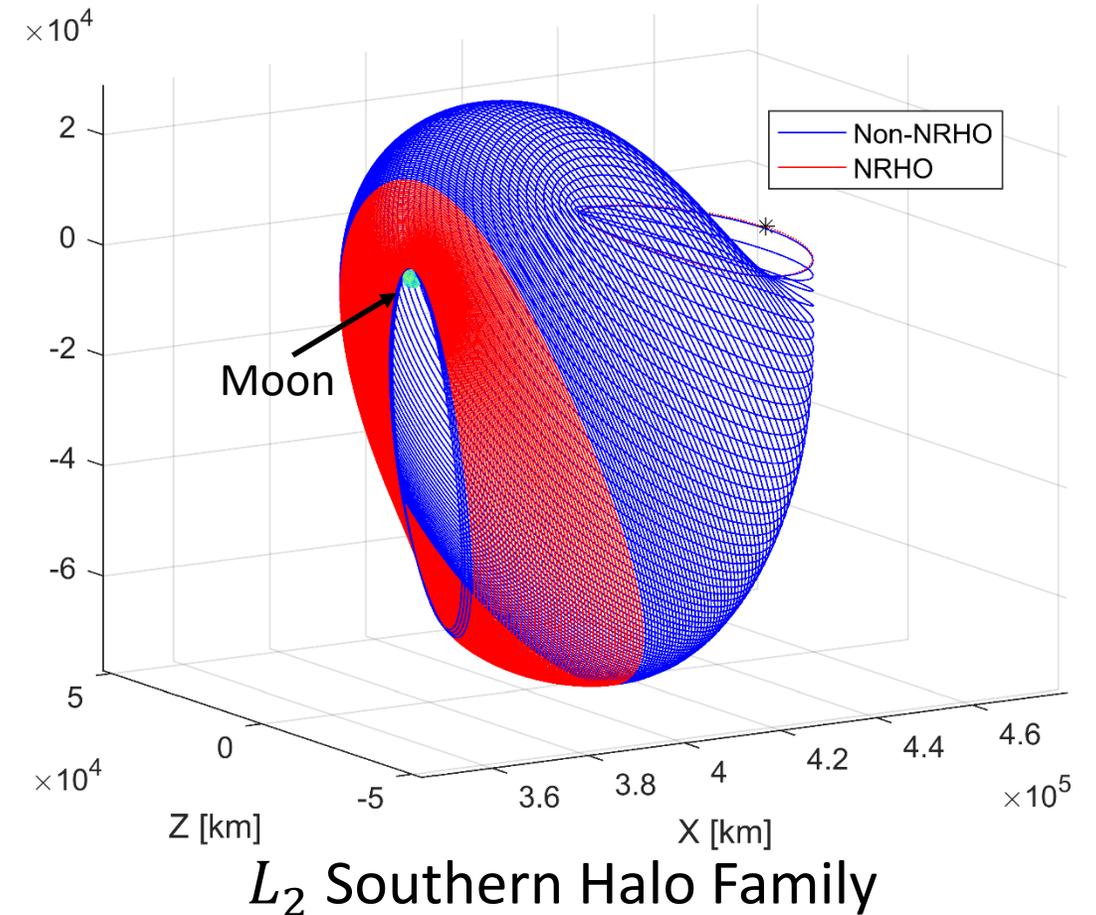
- A variety of stable or near-stable periodic orbits are available in the Circular Restricted Three-Body Problem (CR3BP)
 - Distant Retrograde Orbits (DRO)
 - $L_{4/5}$ Short Period Orbits (SPO)
 - Near-Rectilinear Halo Orbits (NRHO)



L_2 Southern Halo Family

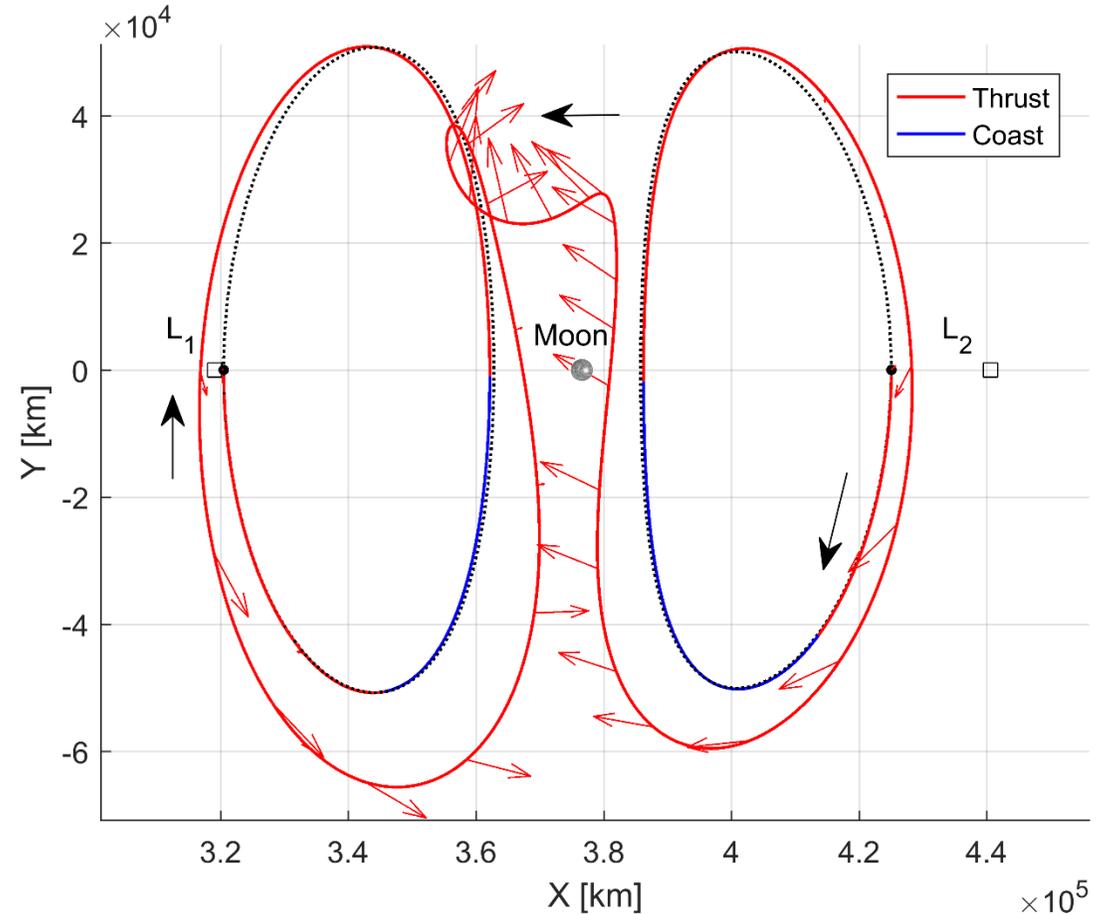
Background

- A variety of stable or near-stable periodic orbits are available in the Circular Restricted Three-Body Problem (CR3BP)
 - Distant Retrograde Orbits (DRO)
 - $L_{4/5}$ Short Period Orbits (SPO)
 - Near-Rectilinear Halo Orbits (NRHO)



Background

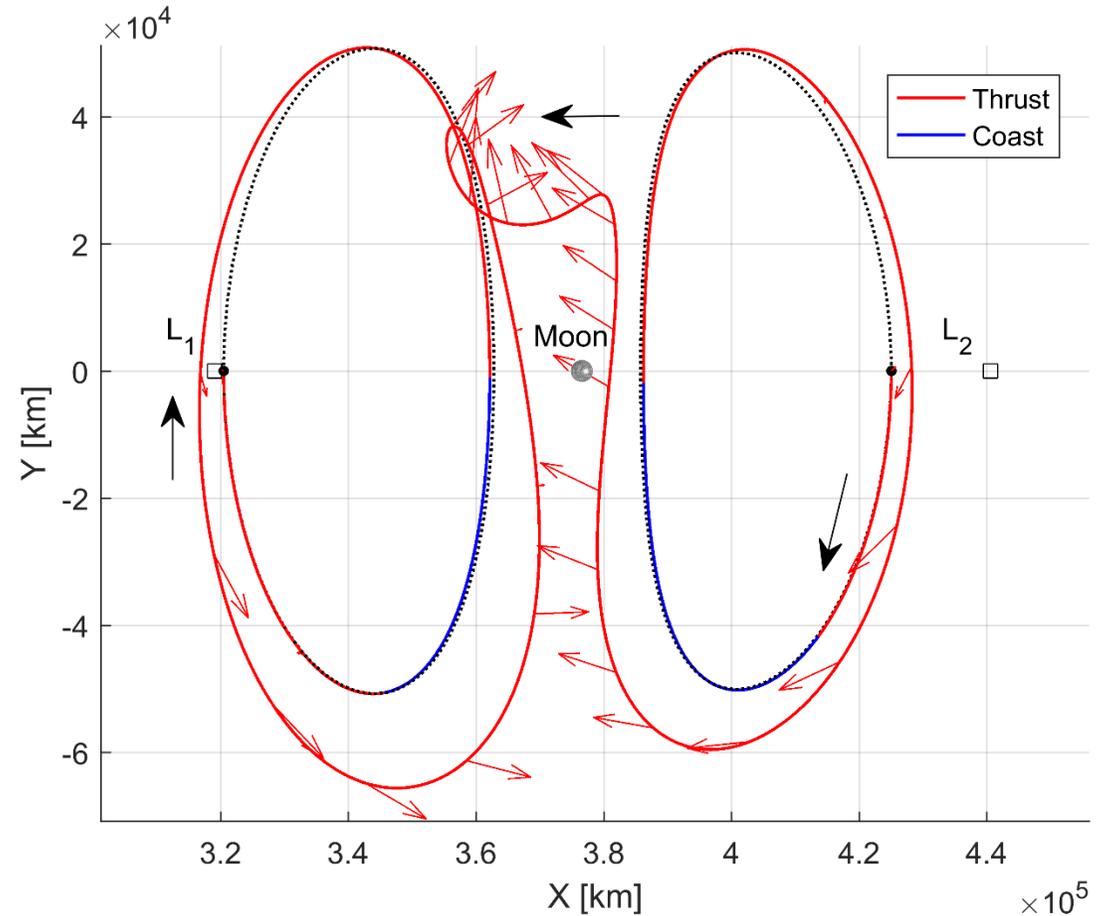
- Low-thrust trajectory design is a continuous optimal control problem, where common objectives are maximization of final mass or minimization of time of flight.
- Direct transcription is a robust method for solving optimal control problems using collocation.
 - Canon, Cullum, and Polak, 1970
 - Hargraves and Paris, 1987
 - Betts and Huffman, 1997



L_2 Halo to L_1 Halo Orbit Transfer

Background

- Two software packages that implement direct transcription for low-thrust trajectory design are employed:
 - Collocation with Optimization for Low-Thrust (COLT)
 - MColl
- MColl enables low-thrust trajectory design within MONTE (Mission Analysis Operation and Navigation Toolkit Environment)

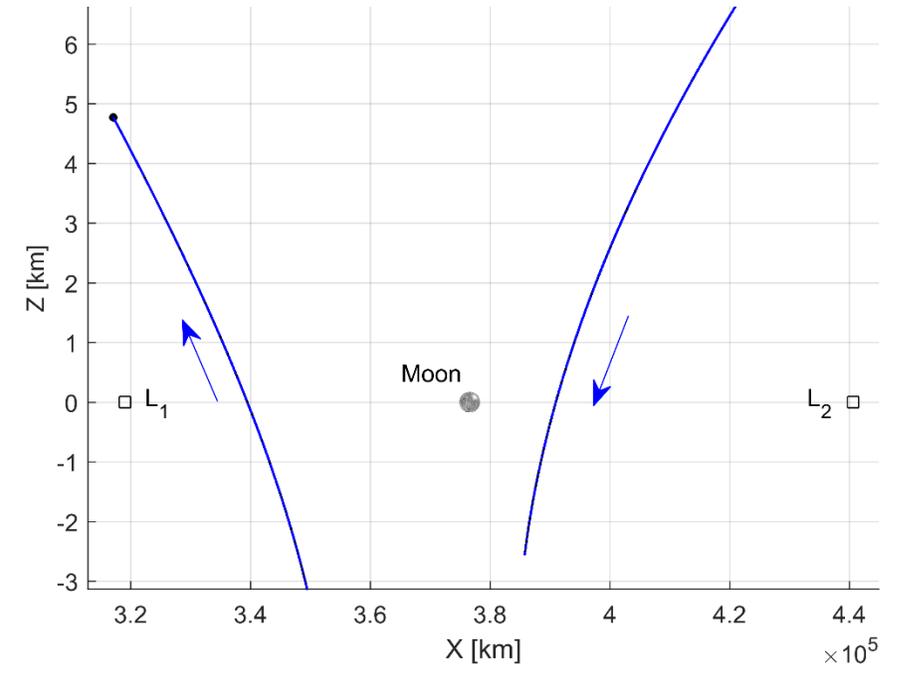
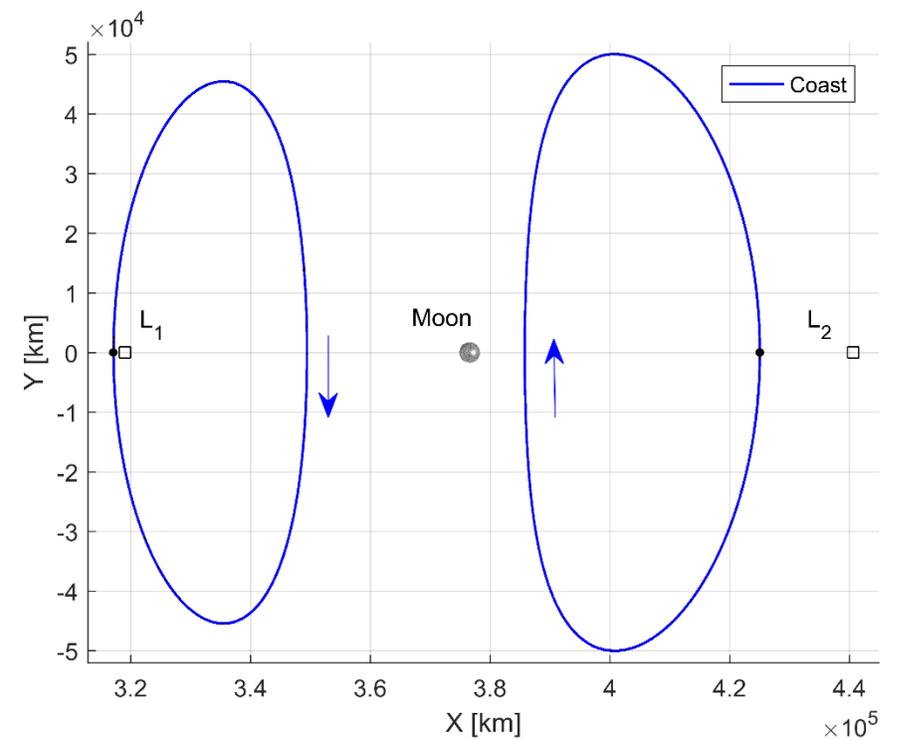


L_2 Halo to L_1 Halo Orbit Transfer

Background

Trajectory Design with Direct Transcription Steps:

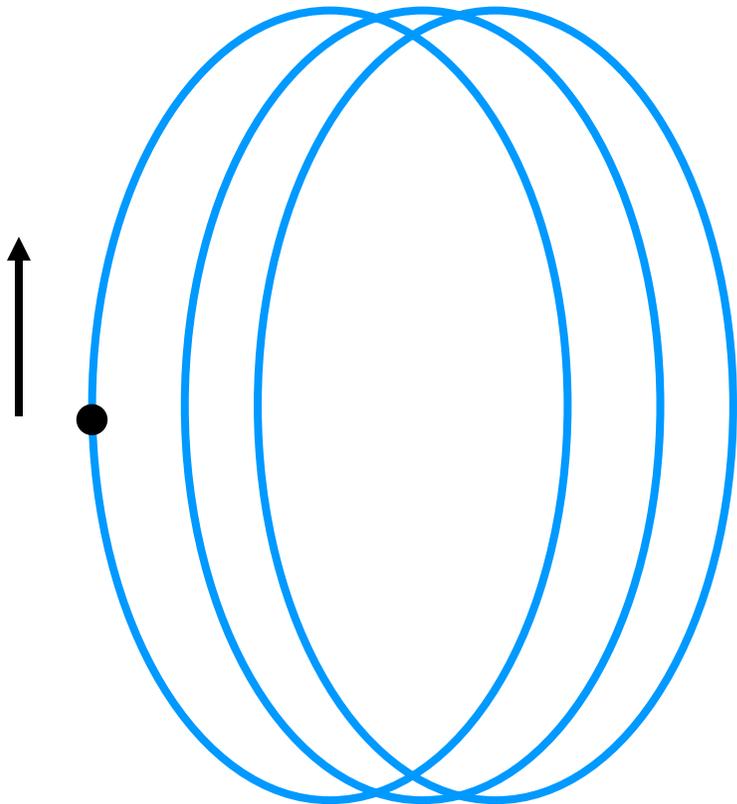
1. Construct initial guess with trajectory stacking technique



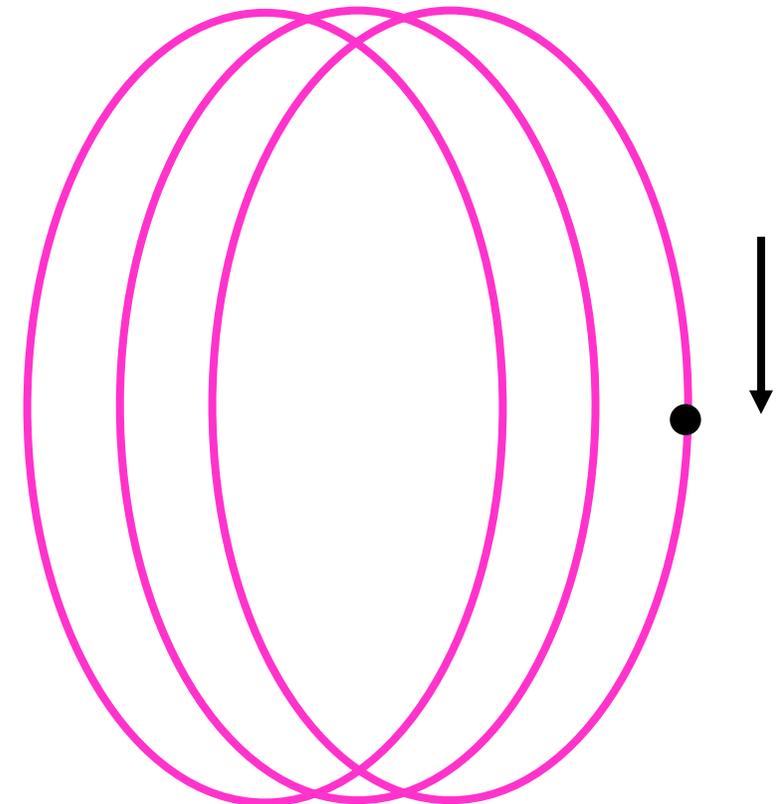
Background

Trajectory Stacking:

$$\text{TOF} = 3 \text{ rev} * \tau_0 + 3 \text{ rev} * \tau_f$$



3 Revolution

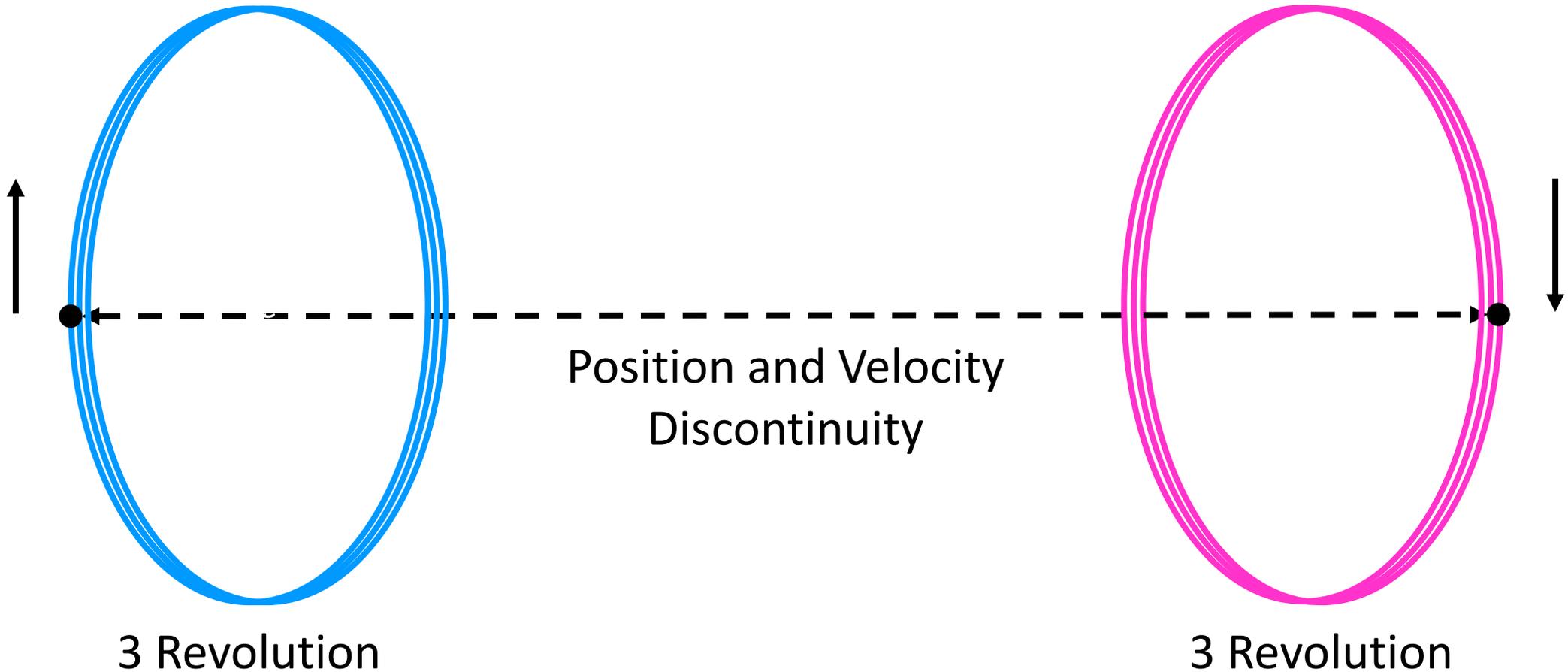


3 Revolution

Background

Trajectory Stacking:

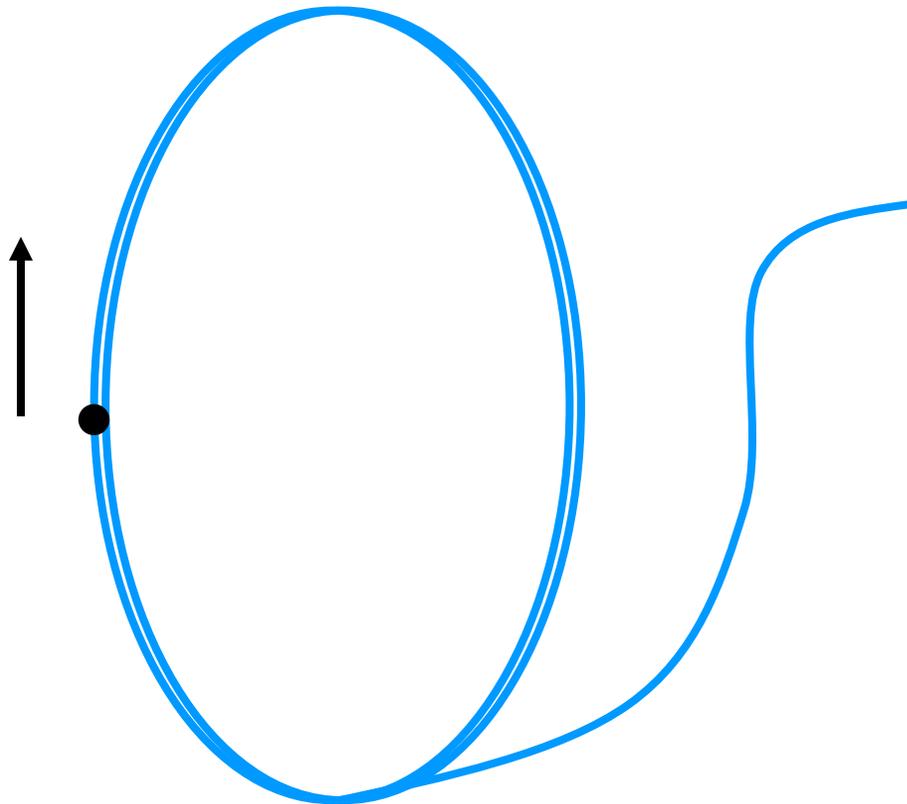
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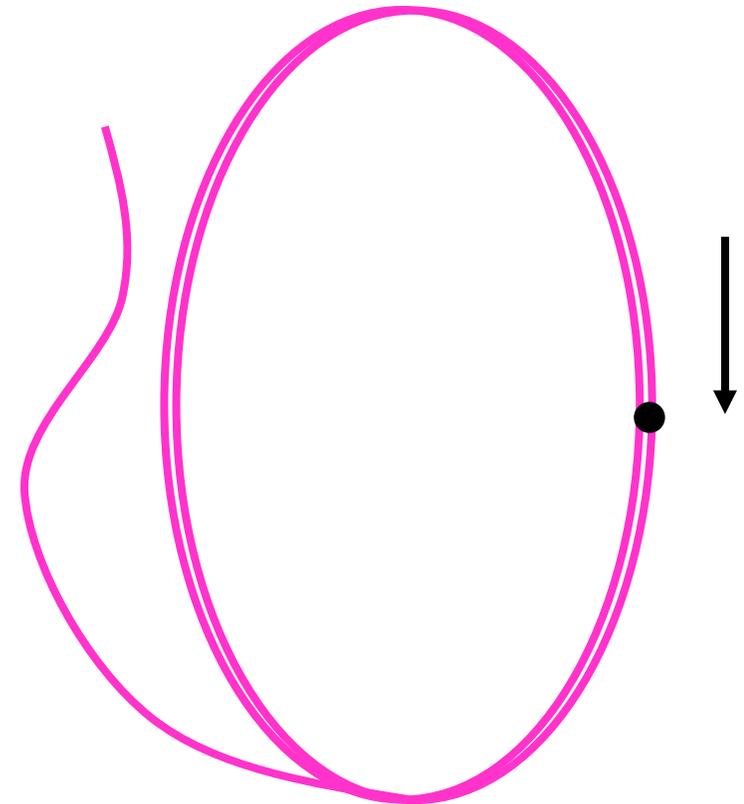
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Trajectory Stacking:

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3 Revolution

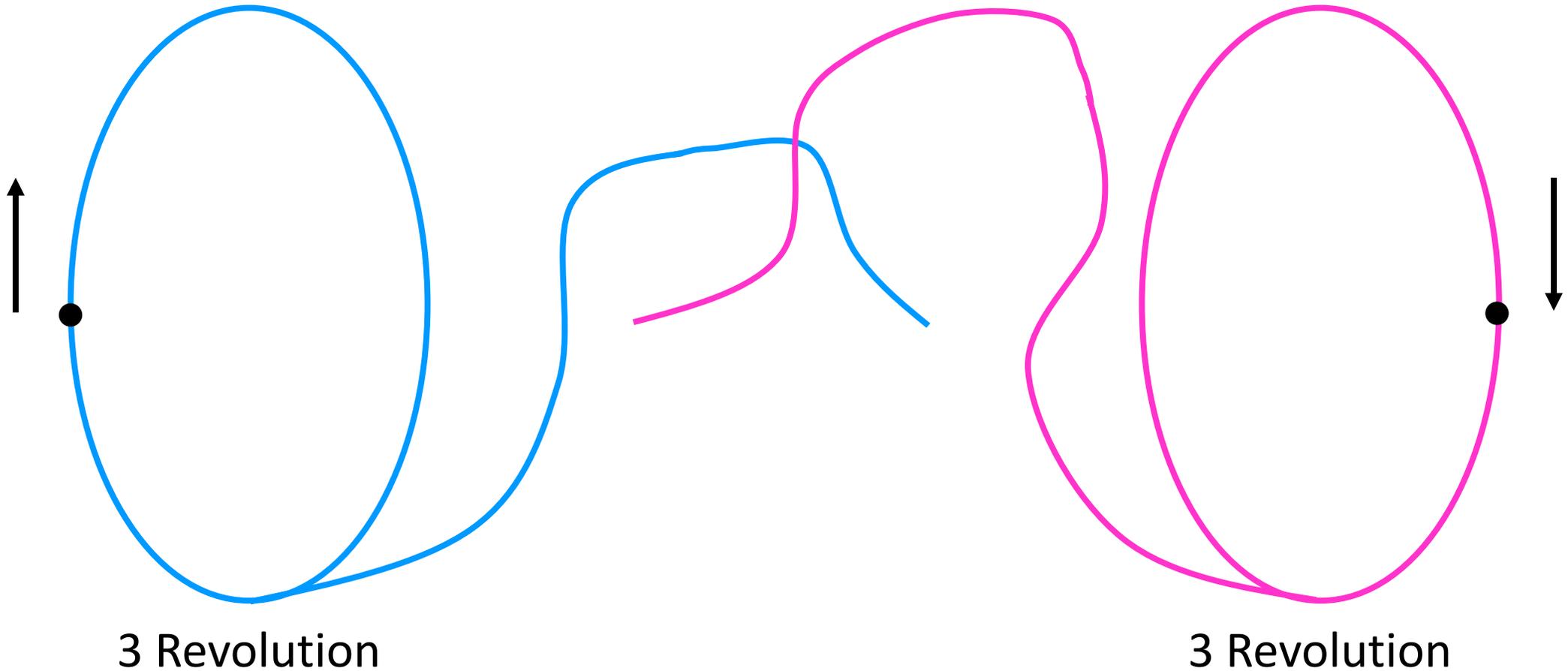


3 Revolution

Background

Trajectory Stacking:

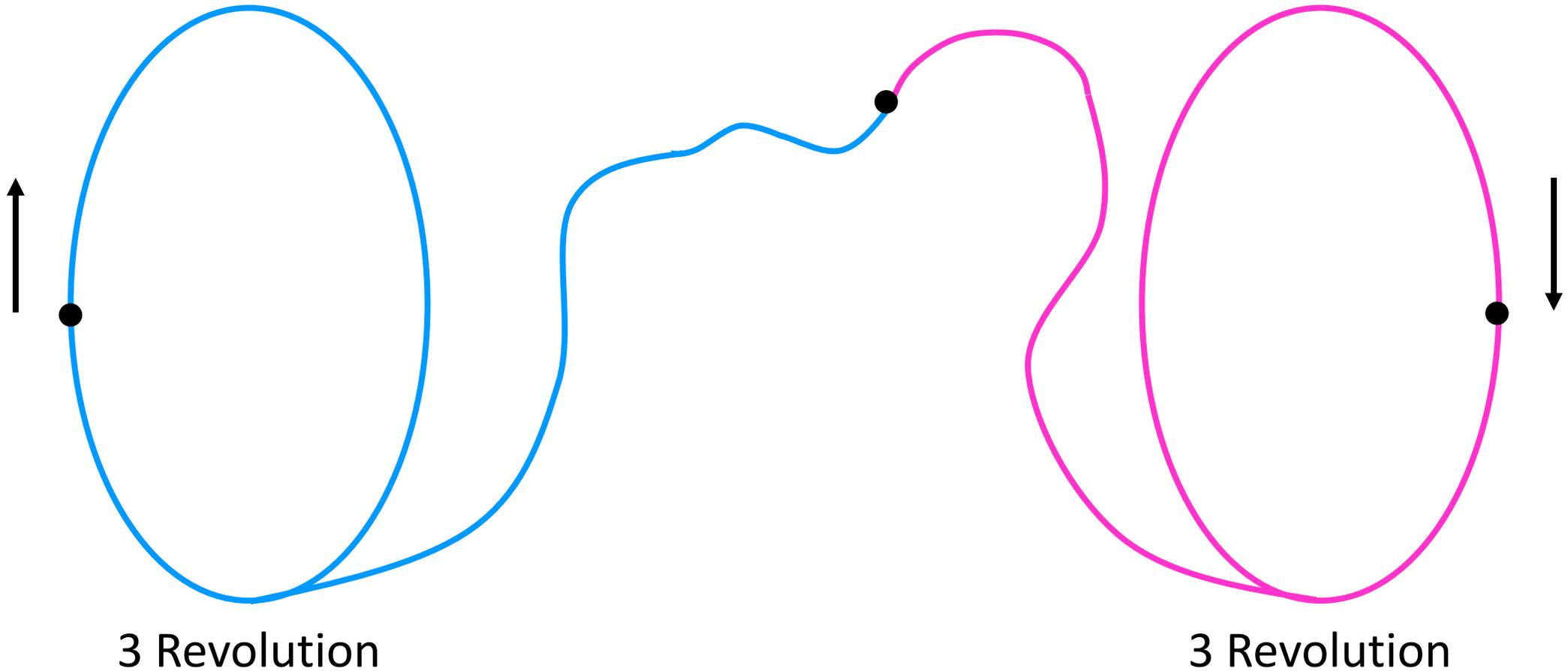
$$\text{TOF} = 3 \text{ rev} * \tau_0 + 3 \text{ rev} * \tau_f$$



Background

Trajectory Stacking:

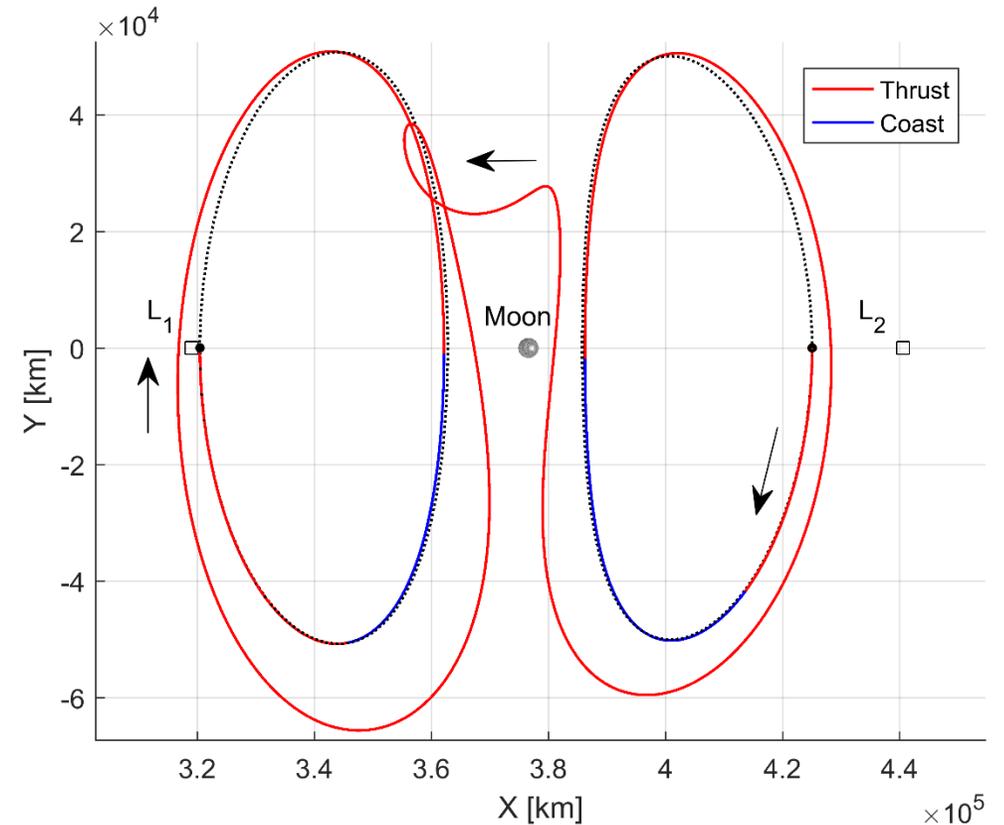
$$\text{TOF} = 3 \text{ rev} * \tau_0 + 3 \text{ rev} * \tau_f$$



Background

Trajectory Design with Direct Transcription Steps:

1. Stack revolutions on departure and arrival orbits to construct initial guess.
2. Converge feasible solution.

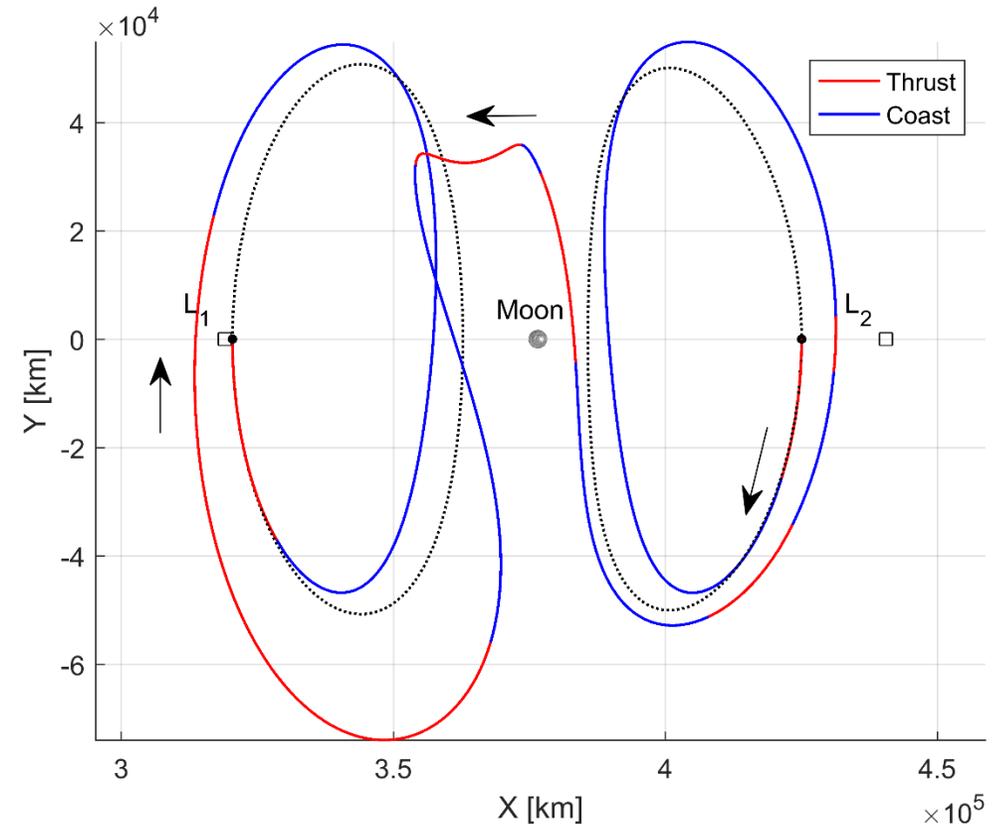


Feasible Transfer

Background

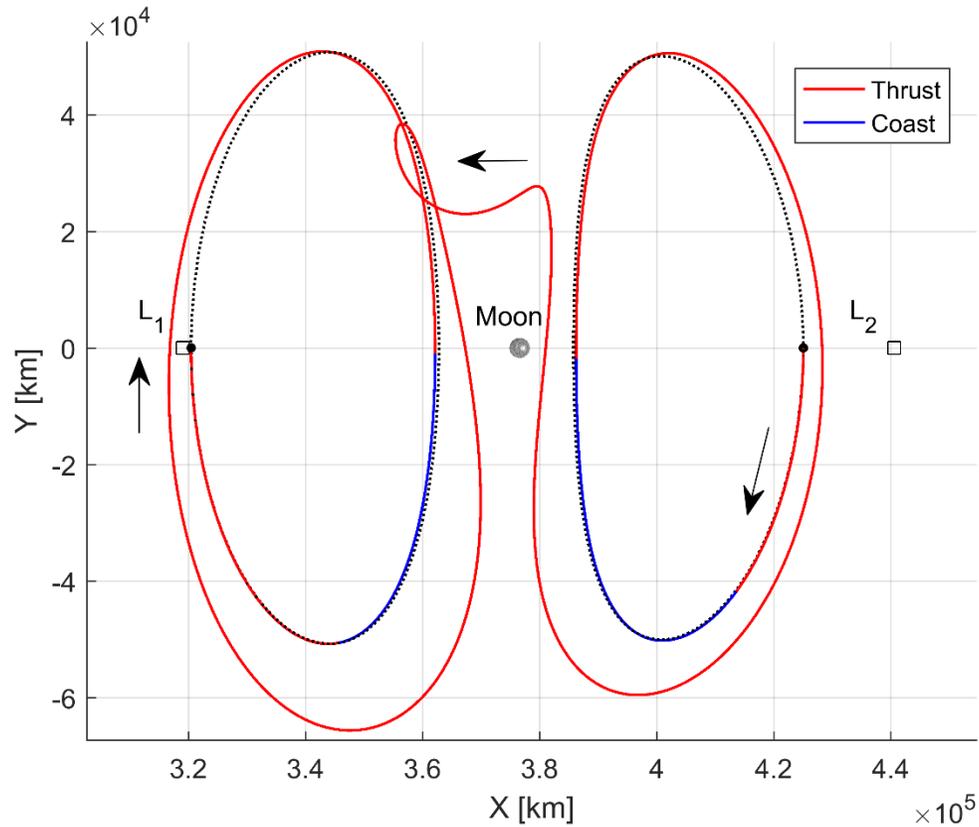
Trajectory Design with Direct Transcription Steps:

1. Stack revolutions on departure and arrival orbits to construct initial guess.
2. Converge feasible solution.
3. Pass feasible solution to optimization algorithm

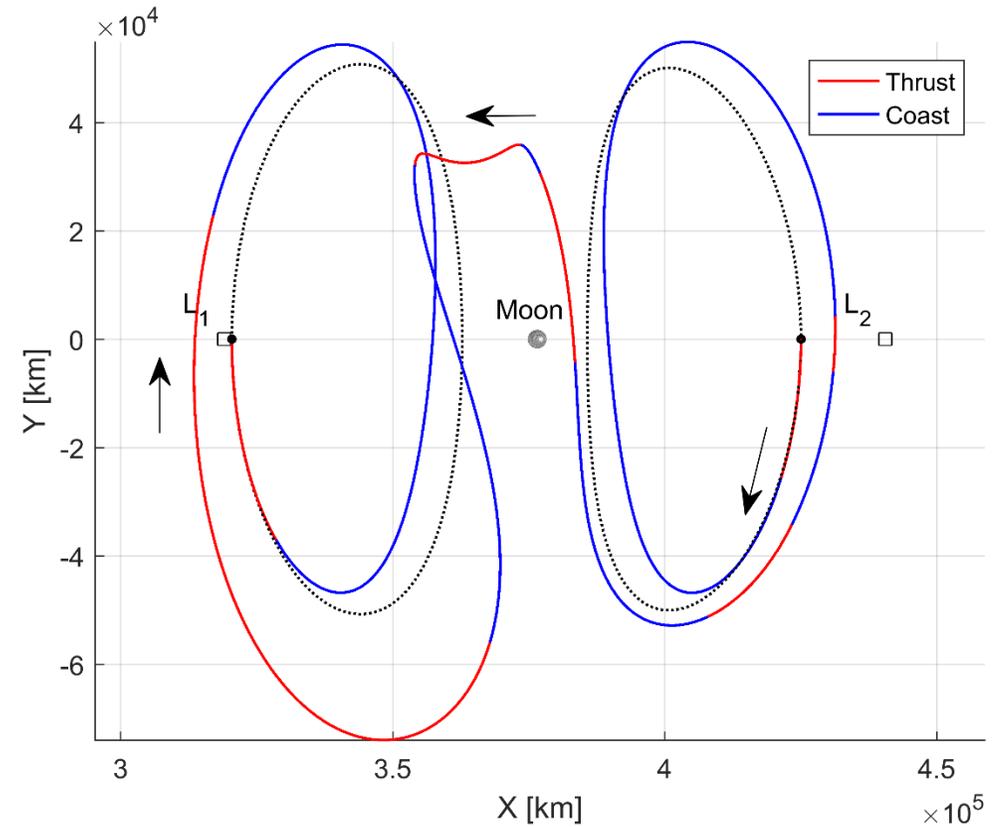


Optimal Transfer

Background

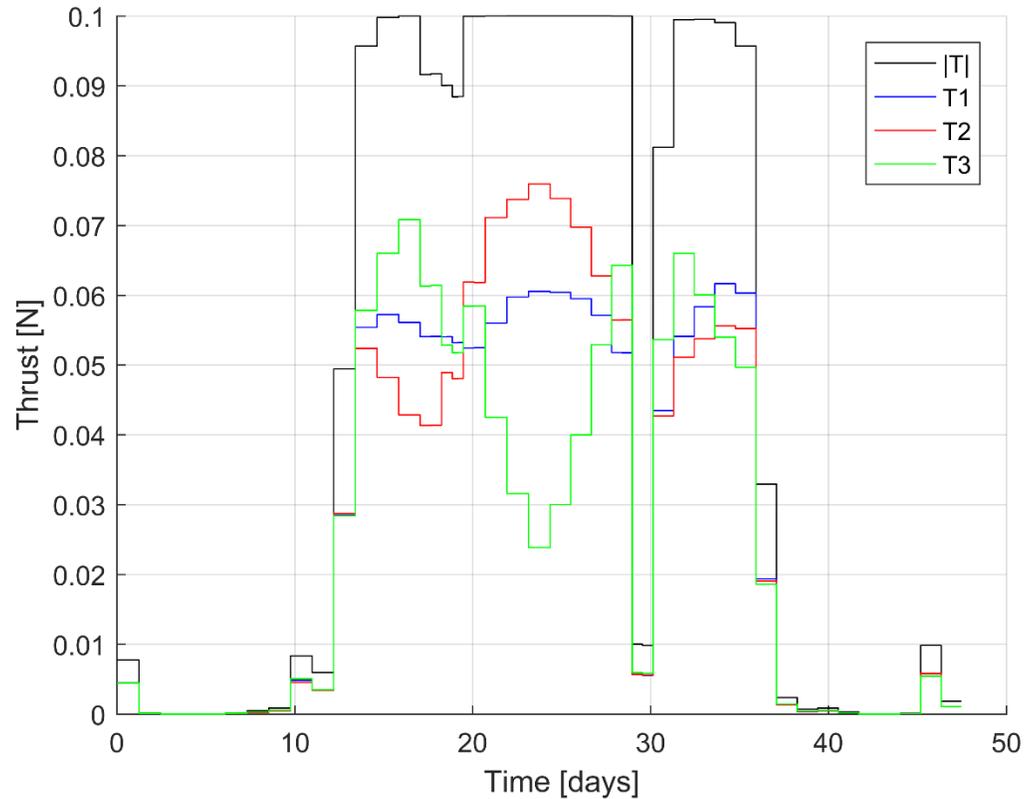


Feasible Transfer
 $m_f/m_0 = 490/500$

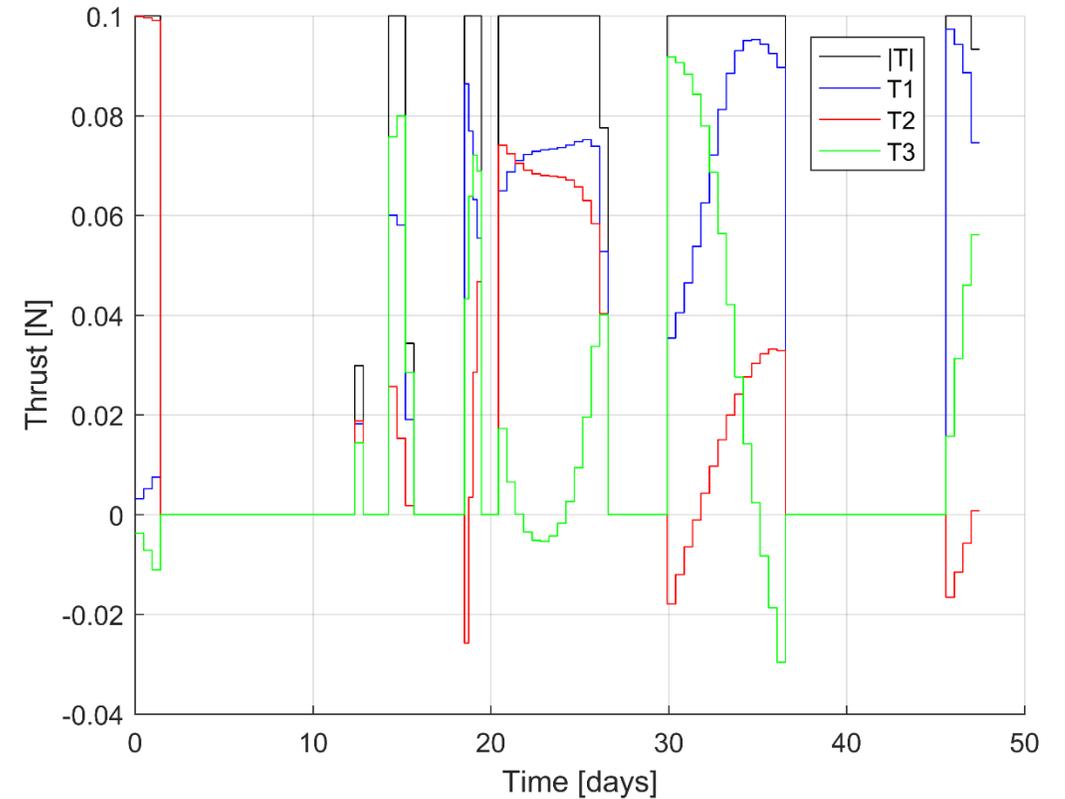


Optimal Transfer
 $m_f/m_0 = 492/500$

Background



Feasible Thrust Profile
 $m_f/m_0 = 490/500$

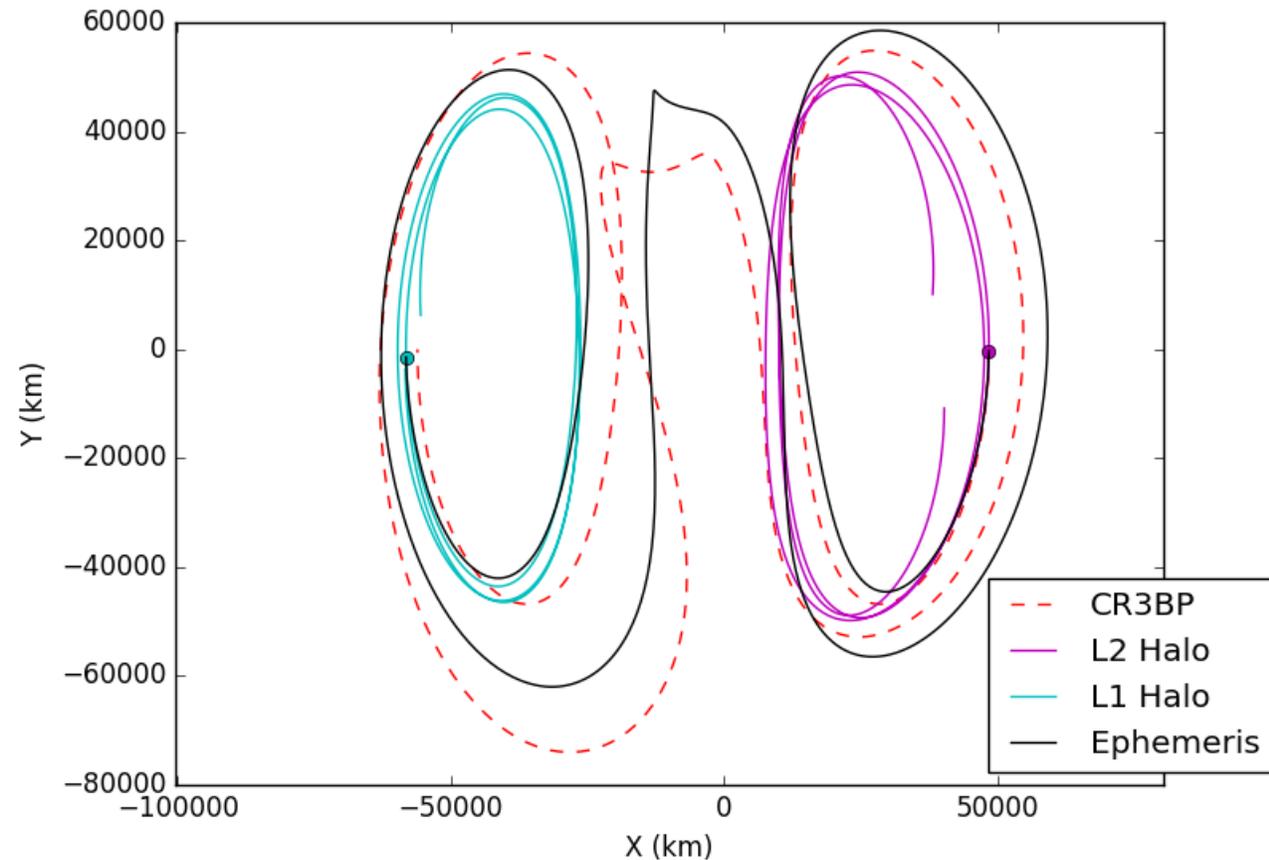


Optimal Thrust Profile
 $m_f/m_0 = 492/500$

Background

Trajectory Design with Direct Transcription Steps:

1. Stack revolutions on departure and arrival orbits to construct initial guess.
2. Converge feasible solution.
3. Pass feasible solution to optimization algorithm
4. Transition optimal CR3BP transfer to a high-fidelity model

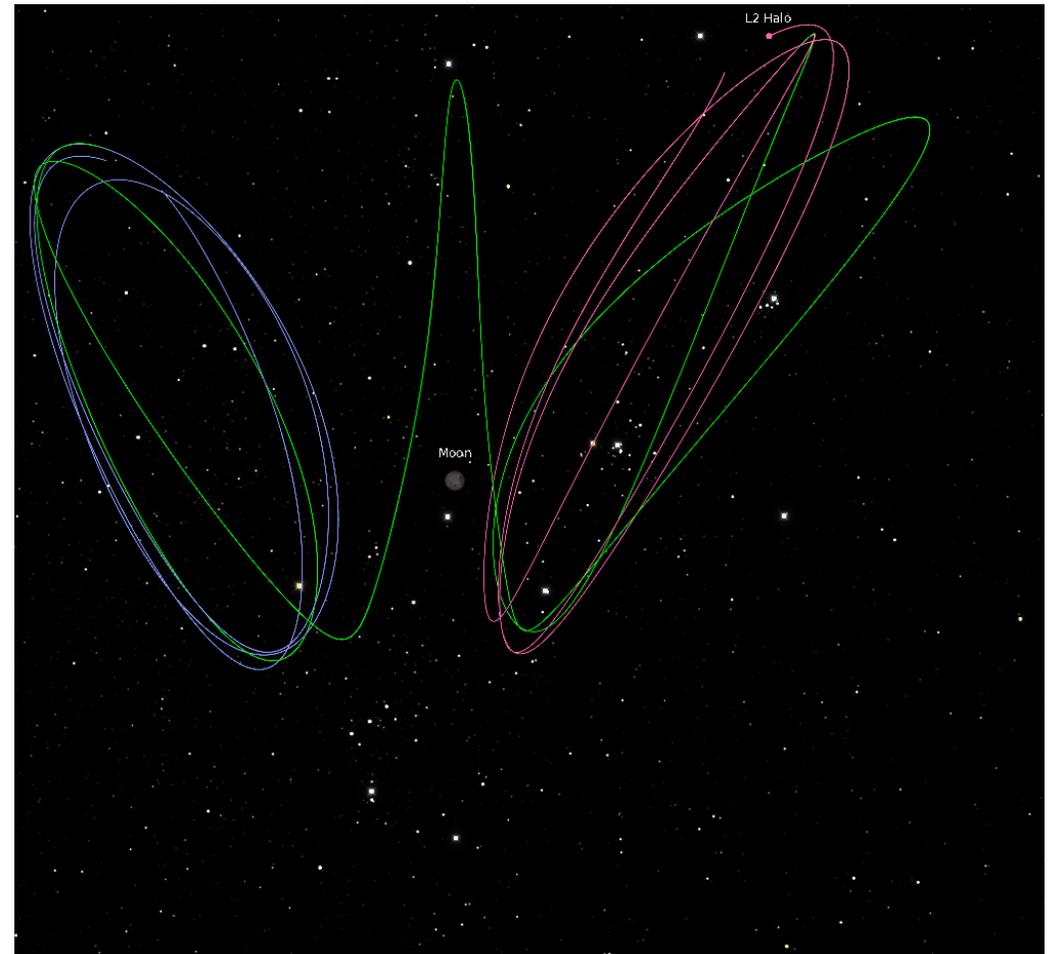


Transfer in a Full Ephemeris Model¹⁷

Background

Trajectory Design with Direct Transcription Steps:

1. Stack revolutions on departure and arrival orbits to construct initial guess.
2. Converge feasible solution.
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I. Trajectory Stacking Technique

II. Orbit Chaining Technique

III. Deep Space Gateway Transfer

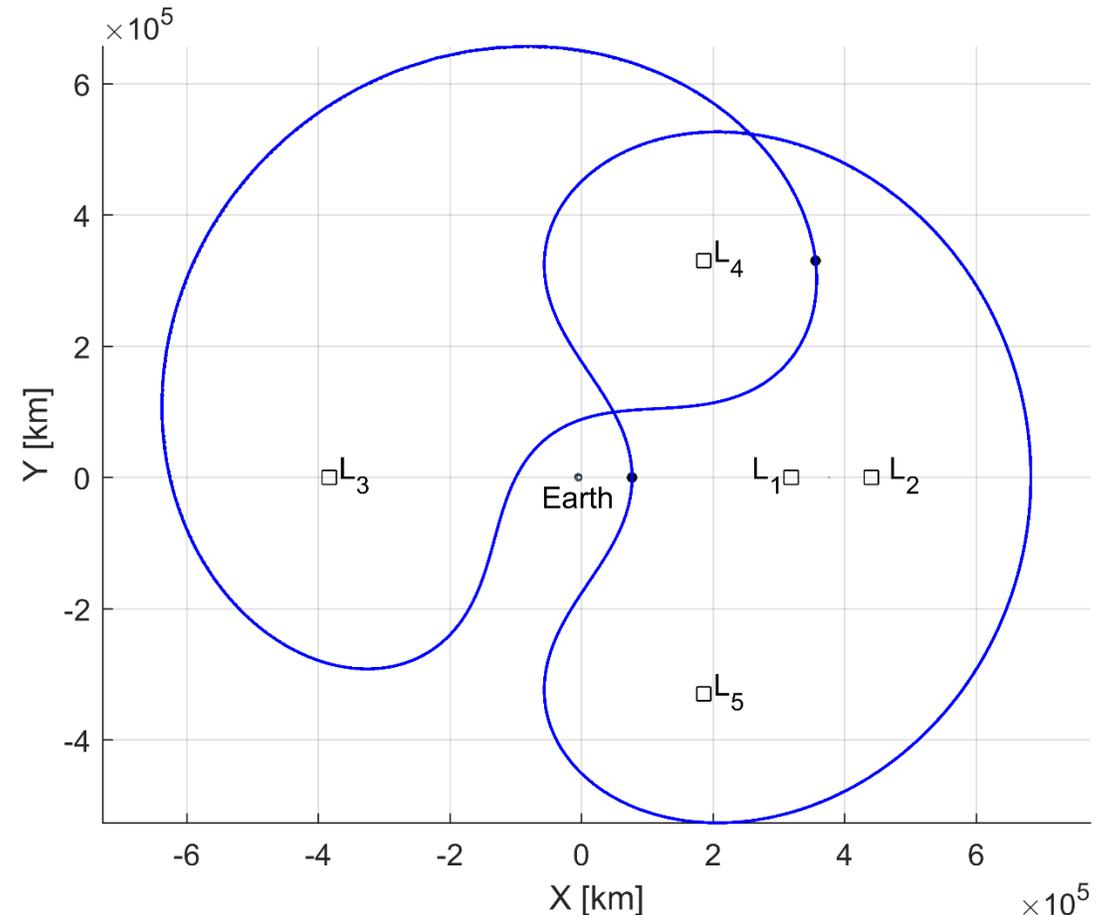
IV. Concluding Remarks

Sample Applications – Trajectory Stacking

Transfer Scenario:

DRO → *L₄ SPO*

- Could facilitate placement of or access to a waystation located at a triangular Lagrange point
- Departure and arrival orbits selected to possess a similar Jacobi constant value



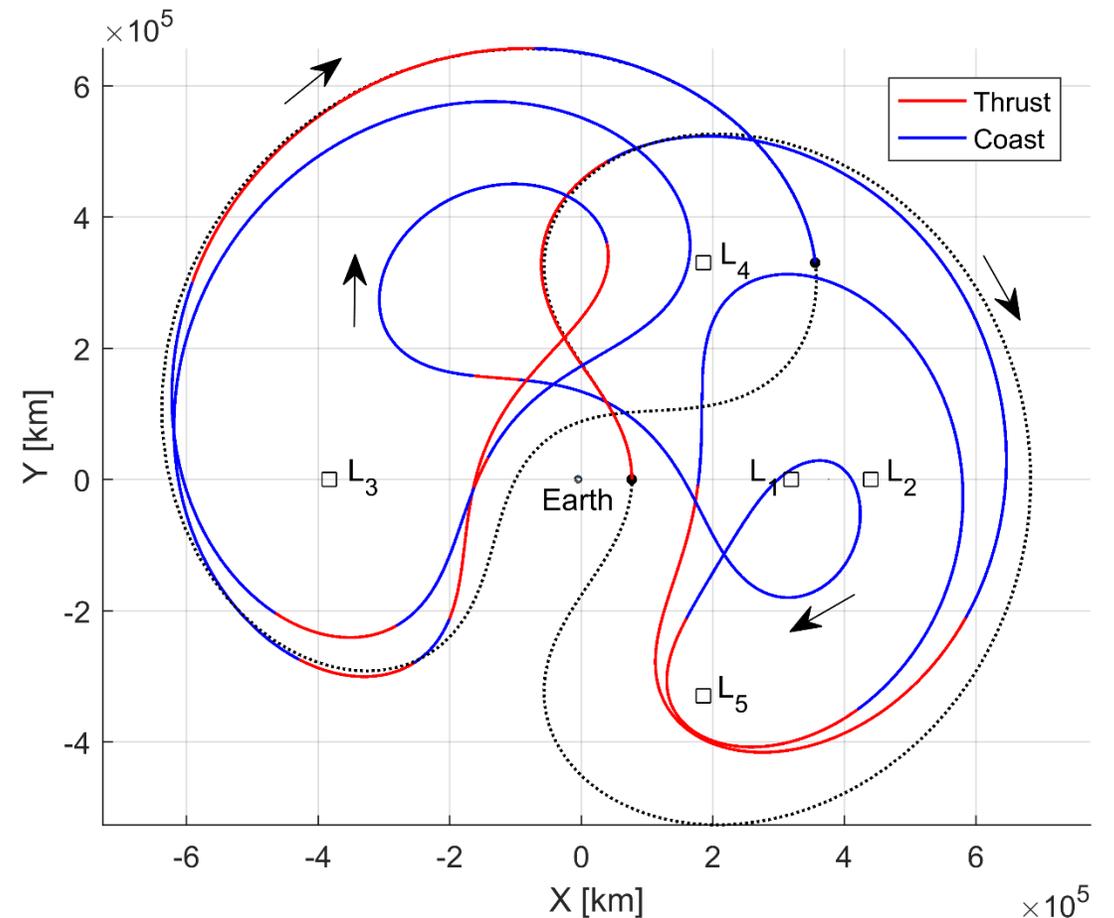
Initial Guess

Sample Applications – Trajectory Stacking

Transfer Scenario:

DRO → *L₄ SPO*

- *DRO 3 Rev* + *L₄ SPO 2 Rev*
- Transfer Time: 134.8 *days*
- Feasible: $m_f/m_0 = 478/500 \text{ kg}$
- Optimal: $m_f/m_0 = 482/500 \text{ kg}$



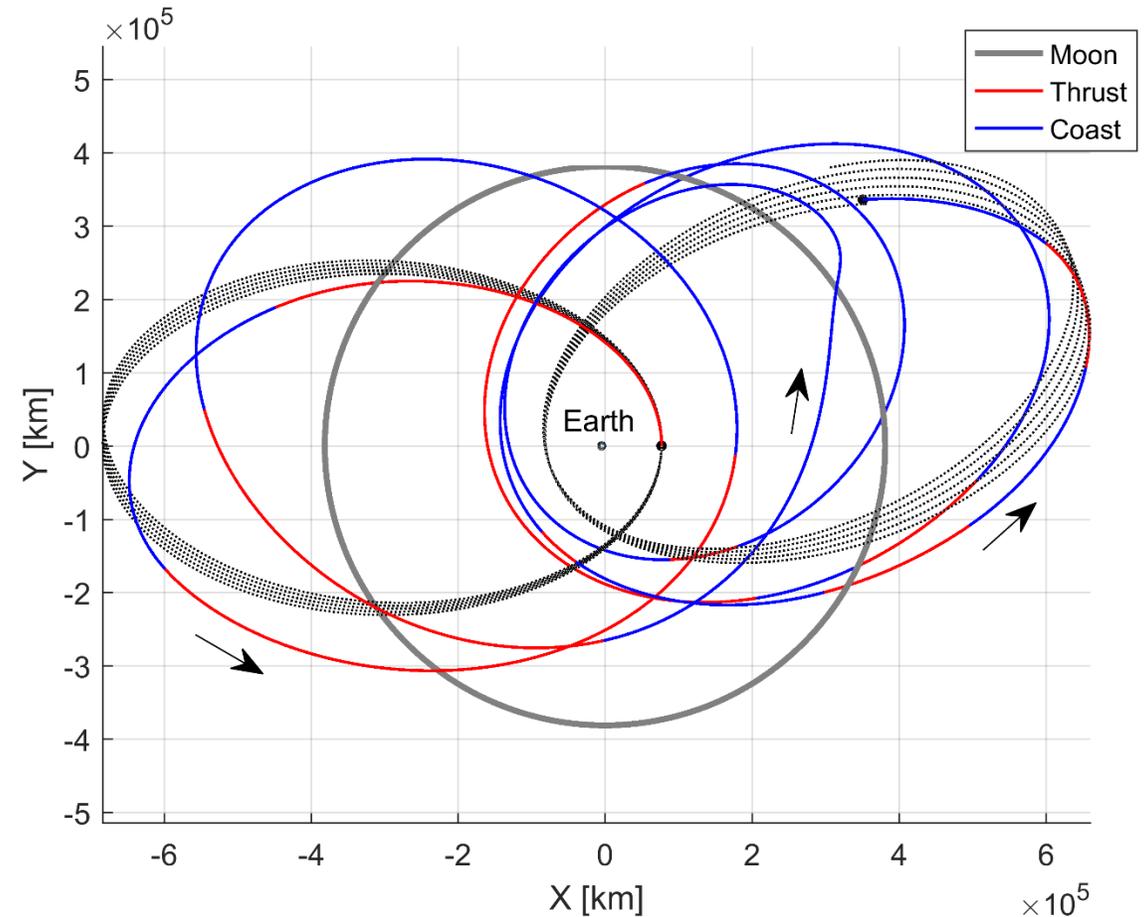
Optimal Transfer – Rotating Frame³²

Sample Applications – Trajectory Stacking

Transfer Scenario:

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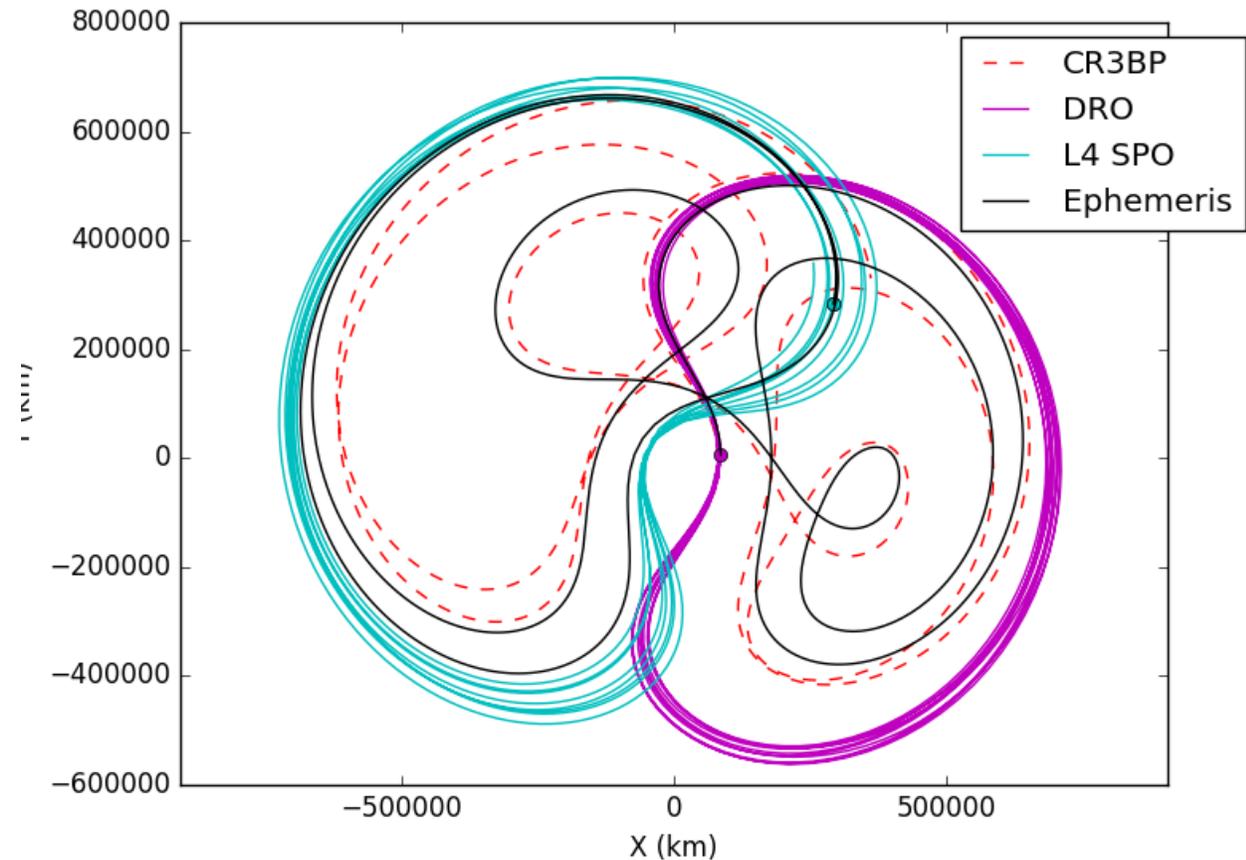
Optimal Transfer – Inertial Frame³³

Sample Applications – Trajectory Stacking

Transfer Scenario:

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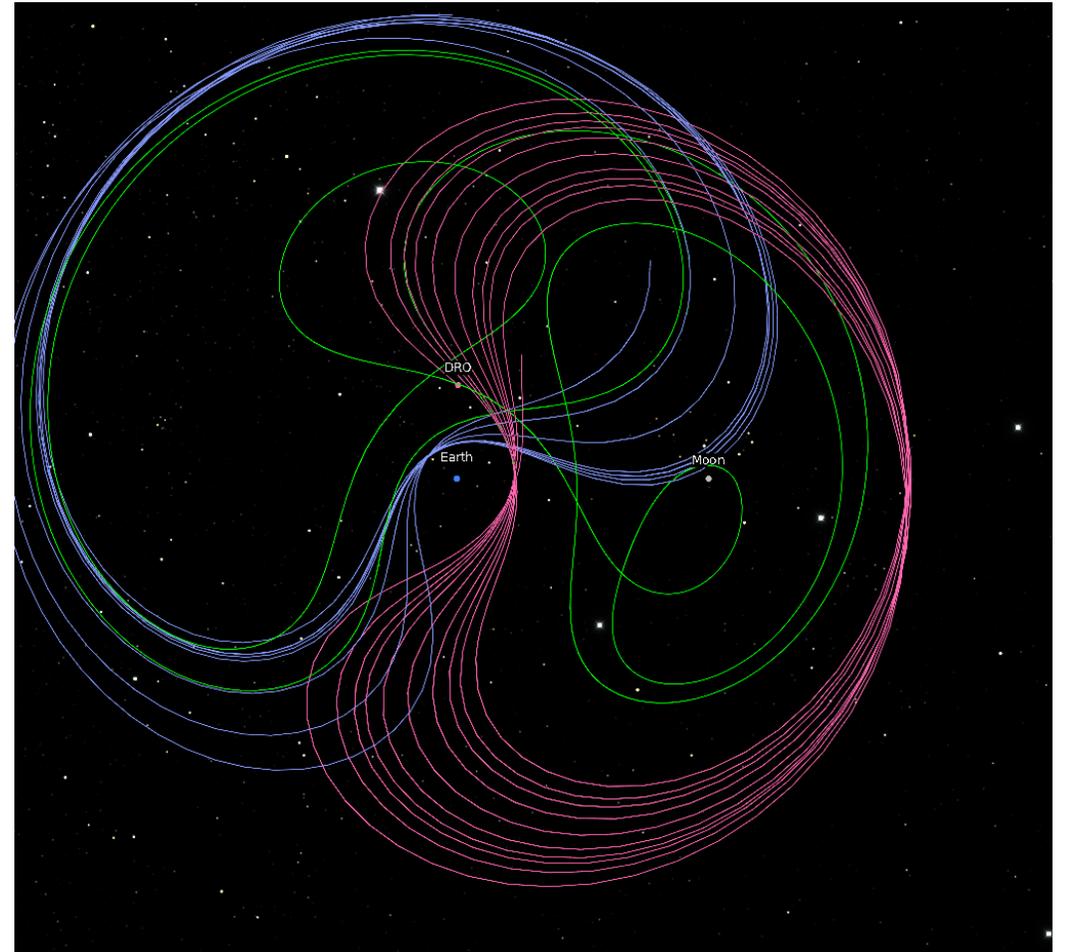


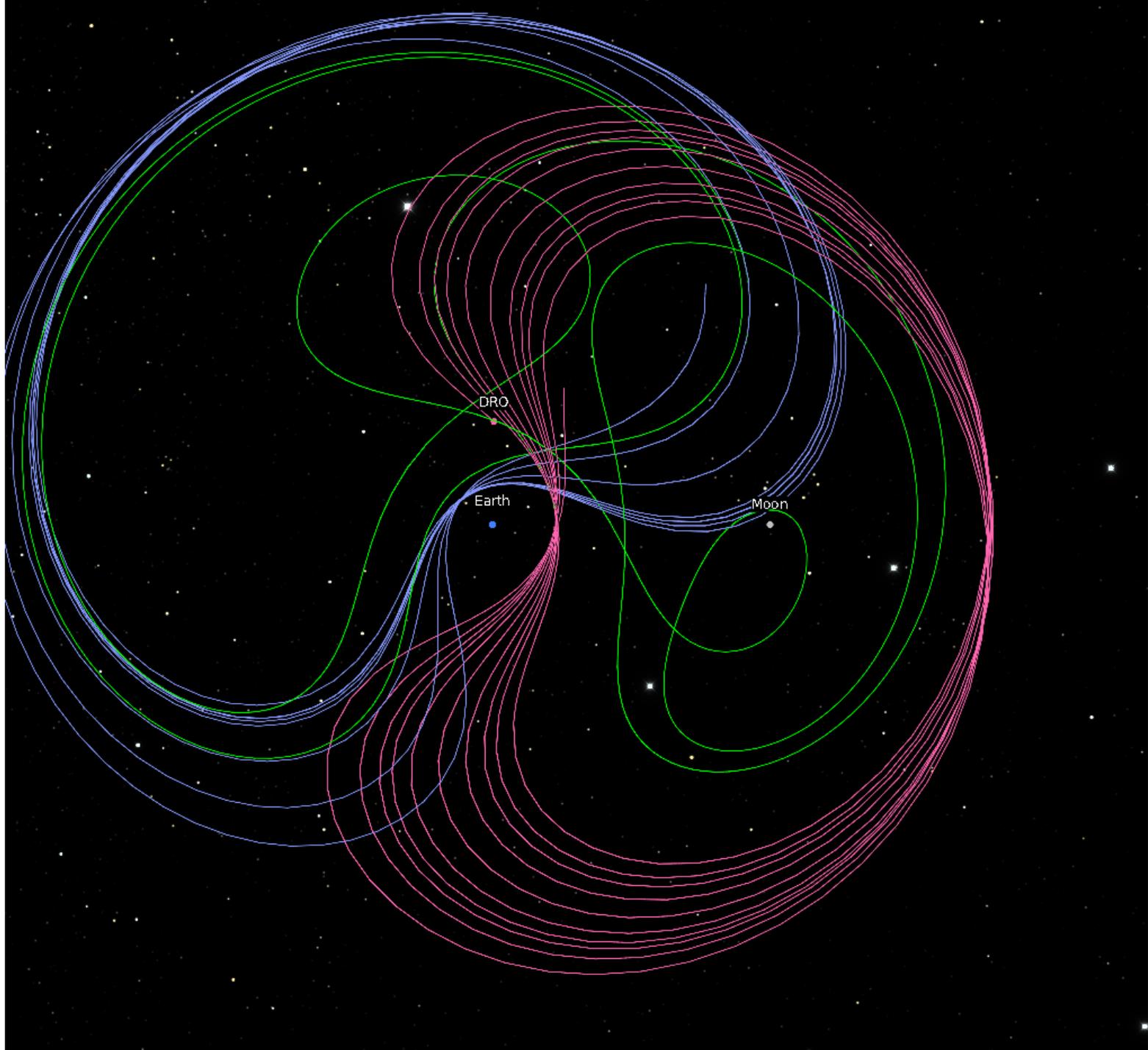
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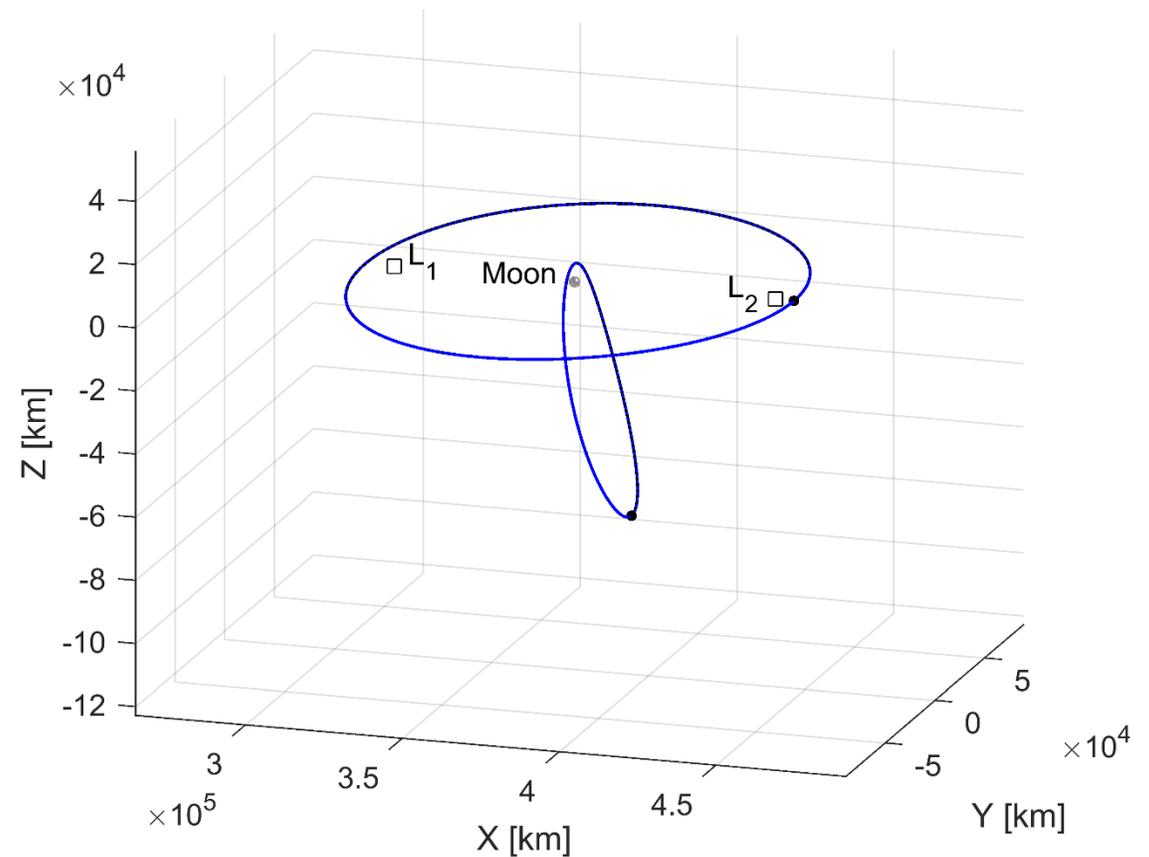


Sample Applications – Trajectory Stacking

Transfer Scenario:

L_2 NRHO \rightarrow DRO

- Could facilitate transfer of space station or communications satellite between two types of lunar orbits
- Orbits have different Jacobi constant values and very different orbital planes.



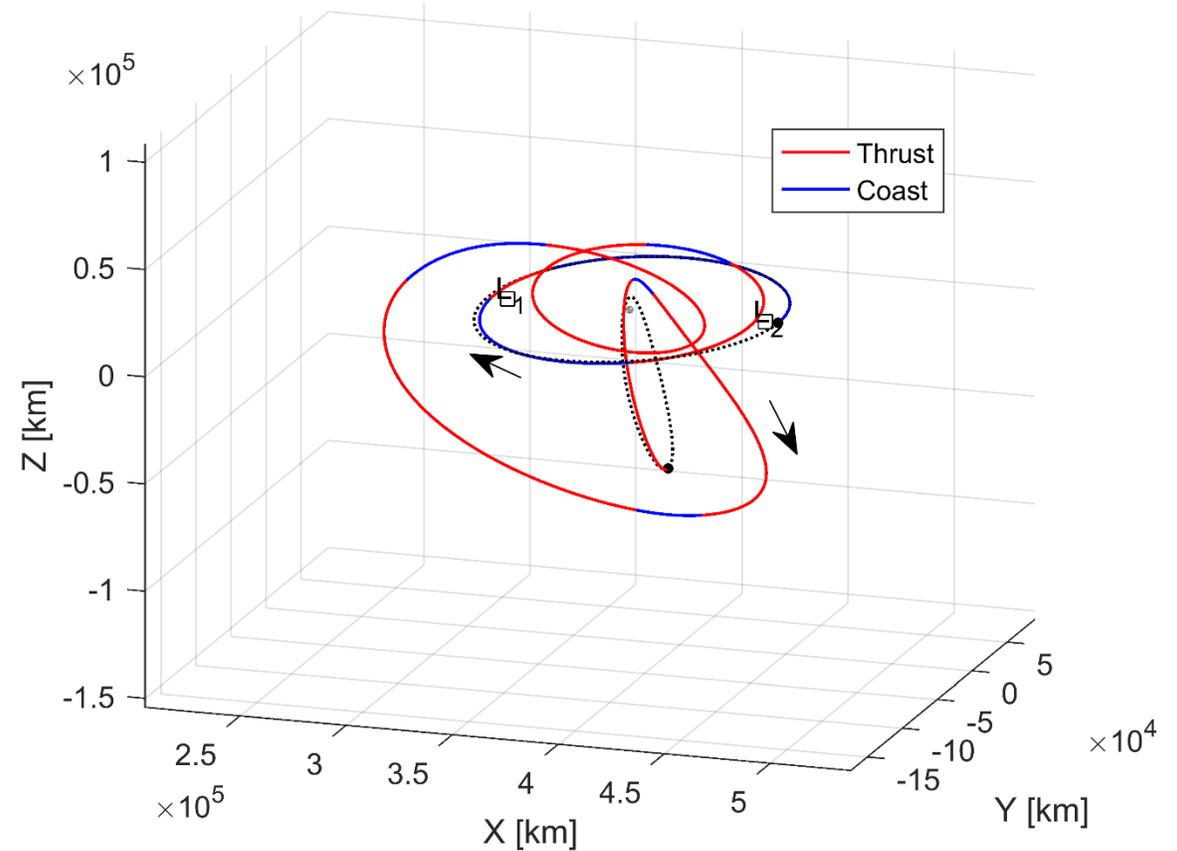
Initial Guess

Sample Applications – Trajectory Stacking

Transfer Scenario:

L_2 NRHO \rightarrow DRO

- NRHO 1 Rev + DRO 3 Rev
- Transfer Time: 46.7 days
- Feasible: $m_f/m_0 = 483/500$ kg
- Optimal: $m_f/m_0 = 486/500$ kg



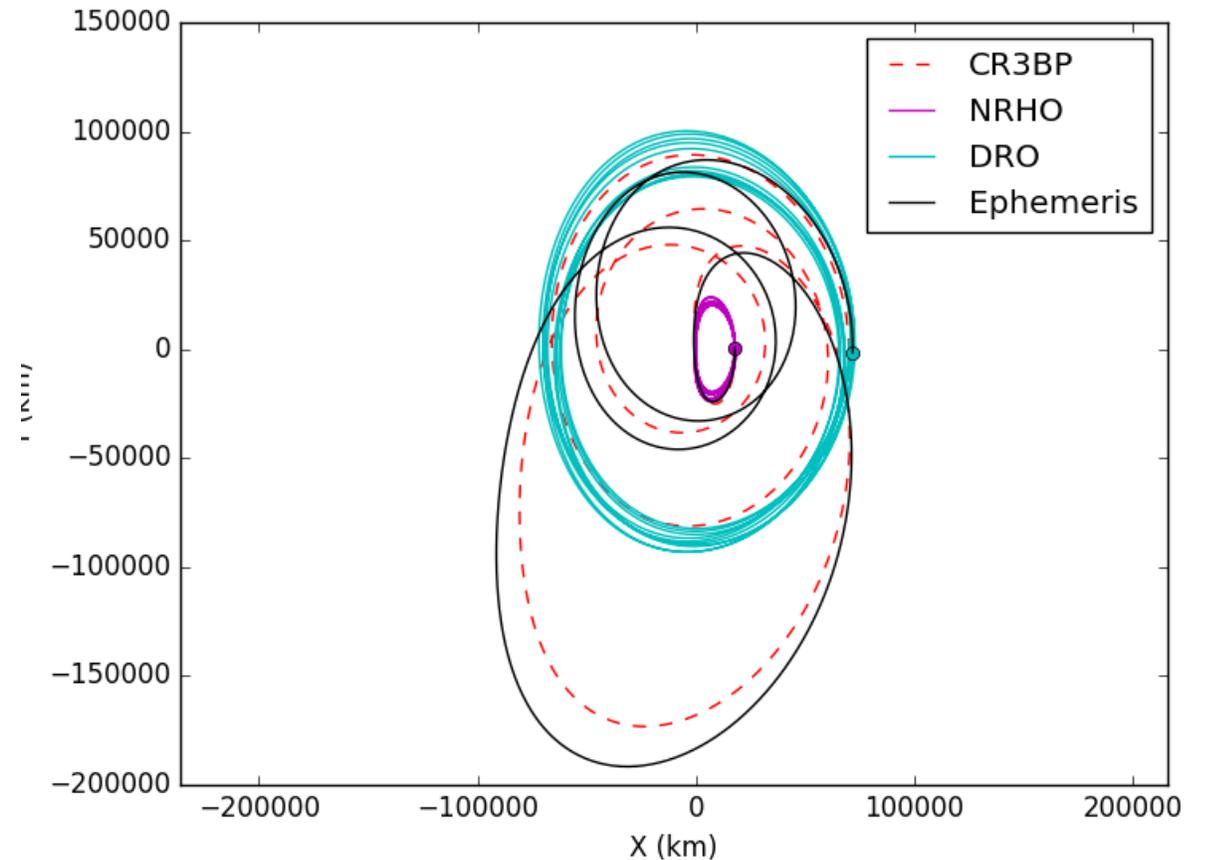
Optimal Transfer – Rotating Frame³⁸

Sample Applications – Trajectory Stacking

Transfer Scenario:

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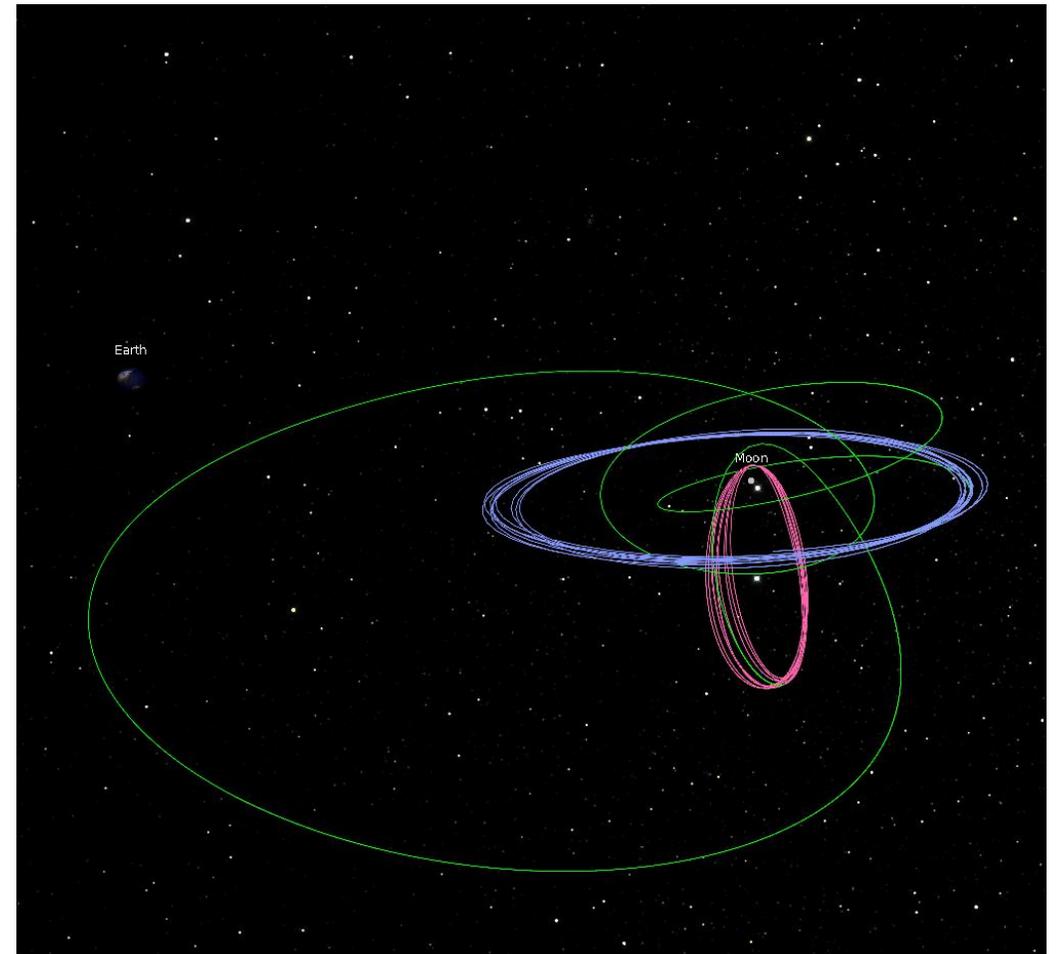
Transfer in a Full Ephemeris Model¹⁹

Sample Applications – Trajectory Stacking

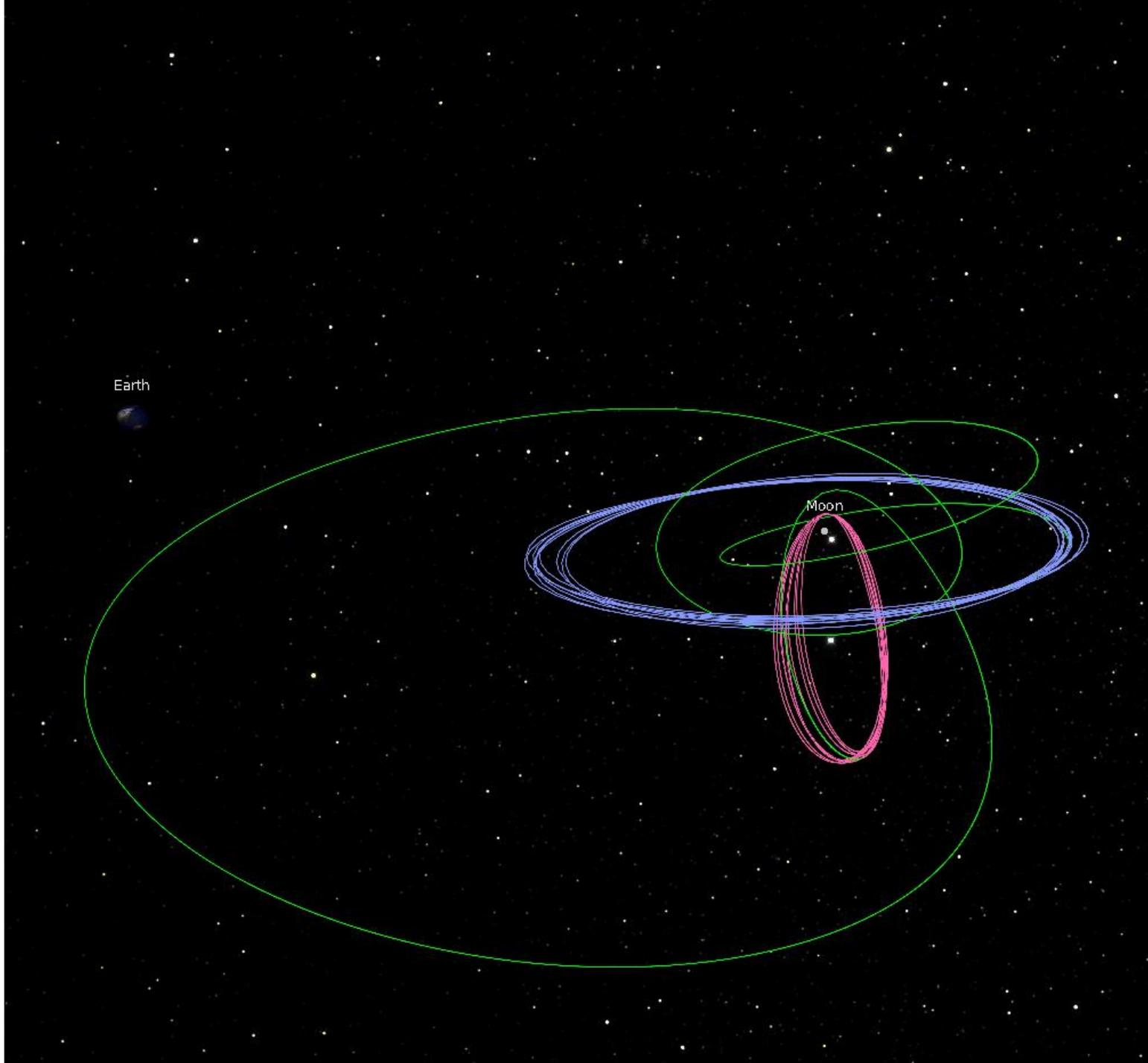
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Transfer in a Full Ephemeris Model¹⁰



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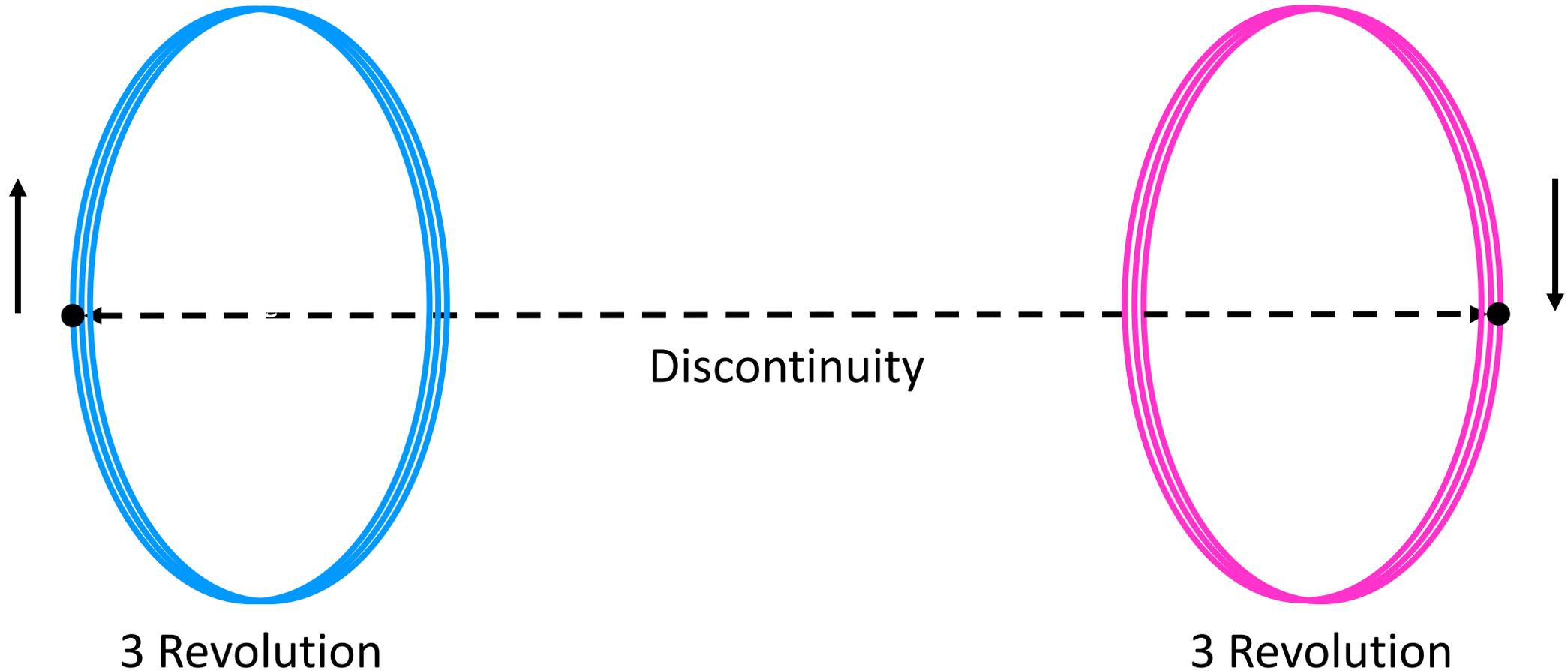
III. Deep Space Gateway Transfer

IV. Concluding Remarks

Sample Applications – Orbit Chaining

Orbit Chaining:

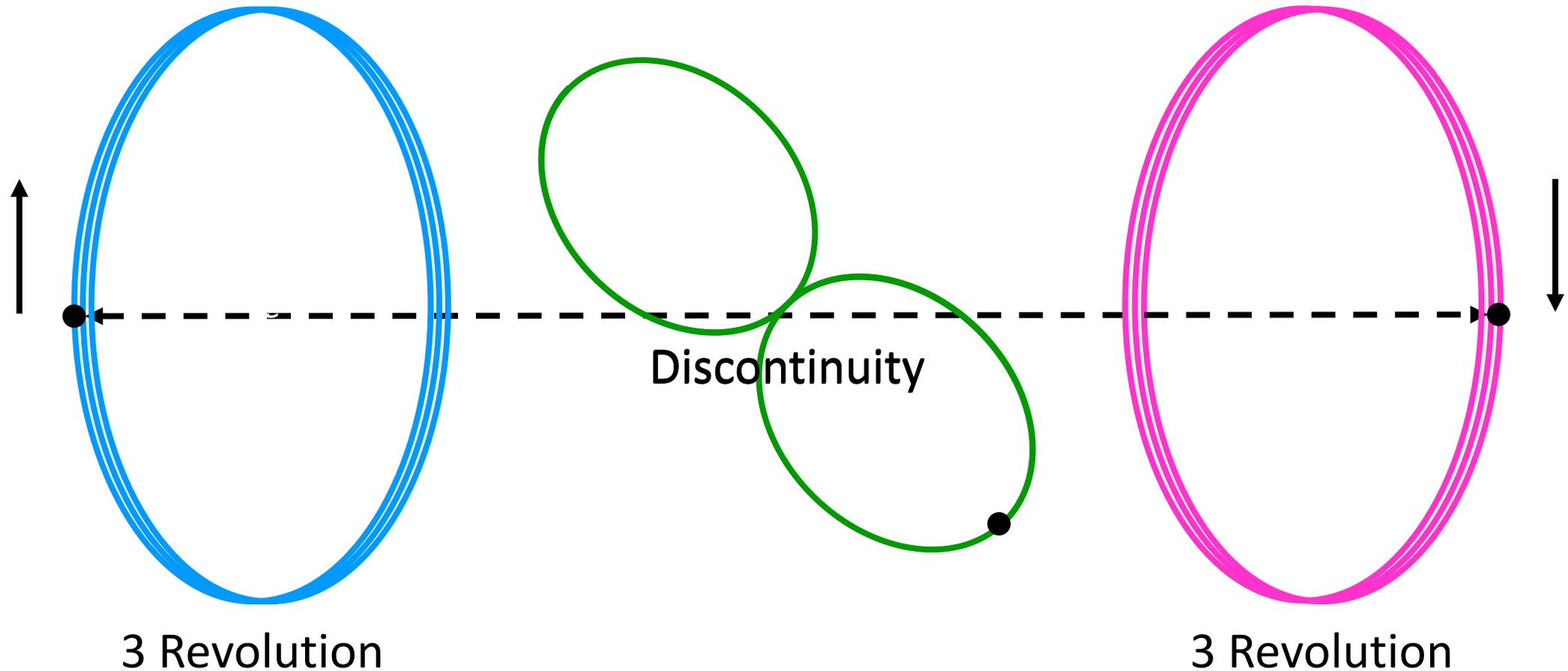
$$\text{TOF} = 3 \text{ rev} * \tau_0 + 3 \text{ rev} * \tau_f$$



Sample Applications – Orbit Chaining

Orbit Chaining:

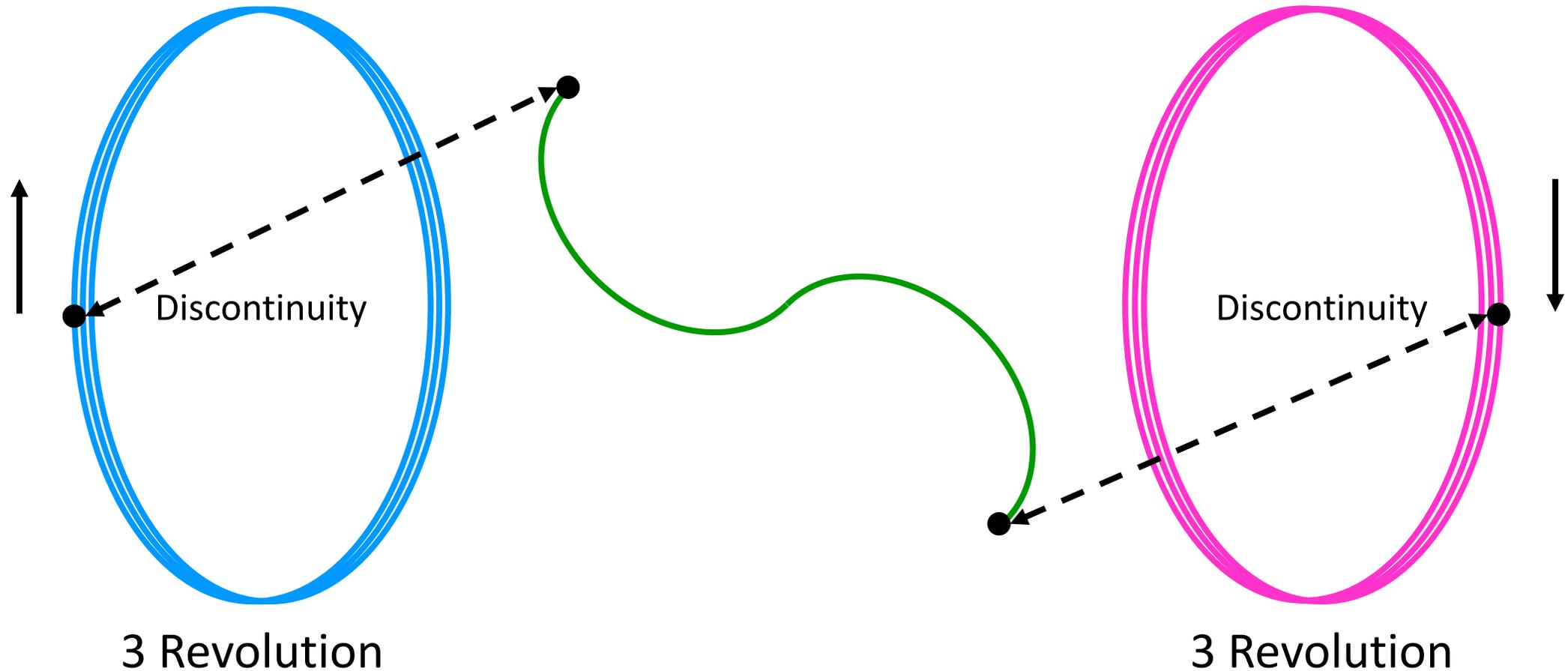
$$\text{TOF} = 3 \text{ rev} * \tau_0 + \tau_{int} + 3 \text{ rev} * \tau_f$$



Sample Applications – Orbit Chaining

Orbit Chaining:

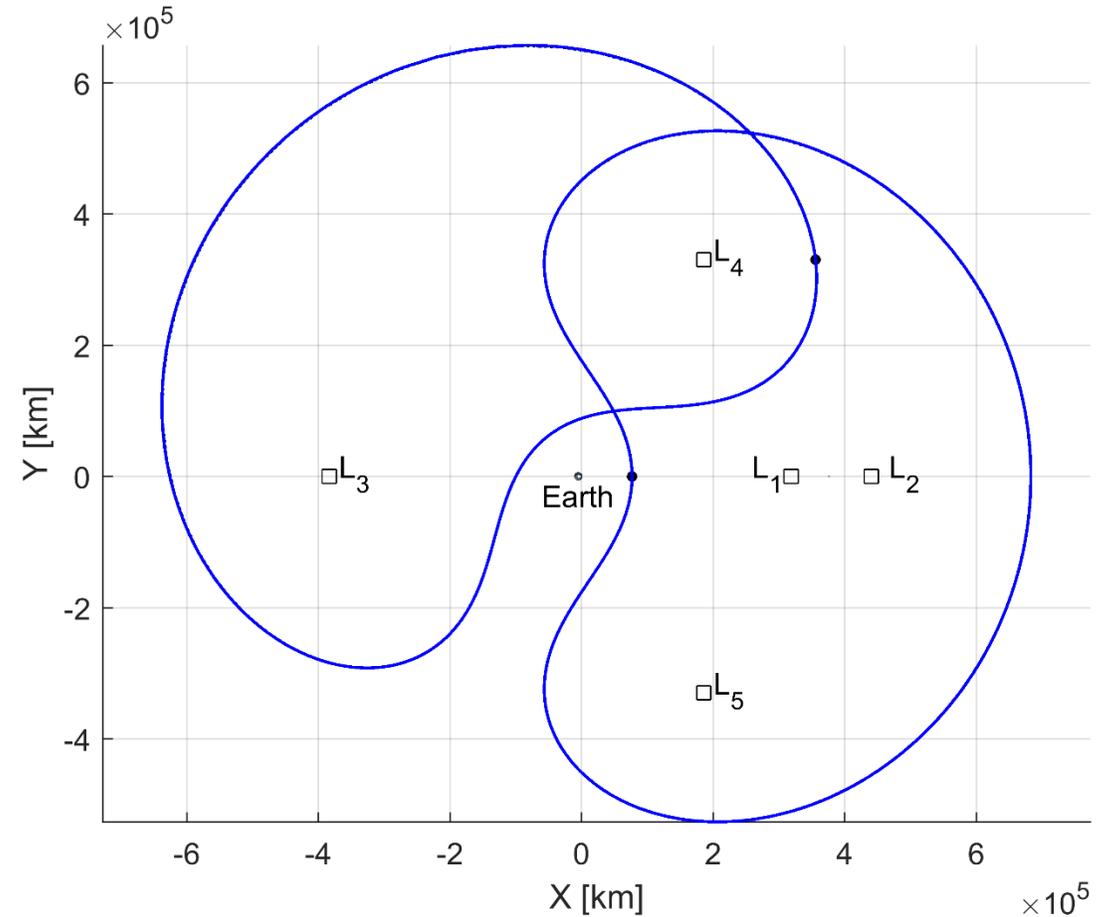
$$\text{TOF} = 3 \text{ rev} * \tau_0 + \tau_{int} + 3 \text{ rev} * \tau_f$$



Sample Applications – Orbit Chaining

Transfer Scenario:

DRO → *???* → *L₄ SPO*

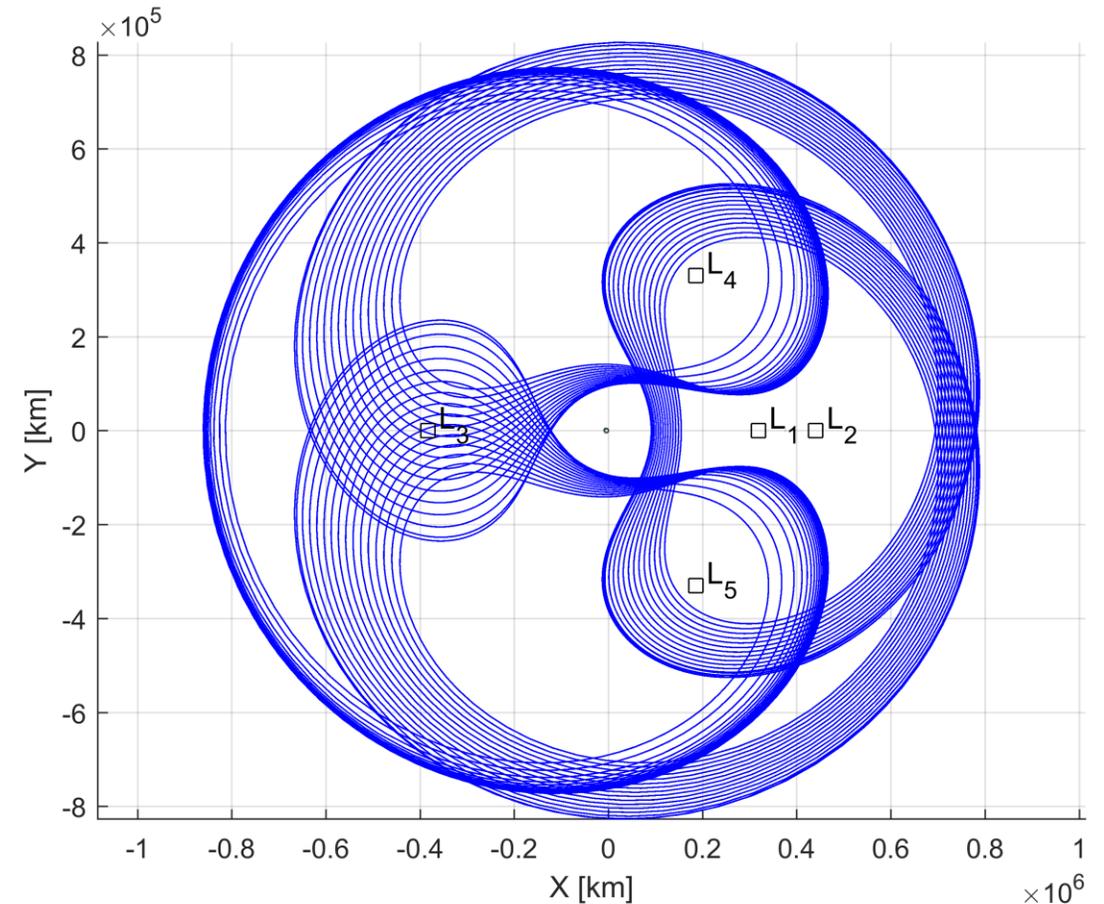


Departure and Arrival Orbits

Sample Applications – Orbit Chaining

Transfer Scenario:

DRO → *3:2 Resonant* → *L₄ SPO*



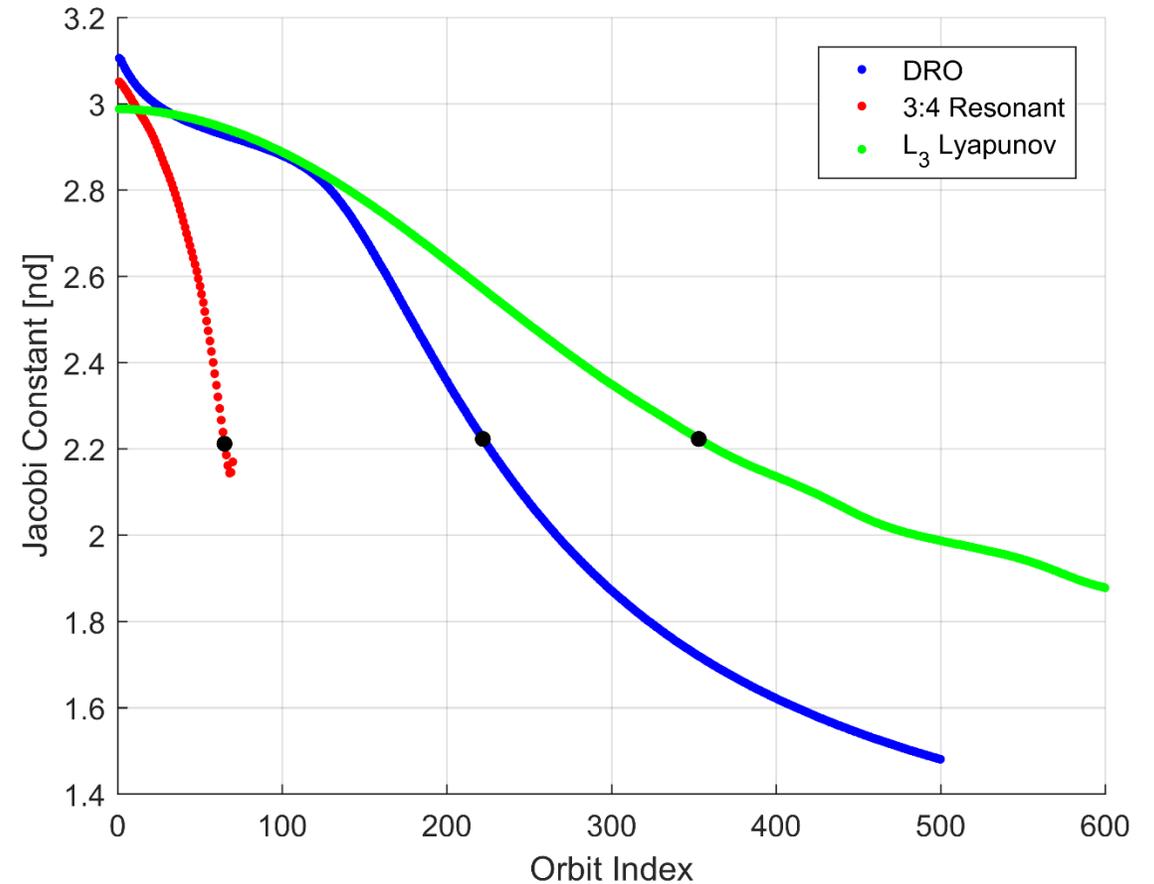
3:2 Resonant Orbit Family

Sample Applications – Orbit Chaining

Transfer Scenario:

DRO → *3:2 Resonant* → *L₄ SPO*

- Select resonant orbit with similar Jacobi constant value



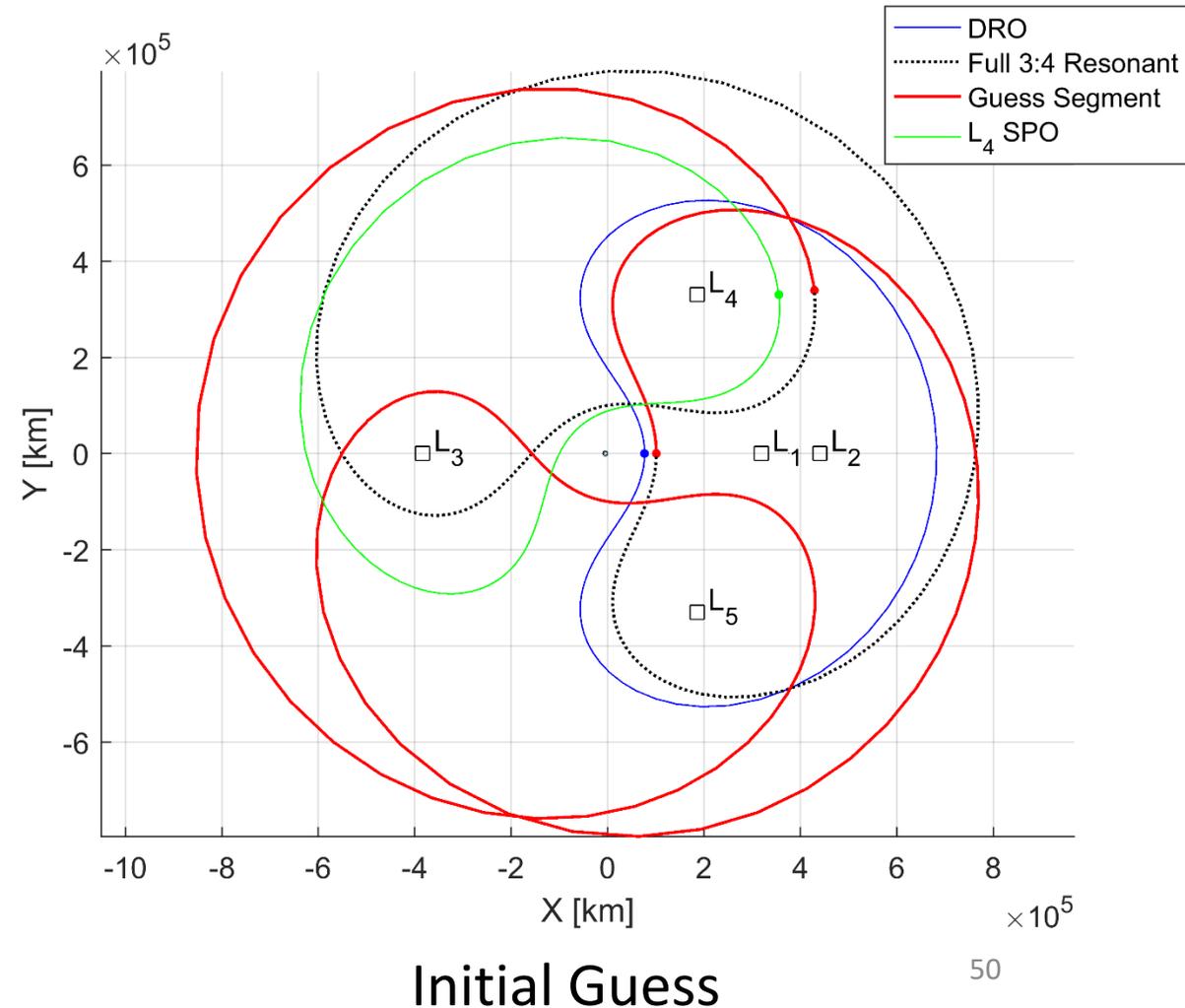
Jacobi Constant vs. Orbit Index ⁴⁹

Sample Applications – Orbit Chaining

Transfer Scenario:

DRO → *3:2 Resonant* → *L₄ SPO*

- Select resonant orbit with similar Jacobi constant value
- Trim resonant orbit near departure and insertion points

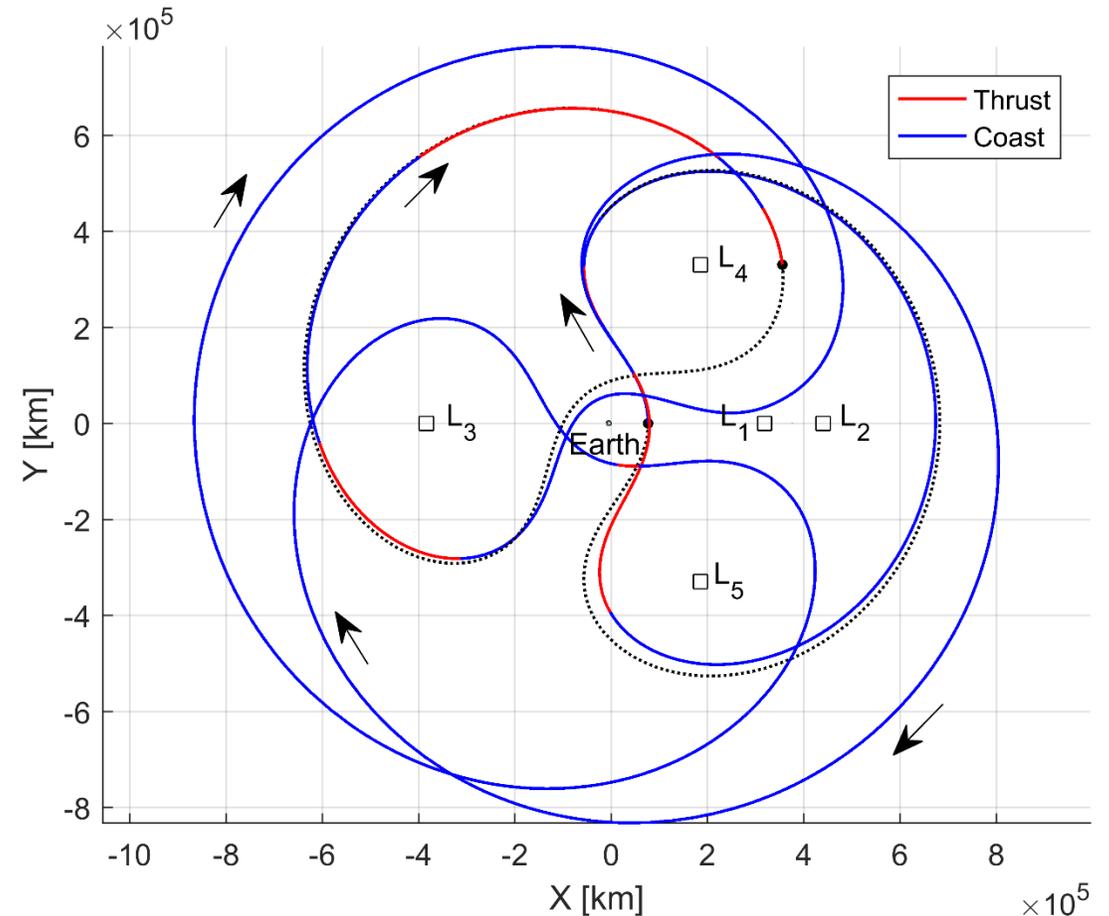


Sample Applications – Orbit Chaining

Transfer Scenario:

DRO → *3:2 Resonant* → *L₄ SPO*

- *DRO 1 Rev* + *3:2 Res 1 Rev* + *L₄ SPO 1 Rev*
- Transfer Time: 121.3 *days*
- Feasible: $m_f/m_0 = 492/500 \text{ kg}$
- Optimal: $m_f/m_0 = 497/500 \text{ kg}$



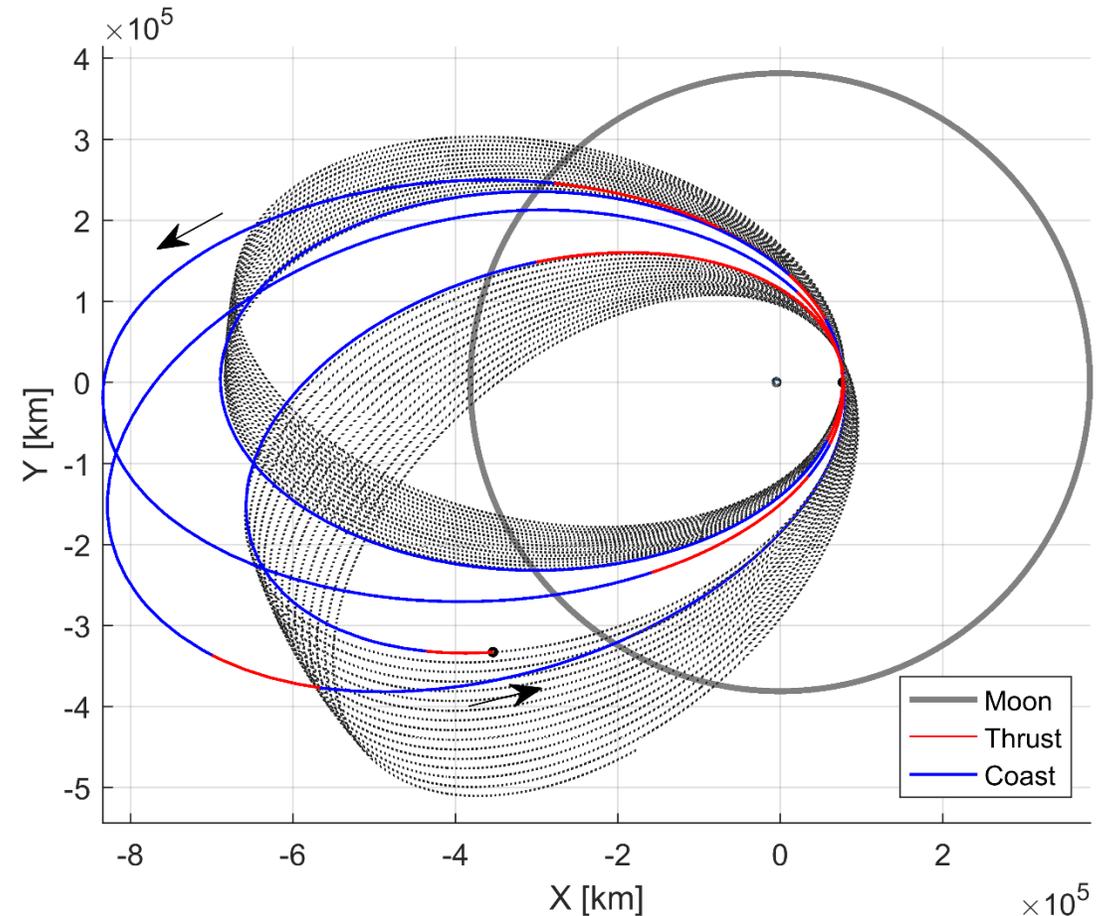
Optimal Transfer

Sample Applications – Orbit Chaining

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- Transfer Time: 121.3 *days*
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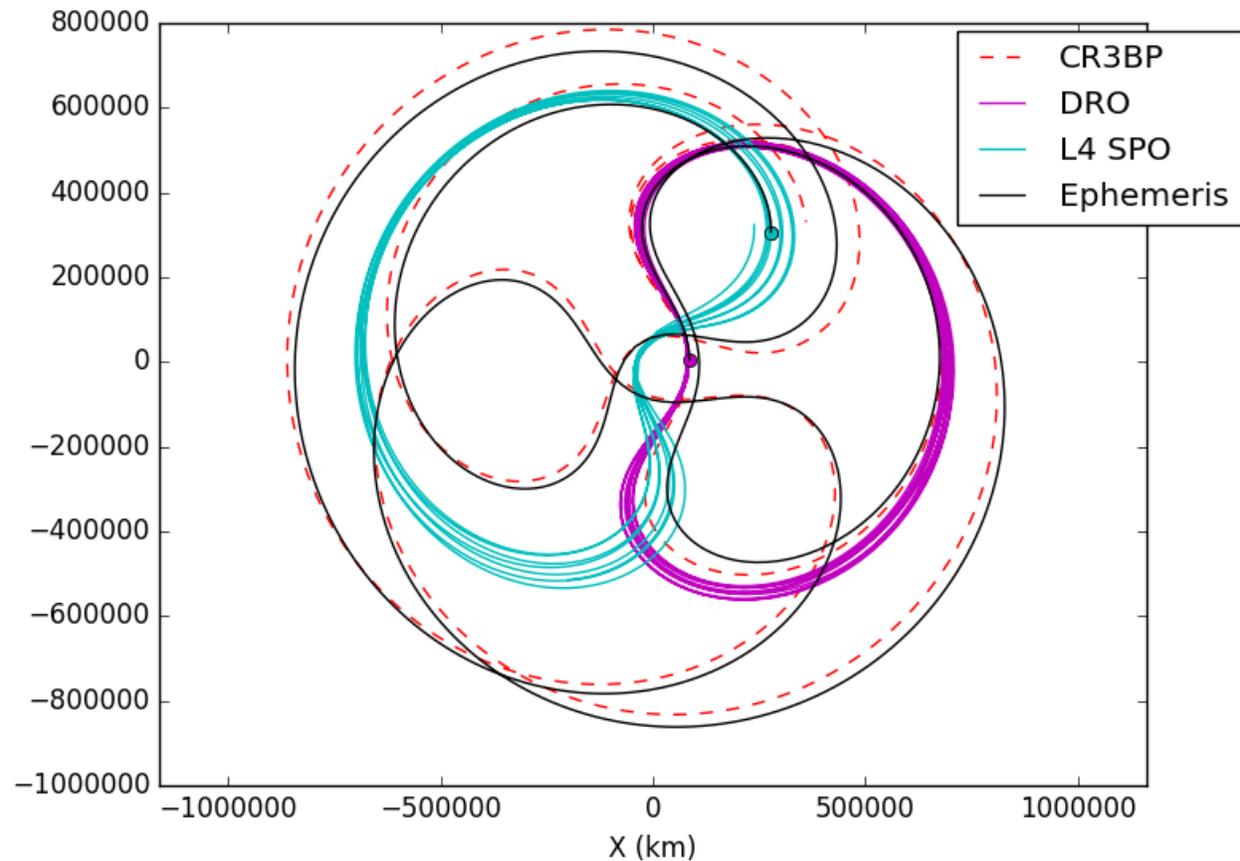
Optimal Transfer – Inertial Frame⁵²

Sample Applications – Orbit Chaining

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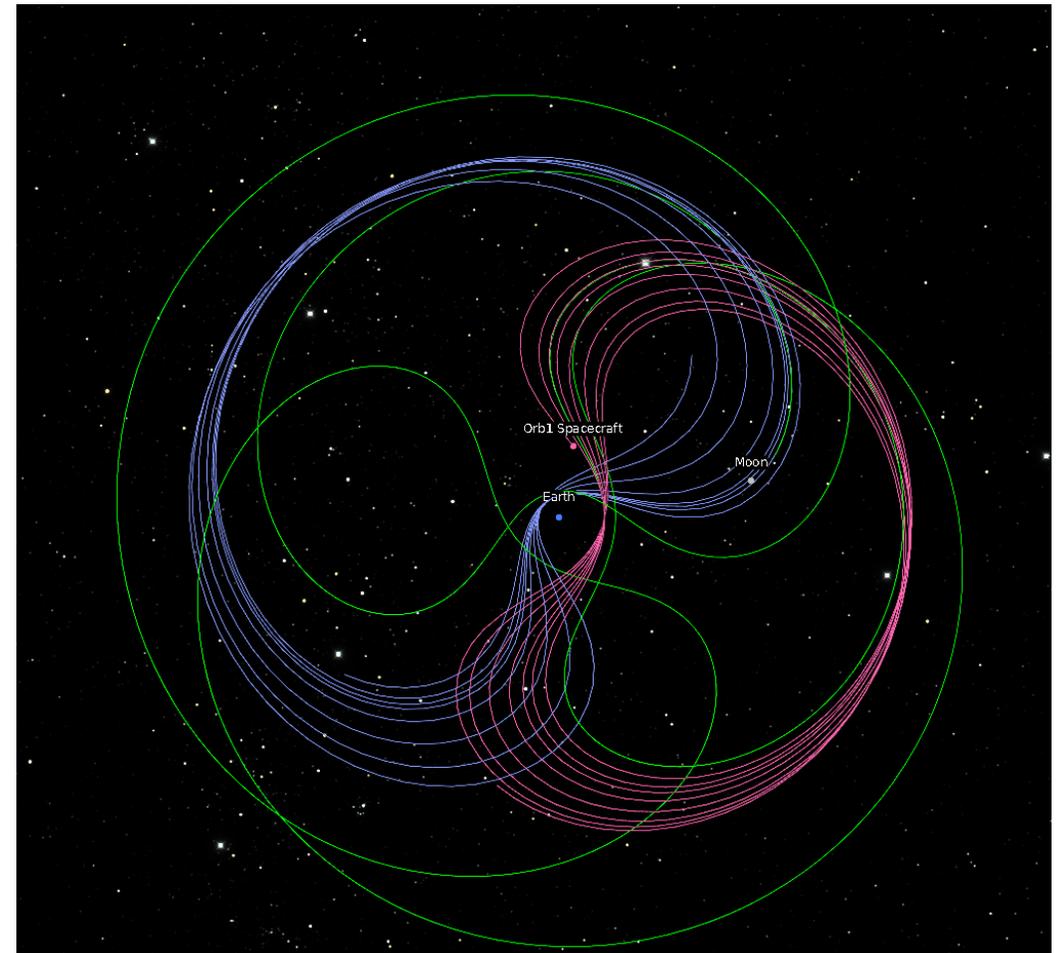
Transfer in a Full Ephemeris Model ⁵³

Sample Applications – Orbit Chaining

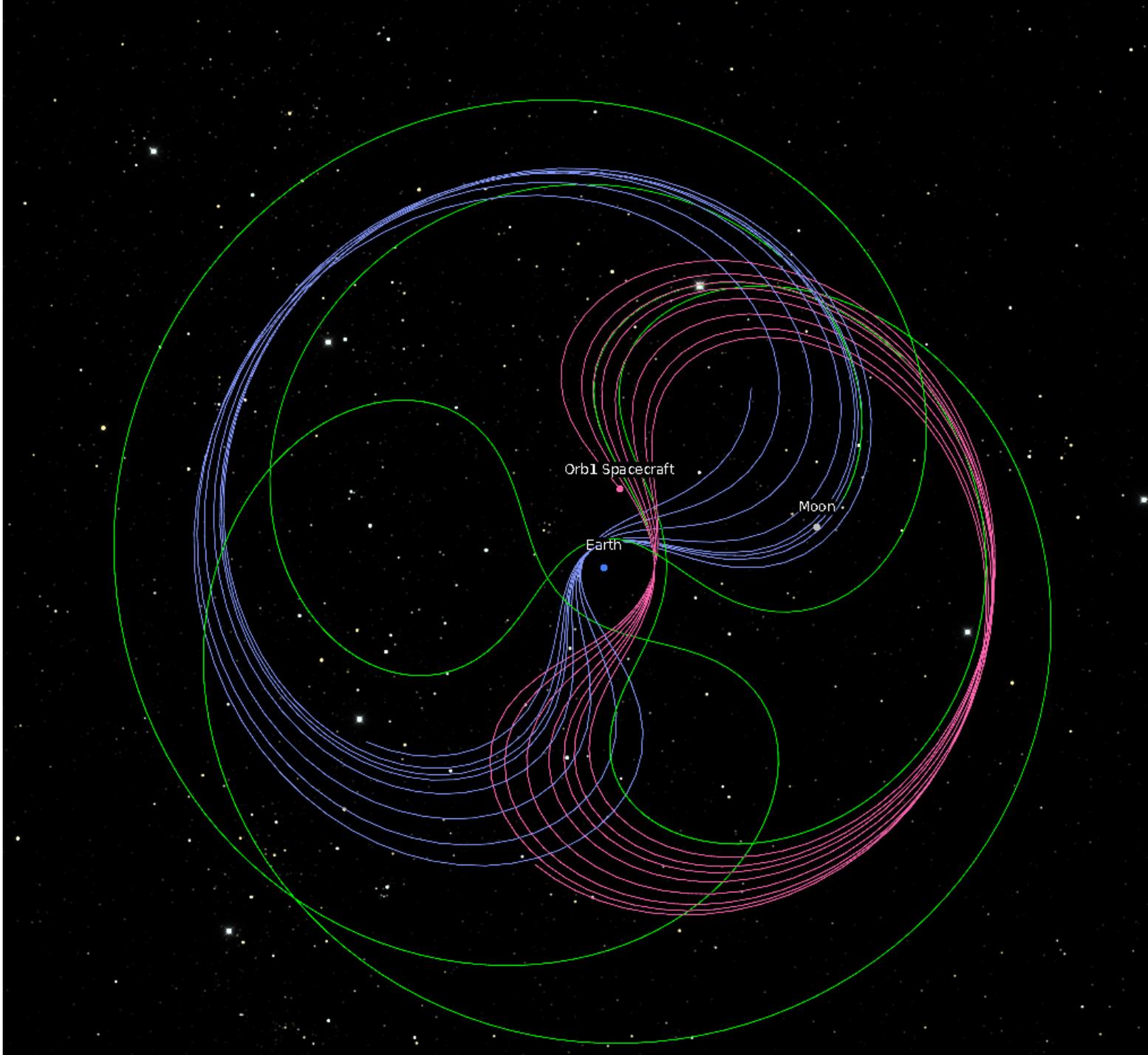
Transfer Scenario:

DRO → *3:2 Resonant* → *L₄ SPO*

- *DRO 1 Rev* + *3:2 Res 1 Rev* +
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- Transfer Time: 121.3 *days*
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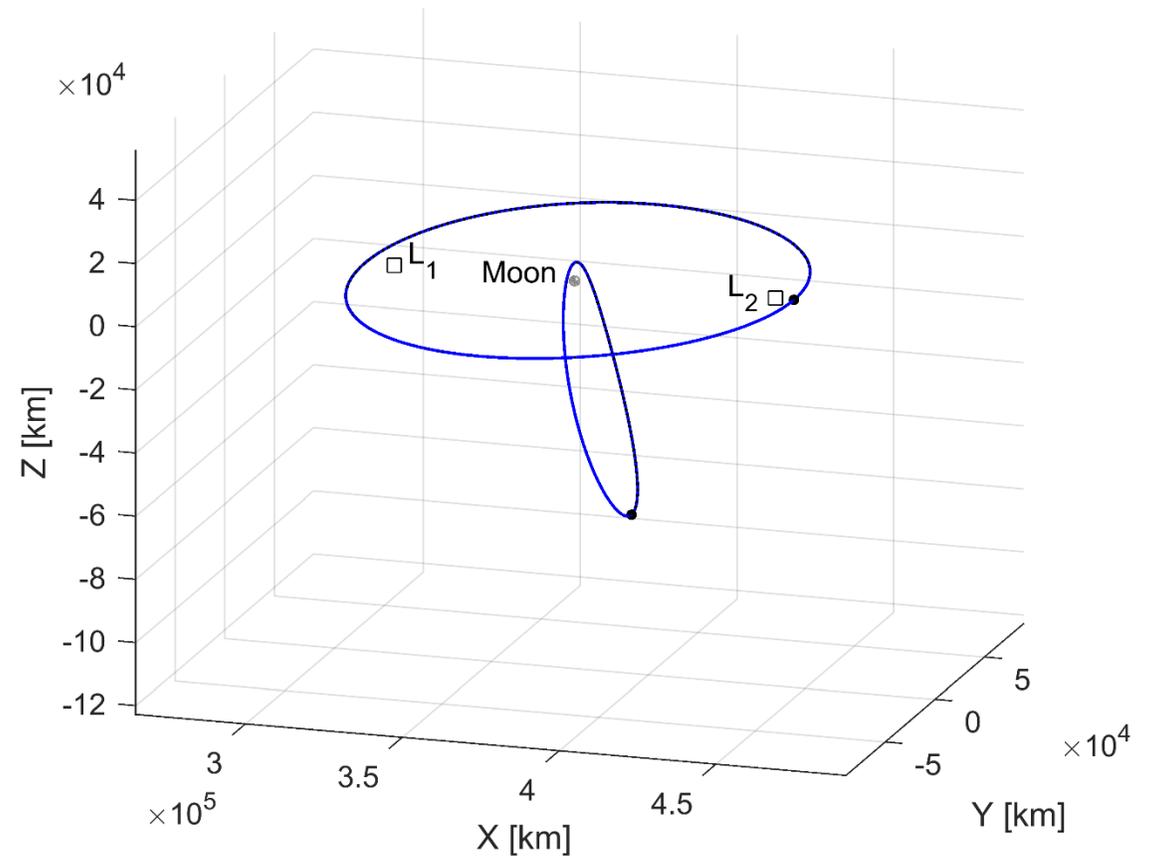
Transfer in a Full Ephemeris Model⁵⁴



Sample Applications – Orbit Chaining

Transfer Scenario:

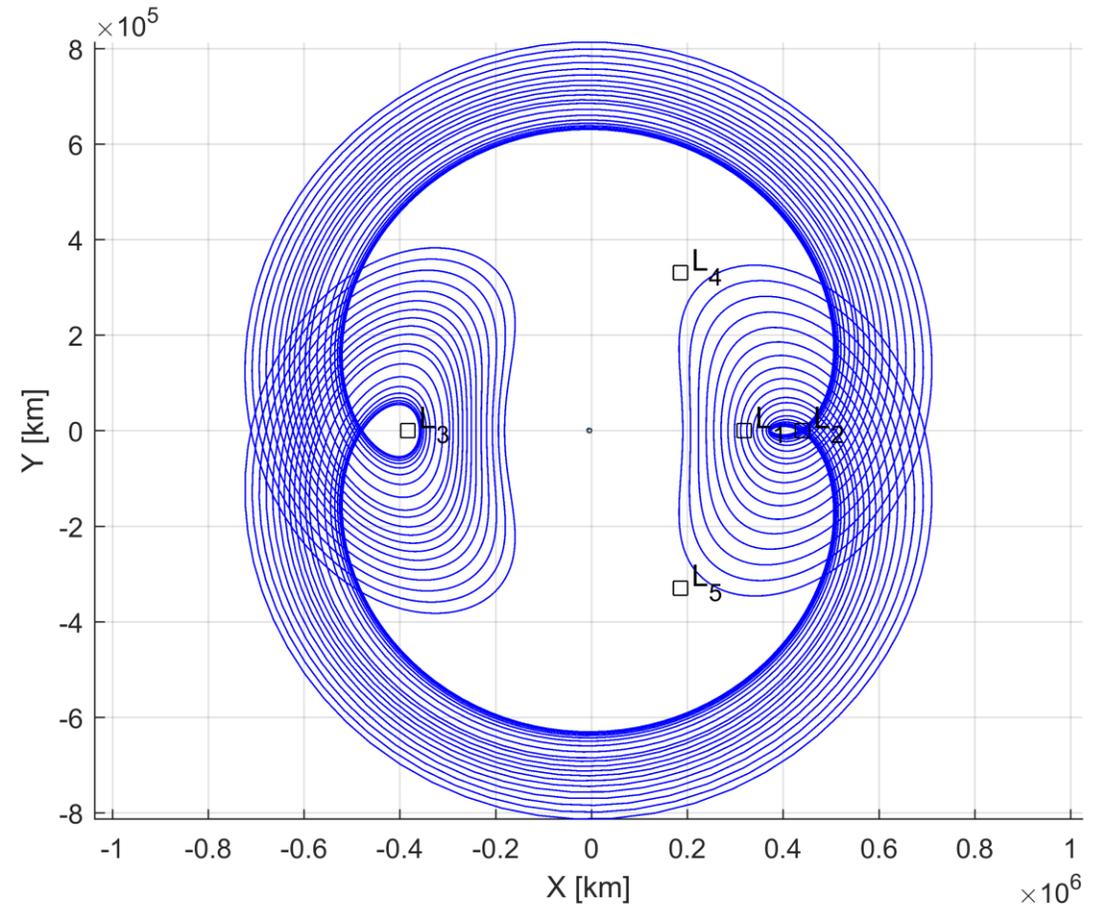
NRHO → *???* → *DRO*



Sample Applications – Orbit Chaining

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*



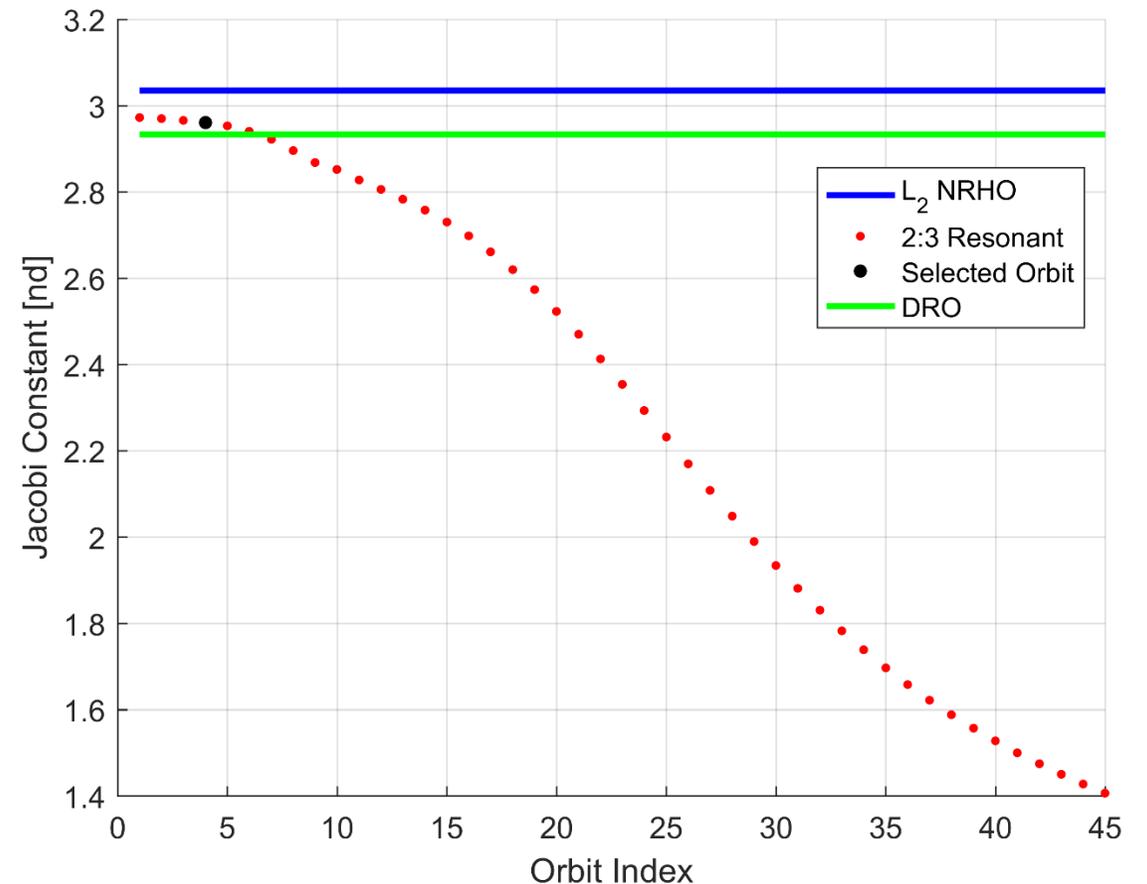
2:3 Resonant Orbit Family

Sample Applications – Orbit Chaining

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

- Select resonant orbit with similar Jacobi constant value



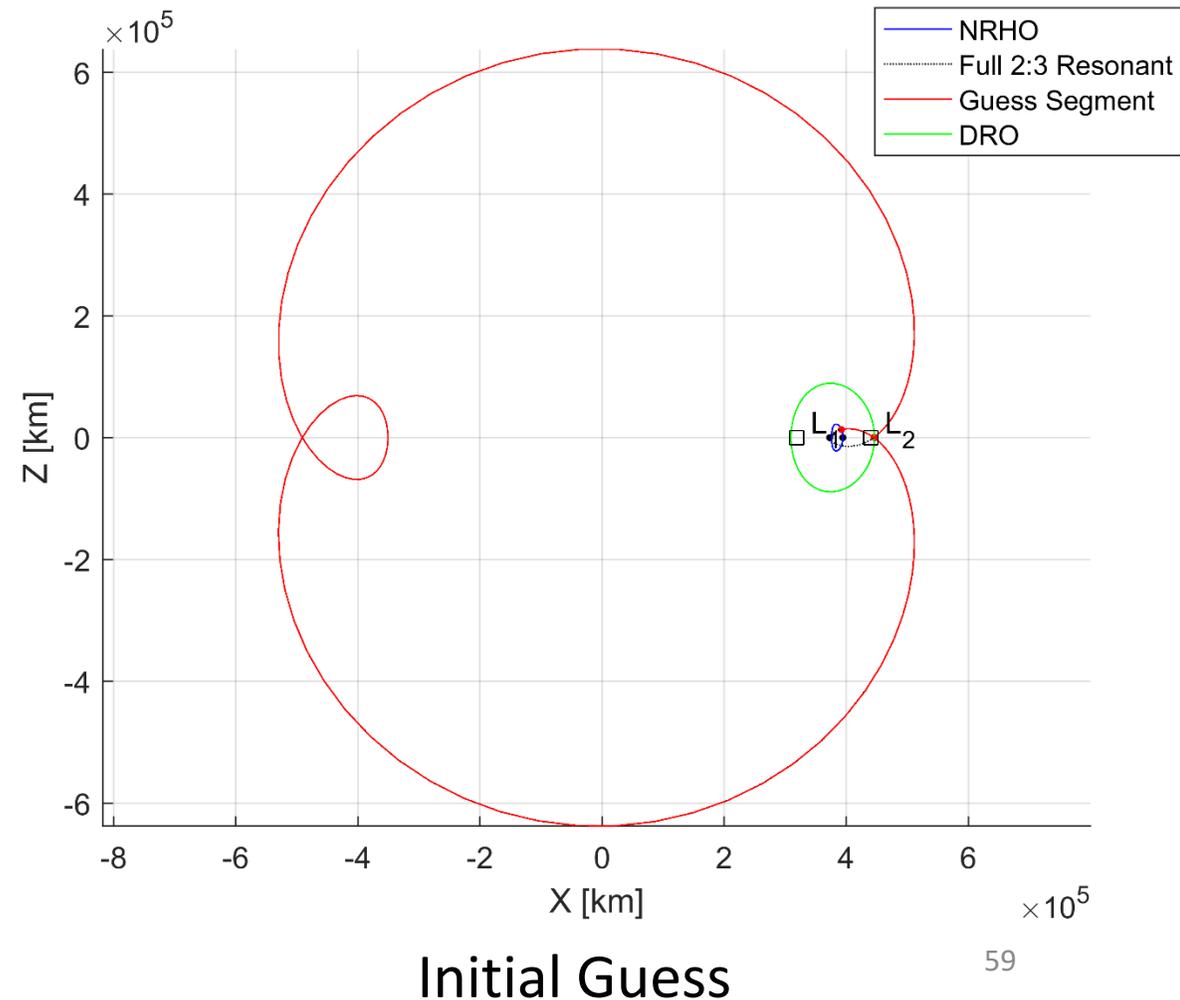
Jacobi Constant vs. Orbit Index ⁵⁸

Sample Applications – Orbit Chaining

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

- Select resonant orbit with similar Jacobi constant value
- Trim resonant orbit near departure and insertion points

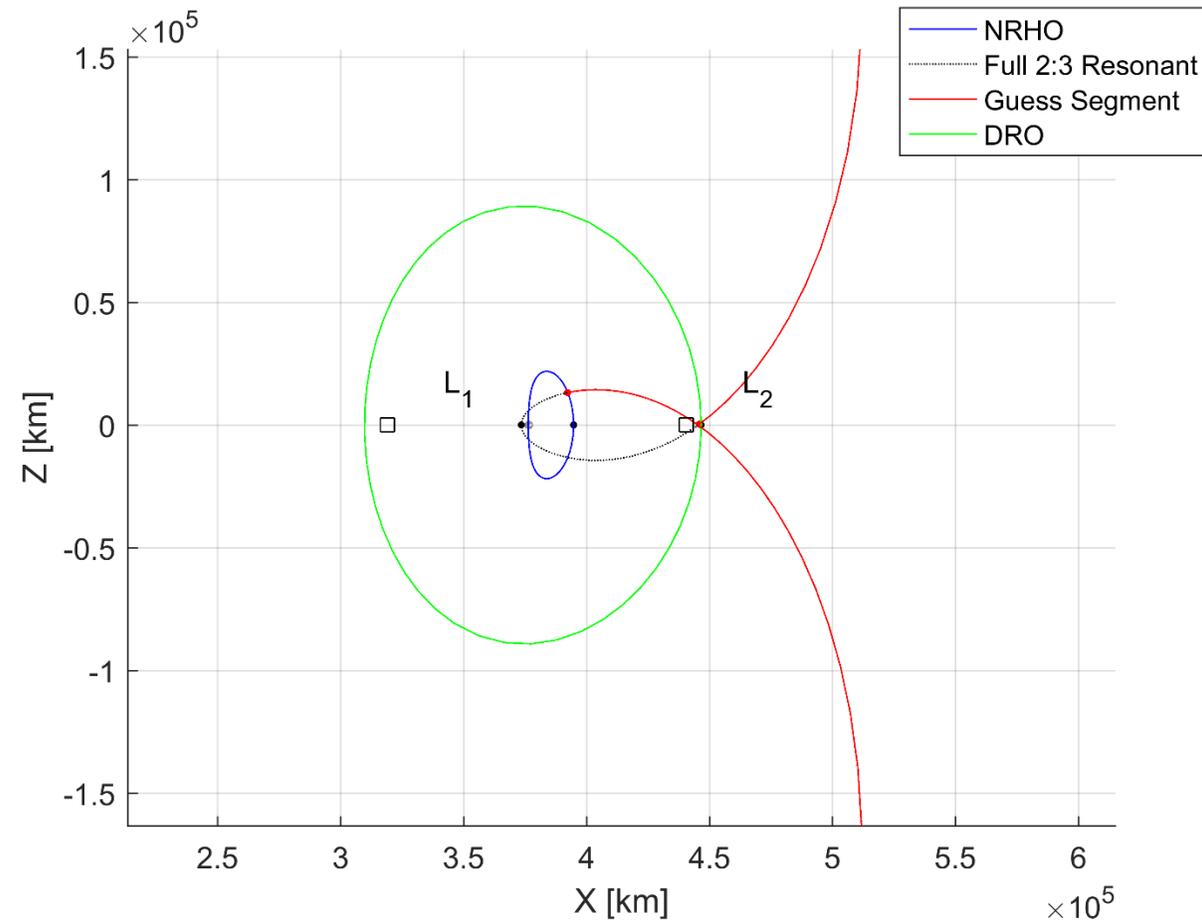


Sample Applications – Orbit Chaining

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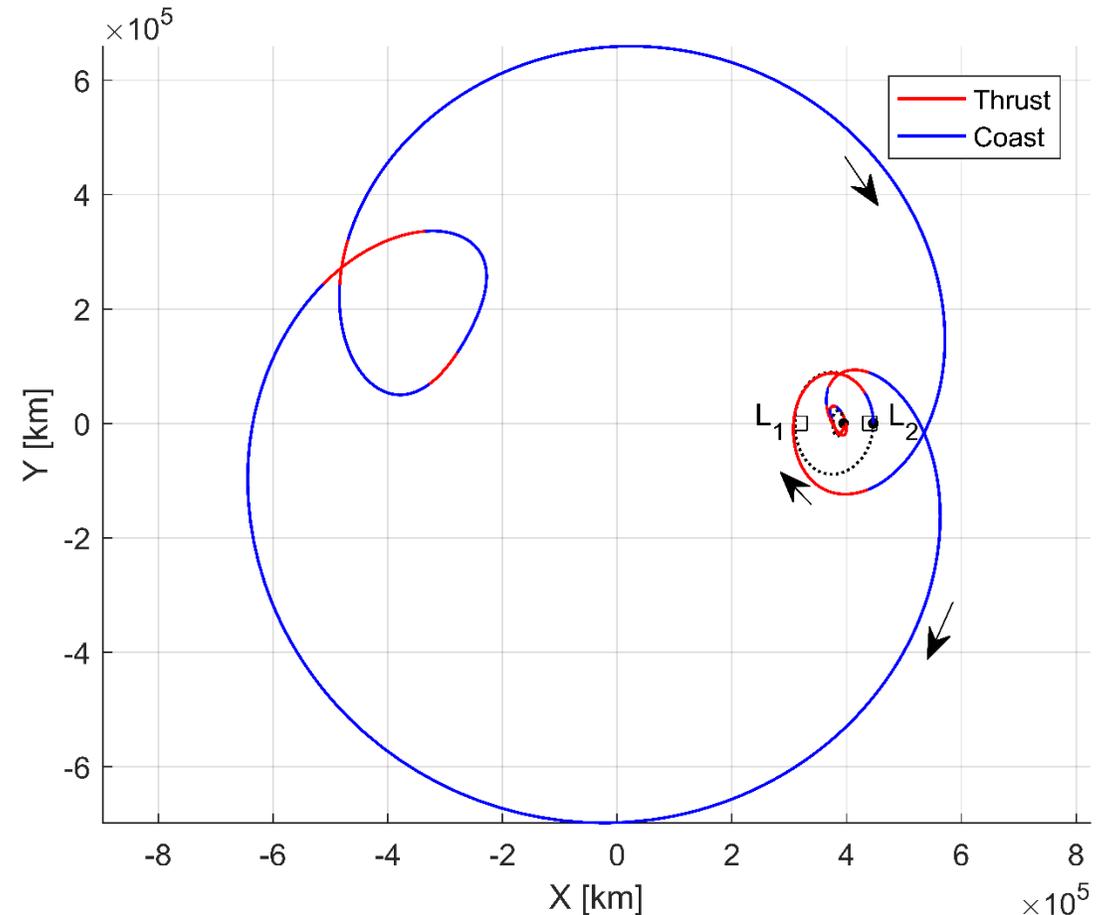
Initial Guess – Zoom View

Sample Applications – Orbit Chaining

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

- *NRHO 1 Rev* + *2:3 Res 1 Rev* + *DRO 1 Rev*
- Transfer Time: 96.8 *days*
- Feasible: $m_f/m_0 = 485/500 \text{ kg}$
- Optimal: $m_f/m_0 = 487/500 \text{ kg}$

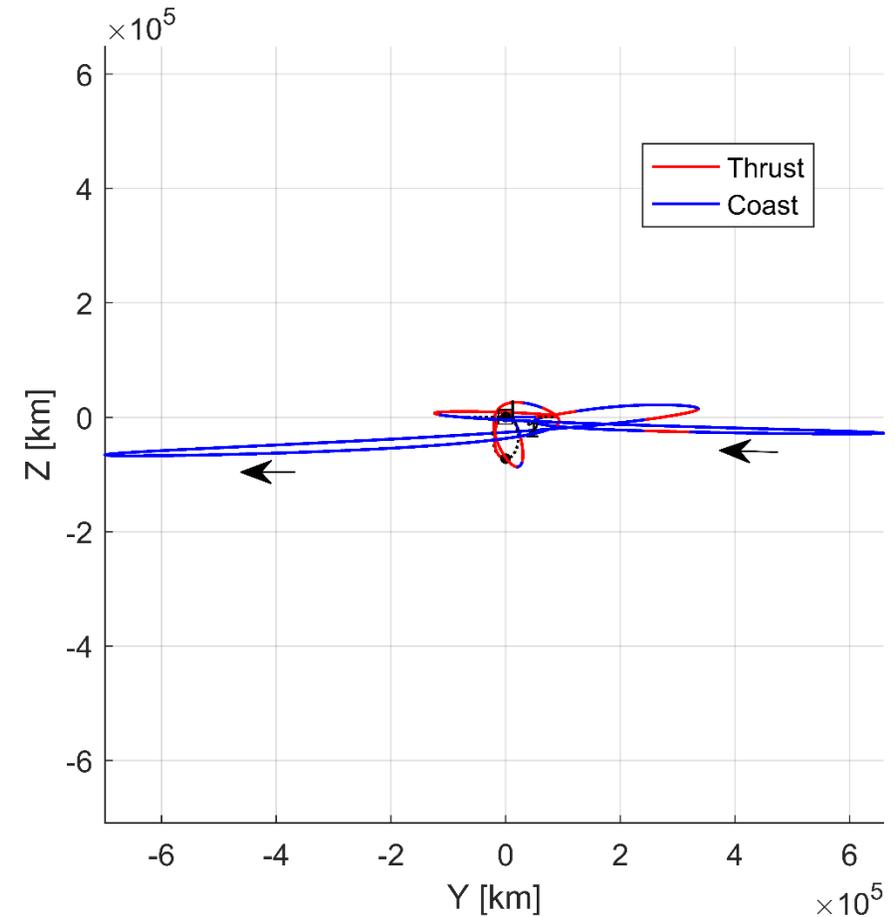


Sample Applications – Orbit Chaining

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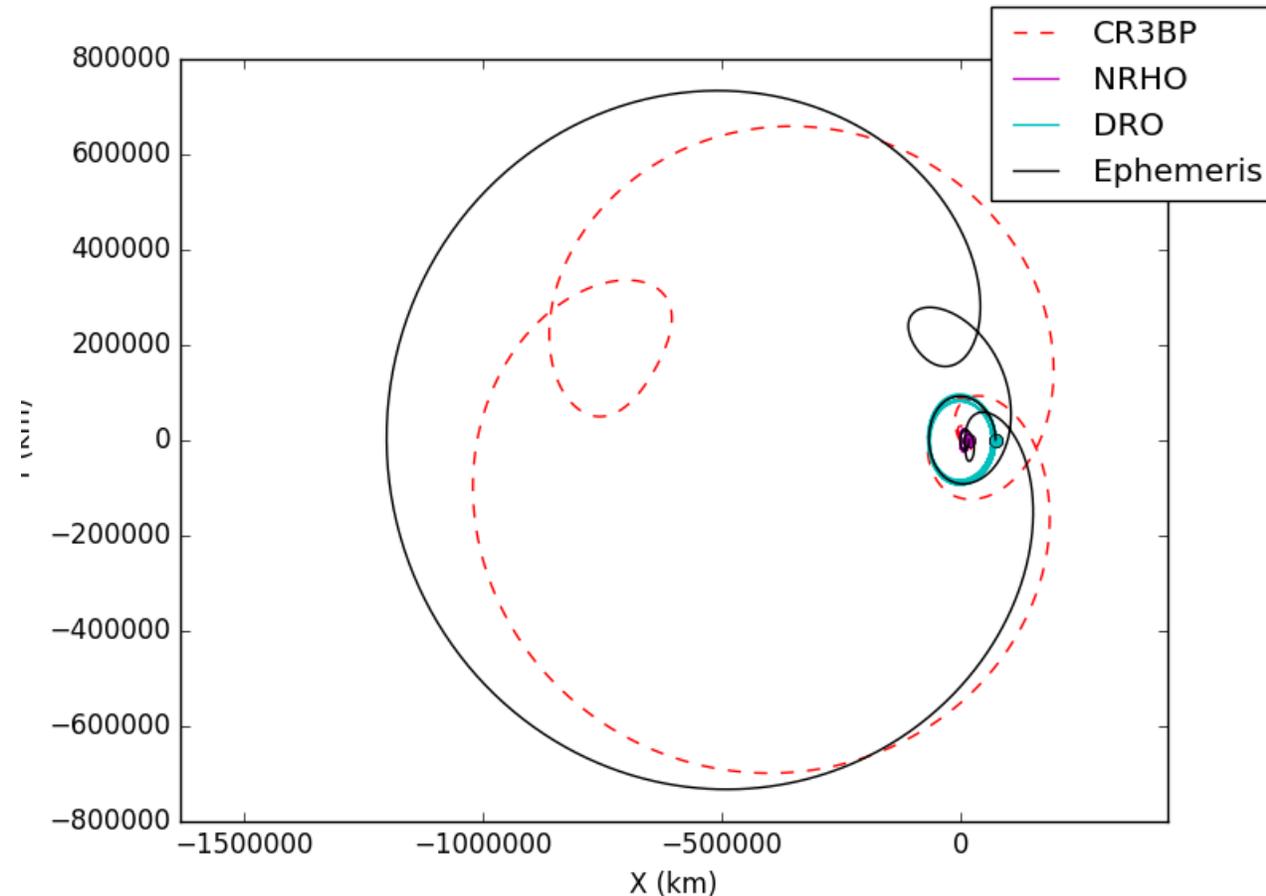
Optimal Transfer

Sample Applications – Orbit Chaining

Transfer Scenario:

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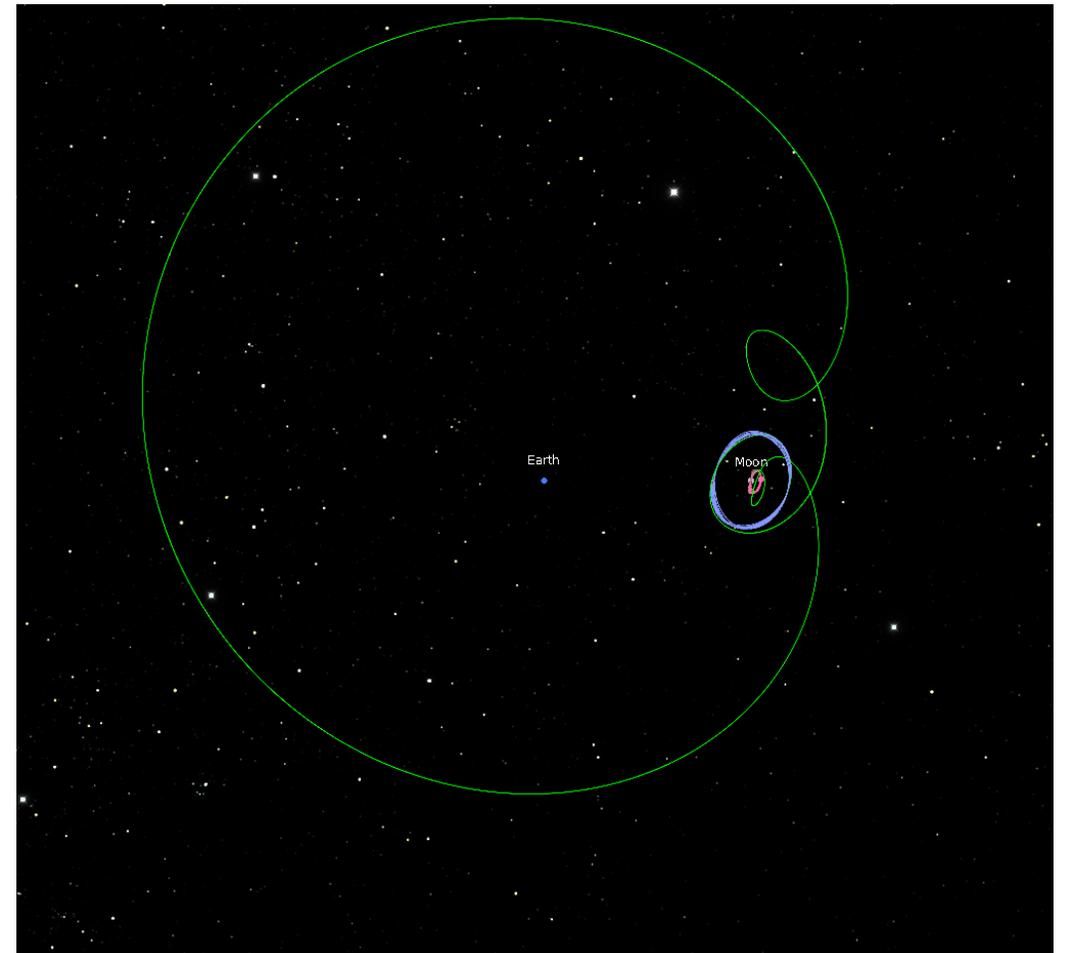
Transfer in a Full Ephemeris Model ⁶³

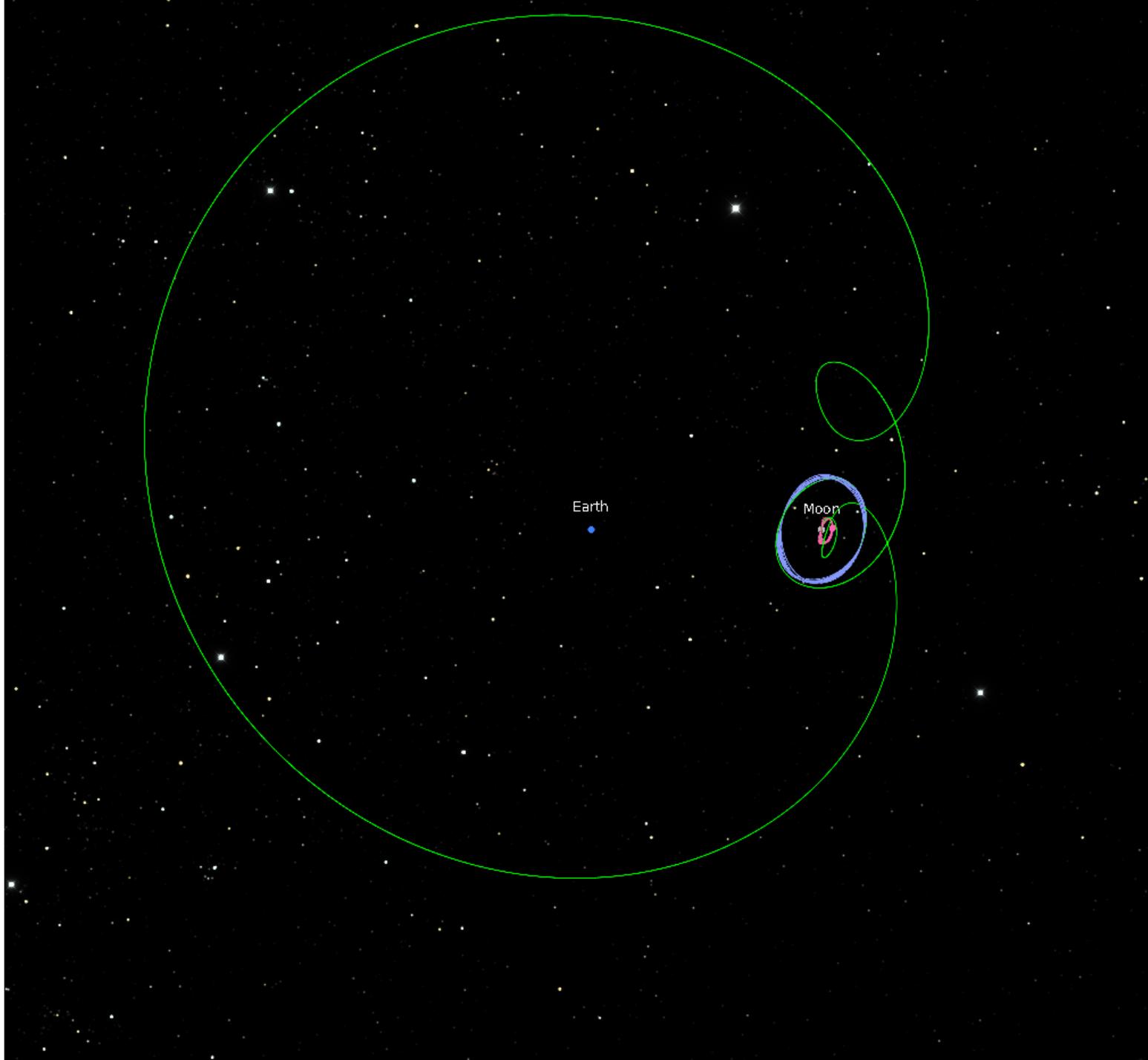
Sample Applications – Orbit Chaining

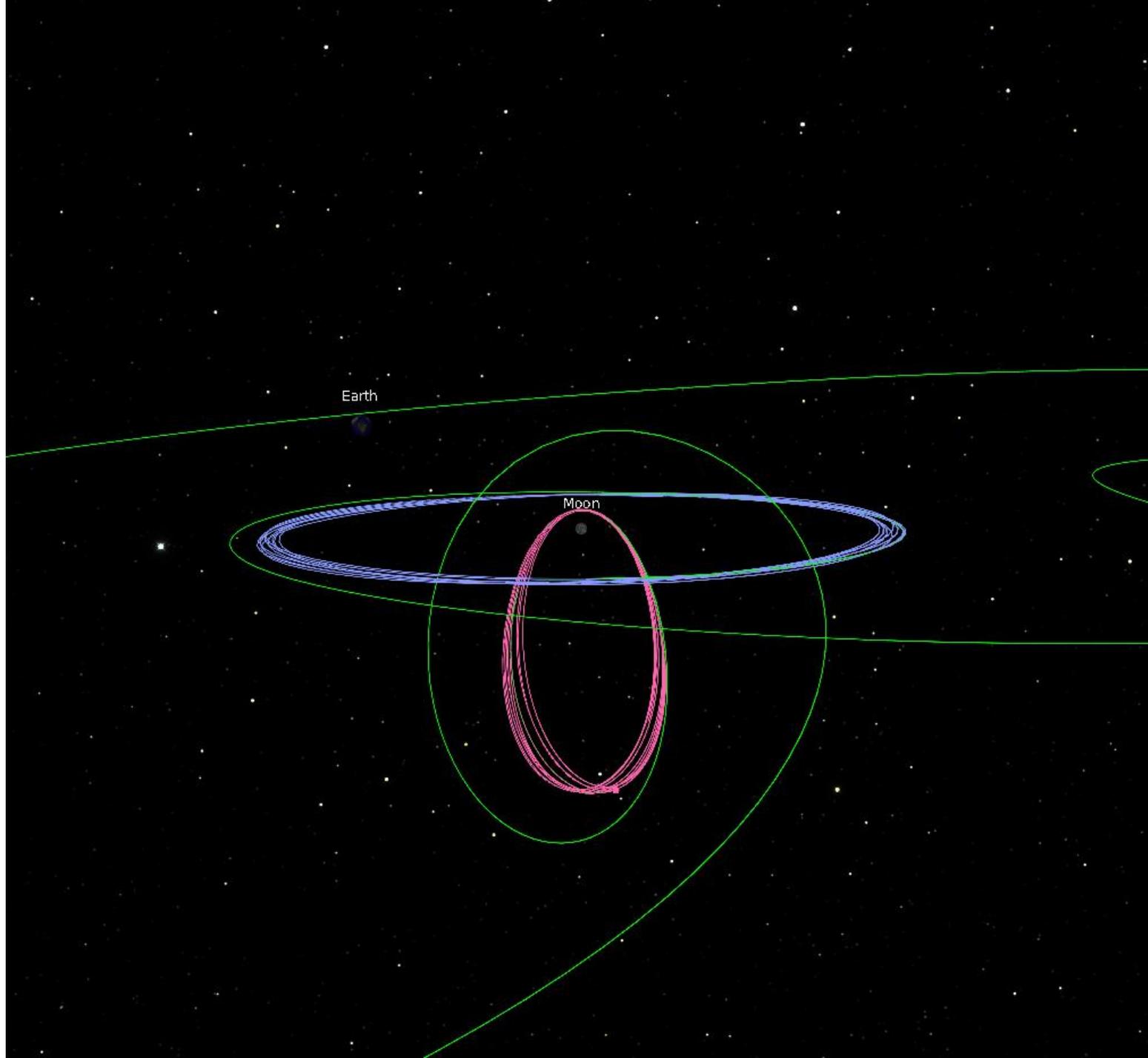
Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

- *NRHO 1 Rev* +
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- Transfer Time: 96.8 *days*
- Optimal: $m_f/m_0 = 487/500$ *kg*







Sample Applications – Orbit Chaining

Transfer Type	CR3BP Model		Ephemeris Model
	<i>TOF (days)</i>	<i>m_f (kg)</i>	<i>m_f (kg)</i>
L_2 Halo \rightarrow L_1 Halo	47.5	491.98	491.73
DRO \rightarrow L_4 SPO	134.8	482.03	*463.83
DRO \rightarrow 3:2 Resonant \rightarrow L_4 SPO	121.3	497.04	*472.75
NRHO \rightarrow DRO	46.7	486.14	488.33
NRHO \rightarrow 2:3 Resonant \rightarrow DRO	96.8	487.15	495.00

*Ephemeris transfer not optimized

Sample Applications – Orbit Chaining

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Outline

I. Motivation

II. Background

III. Sample Applications

I. Trajectory Stacking Technique

II. Orbit Chaining Technique

III. Deep Space Gateway Transfer

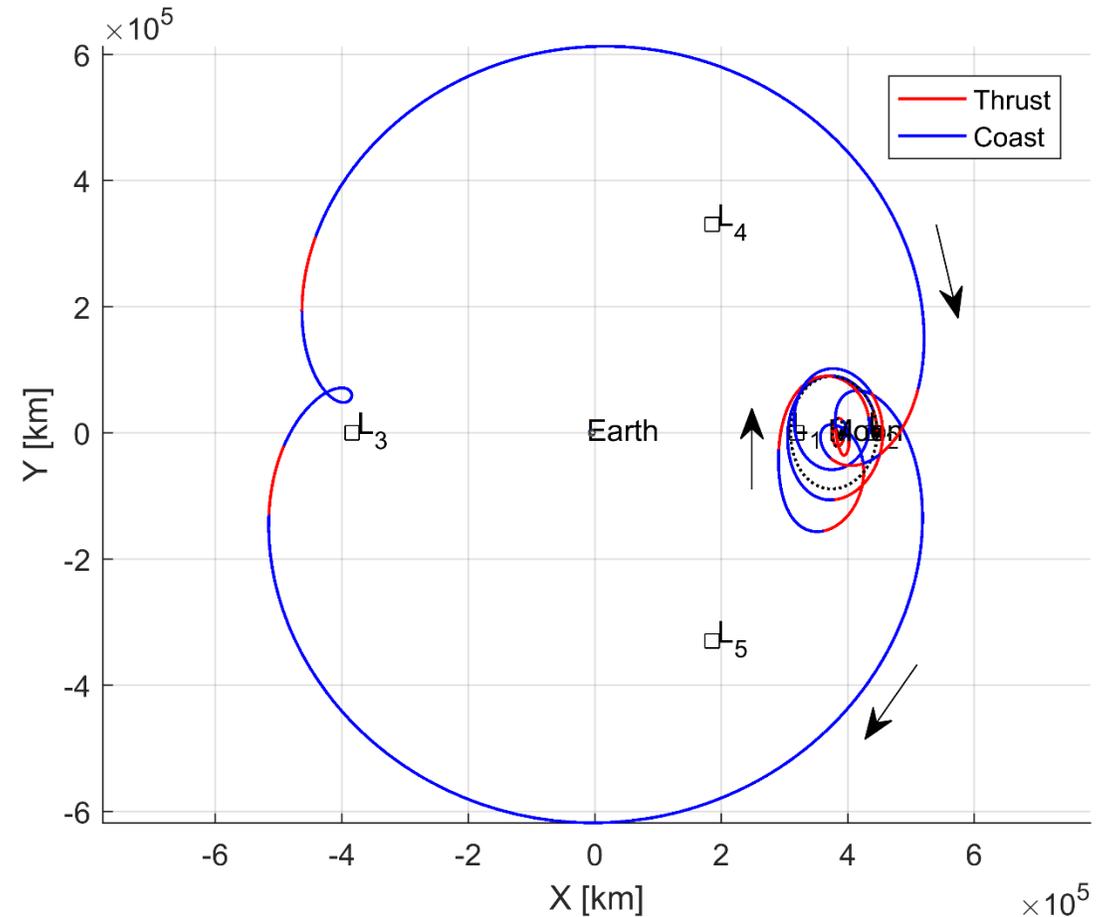
IV. Concluding Remarks

Sample Applications – Deep Space Gateway

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

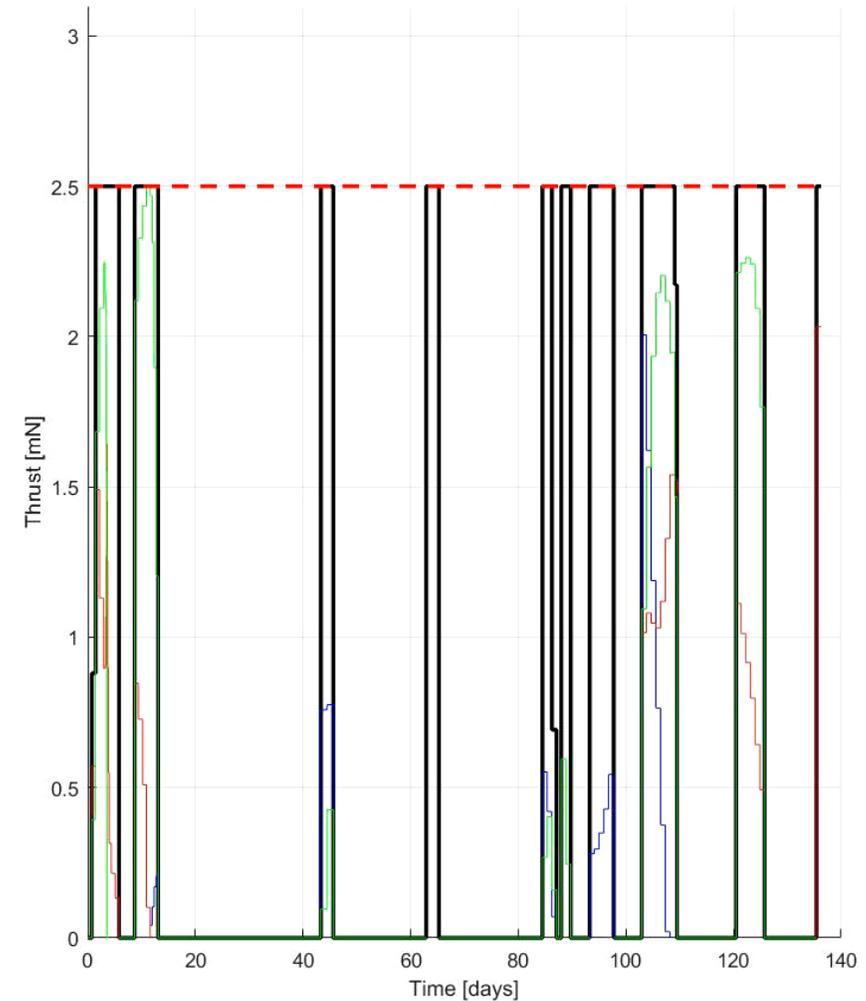
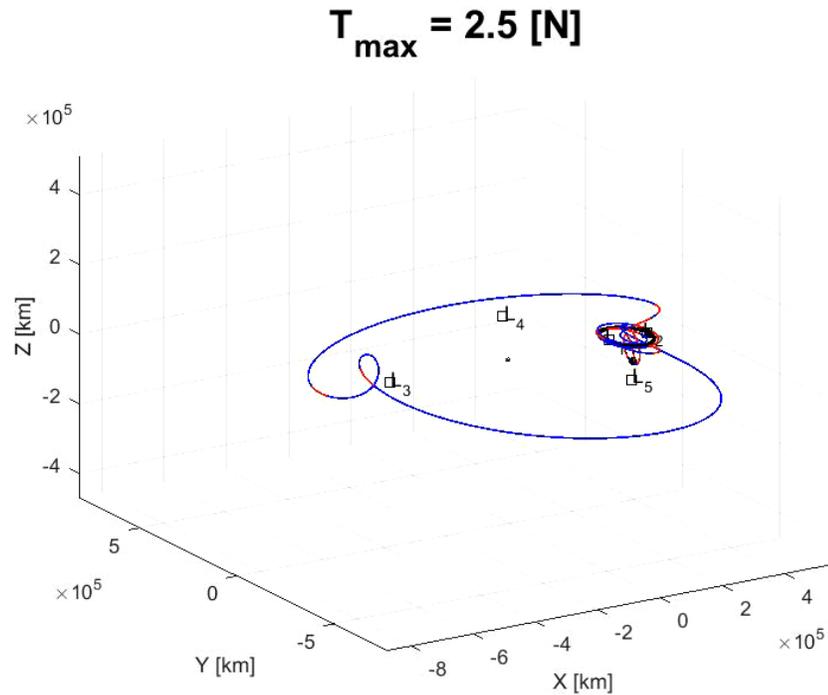
- Now using spacecraft with space station parameters:
 - $m_0 = 30,000 \text{ kg}$
 - $I_{sp} = 3000 \text{ sec}$
- Expected Thrust/Weight ratio:
 - $T/W = 1 \times 10^{-6} \text{ to } 6 \times 10^{-6}$
 - $T_{max} = 0.29 - 1.77 \text{ N}$



Optimal Transfer with $T_{max} = 2.5 \text{ N}$

Sample Applications – Deep Space Gateway

$T_{\max} = 2.5 \text{ [N]}$

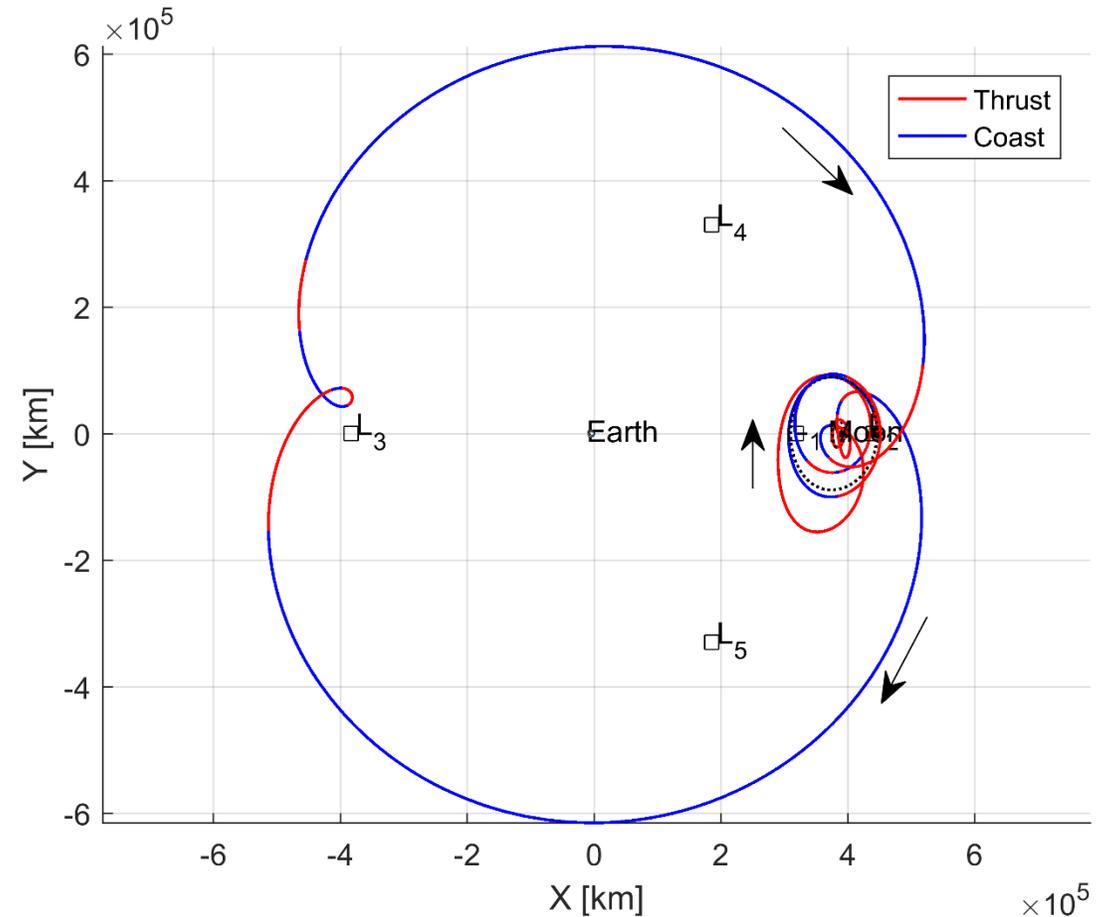


Sample Applications – Deep Space Gateway

Transfer Scenario:

NRHO → *2:3 Resonant* → *DRO*

- *NRHO 1 Rev* + *2:3 Res 1 Rev* + *DRO 1 Rev*
- Transfer Time: 136.24 *days*
- Feasible: $m_f/m_0 = 29.66/30$ *mT*
- Optimal: $m_f/m_0 = 29.69/30$ *mT*



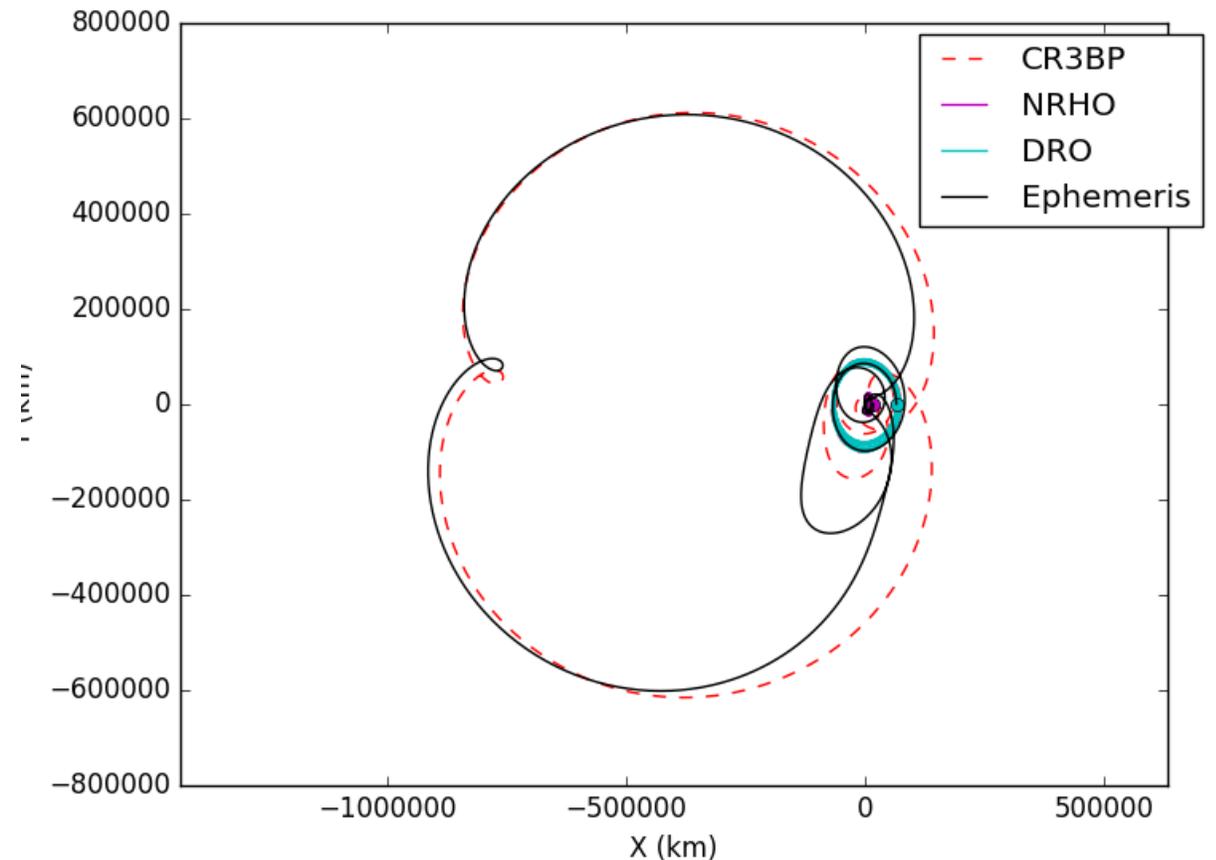
Optimal Transfer with $T_{max} = 1.7$ *N*

Sample Applications – Deep Space Gateway

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NRHO → *2:3 Resonant* → *DRO*

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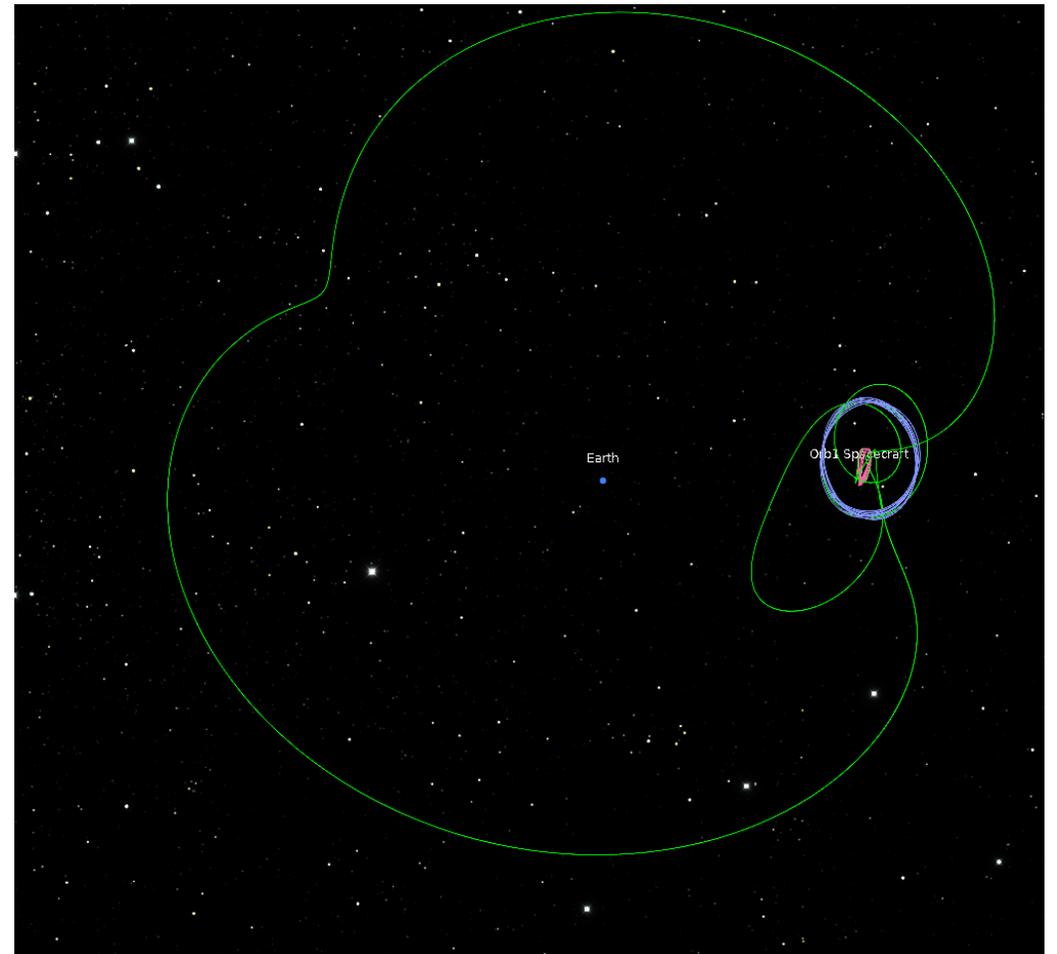
Optimal Transfer with $T_{max} = 1.7 N$

Sample Applications – Deep Space Gateway

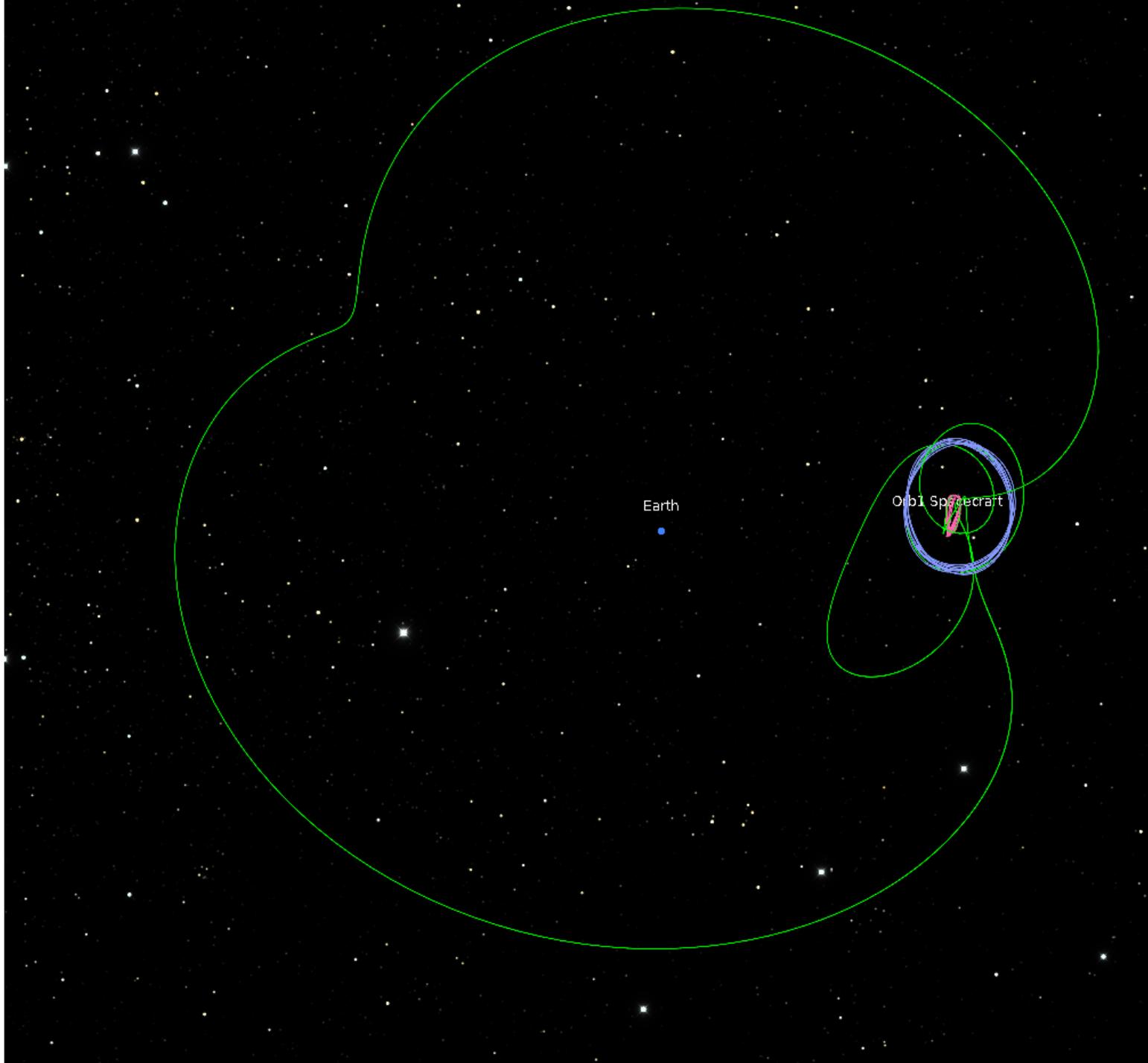
Transfer Scenario:

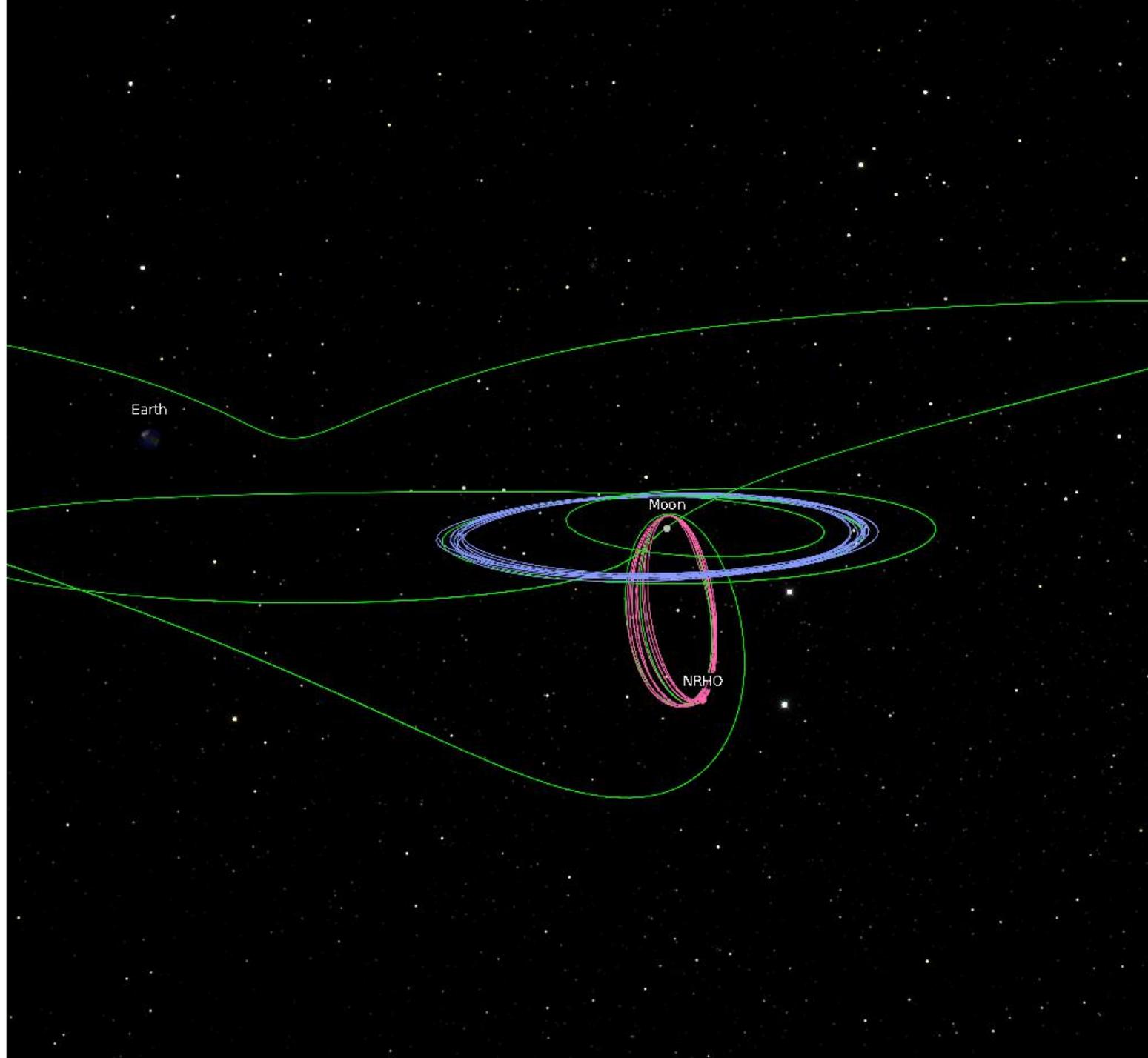
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- Transfer Time: 136.24 *days*
- Optimal: $m_f/m_0 = 29.69/30$ *mT*



Optimal Transfer with $T_{max} = 1.7^7 N$

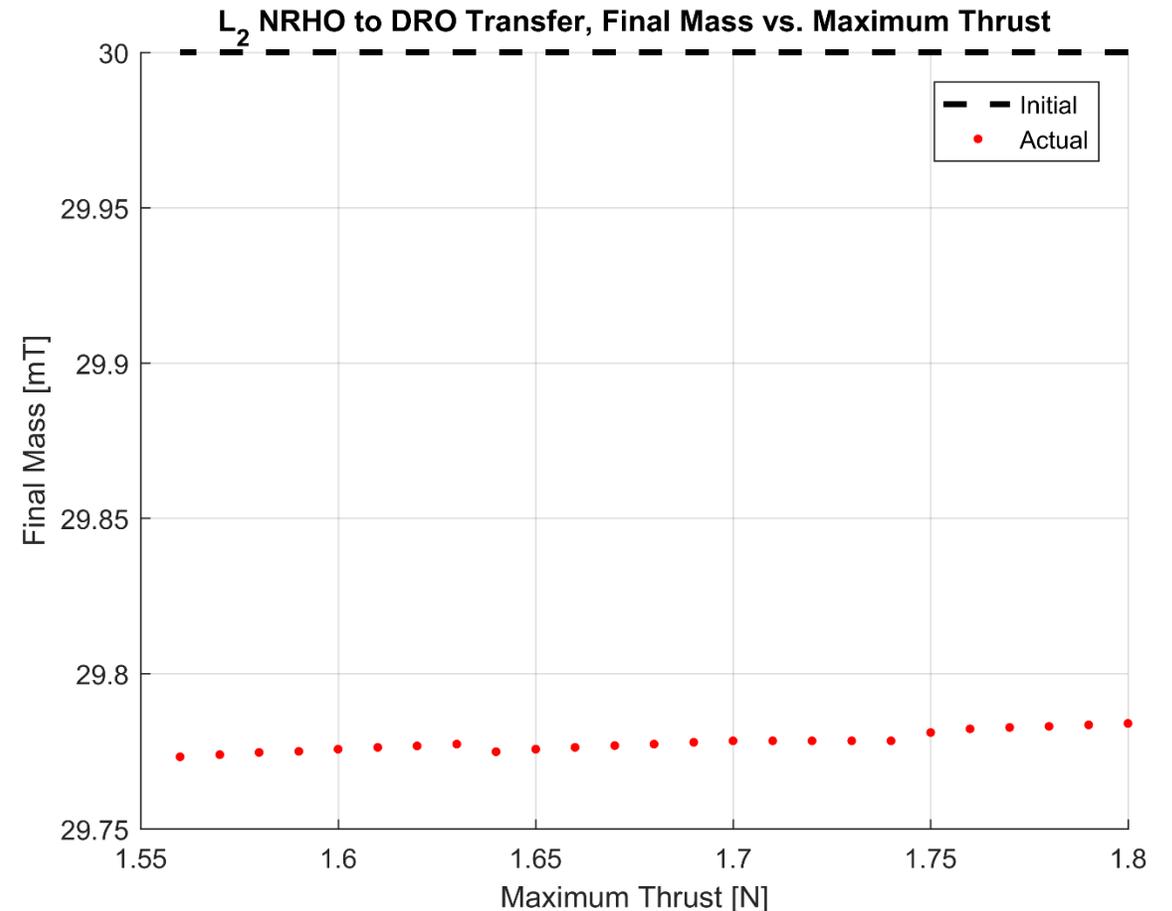




Sample Applications – Deep Space Gateway

Transfer Scenario:

- Can also continue optimal solutions with respect to T_{max} , TOF , or Jacobi of departure and arrival orbits.

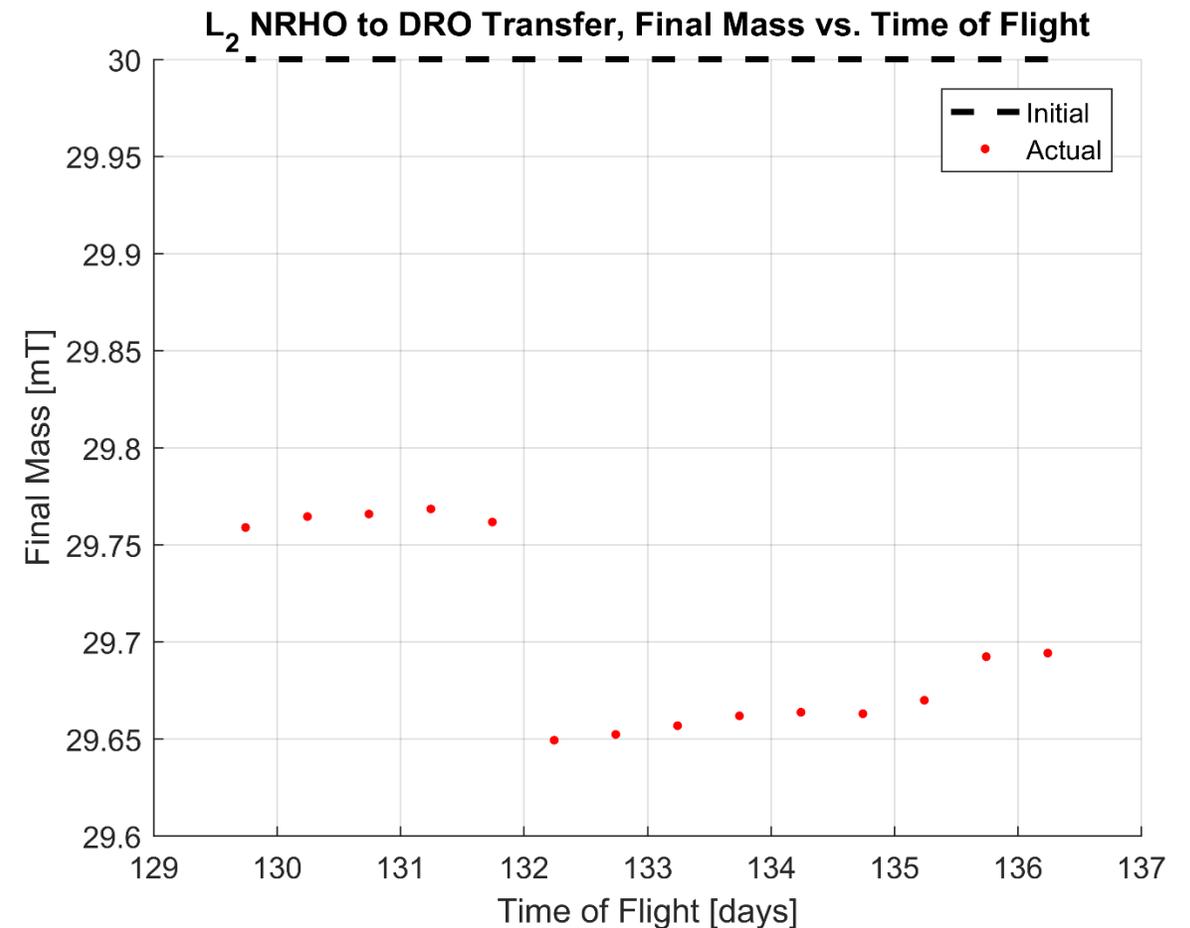


Final Mass vs. Maximum Thrust⁷⁷

Sample Applications – Deep Space Gateway

Transfer Scenario:

- Can also continue optimal solutions with respect to T_{max} , TOF , or Jacobi of departure and arrival orbits.



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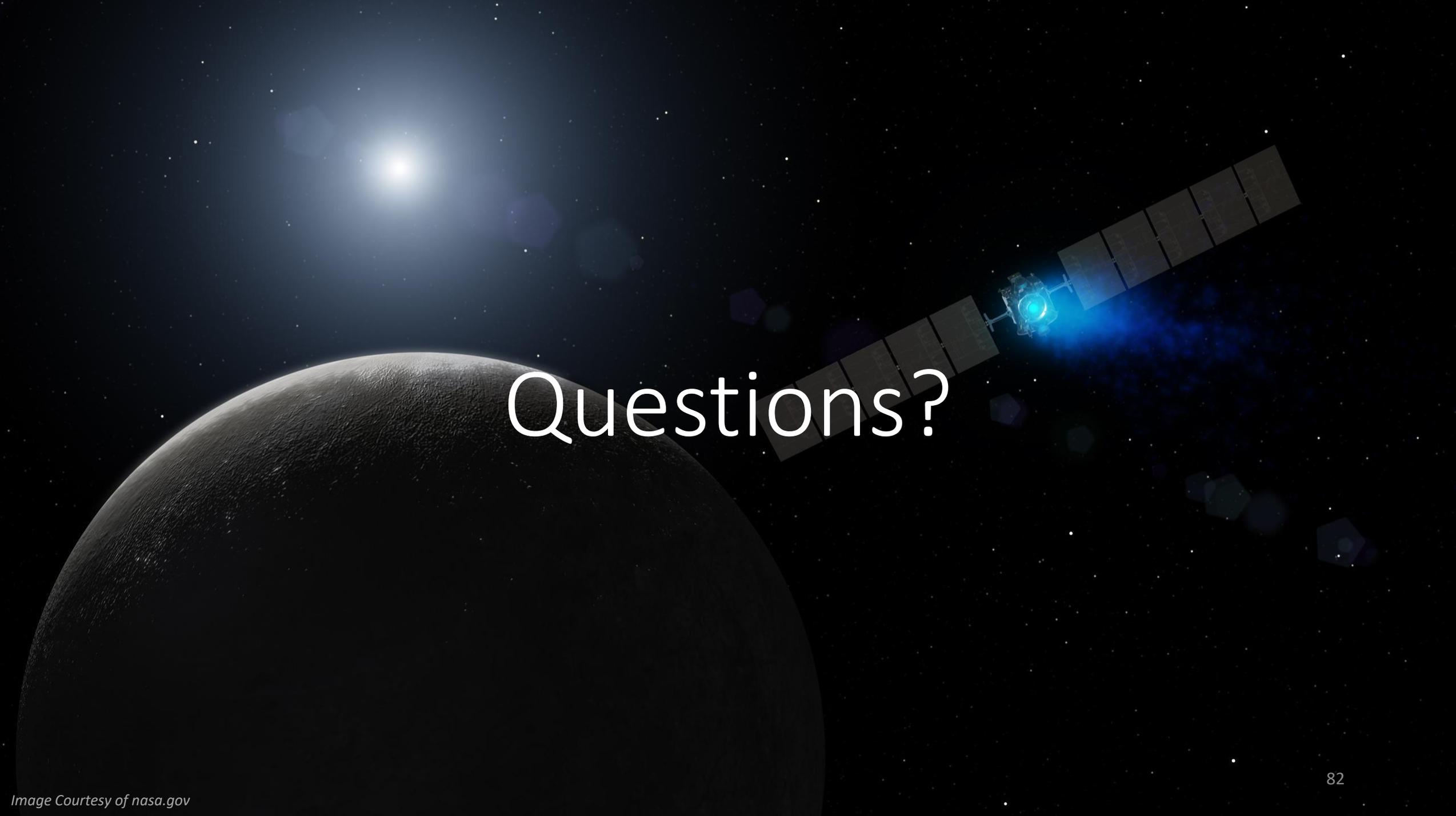
IV. Concluding Remarks

Concluding Remarks

- Demonstrated techniques for conducting low-thrust trajectory design that leverage collocation to compute optimal trajectories.
- Showed how natural dynamical structures in the CR3BP may be “chained” together to construct an initial guess.
- Applied these strategies in a continuation process to compute a transfer for a space station sized vehicle.
- Future Work:
 - Further develop orbit chaining technique
 - Explore application of these techniques to other scenarios where little information is available to construct an initial guess
 - Improve efficiency of direct transcription algorithms

Thank You!

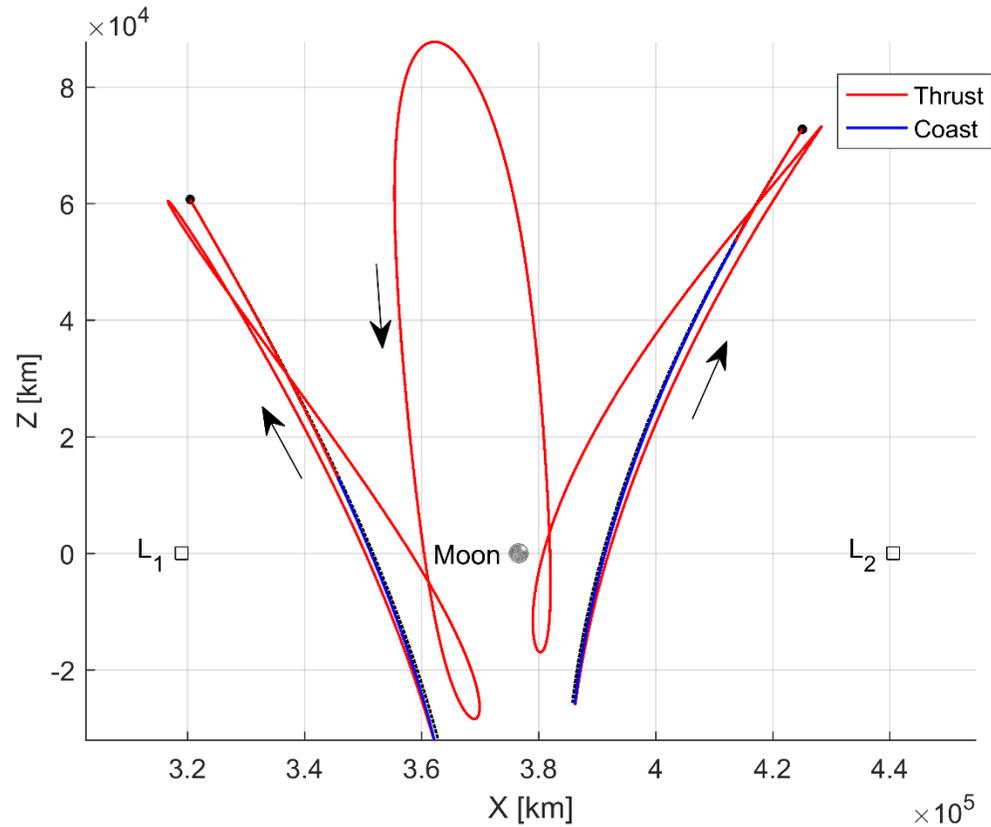
- Dr. Daniel Grebow and Dr. Thomas Pavlak
- Professor Kathleen Howell
- Dr. Juan Senent for SNOPT assistance
- Dr. Jeff Stuart for SparQ assistance
- Emily Zimovan for intermediate trajectory arc discussions
- Section 392 and JPL
- This work was supported by a NASA Space Technology Research Fellowship

A satellite with solar panels is shown in space, emitting a blue glow. In the background, a large planet is visible on the left, and a bright star is in the upper left. The word "Questions?" is written in white text across the center.

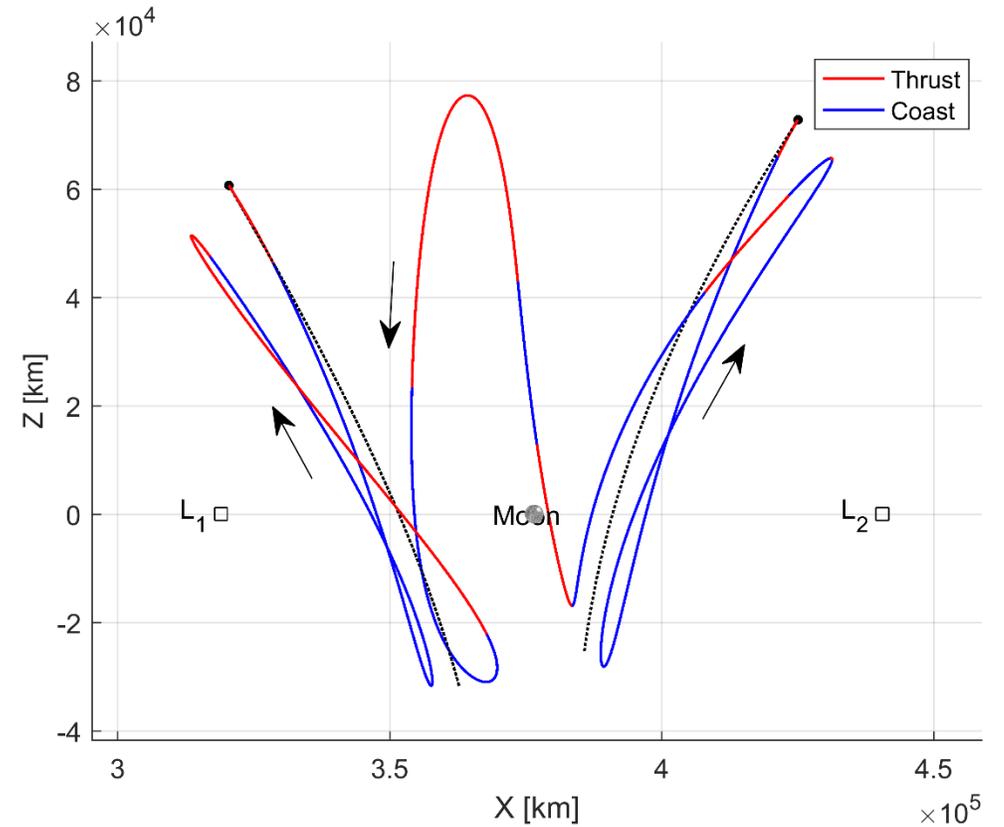
Questions?

Backup Slides

Background



Feasible Transfer
 $m_f/m_0 = 490/500$



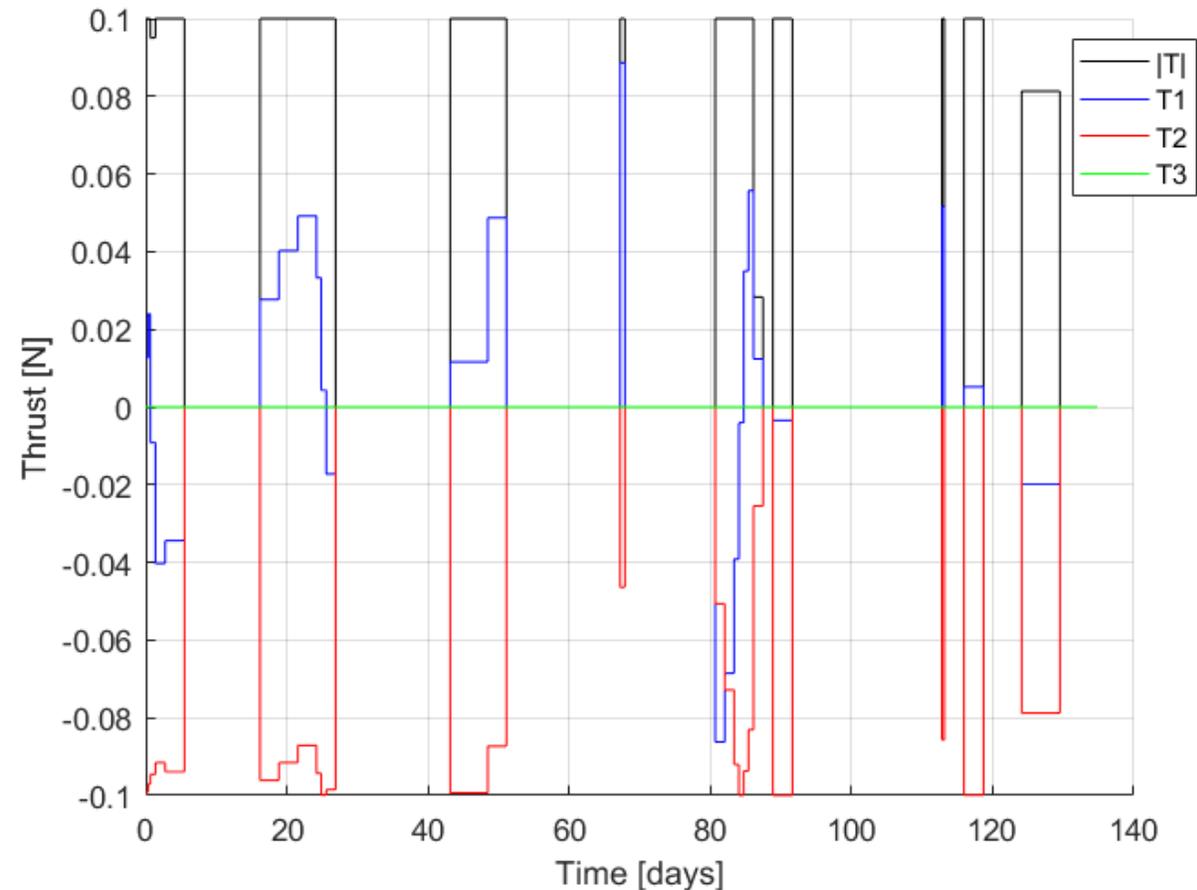
Optimal Transfer
 $m_f/m_0 = 492/500$

Sample Applications – Trajectory Stacking

Transfer Scenario:

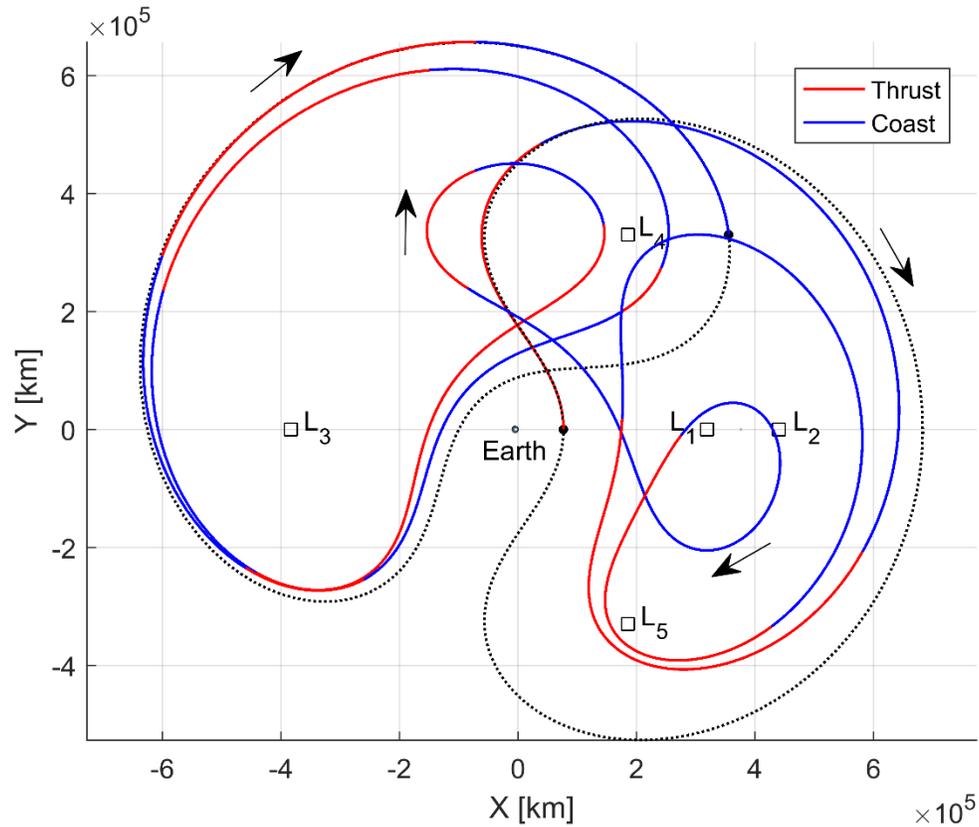
DRO → *L₄ SPO*

- *DRO 3 Rev* + *L₄ SPO 2 Rev*
- Transfer Time: 134.8 *days*
- Optimal: $m_f/m_0 = 482/500 \text{ kg}$

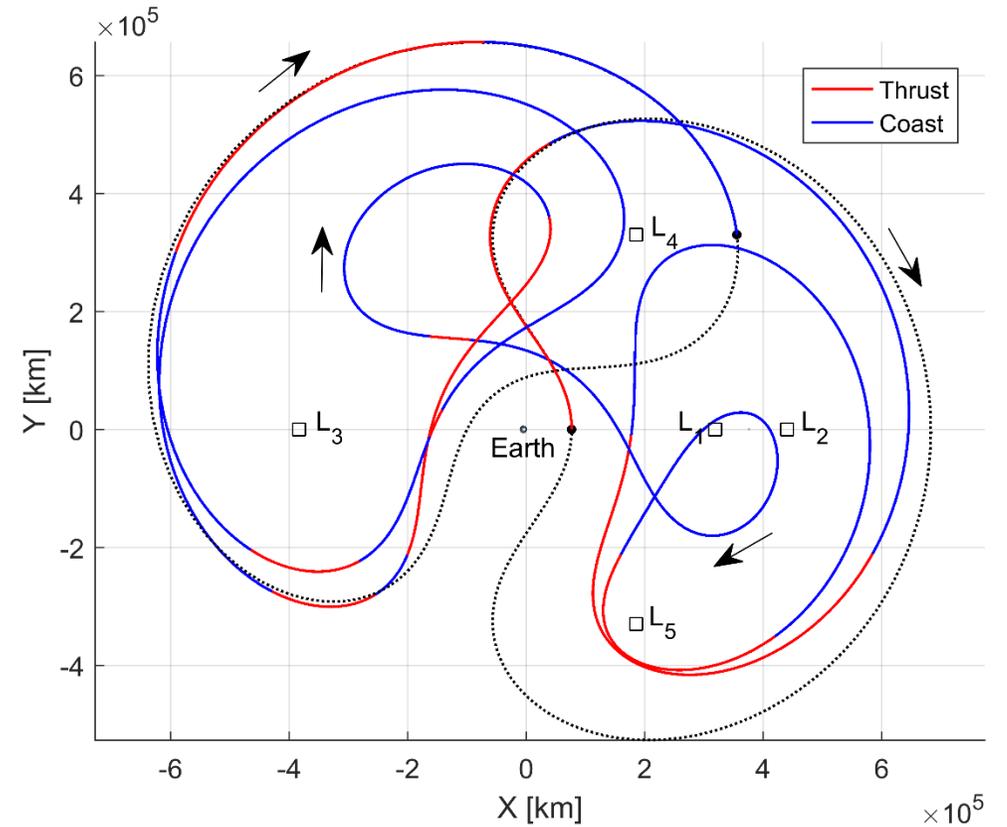


Optimal Thrust Profile

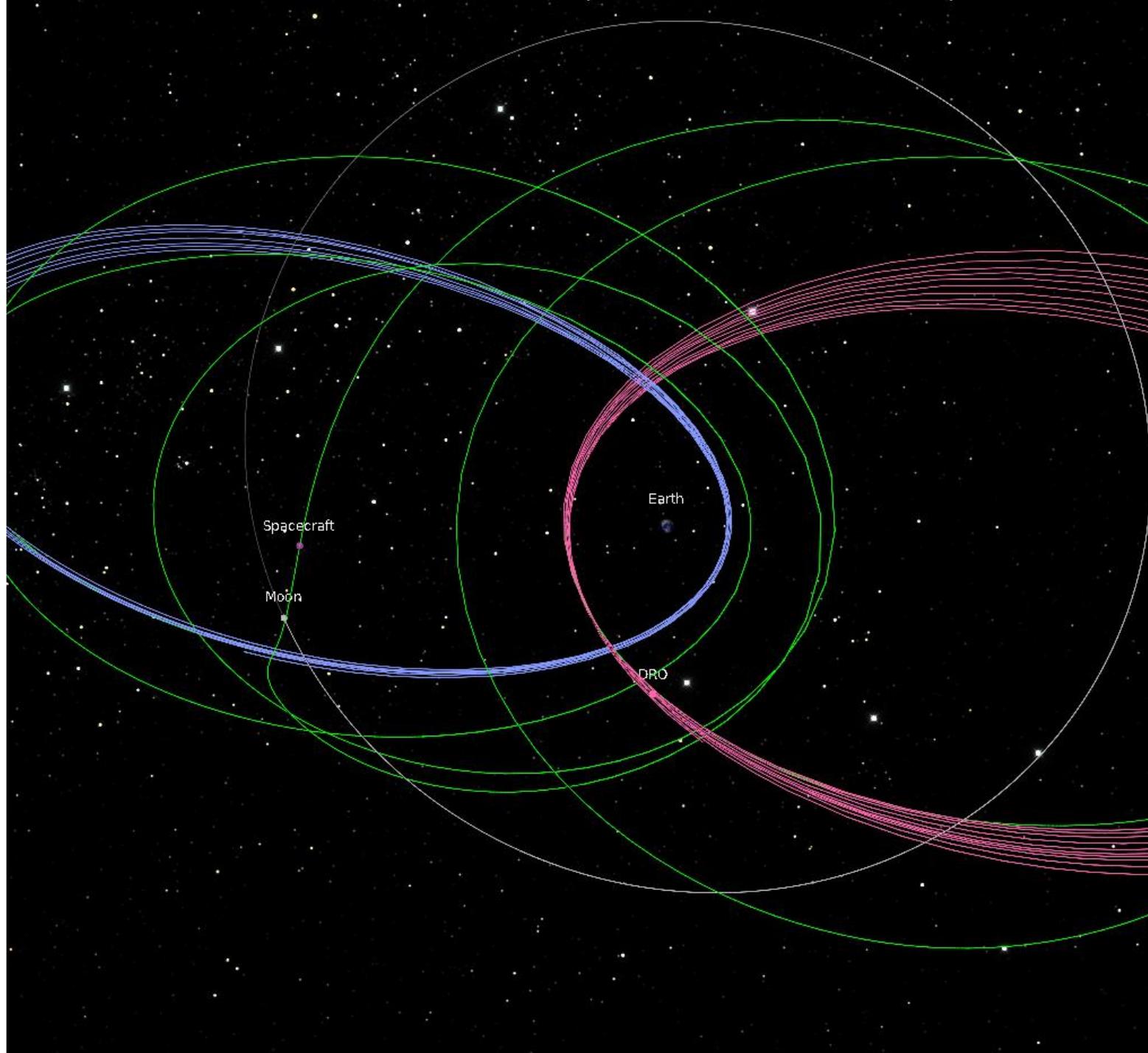
Background



Feasible Transfer
 $m_f/m_0 = 478/500$



Optimal Transfer
 $m_f/m_0 = 482/500$

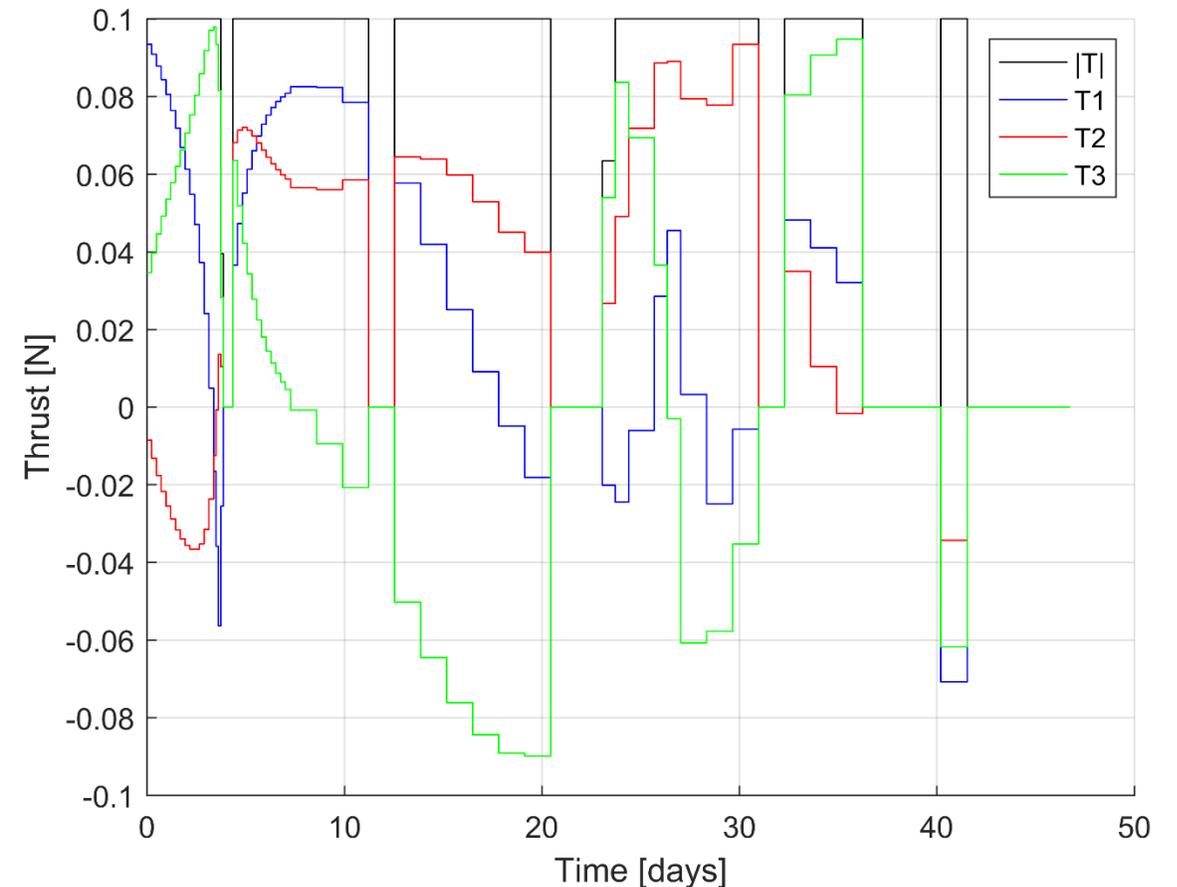


Sample Applications – Trajectory Stacking

Transfer Scenario:

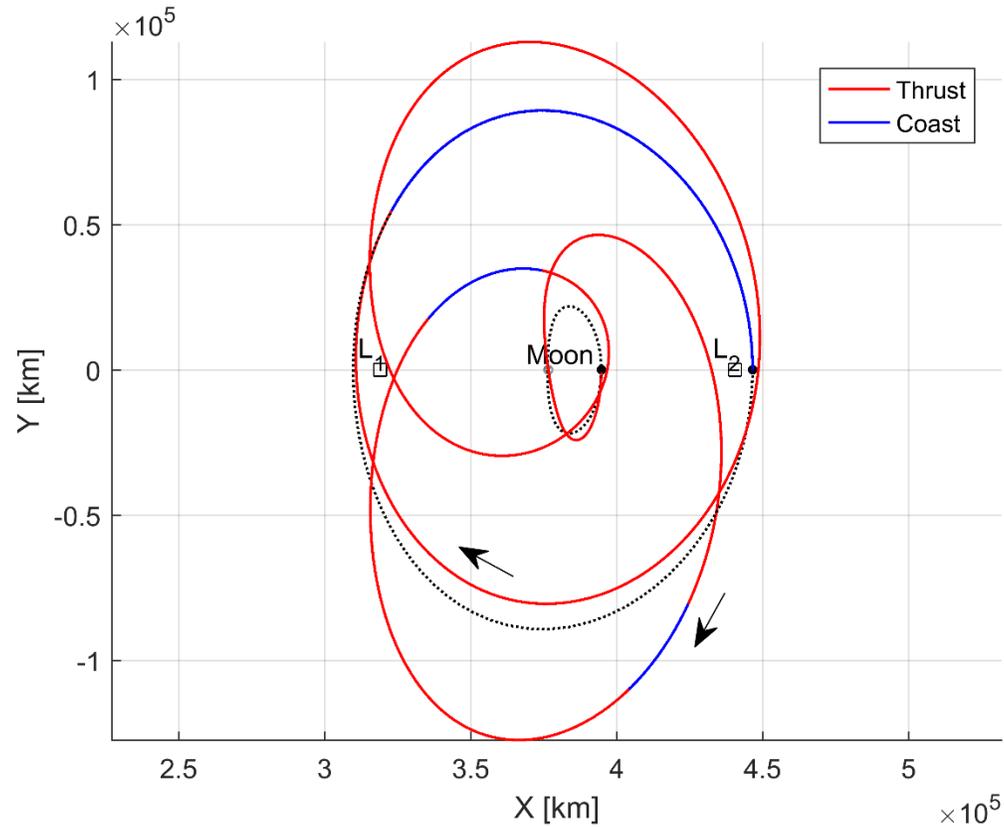
L_2 NRHO \rightarrow DRO

- NRHO 1 Rev + DRO 3 Rev
- Transfer Time: 46.7 days
- $m_f/m_0 = 486/500$ kg

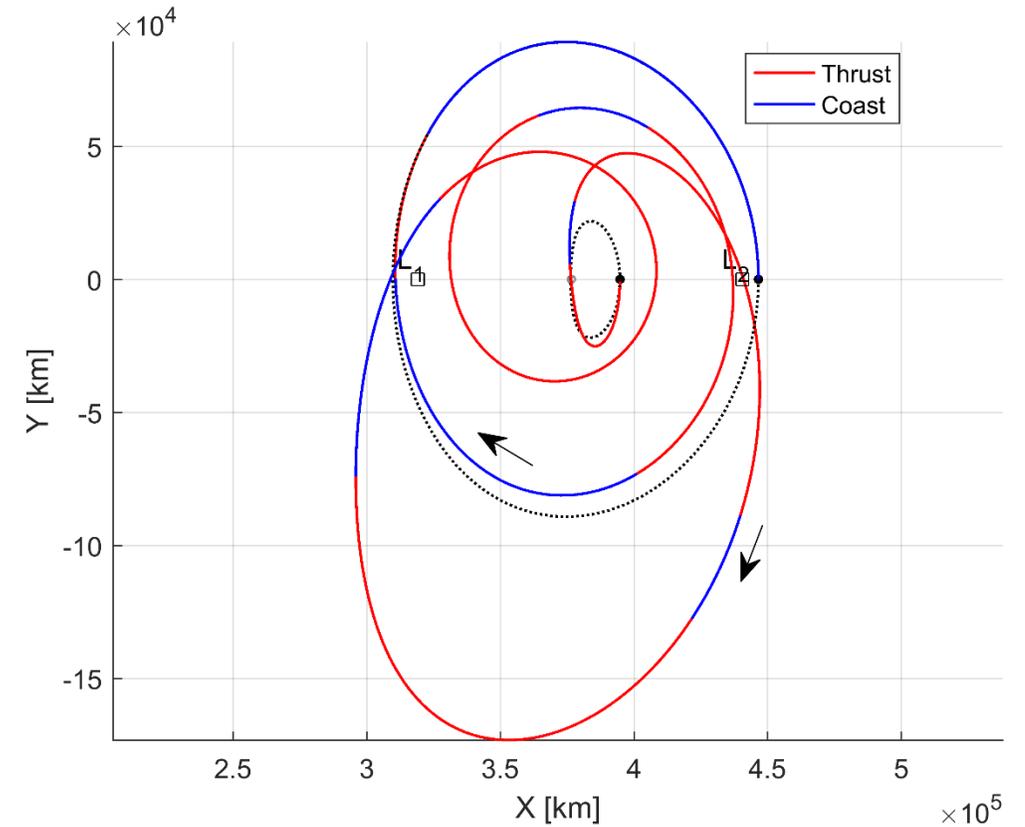


Optimal Thrust Profile

Background

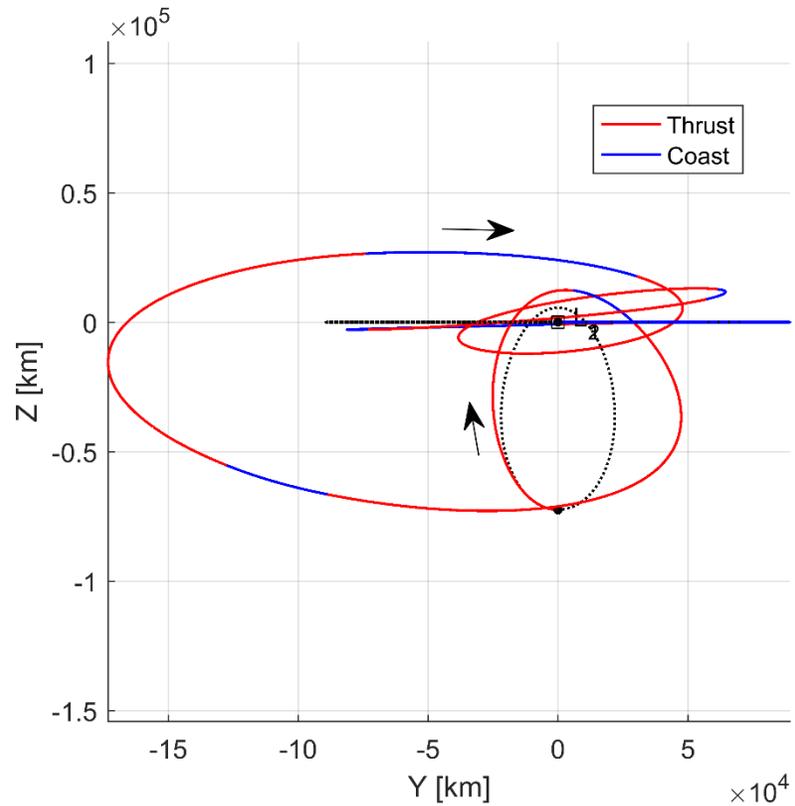


Feasible Transfer
 $m_f/m_0 = 483/500$

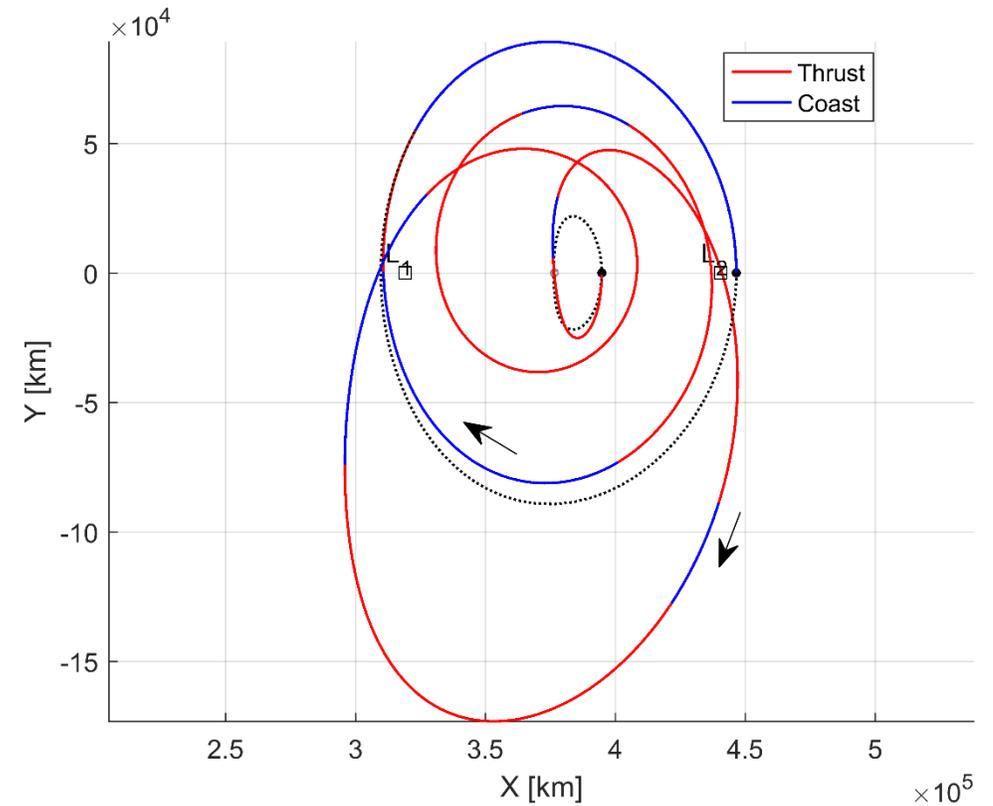


Optimal Transfer
 $m_f/m_0 = 486/500$

Sample Applications – Trajectory Stacking

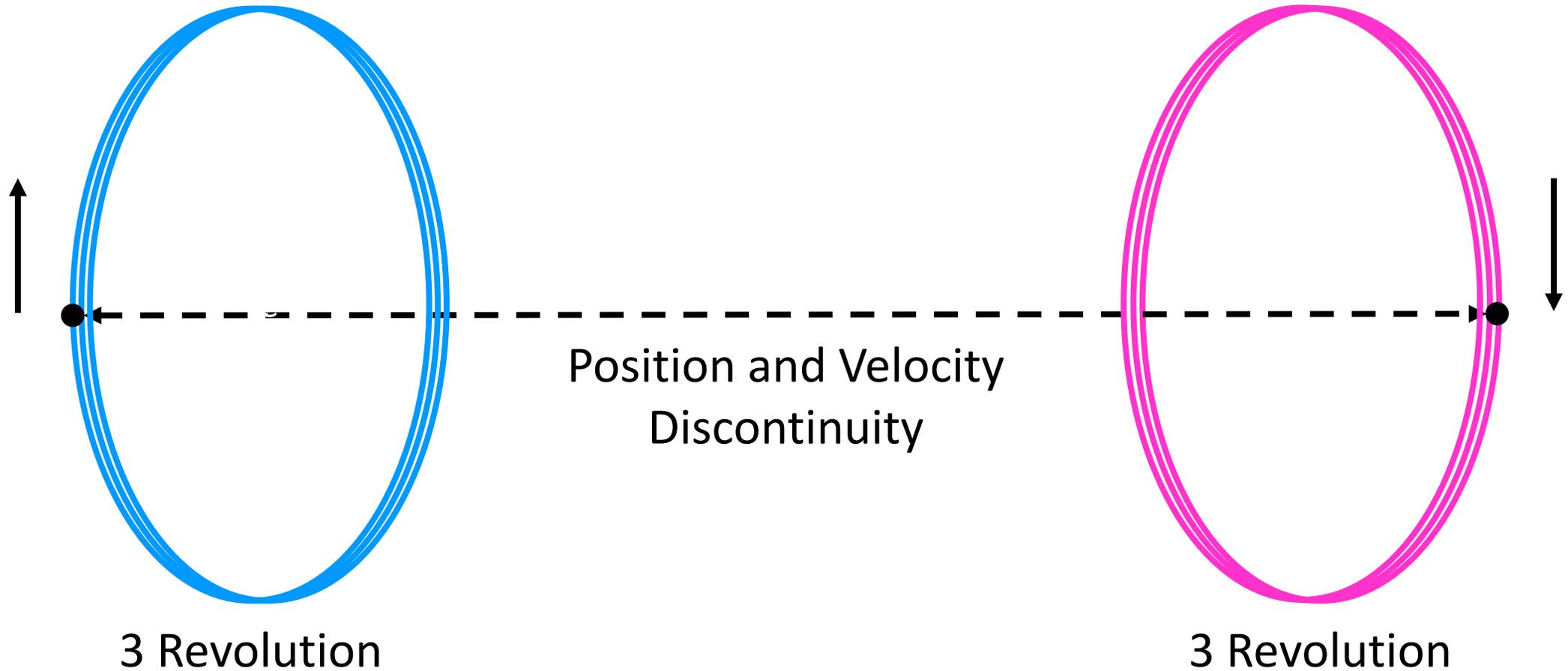


YZ View



XY View

Background



Background

