A historical perspective on the development of the PWM switch model

Vatché Vorpérian

Jet Propulsion Laboratory
California Institute of Technology

Abstract

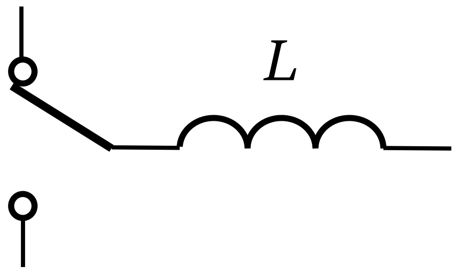
This review begins with the earliest appearance, in the published literature, of a sub-circuit of a dc-to-dc converter which is common to most converters and concludes with later works in which this sub-circuit is treated analytically and ultimately reduced to its final form known today as the PWM switch. Since this is my historical perspective, it is limited to my knowledge of the published literature and as such its scope may neither be sufficiently extensive nor inclusive. I will review the works of Cuk, Landsman, Tymerski, Meares up to the emergence of the PWM switch as the smallest sub-circuit of a converter with invariant structural and electrical terminal characteristics which bestow it with an invariant equivalent circuit model. Historically, the model of the PWM switch was promoted as a pedagogical tool by pointing out its similarity to the model of the transistor and its use in amplifier circuit analysis in order to render the analysis and simulation of PWM converters easily accessible to students of power electronics. It appears that this analogy to the model of the transistor has caught on well and many well known textbooks of power electronics have already adopted it.

Overview

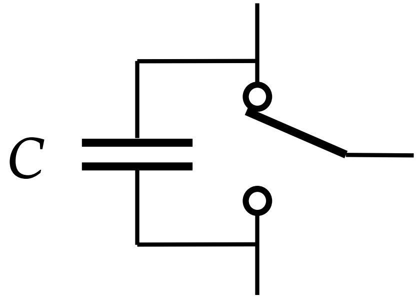
- Cuk's topological analysis.
- Landsman's canonical cell.
- Tymerski's extension of Landsman's cell.
- Meares' identification of the PWM cell and its transformer model.
- Vorpérian's identification of the PWM switch, its model in continuous and discontinuous conduction modes and refinement of its model to account for parasitic elements.
- Tymerski's extension of the PWM switch model to harmonic analysis of switching power amplifiers using Taylor series vs. Volterra functional series approach.
- Ridley's application to current mode control.
- Does the model of the PWM switch work for all topologies?

Cuk's topological analysis[1]

➢ Cyclical rotation of an inductor connected in series with a single-pole-double-throw switch.



➢ Cyclical rotation of a capacitor connected in parallel with a single-pole-double-throw switch



Reference

[1]Slobodan Cuk and R.D. Middlebrook, "A New Optimum Topology Switching dc-to-dc Converter," Proceedings of the 1977 IEEE Power Electronics Specialists Conference, pp. 160-179.

Cuk's topological analysis (*Cont.*)

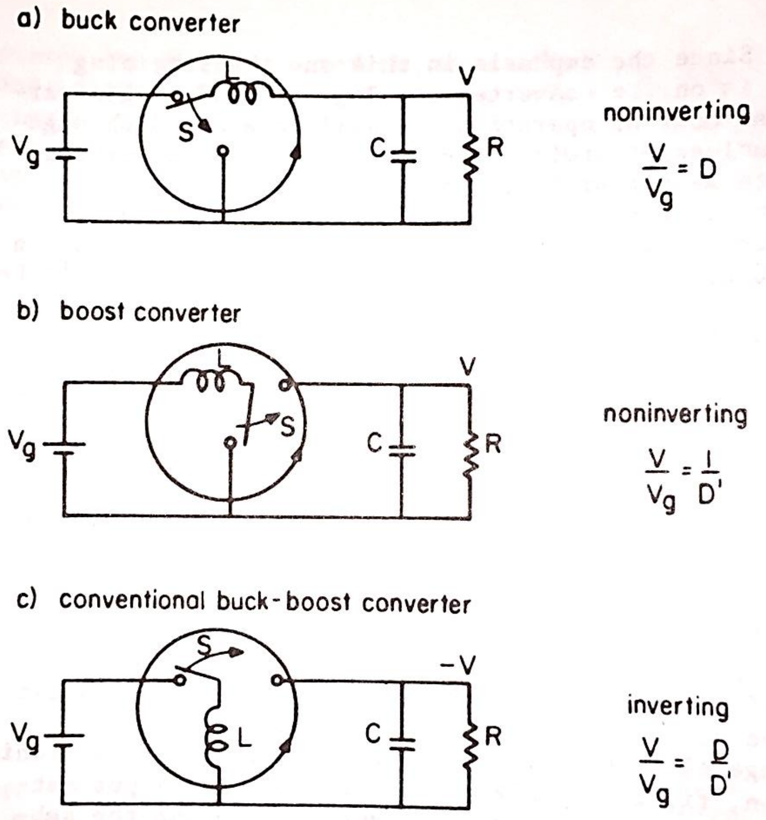


Fig. 5. Generation of the buck, boost and buck-boost converters by cyclic rotation of the series connection of inductance L and switch S.

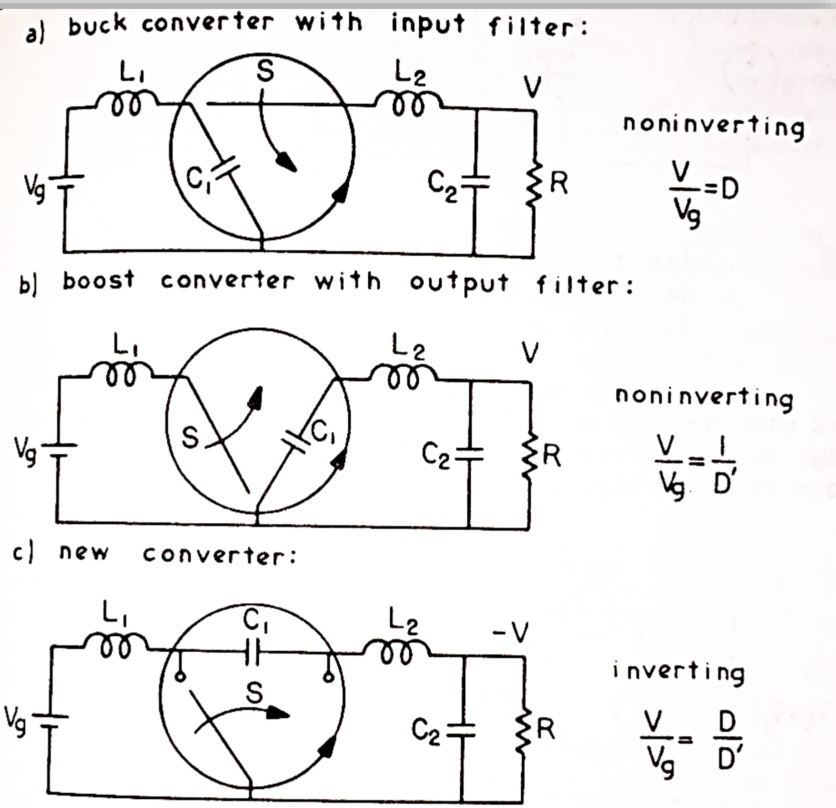


Fig. 16. Generation of the three converters: buck with input filter, boost with output filter, and new converter by cyclic rotation of the parallel connection of capacitance C and switch S.

Cuk's topological analysis (*Cont.*)

- No mathematical treatment of either sub-circuit was given, because Cuk had already invented the systematic mathematical analysis of PWM converters using the technique of State Space Averaging.

- These two sub-circuits were shown to have structural invariance only.

- Other sub-circuits were to follow later (Tymerski, Landsman)

Landsman’s canonical cell[2]

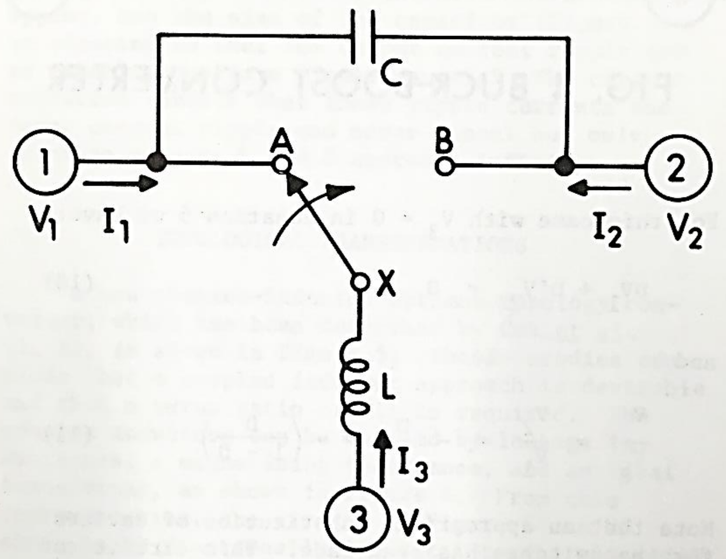


FIG. 1 CANONICAL SWITCHING CELL

Reference

[2]Emanual .E. Landsman, “A Unifying Derivation of Switching dc-to-dc Converter Topologies,” Proceedings of the 1979 IEEE Power Electronics Specialists Conference, pp. 239-243.

Landsman's canonical cell (*Cont.*)

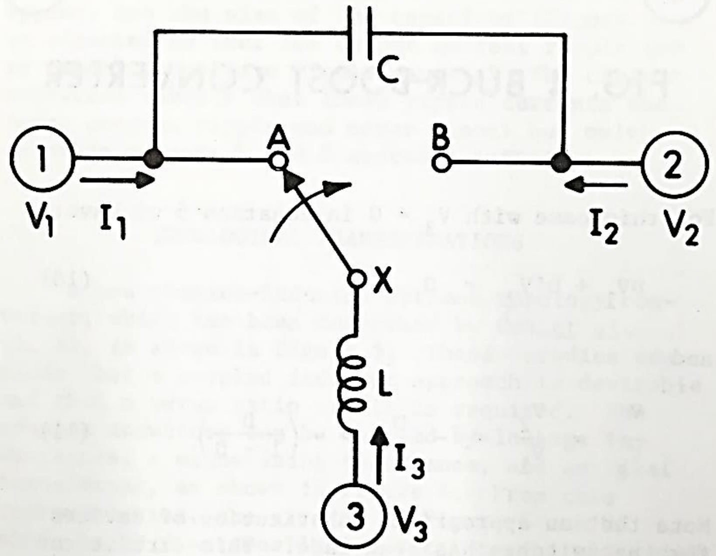


FIG. 1 CANONICAL SWITCHING CELL

Structural and quantitative invariance shown

In order to analyze this three-terminal network in the most general way, it will be assumed that none of the terminals is grounded. Voltages will be measured with respect to an external ground reference. The switch is assumed to be in position A for time DT and in position B for time D'T, where T is one period of operation and D + D' = 1. Conservation of charge and power yield:

$$I_1 + I_2 + I_3 = 0 \quad (1)$$

$$V_1I_1 + V_2I_2 + V_3I_3 = 0 \quad (2)$$

Lossless components have been assumed and voltages and currents are steady-state DC values with no superimposed ripple. Averaging the currents yield:

$$I_1 = -DI_3 \quad (3)$$

$$I_2 = -D'I_3 \quad (4)$$

Substituting (3) and (4) into (2) yields:

$$-DV_1 - D'V_2 + V_3 = 0 \quad (5)$$

from which all voltage transfer functions can be derived. Substitution of (3) and (4) into (1) yield:

$$- D - D' + 1 = 0 \quad (6)$$

which checks the equations.

Landsman's canonical cell (*Cont.*)

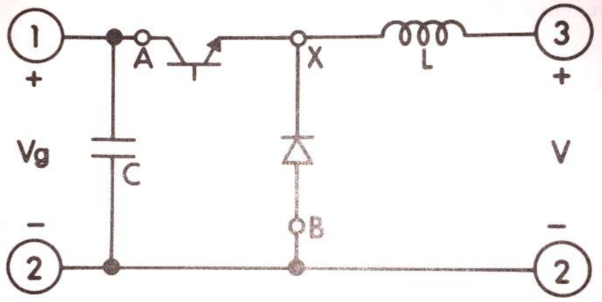


FIG. 2 DOWN-CONVERTER

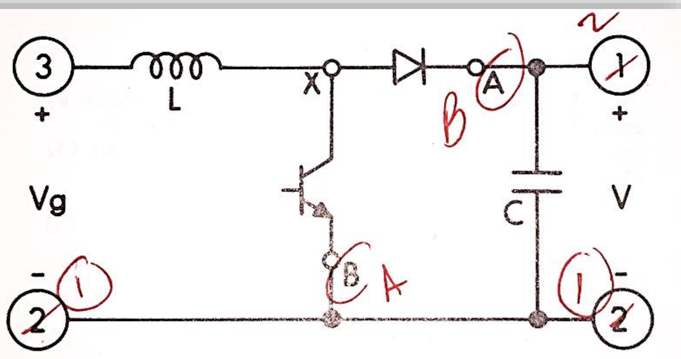


FIG. 3 UP-CONVERTER

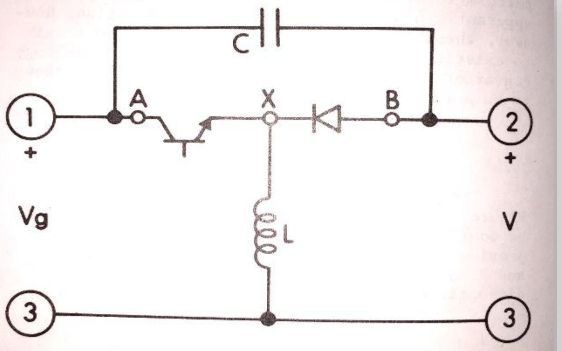


FIG. 4 BUCK-BOOST CONVERTER

Landsman's canonical cell (*Cont.*)

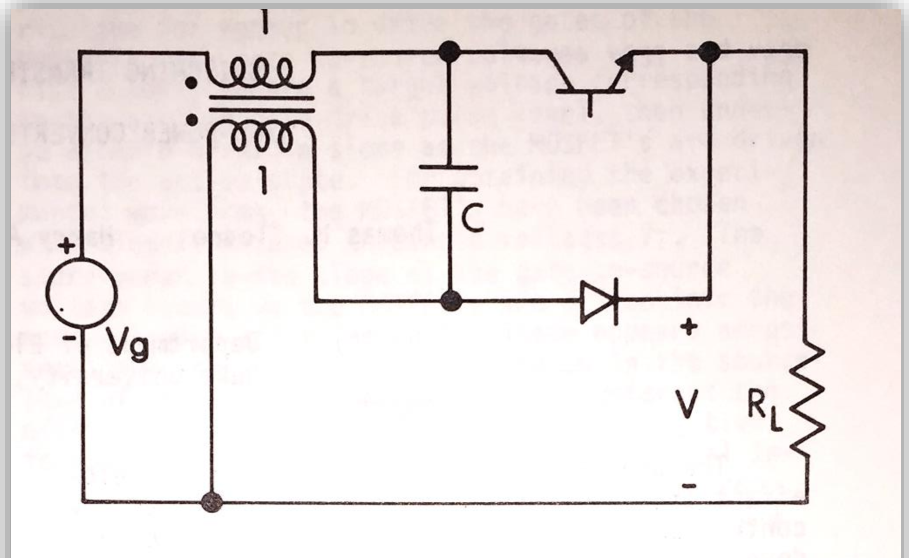


FIG. 10 DOWN–CONVERTER

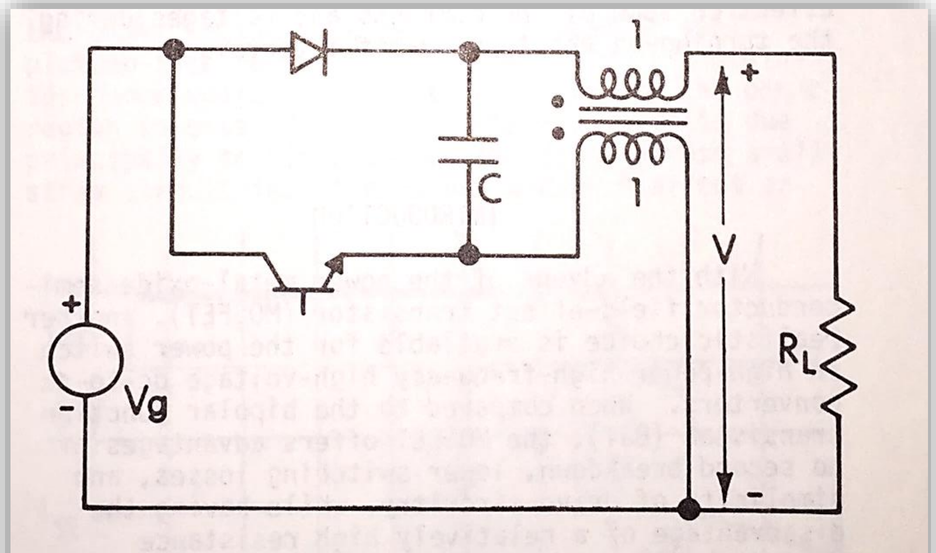


FIG. 11 UP–CONVERTER

Tymerski's generalization of Landsman's canonical cell[3,4]

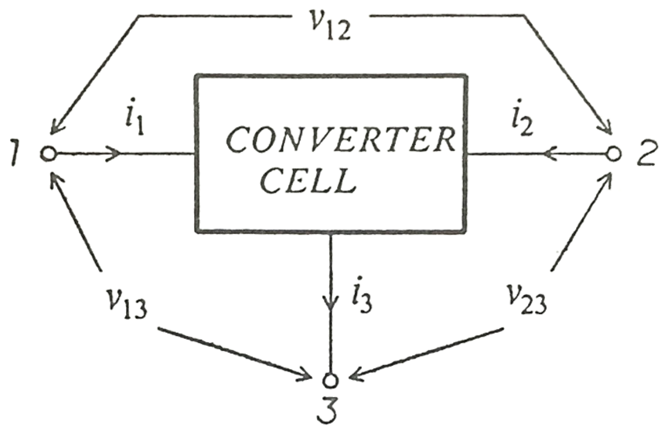


Fig. 2.2 *A three-terminal converter cell with designated terminal voltages and currents.*

Table 2.1 *The six ways of configuring a three-terminal converter cell to the input source and output sink. The table entries represent the terminal number of the converter cell.*

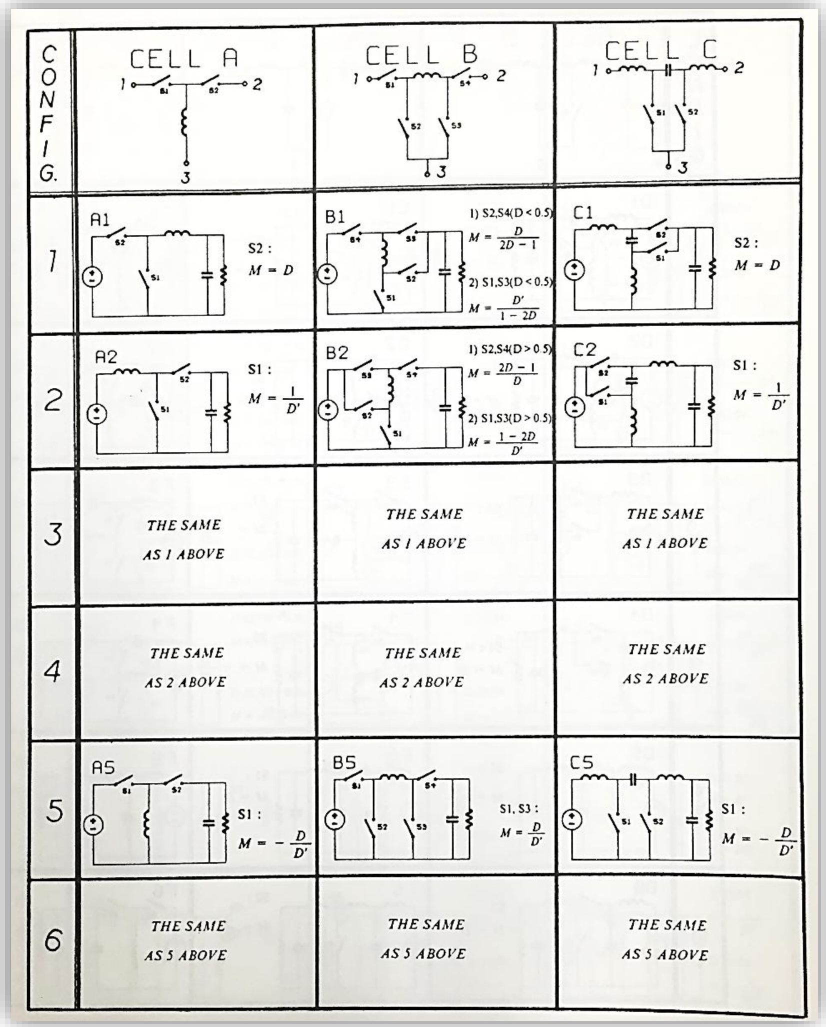
CONFIGURATION NUMBER	1	2	3	4	5	6
COMMON	1	1	2	2	3	3
INPUT	2	3	1	3	1	2
OUTPUT	3	2	3	1	2	1

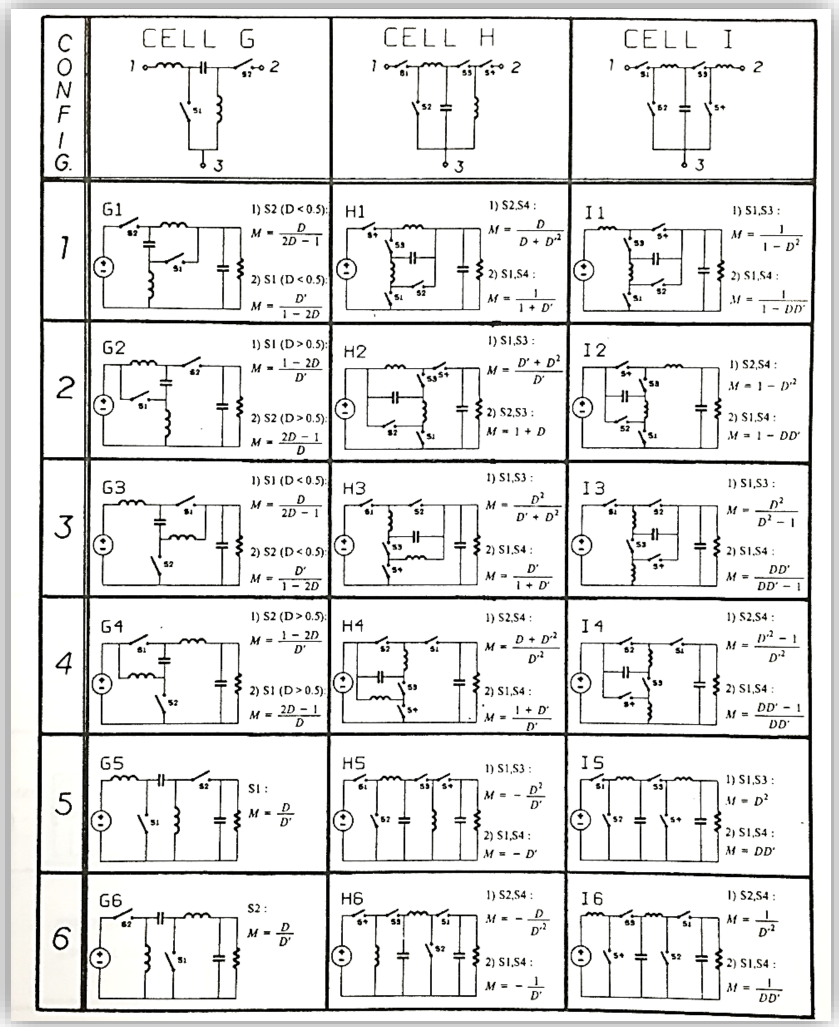
Reference

[3]Richard P.E. Tymerski and V. Vorpérian, "Generation, Classification and Analysis of Switched-Mode dc-to-dc Converters by the Use of Converter Cells," Proceedings of the 1986 IEEE International Telecommunication Energy Conference, pp. 181-195.

[4]Richard P.E. Tymerski, "Topology and Analysis in Power Conversion and Inversion," Ph.D Dissertation, Virginia Tech, 1988.

Tymerski's generalization of Landsman's canonical cell (*cont.*)





Many new topologies were revealed

Meares' switching cell[5]

The smallest invariant subcircuit is identified both structurally and quantitatively.

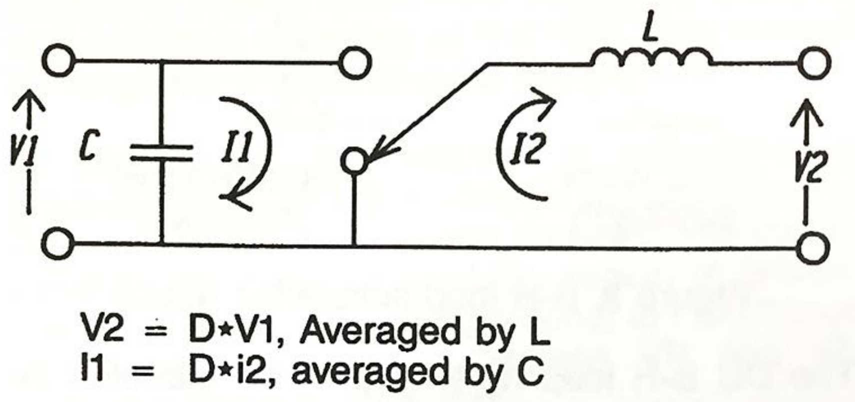


Figure 7, Buck regulator

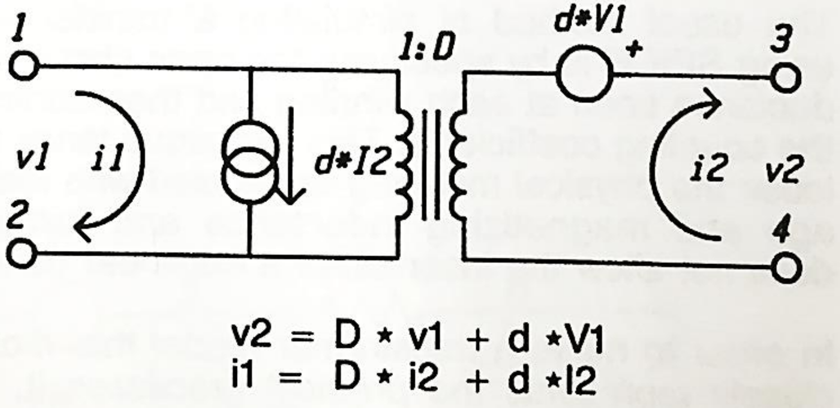


Figure 8, Small signal Buck regulator model

This caption should have read: Small-signal model of switching cell.

Reference

[5]L.G. Meares, "New Simulation Techniques Using Spice," Proceedings of the 1986 IEEE Applied Power Electronics Conference, pp. 198-205.

Meares' PWM cell[5]

The PWM cell is identified and modeled as
an auto-transformer with an adjustable turns ratio
equal to the duty cycle.

A Spice simulation model is developed which automatically
linearizes the PWM cell at a dc operating point numerically to
determine the dc and small signal characteristics of the
dasic converters.

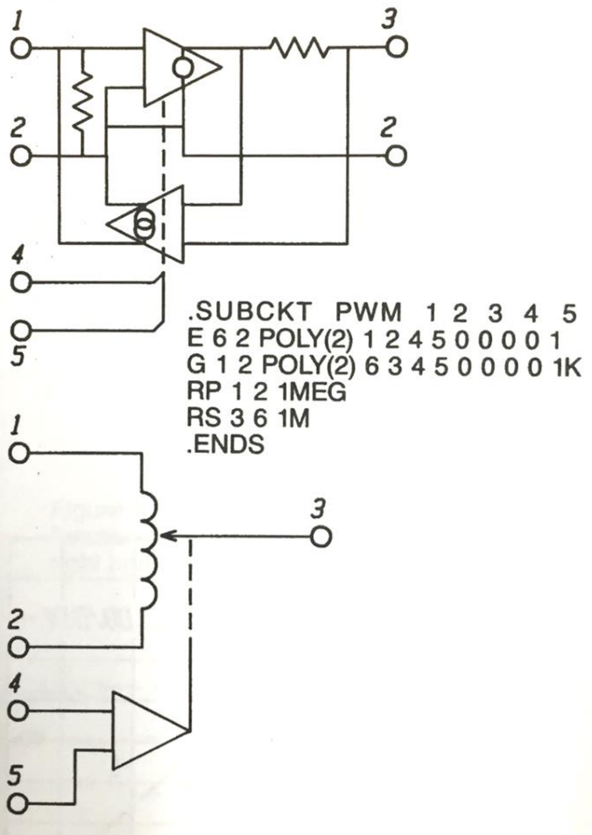
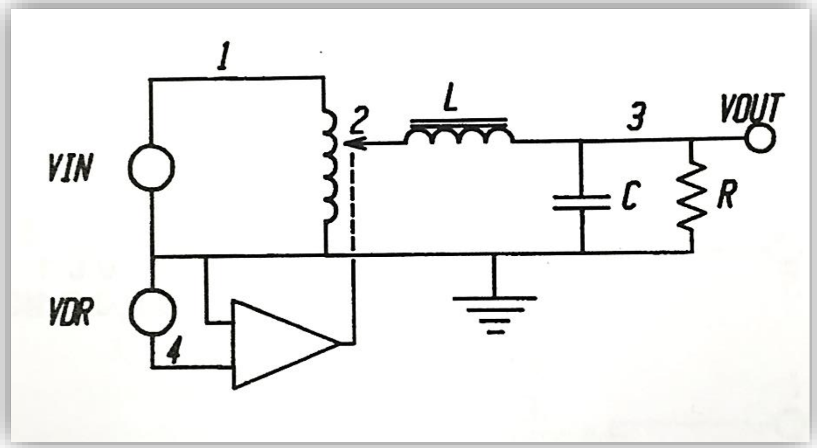


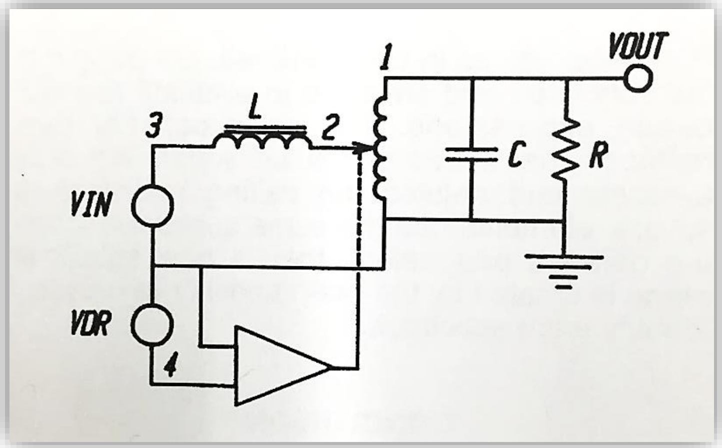
Figure 9, SPICE Pulse Width Modulator

Reference

[5]L.G. Meares, "New Simulation Techniques Using Spice," Proceedings of the 1986 IEEE Applied Power Electronics Conference, pp. 198-205.

Meares' PWM cell[5]



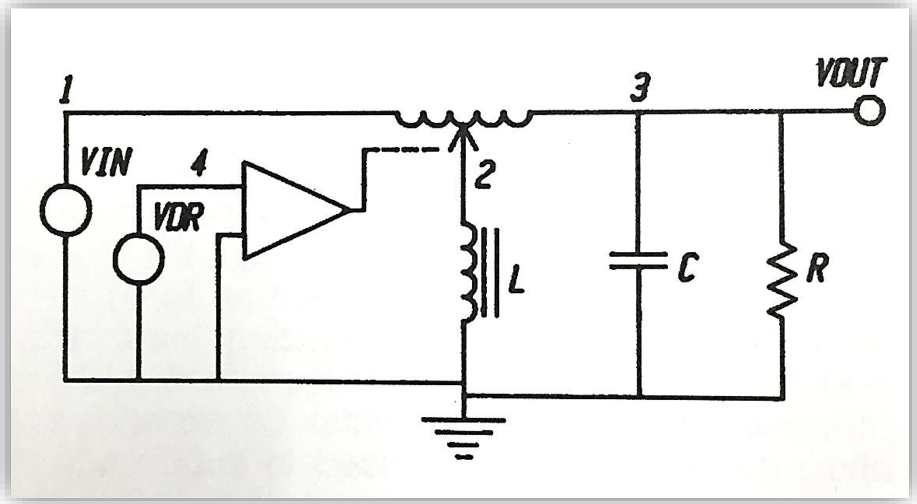


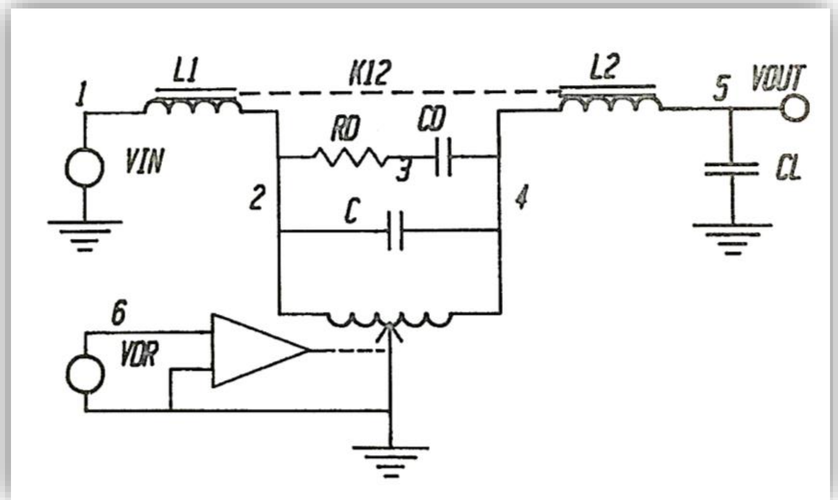
The PWM cell model is used to analyze buck and boost converters

Reference

[5]L.G. Meares, "New Simulation Techniques Using Spice," Proceedings of the 1986 IEEE Applied Power Electronics Conference, pp. 198-205.

Meares' PWM cell[5]





The PWM cell model is used to analyze buckboost and Cuk converters

Reference

[5]L.G. Meares, "New Simulation Techniques Using Spice," Proceedings of the 1986 IEEE Applied Power Electronics Conference, pp. 198-205.

The model of the PWM switch[6]

- The PWM switch and its equivalent circuit model were discovered independently by Vorperian[7] at Virginia Tech in the same year as Meares published his APEC paper in 1986.

- The model included and accounted for the subtle effect of parasitic resistances in any two-switched PWM converter.

- The model of the PWM switch in discontinuous conduction mode operation was also given.

Reference

[6]Vatche Vorperian, “Simplified Analysis of PWM Converters Using the Model of the PWM Switch: Parts I and II,” IEEE Transactions on Aerospace and Electronic Systems,” Vol. 26, No 3. pp. 490-505.

[7]Vatche Vorperian, “Model of the single-pole double throw switch”

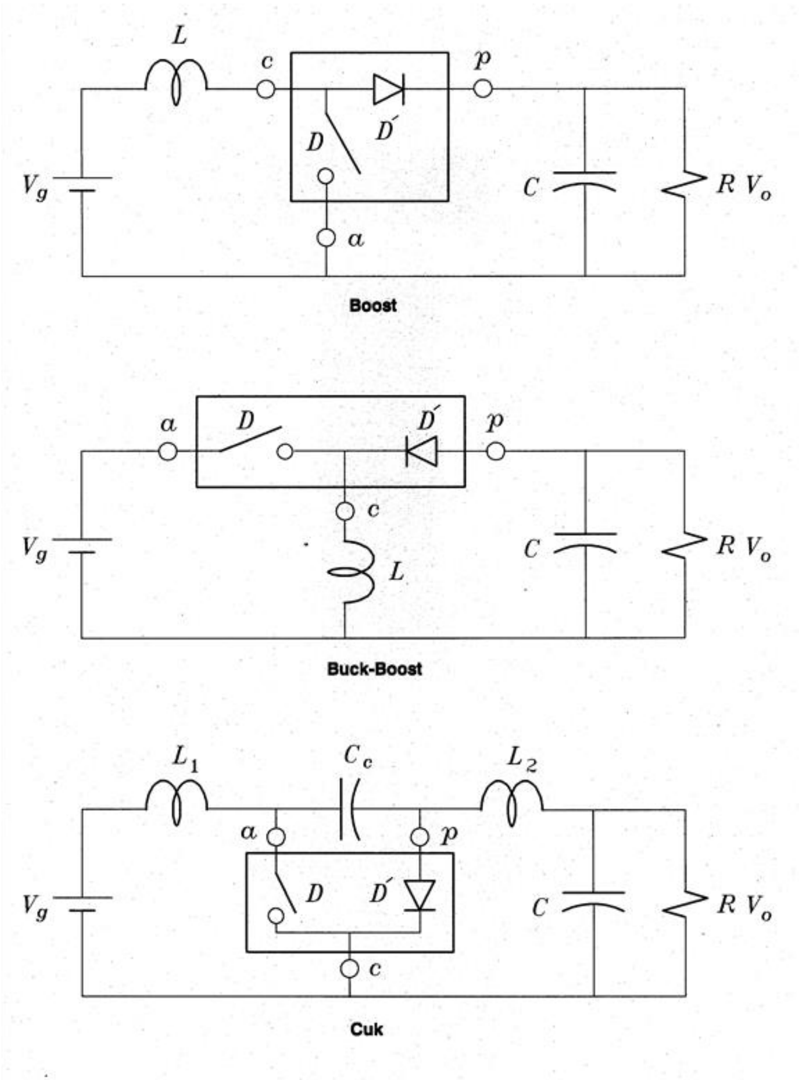
The model of the PWM switch (*Cont.*)

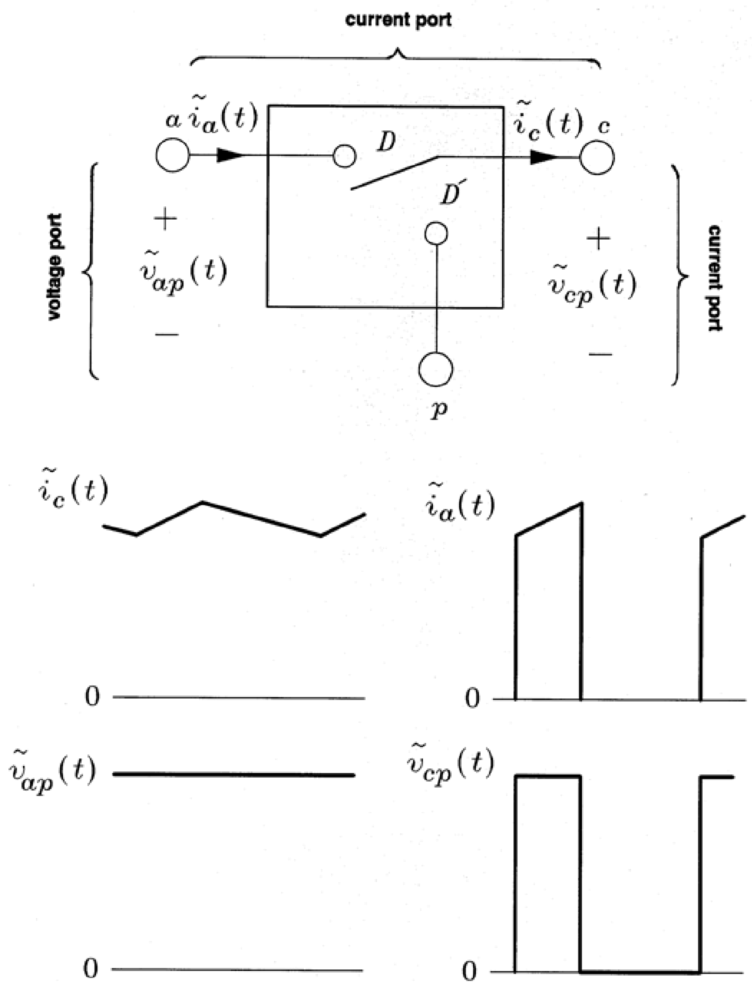
- It has been shown that the model of the PWM switch can be identified in any two-state switching dc-to-dc converter, such as those synthesized by D. Maksimovich[8]
- The model has been extended to charge control and peak current mode control with constant frequency, constant on-time, and constant off-time.
- Simulation models were developed by various software companies such as MicroSim Pspice, Analog Workbench, and many others.

Reference

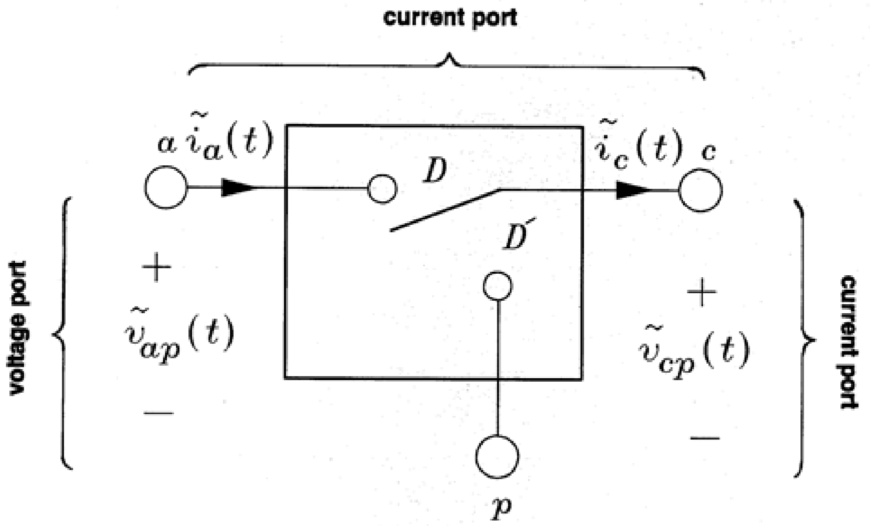
[8]Dragan Maksimovich and Slobodan Cuk, "General Unified Properties and Synthesis of PWm Converters," *Proceedings of the 1989 IEEE Power Electronics Specialist Conference*, PESC 89 Record

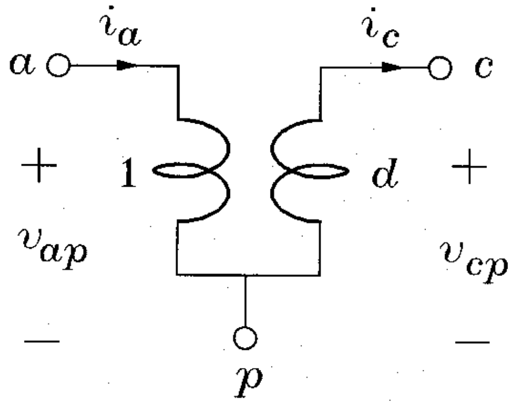
The model of the PWM switch (*Cont.*)



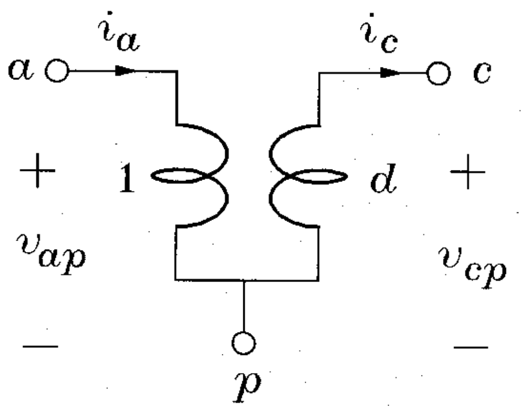


The model of the PWM switch (*Cont.*)

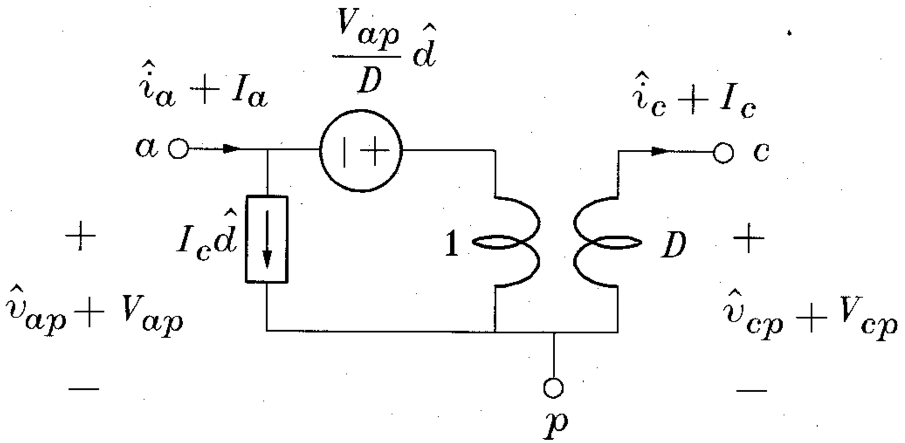




The model of the PWM switch (*Cont.*)



Average model of the PWM switch



DC and small-signal model of the PWM switch

The model of the PWM switch (*Cont.*)

Three immediate and significant milestones followed the development of the model of the PWM switch.

1. ***Simplified analysis of PWM converters***[6]: It was shown that the dynamical analysis of PWM dc-to-dc converters was significantly simplified once the equivalent circuit of a converter was obtained by substituting the model of the PWM switch and Middlbrook's techniques of Design-Oriented Analysis were applied

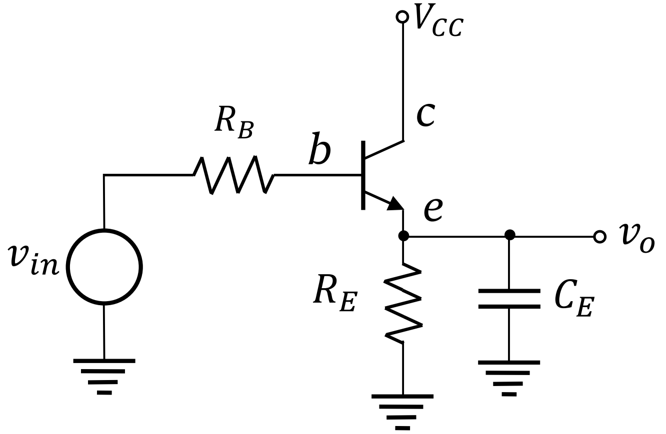
   The model of the PWM switch was promoted as a pedagogical tool by pointing out its similarity to the model of the transistor and its use in amplifier circuit analysis. Thus the analysis and simulation of PWM converters became easily accessible to students of power electronics using standard and familiar tools. Many textbooks have already adopted this technique.

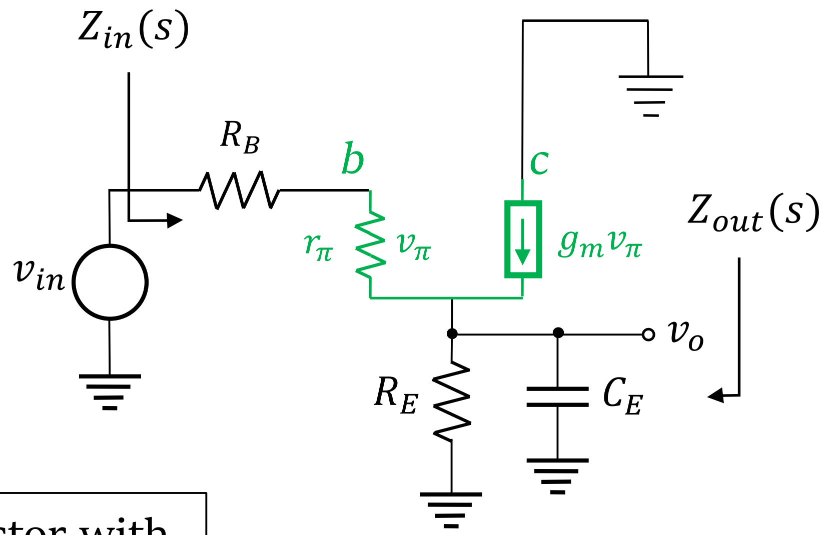
Reference

[6]Vatche Vorperian, "Simplified Analysis of PWM Converters Using the Model of the PWM Switch: Parts I and II," IEEE Transactions on Aerospace and Electronic Systems," Vol. 26, No 3. pp. 490-505.

The model of the PWM switch (*Cont.*)

Example: *Determination of the small-signal characteristics of the emitter follower*



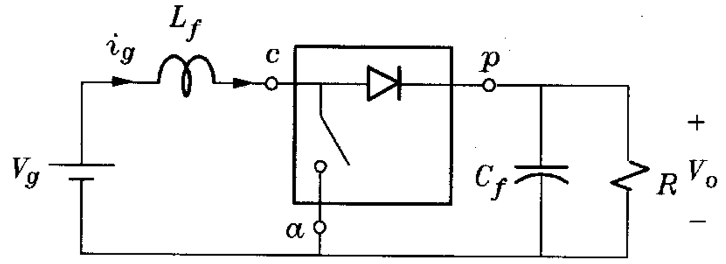


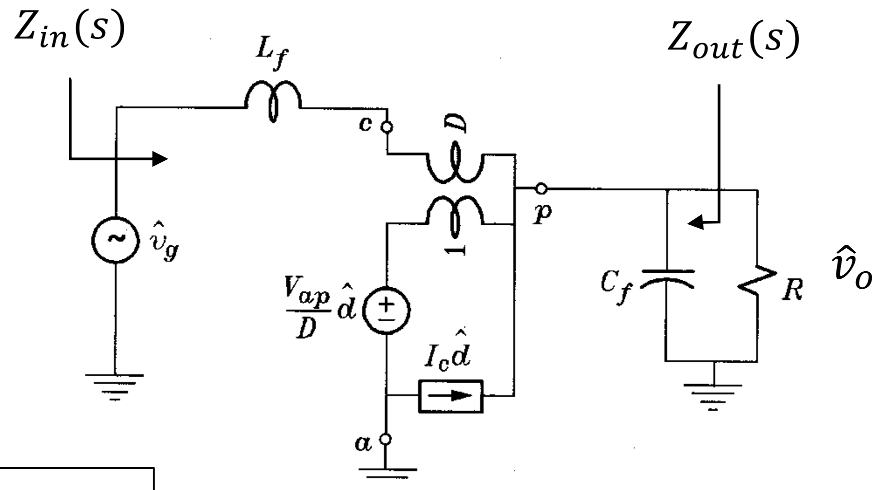
Replace the transistor with its equivalent circuit model

$$\frac{\hat{v}_o}{\hat{v}_{in}} \equiv voltage\ gain\ TF$$

The model of the PWM switch (*Cont.*)

Example: *Determination of the dc and small-signal characteristics of the boost converter*





Replace the PWM switch with its equivalent circuit model

$$\frac{\hat{v}_o}{\hat{d}} \equiv \textit{Control-to-output TF}$$

$$\frac{\hat{v}_o}{\hat{v}_g} \equiv \textit{Line-to-output TF}$$

The model of the PWM switch (*Cont.*)

2. ***Non-linear dynamical analysis of PWM converters*[4]*:*** Richard Tymerski, a Ph.D. candidate at that time at Virginia Tech at that same time, recognized that he could immensely simplify the non-linear analysis of PWM switching amplifiers by replacing the Volterra functional series technique, first worked out by Robert Ericsson for PWM converters, with a simple Taylor's series expansion of the model of the PWM switch.

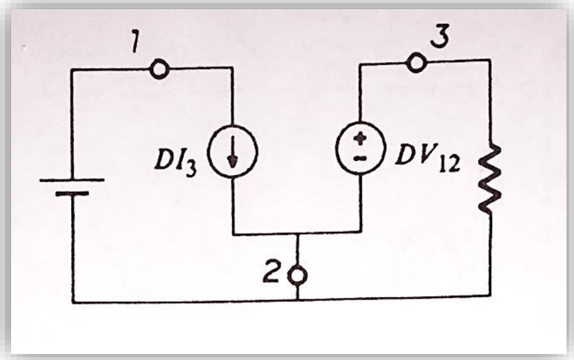
Reference

[4]Richard P.E. Tymerski, "Topology and Analysis in Power Conversion and Inversion," Ph.D Dissertation, Virginia Tech, 1988.

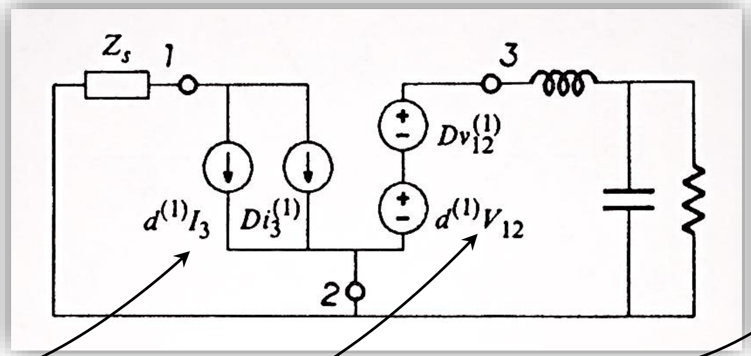
The model of the PWM switch (*Cont.*)

2. ***Non-linear dynamical analysis of PWM converters***[4]

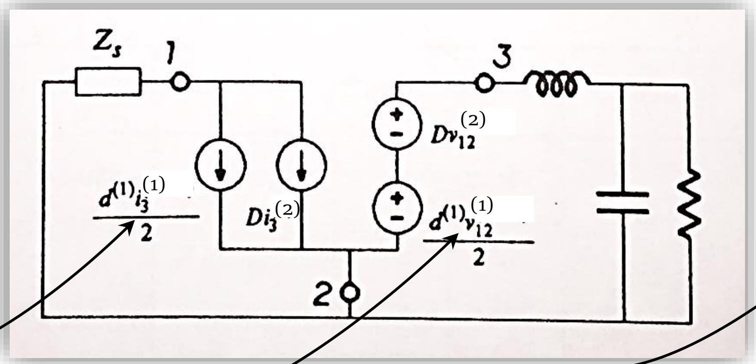
Dc analysis



1st harmonic



2nd harmonic



Solve for: I_3 and V_{12} (OR I_a and V_{ap})

Solve for: $i_3^{(1)}$ and $v_{12}^{(1)}$ (OR $i_a^{(1)}$ and $v_{ap}^{(1)}$)

Solve for: $i_3^{(2)}$ and $v_{12}^{(2)}$ (OR $i_a^{(2)}$ and $v_{ap}^{(2)}$)

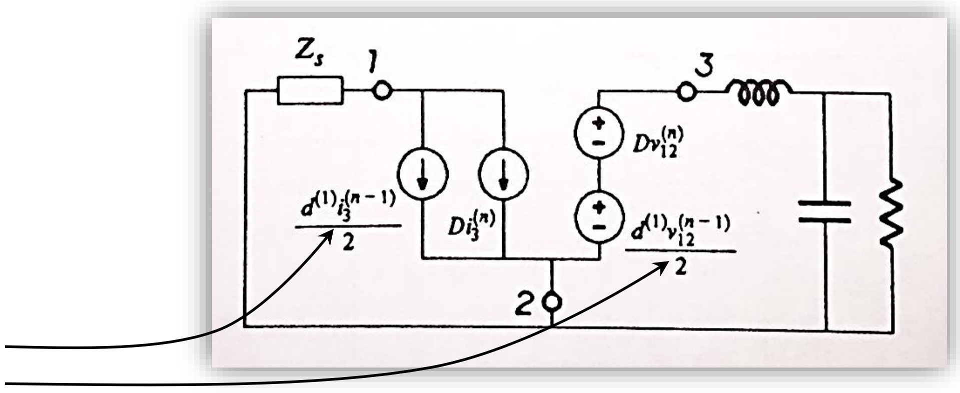
Reference

[4]Richard P.E. Tymerski, "Topology and Analysis in Power Conversion and Inversion," Ph.D Dissertation, Virginia Tech, 1988.

The model of the PWM switch (*Cont.*)

2. ***Non-linear dynamical analysis of PWM converters***[4]

N^{th} harmonic



Reference

[4]Richard P.E. Tymerski, "Topology and Analysis in Power Conversion and Inversion," Ph.D Dissertation, Virginia Tech, 1988.

The model of the PWM switch (*Cont.*)

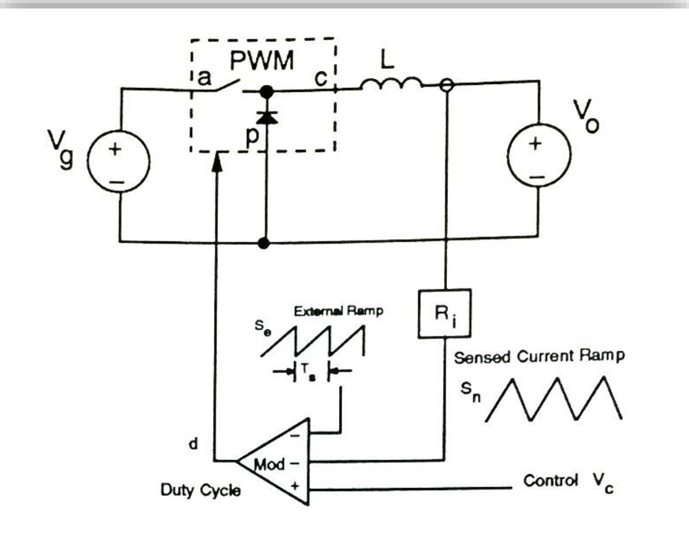
3. ***Model of current mode control***[7]: Ray Ridley, another Ph.D. candidate at that time at Virginia Tech, immediately recognized that he could derive a continuous-time model of converters in current-mode control simply by deriving an invariant, continuous-time model of the current feedback loop only and applying it to the model of converter obtained with the PWM switch.

Reference

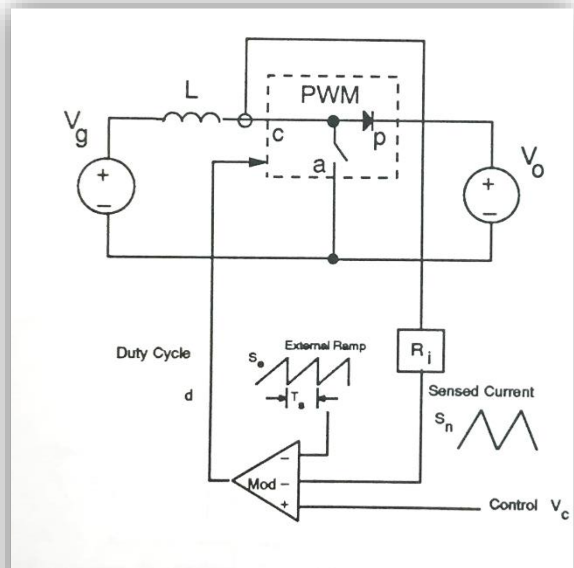
[7]Raymond B. Ridley, "A New Small-Signal Model for Current-Mode Control," Ph.D dissertation, Virginia Tech, November 27, 1990.

The model of the PWM switch (*Cont.*)

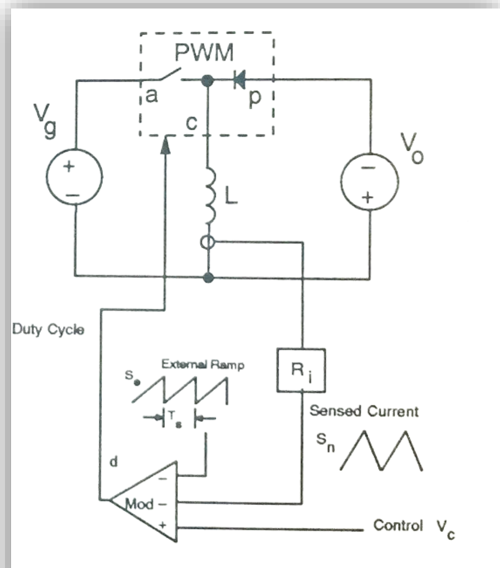
3. ***Model of current mode control***[7]:



(a) Buck



(b) Boost



(c) Buck-Boost

Reference

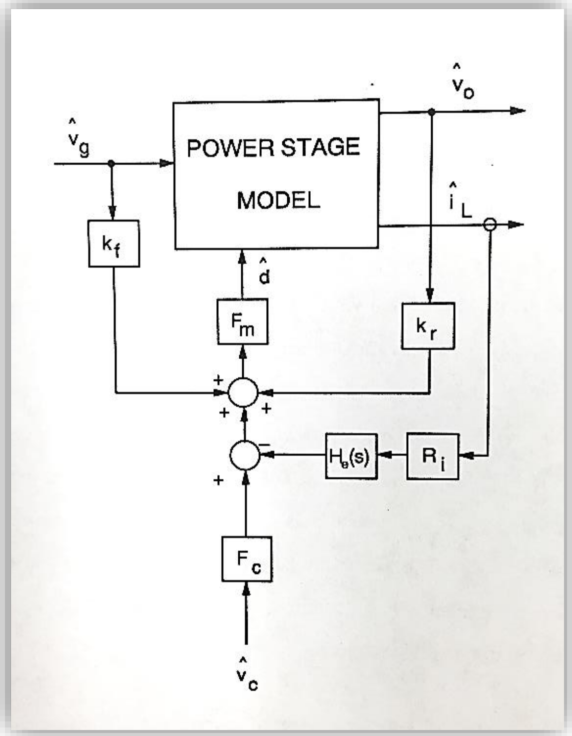
[7]Raymond B. Ridley, "A New Small-Signal Model for Current-Mode Control," Ph.D dissertation, Virginia Tech, November 27, 1990.

The model of the PWM switch (*Cont.*)

3. ***Model of current mode control***[7]:

TABLE 4.3
Feedforward Gains for Constant-Frequency, Trailing-Edge Control

	Buck	Boost	Buck-Boost
k_f	$-\frac{DT_s R_i}{L}\left[1-\frac{D}{2}\right]$	$-\frac{T_s R_i}{2L}$	$-\frac{DT_s R_i}{L}\left[1-\frac{D}{2}\right]$
k_r	$\frac{T_s R_i}{2L}$	$\frac{D'^2 T_s R_i}{2L}$	$\frac{D'^2 T_s R_i}{2L}$

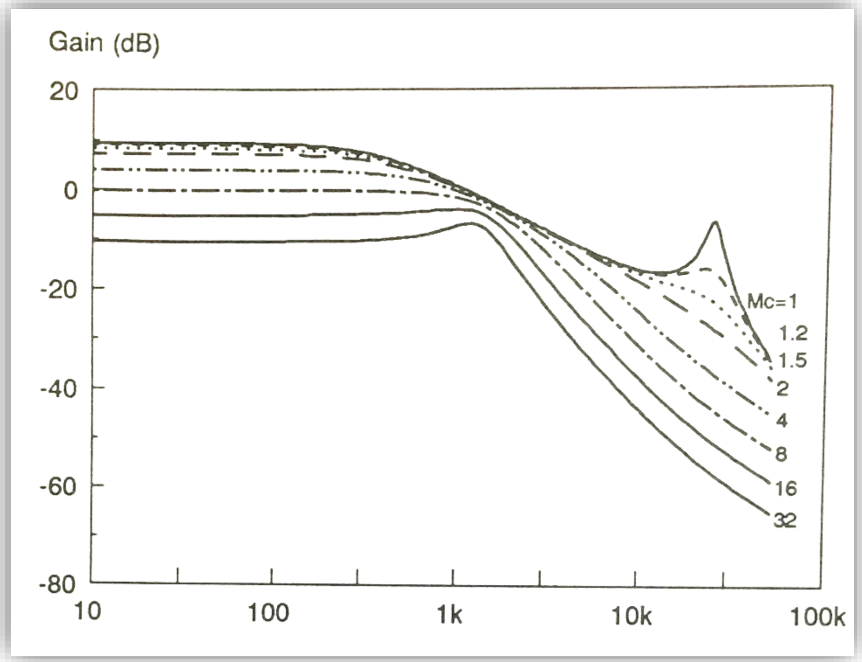


Reference

[7]Raymond B. Ridley, "A New Small-Signal Model for Current-Mode Control," Ph.D dissertation, Virginia Tech, November 27, 1990.

The model of the PWM switch (*Cont.*)

3. ***Model of current mode control***[7]:



The control-to-output transfer function

Shows a resonant peak at half the switching

Frequency whose damping is proportional

The external ramp and duty cycle. This correctly

Predicts the subharmonic oscillation of current

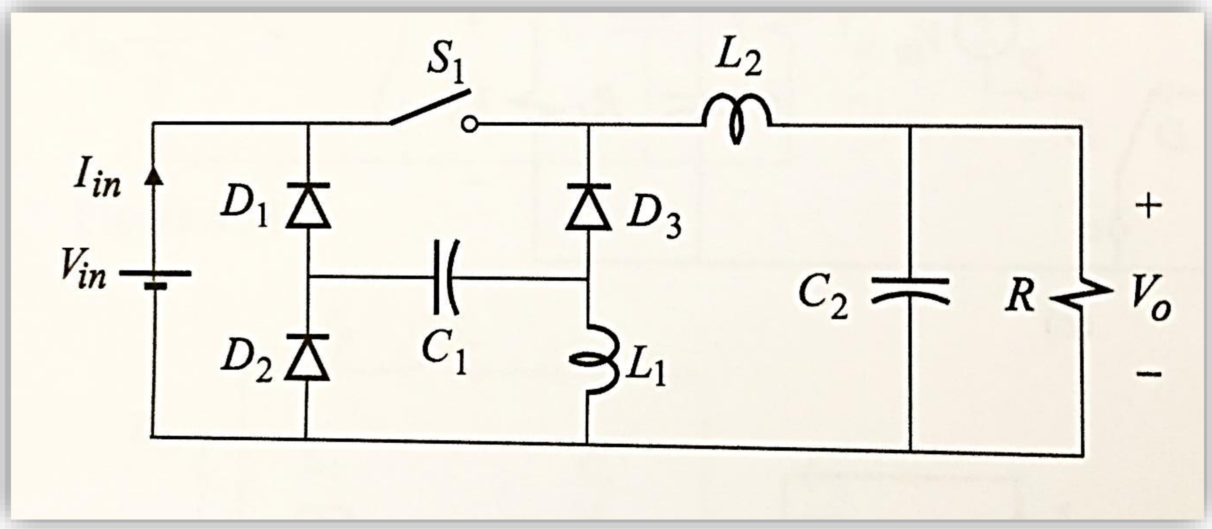
Mode control.

Reference

[7]Raymond B. Ridley, "A New Small-Signal Model for Current-Mode Control," Ph.D dissertation, Virginia Tech, November 27, 1990.

The model of the PWM switch (*Cont.*)

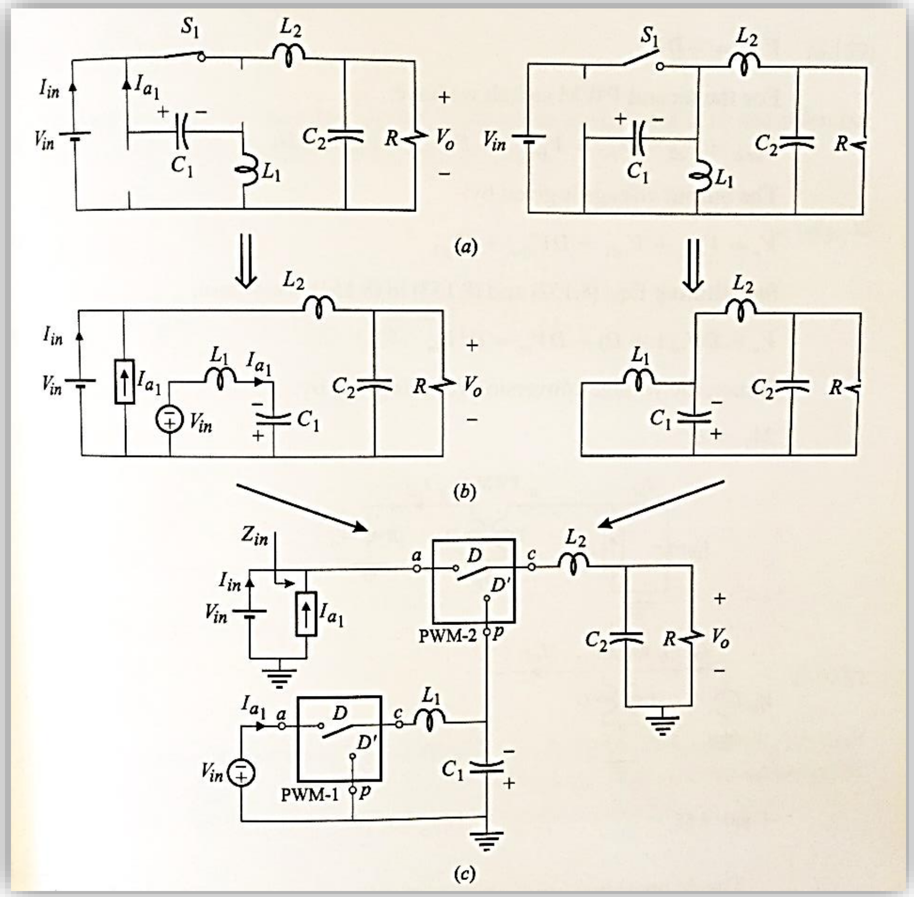
Does the model of the PWM switch work for all topologies?



The model of the PWM switch (*Cont.*)

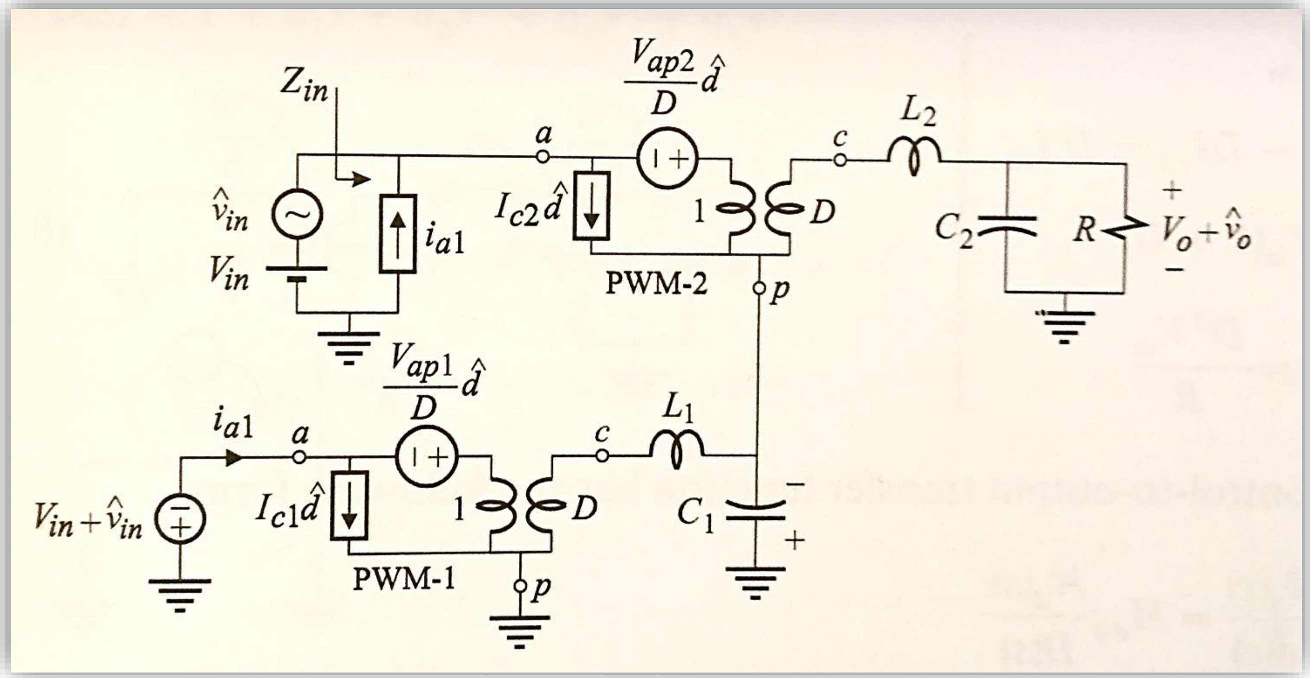
Does the model of the PWM switch work for all topologies?

Examine the two states of the converter and identify two PWM swithces



The model of the PWM switch (*Cont.*)

Does the model of the PWM switch work for all topologies?



Small signal model of the converter