Improvements in GRACE Gravity Field Determination through Stochastic Observation Modeling

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G11C-04

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Introduction

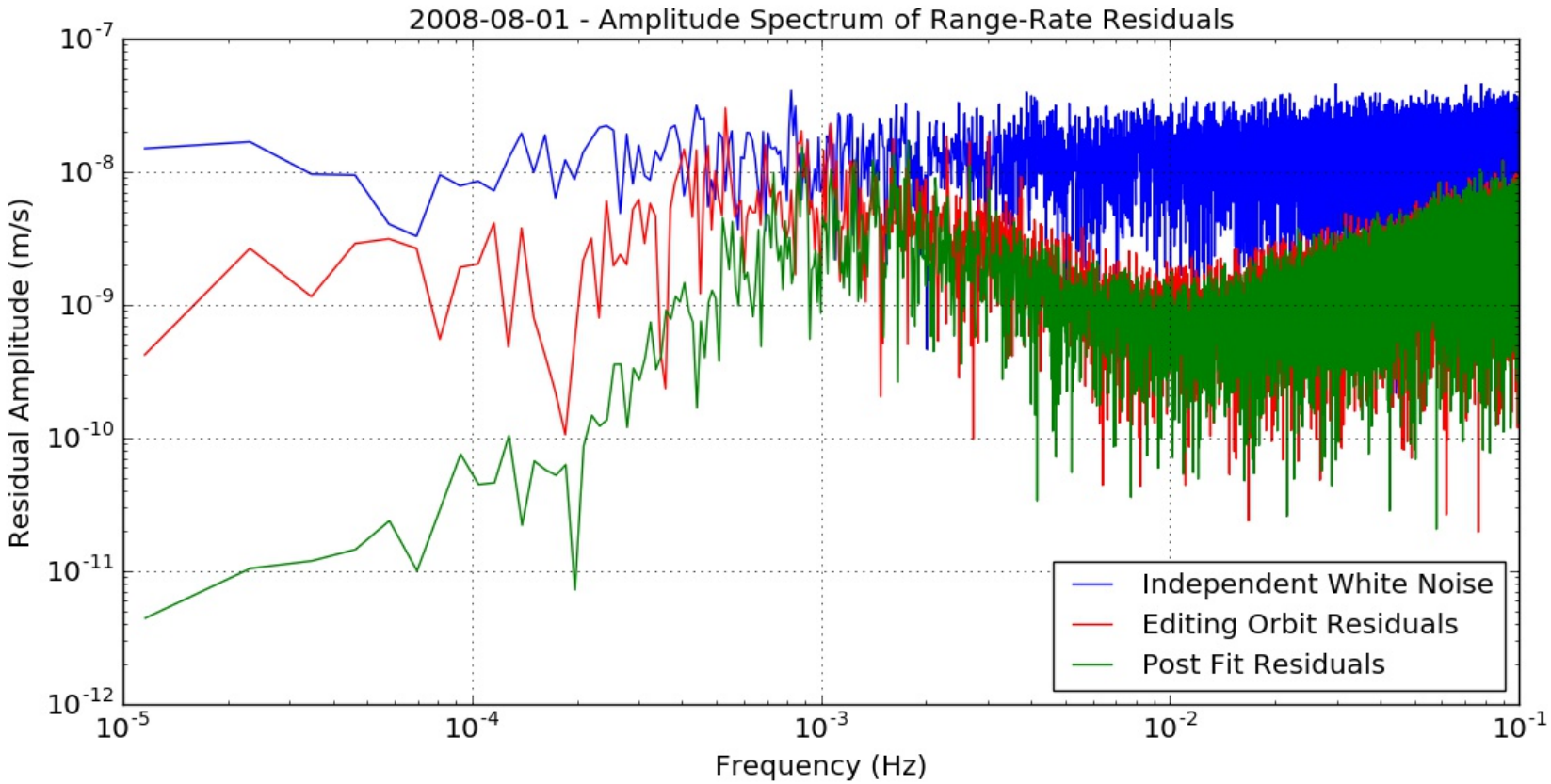
- Gravity Recovery and Climate Experiment (GRACE)
  - Approaching 15 year lifetime
  - Providing unprecedented observations of Earth's time variable gravity
- Unconstrained solutions still contain significant errors – stripes
- Revisit some implicit assumptions in the estimation problem

Nonlinear Estimation Problem

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\epsilon} \qquad E\left[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T\right] = \boldsymbol{R} = \sigma^2 \boldsymbol{I}$$

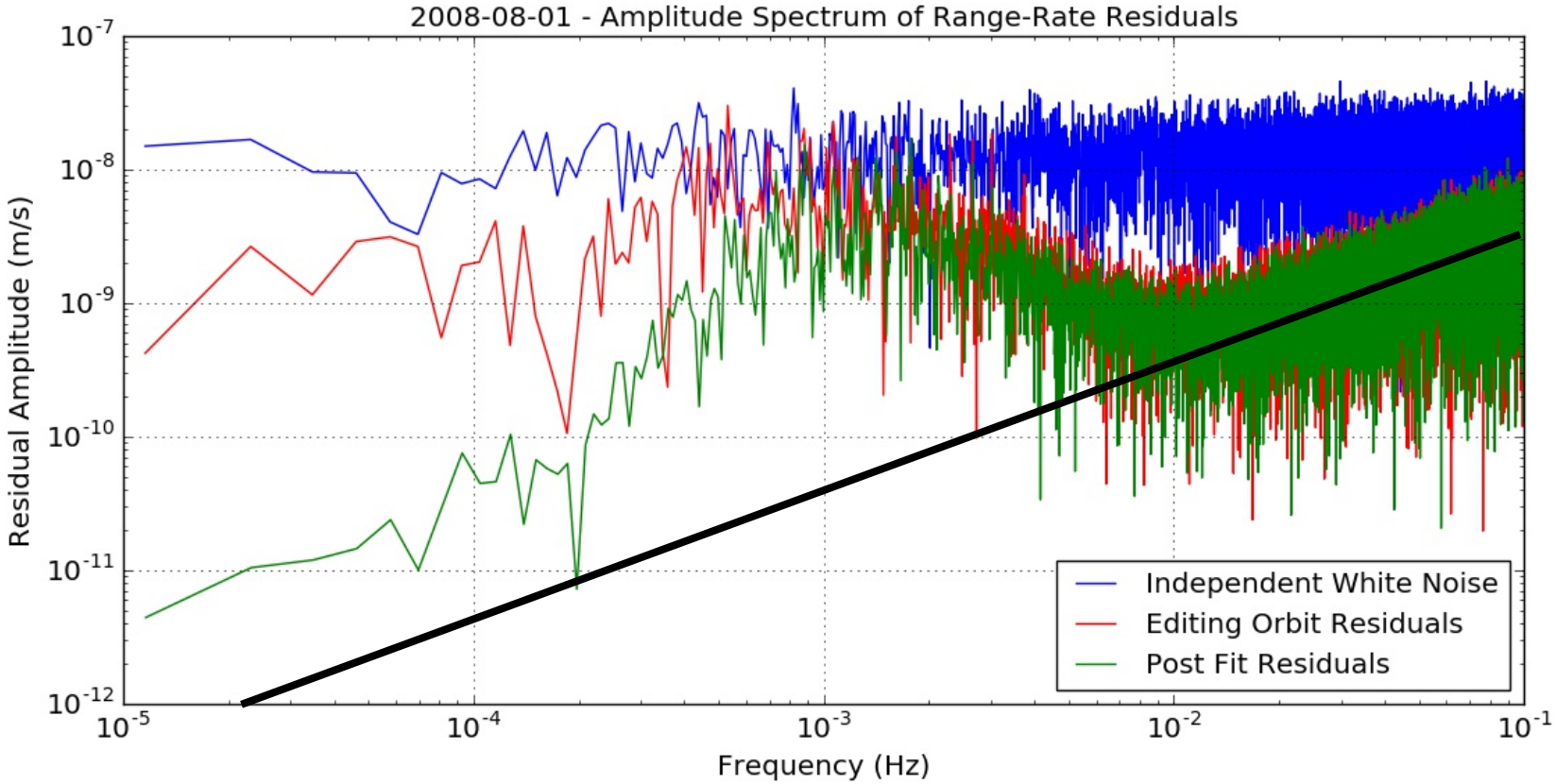
- GRACE Level 2 processing centers assume independent Gaussian noise
- Gravity science is beginning to explore correlated observation error (Ditmar, 2007)
  - GOCE processing (Pail et. al., 2010, 2011)
  - TU Graz – ITSG-GRACE2016 (Mayer-Gürr et. al., 2016)

Correlated Observation Error



GRACE inter-satellite range-rate residuals show substantial correlation – due to imperfections in the force/observation modeling

Correlated Observation Error



f^2 dependence – "blue noise"

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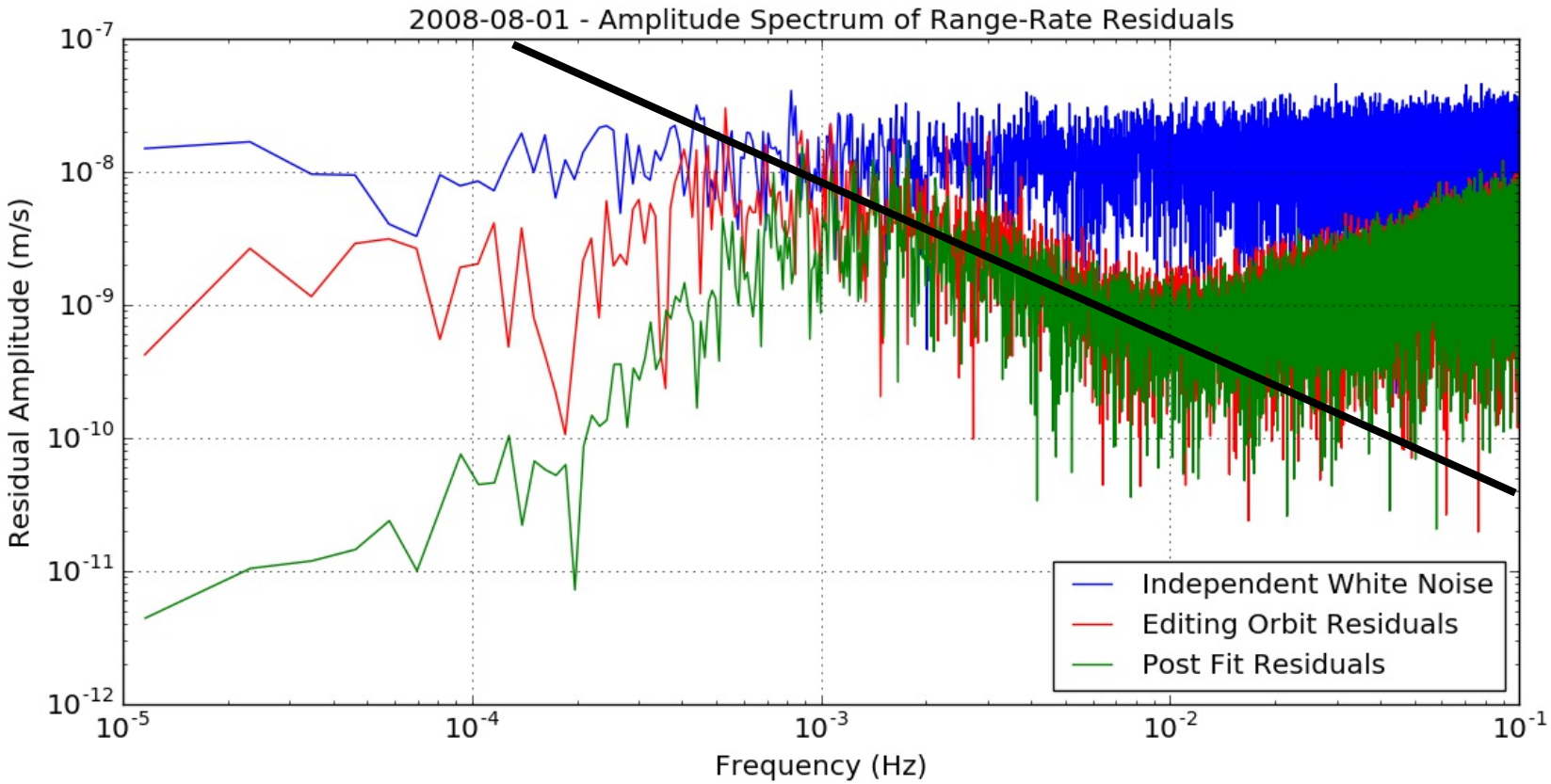
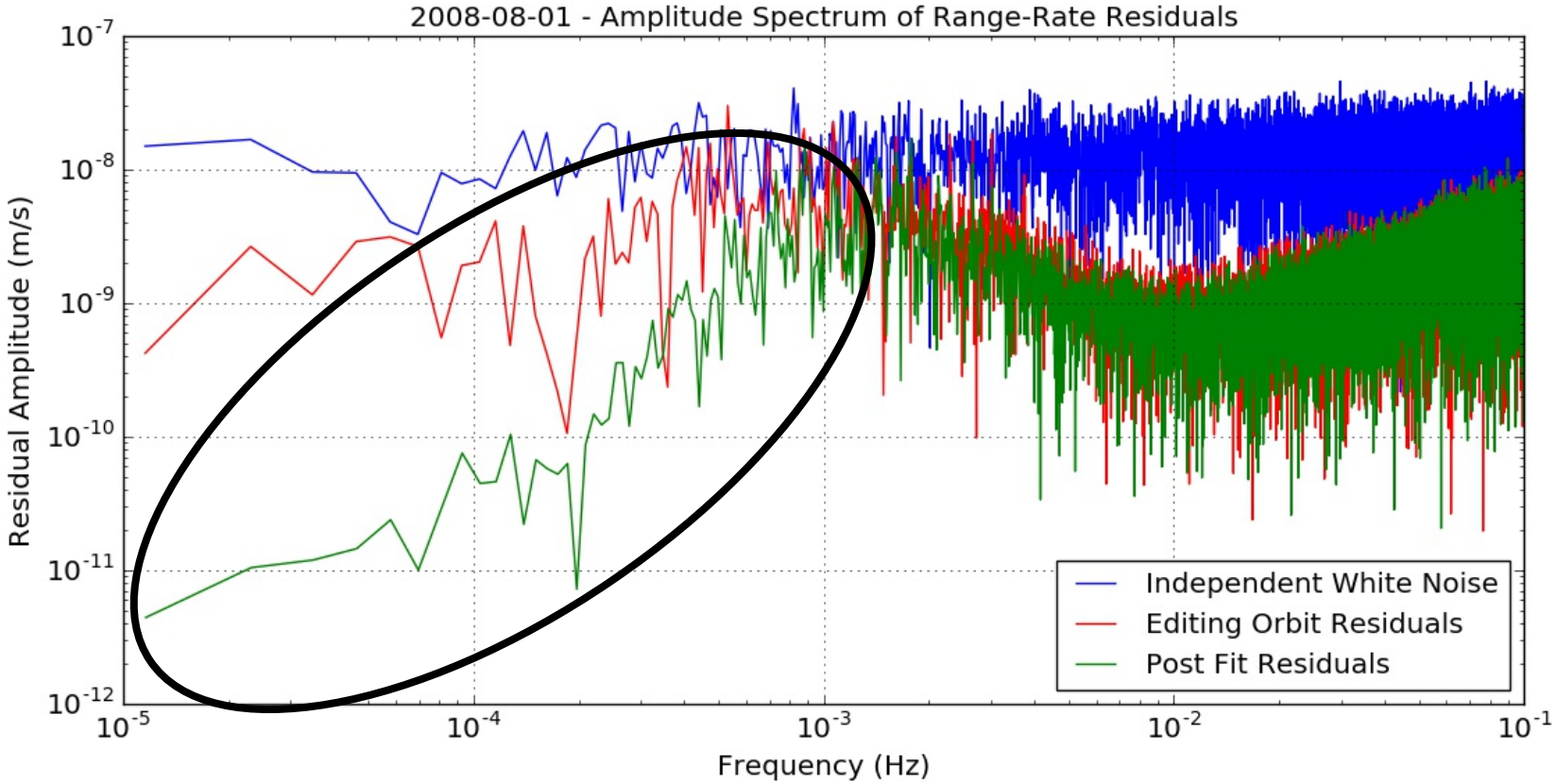
Correlated Observation Error



$1/f^2$ dependence – "red noise"

GRACE inter-satellite range-rate residuals show substantial correlation – due to imperfections in the force/observation modeling

Correlated Observation Error



Low frequency highly dependent on parameterization and modeling deficiencies – digression

GRACE inter-satellite range-rate residuals show substantial correlation – due to imperfections in the force/observation modeling

Digression – Empirical Parameters

$$\boxed{\begin{aligned} \ddot{r} - 2n\dot{\tau} - 3n^2 r &= f_r \\ \ddot{\tau} + 2n\dot{r} &= f_\tau \\ \ddot{\nu} + n^2 \nu &= f_\nu \end{aligned}}$$

$$f_i = P_i \cos \omega t + Q_i \sin \omega t,$$
$$\omega \neq n,$$
$$i = r, \tau, \nu$$

- Hill's equations – describe deviations of linearized motion under of the influence of external perturbative forces
- Harmonic perturbations give rise to motion at distinct frequencies: the perturbing frequency, the orbital period, and long period drift (2 of which are common to all perturbations

$$x_i(t) = A_i + B_i t + C_i t^2 + D_i \cos nt + E_i \sin nt + F_i \cos \omega t + G_i \sin \omega t, \quad i = r, \tau, \nu$$

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Digression – Empirical Parameters

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Digression – Empirical Parameters

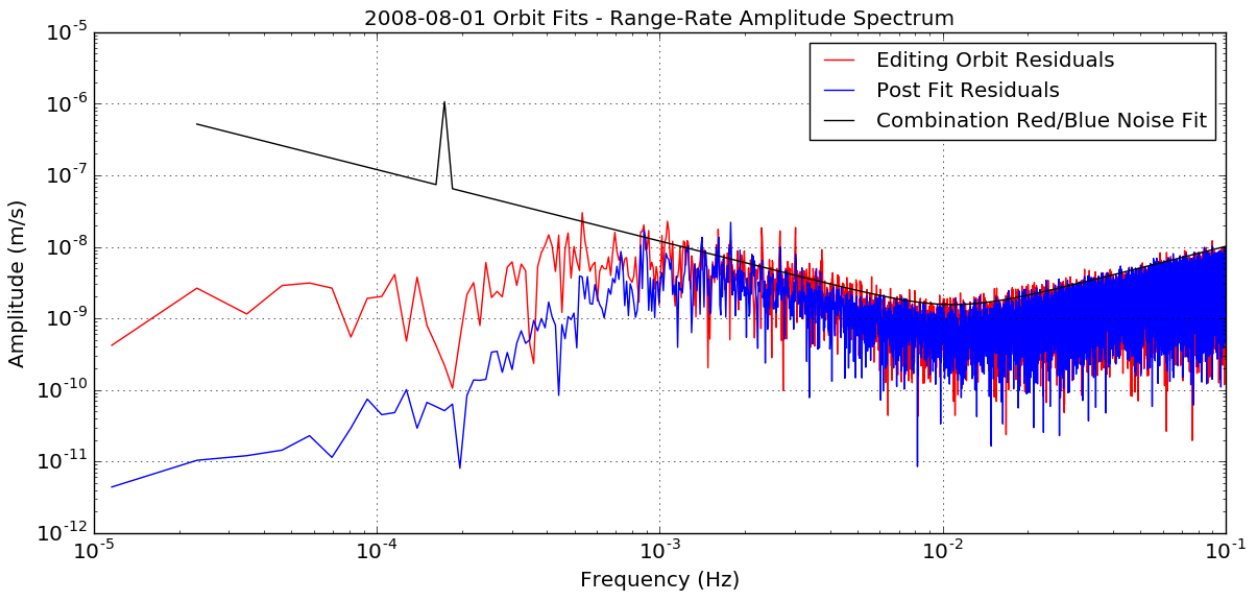
- This application of Hill's equations has been generalized to inter-satellite range-rate for applications related to GRACE (Kim, 2000)

$$\delta\dot{\rho}\left(t\right) = A + Bt + Ct^2 + \left(D + Et\right)\cos nt + \left(F + Gt\right)\sin nt$$

- Typically some subset of the constant multipliers are estimated to account for deficiencies in the underlying force/observation models
- Alternatively, the observation covariance may be used to stochastically model errors of this form

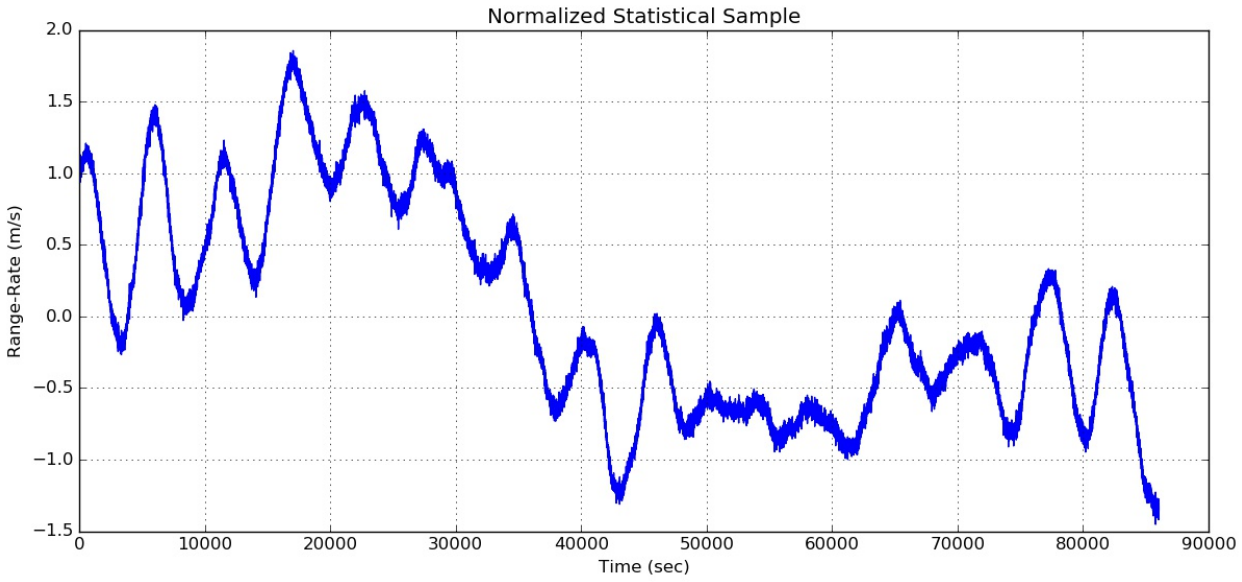
Modeling the Covariance from a Spectrum

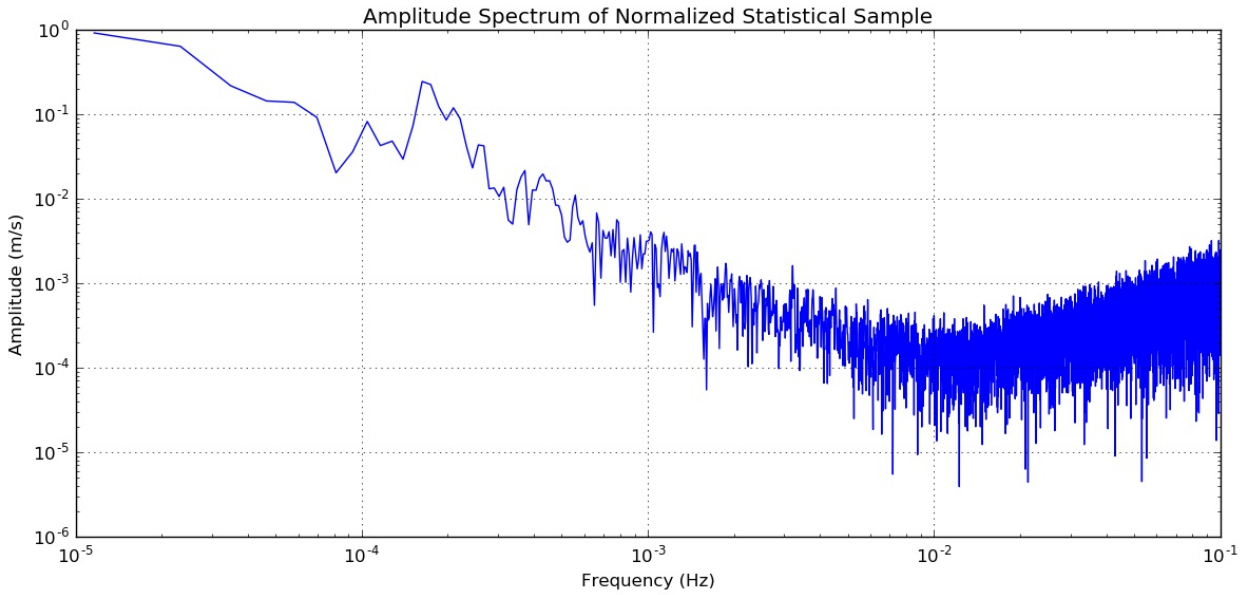
$$\sigma^2 \begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{N-1} \\ \rho_1 & 1 & \cdots & \rho_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N-1} & \rho_{N-2} & \cdots & 1 \end{bmatrix}$$



- A discrete, stationary process may be described by an NxN covariance matrix and is defined by N parameters – 1 variance and N-1 correlations
- Given a spectrum/time series, the variance and autocorrelation function is easily computed – fully defining the covariance matrix – the example shown is for August 1, 2008

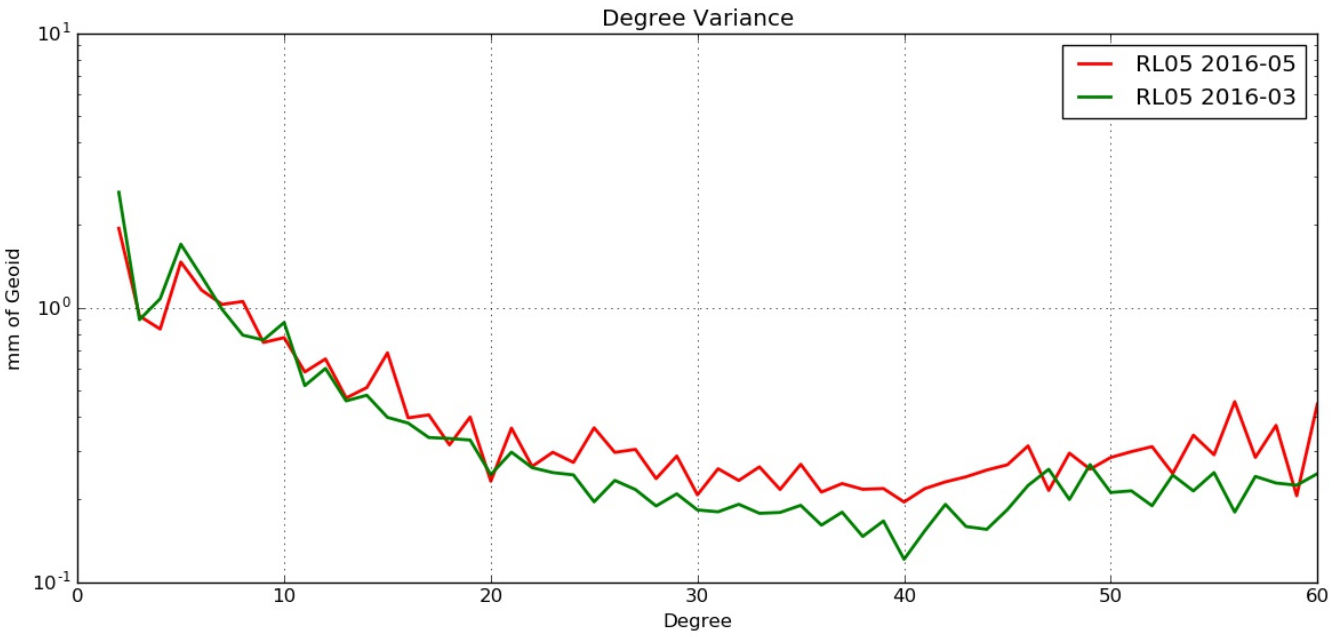
Sampling from the Covariance

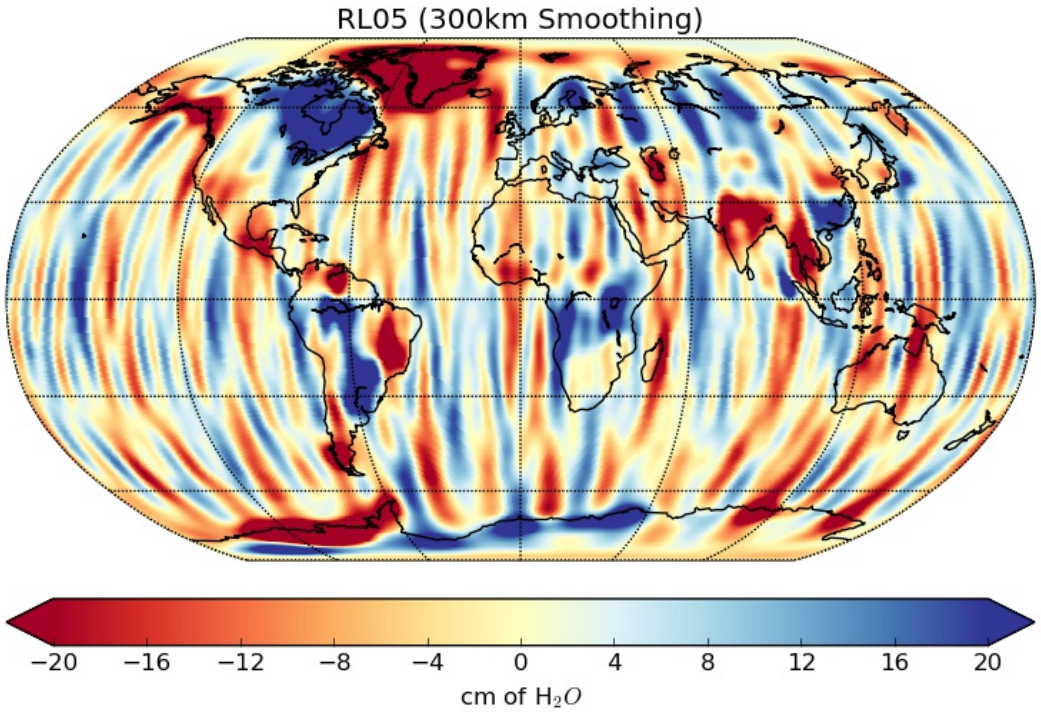




- Sampling from the modeled covariance yields the time series and spectrum shown, matching the desired characteristics of the input spectral model – the example shown is from August 1, 2008

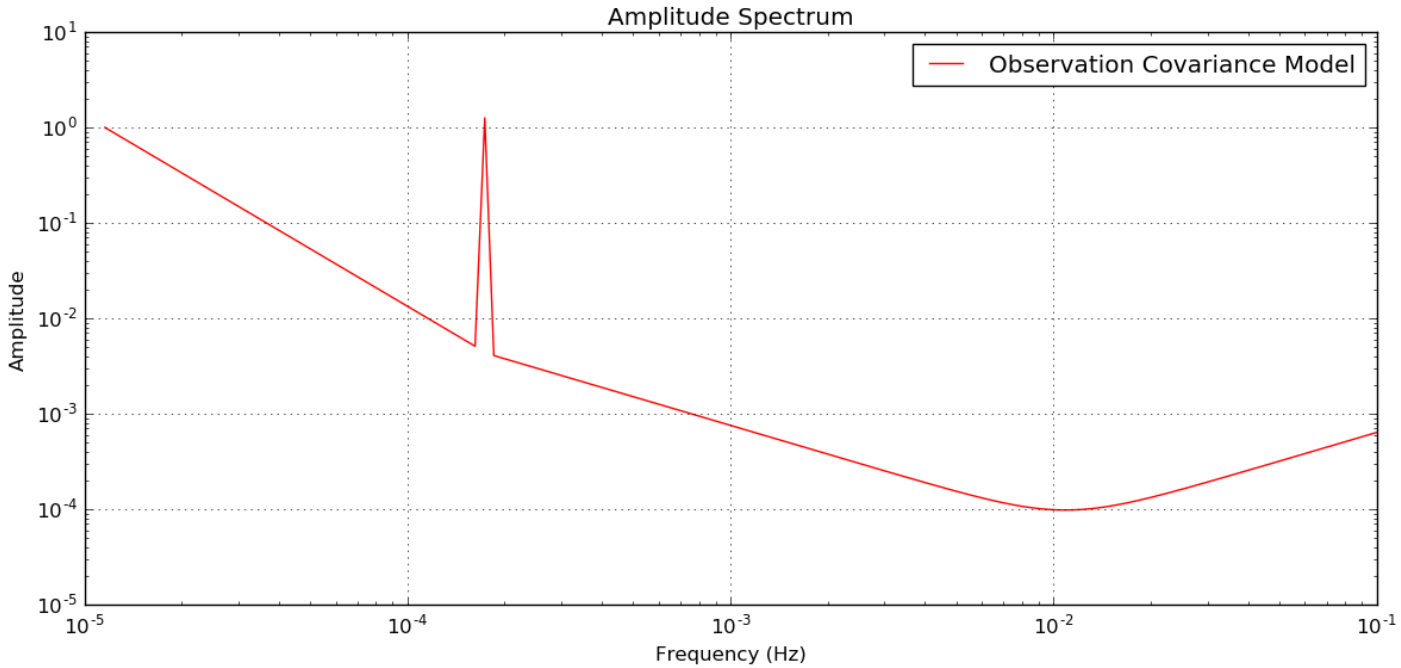
Monthly Solution – May 2016





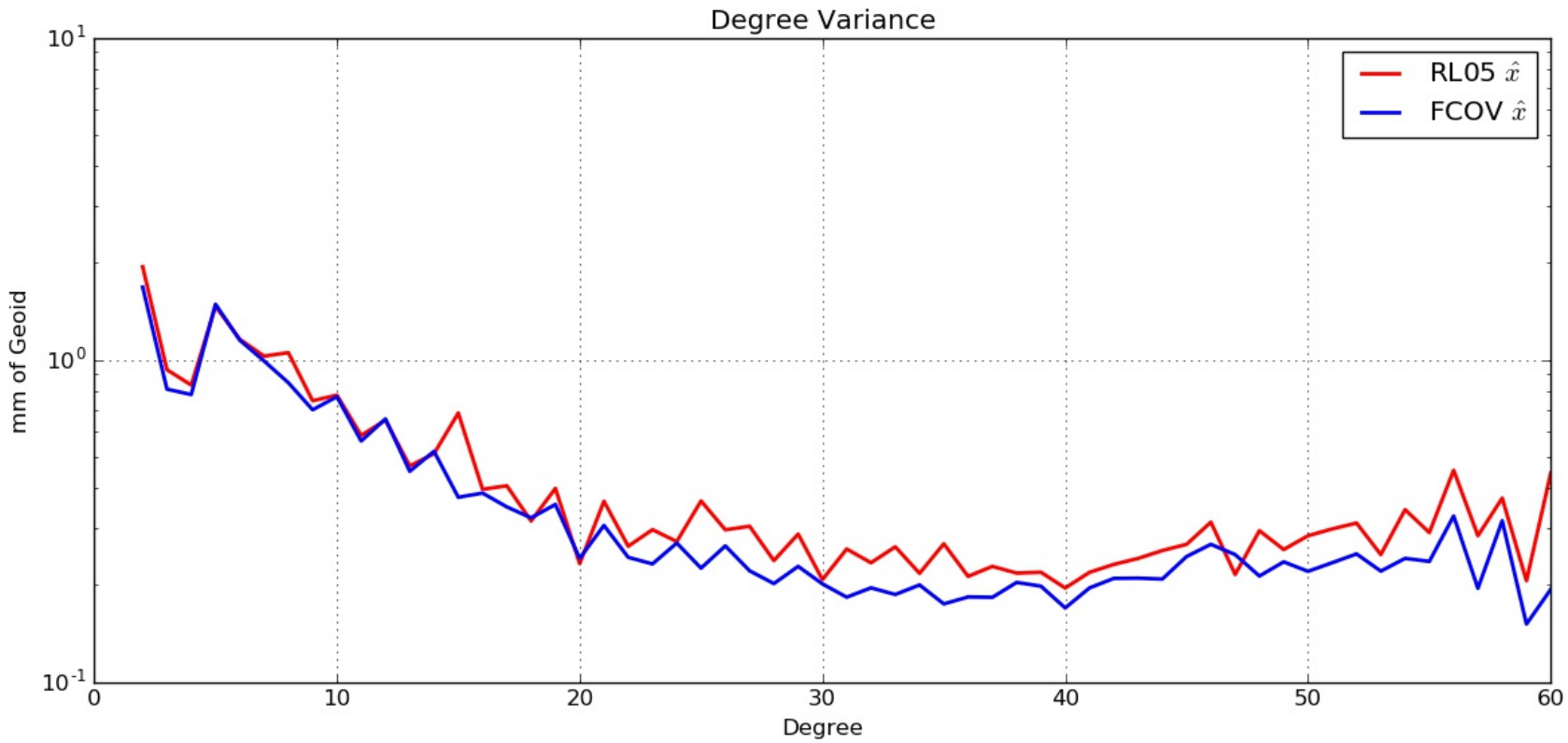
- CSR RL05 solutions (mean removed) – comparison of recent months
- May solution displays abnormally large errors – resonant degrees and large stripes

Monthly Solution – May 2016



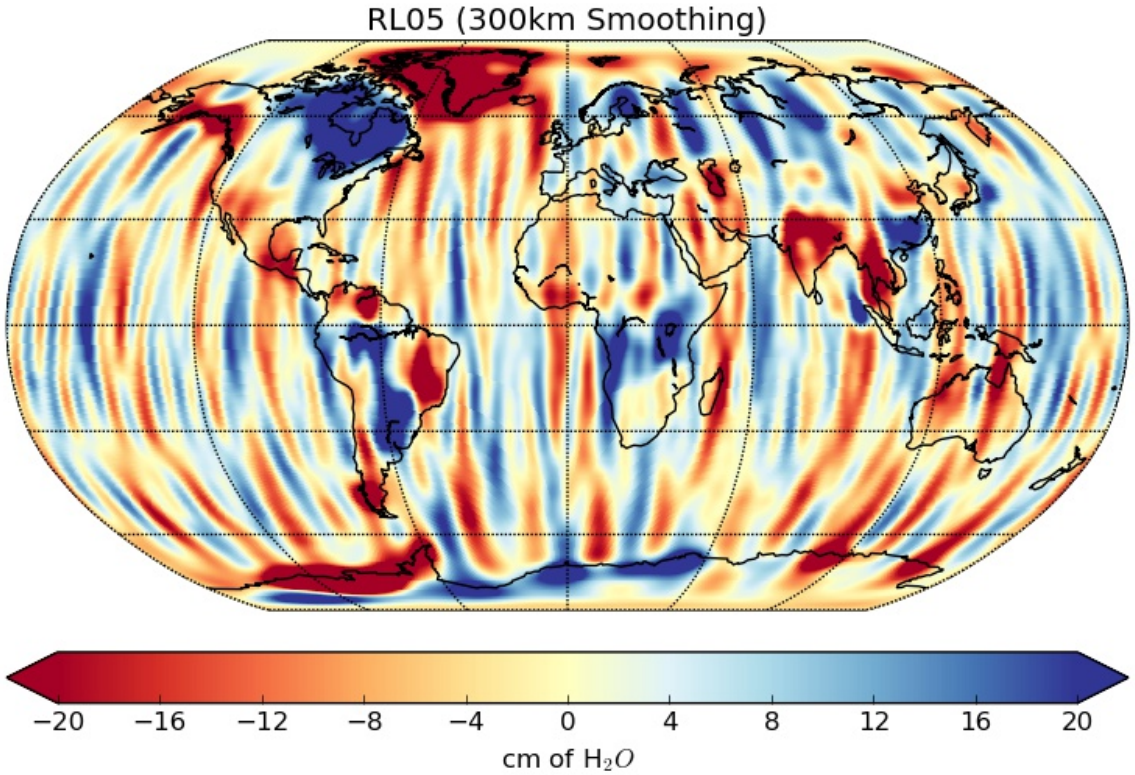
- Large errors at resonant degrees hint at large uncertainty at low frequency and 1 cpr
- Hence, the observation covariance is modeled as shown with a large peak at 1cpr and $1/f^4$ slope in the low frequency

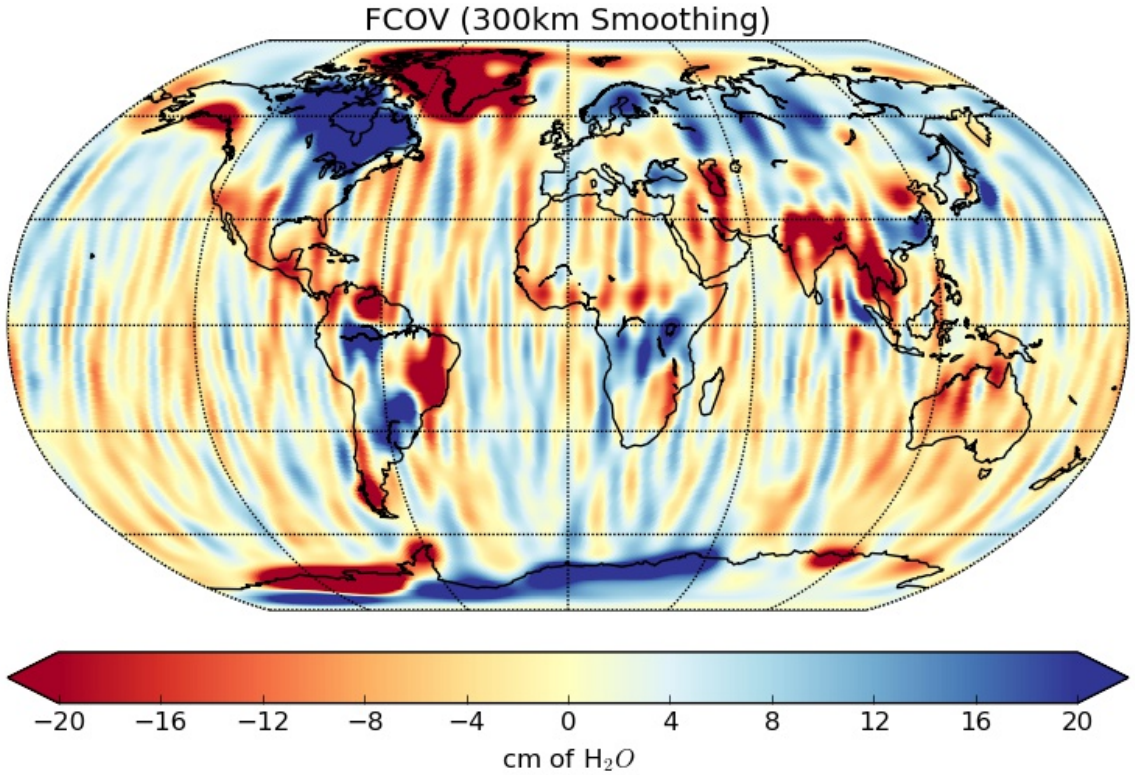
Monthly Solution – May 2016



- FCOV denotes the use of the observation covariance model
- The improvement is apparent at all degrees

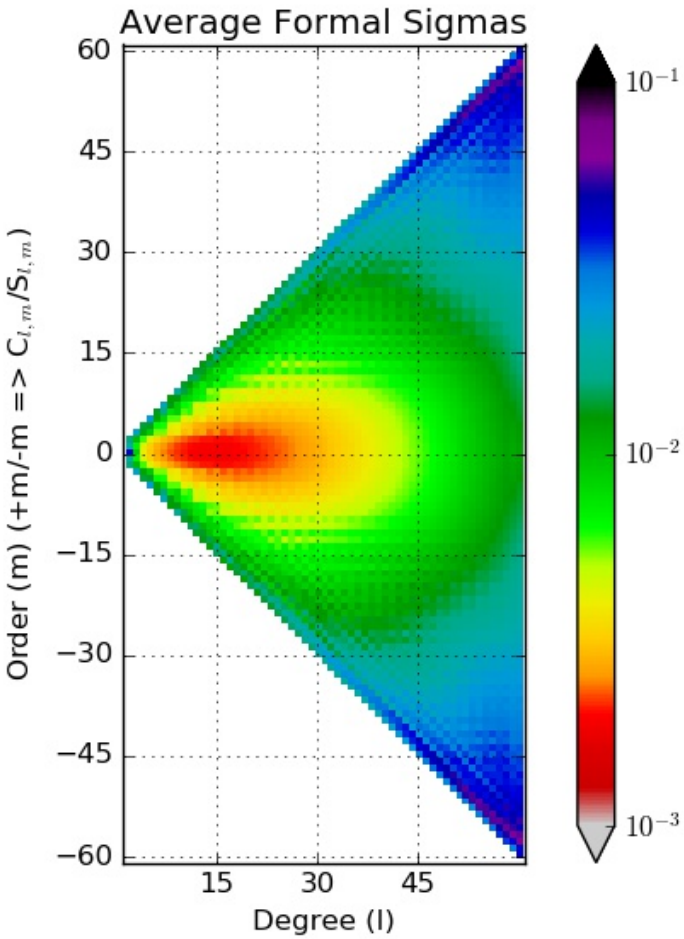
Monthly Solution – May 2016



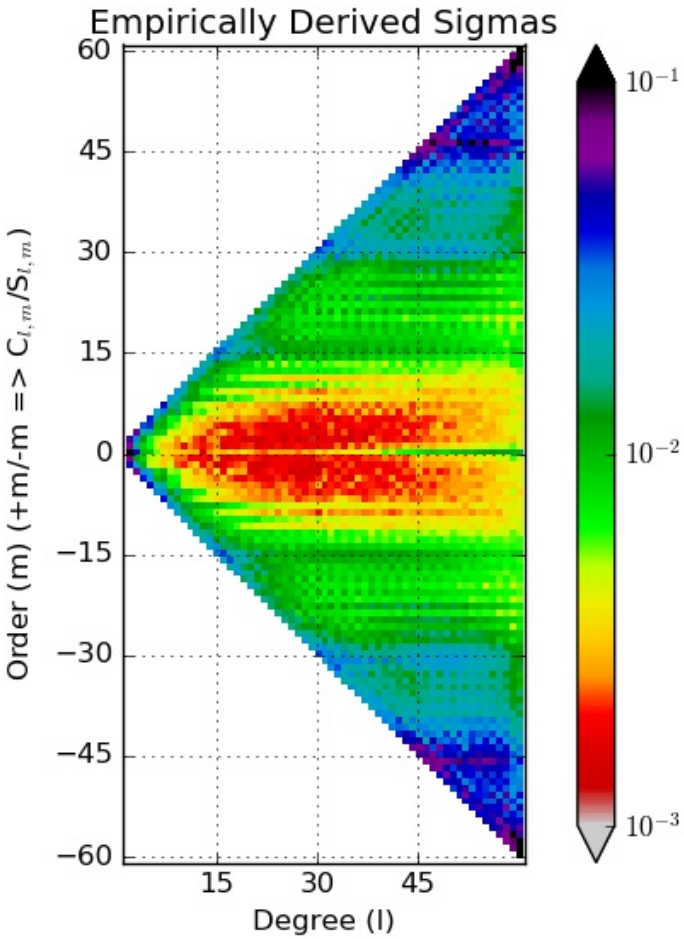


- FCOV denotes the use of the observation covariance model
- Stripes have been greatly reduced while preserving geophysical signal

Formal Error Statistics



RL05 – Formal Error

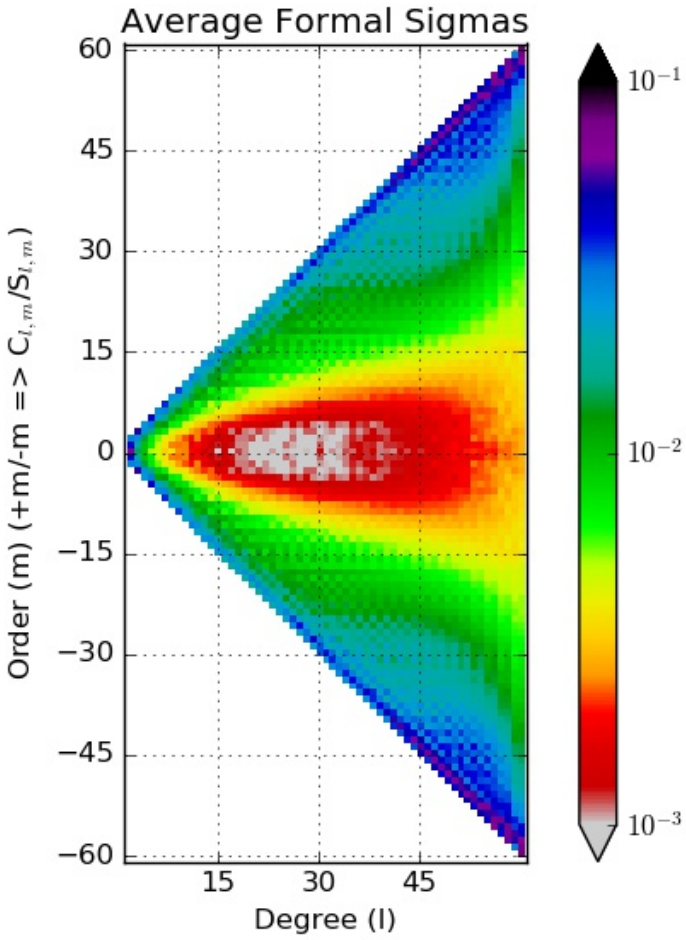


Empirical Error Estimates

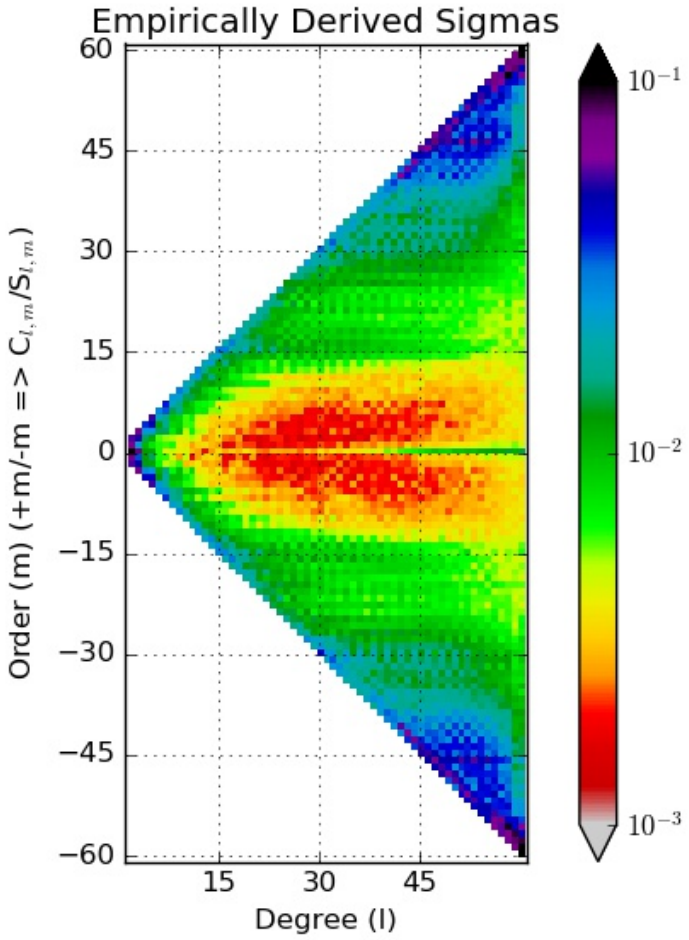
- Figures show diagonal values of the formal CSR RL05 error covariance (left) and empirical estimates of GRACE spherical harmonic error (right – derived via comparison with mascon solutions)
- Time span 2004-2010
- Formal errors are not representative of GRACE error characteristics

Formal Error Statistics

- Figures show diagonal values of the formal error when using the full observation covariance (left) and empirical estimates of GRACE spherical harmonic error (right – derived via comparison with mascon solutions)
- Time Span 2004-2010
- Modeling of low frequency error improves agreement of formal error characteristics with empirical derivations



Full Observation Covariance – Formal Error



Empirical Error Estimates

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Summary

- The observation error covariance may be used to model random measurement noise and systematic effects caused by deficiencies in the underlying force models

- Using a spectral model, more representative observation covariance matrices may be derived

- Utilizing these in the estimation problem, improves estimates of gravity field and drastically improves formal error characterization

Thank you!!

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References

1. Ditmar, P., Klees, R., and Liu, X., “Frequency-Dependent Data Weighting in Global Gravity Field Modeling from Satellite Data Contaminated by Non-Stationary Noise,” *Journal of Geodesy*, Vol. 81, No. 1, 2007, pp. 81-96, doi: 10.1007/s00190-006-0074-4.
2. Pail, R., et. al., “Combined Satellite Gravity Field Model GOCO01S Derived from GOCE and GRACE,” *Geophysical Research Letters,* Vol. 37, No. L20314, 2010, doi: 10.1029/2010GL044906.
3. Pail, R., et. al., “First GOCE Gravity Field Models Derived by Three Different Approaches,” *Journal of Geodesy*, Vol. 85, November 2011, pp. 819-843, doi: 10.1007/s00190-011-0467-x.
4. Mayer-Gürr, T., et. al., ITSG-GRACE2016 – Monthly and Daily Gravity Field Solutions from GRACE. GFZ Data Services. http:/doi.org/10.5880/icgem.2016.007.
5. Kim, J. R., *A Low-Low Satellite-to-Satellite Tracking Mission*, Ph.D. Thesis, The University of Texas at Austin, Austin, TX, May 2000.

Back-up

FFT Based Covariance Modeling

Discrete Fourier Transforms

$$X_k = \sum_{n=0}^{N-1} x_n e^{\frac{-2\pi jkn}{N}} \qquad x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{2\pi jkn}{N}}$$

Variance and Correlation in Terms of Fourier Coefficients

$$\sigma^2 = \frac{1}{N-1}\frac{1}{N}\sum_{k=0}^{N-1} X_k \bar{X}_k \qquad \rho_\tau \sigma^2 = \frac{1}{N-1}\frac{1}{N}\sum_{k=0}^{N-1} X_k \bar{X}_k e^{\frac{2\pi jk\tau}{N}}$$

Resulting Covariance is Circulant

$$\sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_3 & \rho_2 & \rho_1 \\ \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 & \rho_2 \\ \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_2 & \rho_3 & \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_1 & \rho_2 & \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}$$

Time Series Based Covariance Modeling

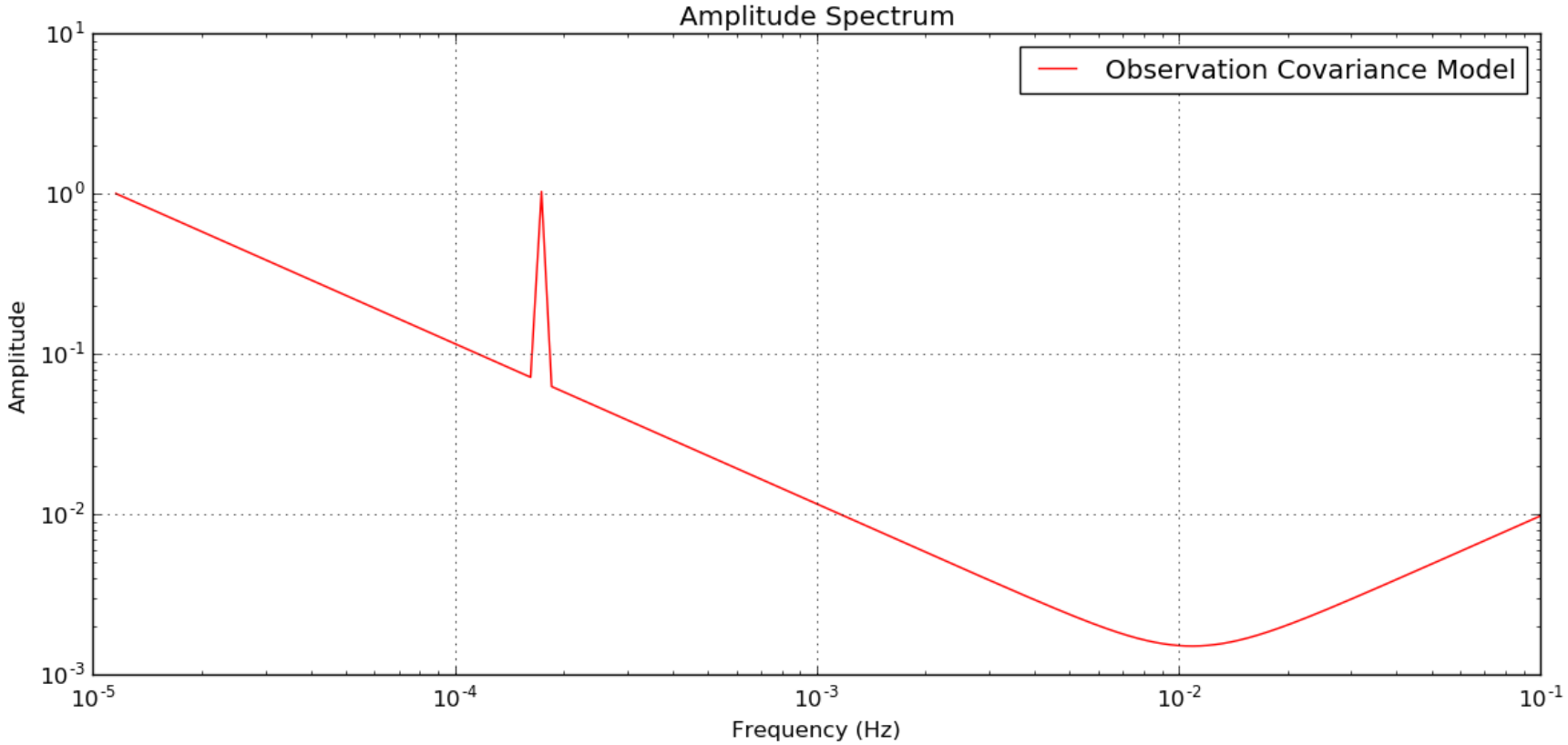
Variance and Correlation in Terms of the Discrete Time Series

$$\sigma^2 = \frac{1}{N-1}\sum_{n=0}^{N-1} x_n \bar{x}_n \qquad \rho_\tau \sigma^2 = \frac{1}{N-1}\sum_{n=0}^{N-1-\tau} x_n \bar{x}_{n+\tau}$$

Correlations in the Covariance Matrix Decay Naturally with Time

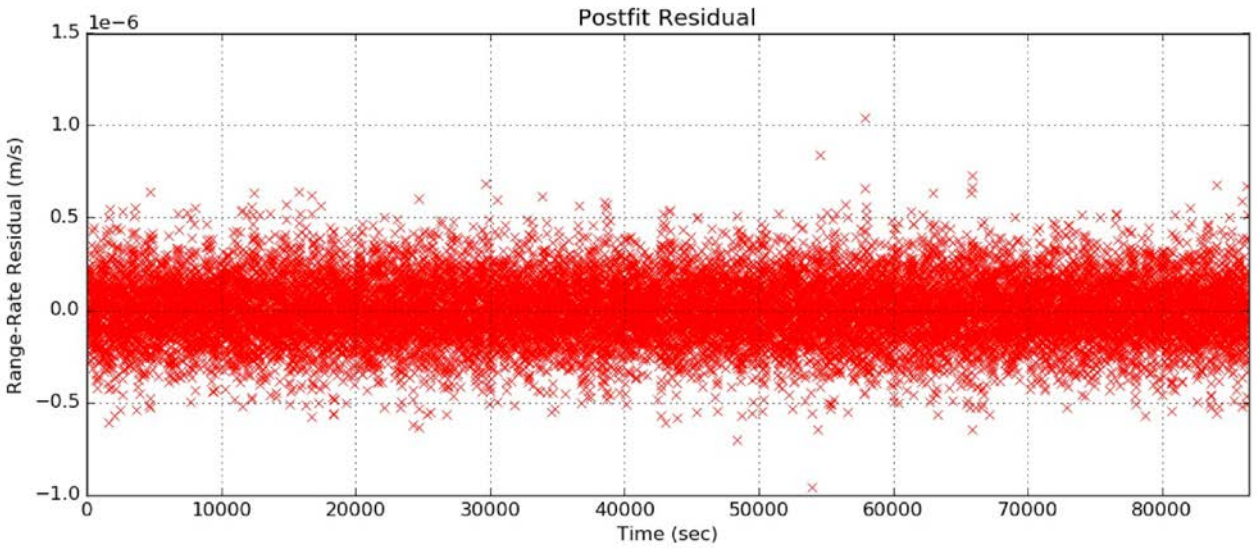
$$\sigma^2 \begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{N-1} \\ \rho_1 & 1 & \cdots & \rho_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N-1} & \rho_{N-2} & \cdots & 1 \end{bmatrix}$$

August 2008 – Spectral Model



Spectral model for August 2008 differs from May 2016 – reduced 1 cpr spike and slope is 1/f2 in the low frequency

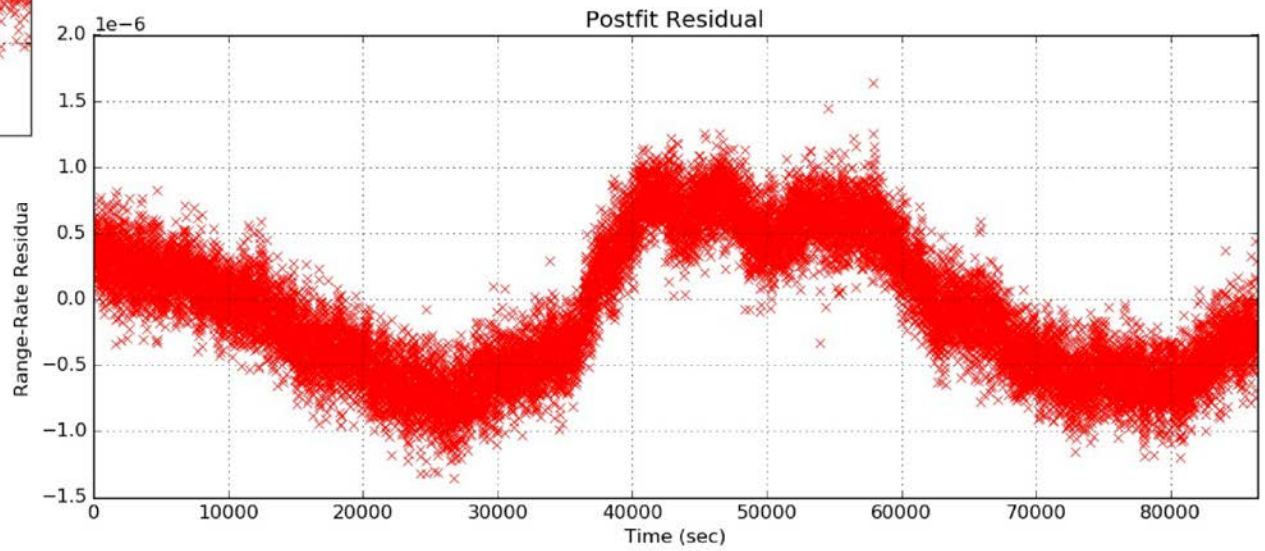
Post-Fit Residual Comparison – August 2008



RL05

Full observation covariance (FCOV) post-fits show measurement noise combined with some dynamical/observation model error.

FCOV



RL05 post-fits show primarily instrument noise – the dynamical/observation model error is absorbed by the estimated parameters (empiricals, dynamical, and gravity field).

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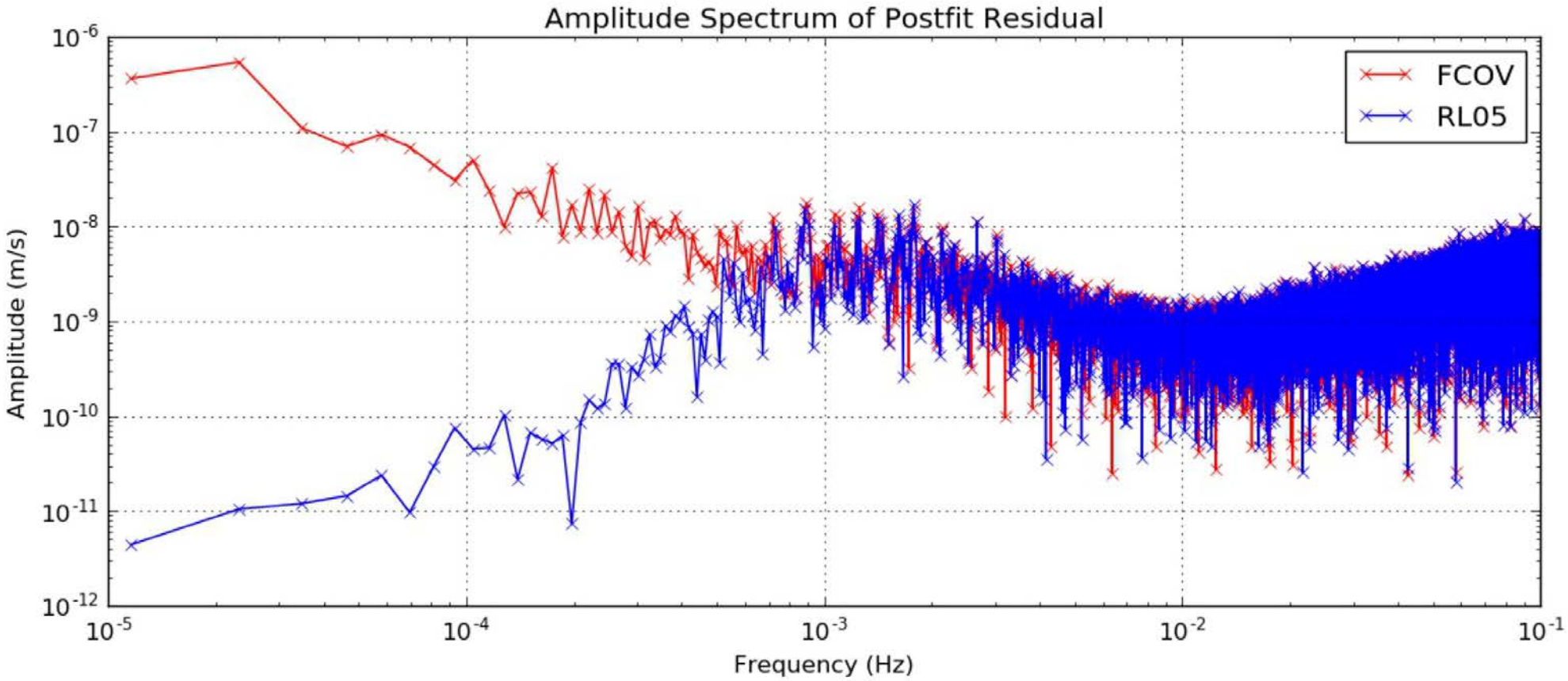
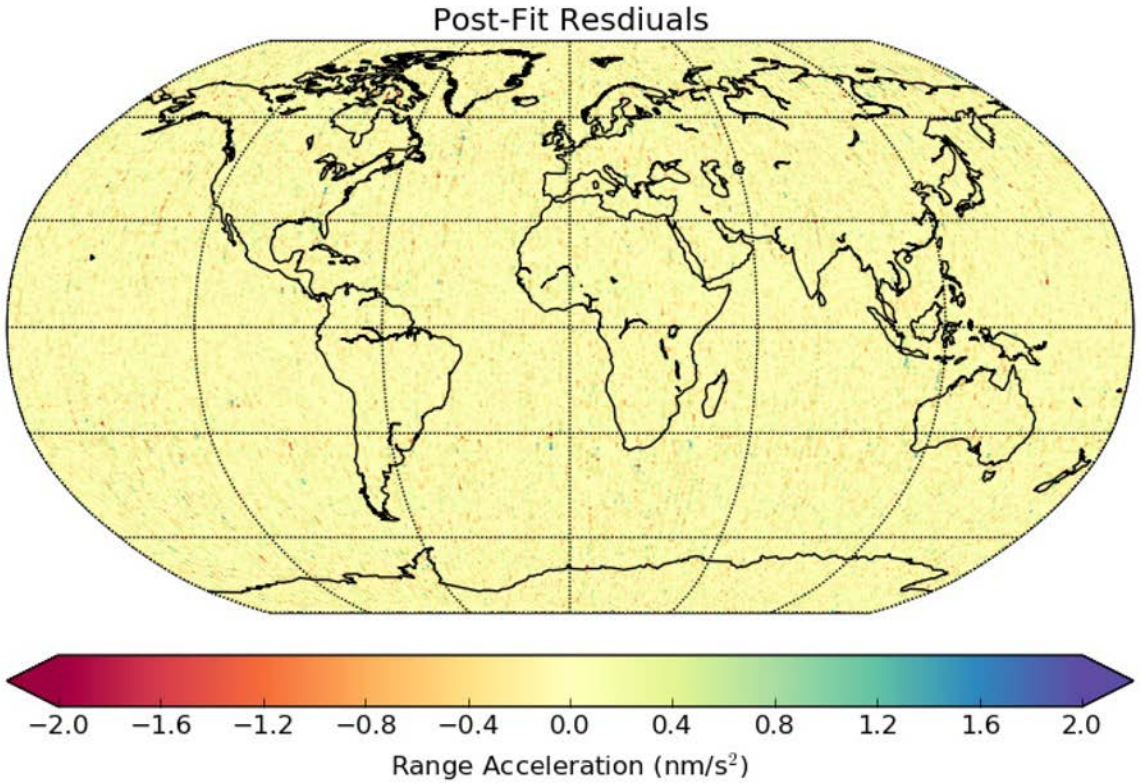


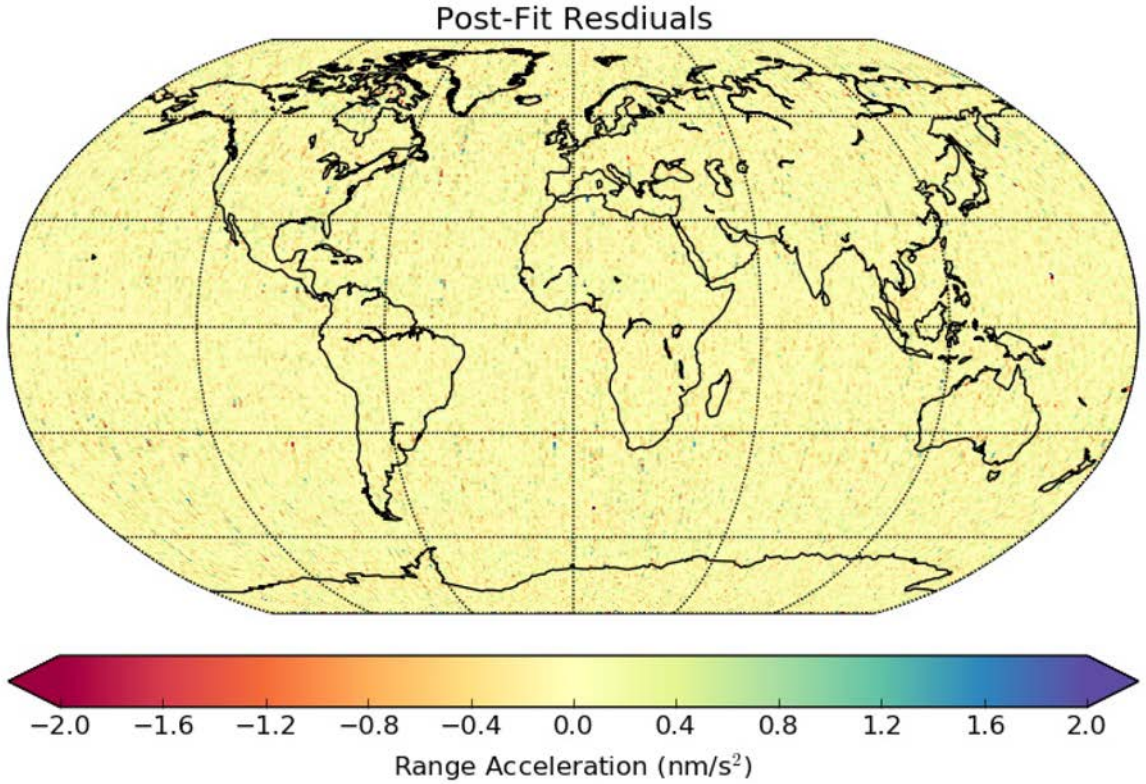
Post-fits are roughly equivalent in the frequency band of interest

Post-Fit Residual Maps – August 2008

RL05



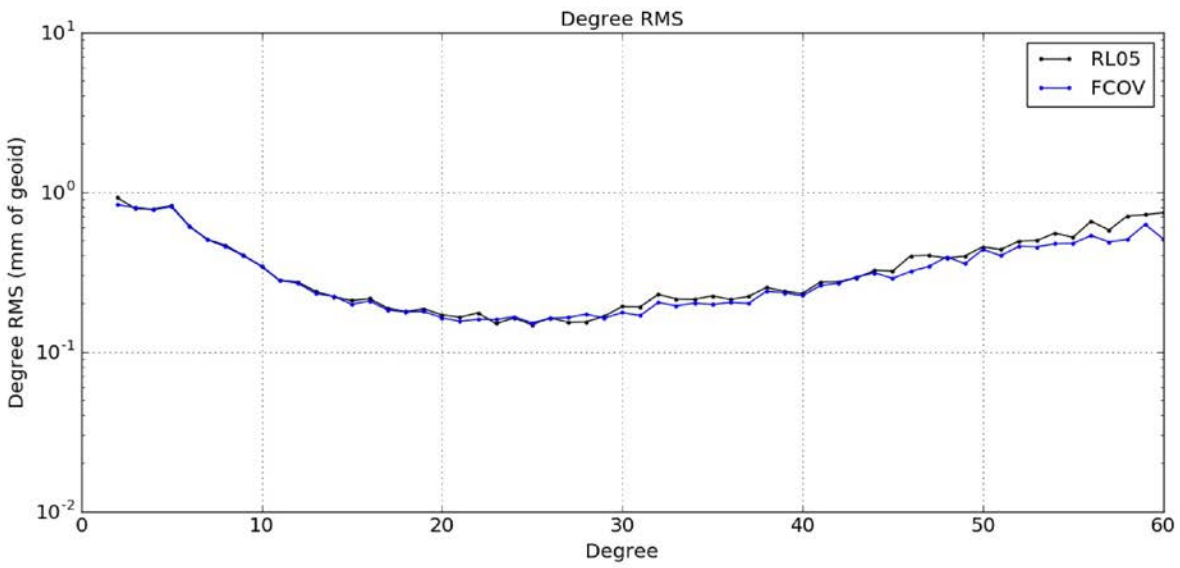
FCOV



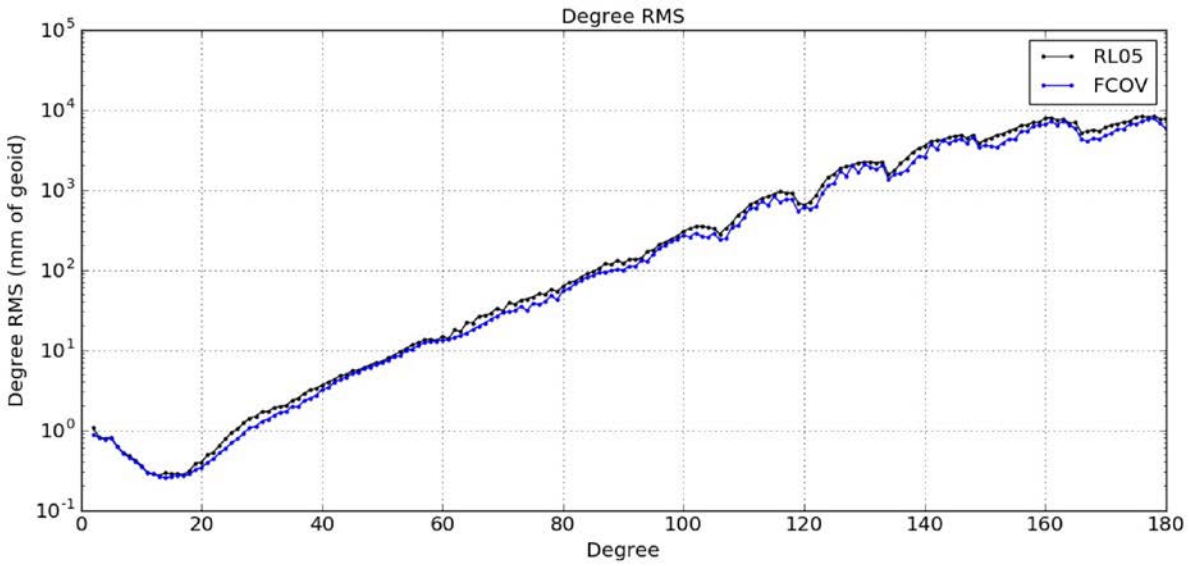
No significant geographically located signal in the post-fit – i.e. the observation covariance does not hinder gravity field signal capture

Degree Variance Comparisons – 2002-2003

60x60



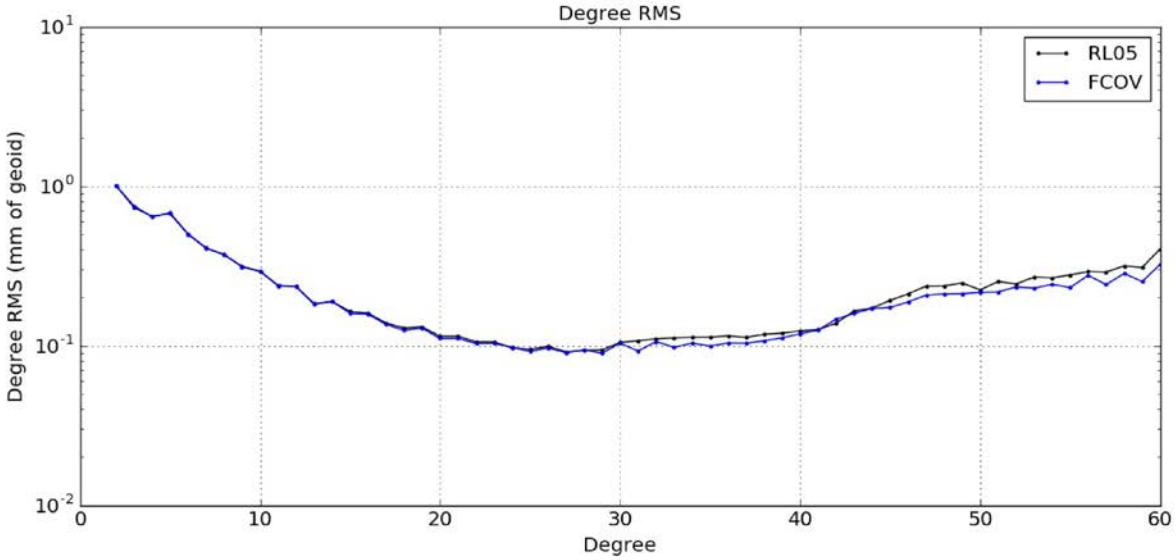
180x180



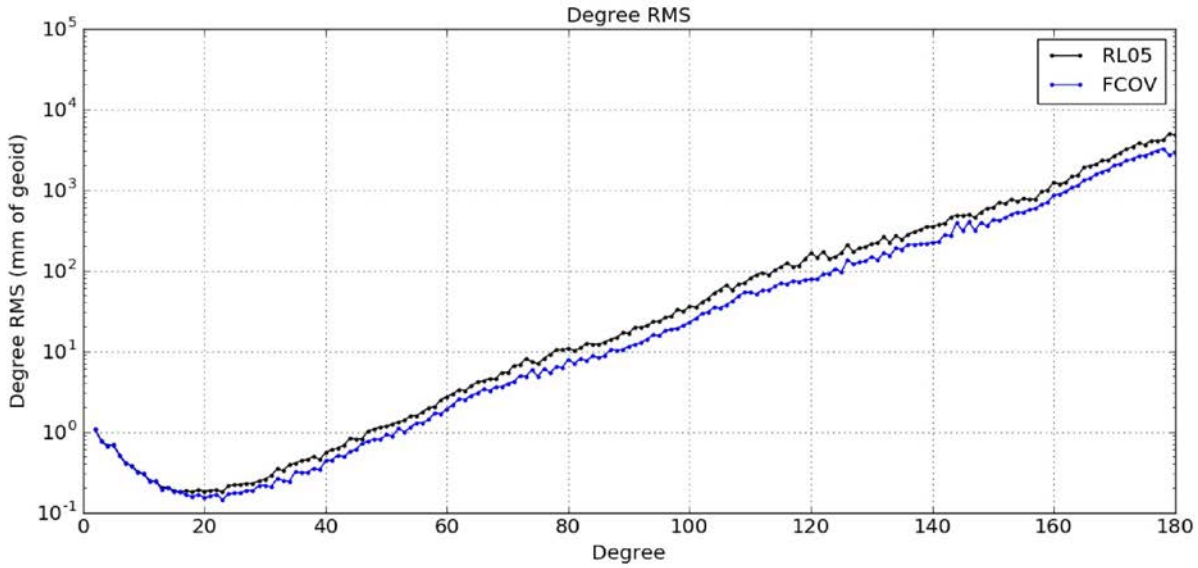
- Power in the low degrees – dominated by signal – are equivalent, while higher degrees – dominated by noise – have been reduced
- Note: To facilitate analysis, the observation covariance is modeled using the August 2008 model – rendering the solutions sub-optimal

Degree Variance Comparisons – 2004-2010

60x60



180x180



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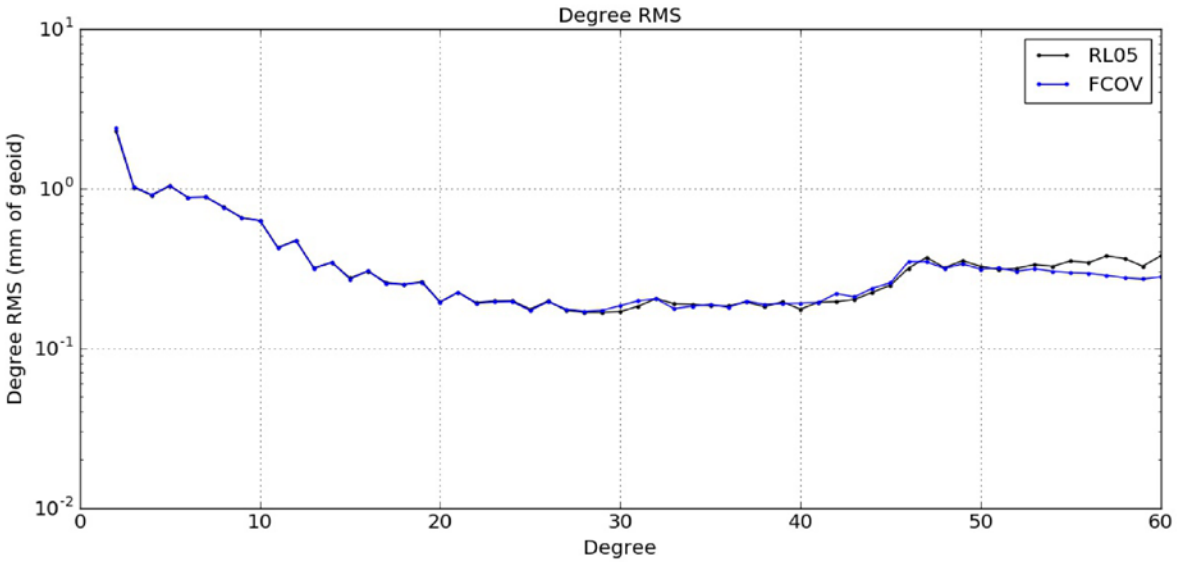
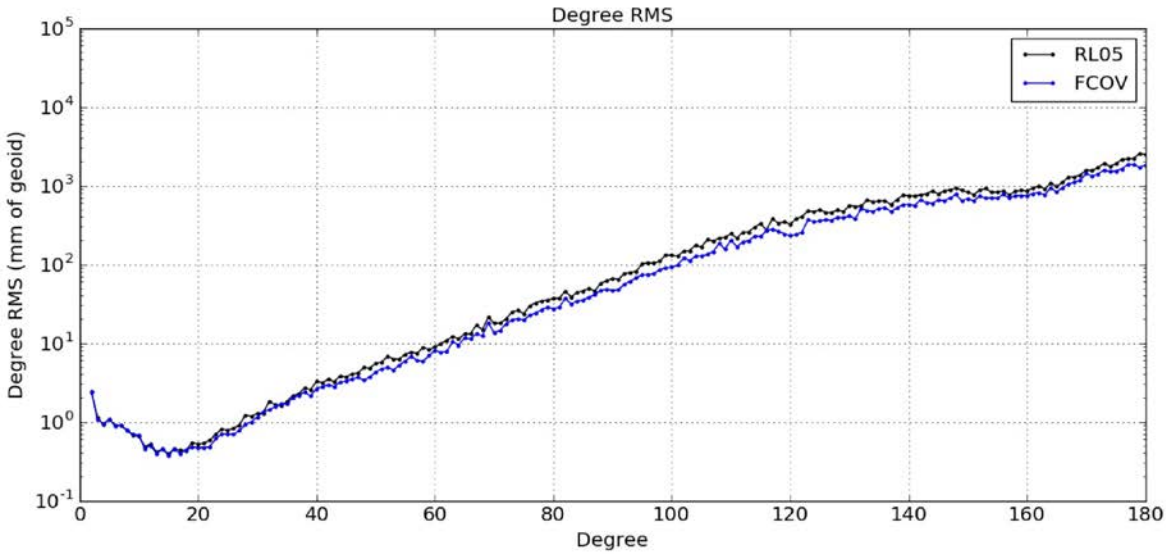
Degree Variance Comparisons – 2011-2016

60x60



180x180



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