Introduction to Radiative Transfer

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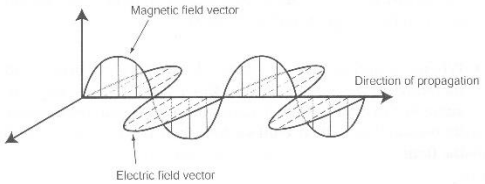
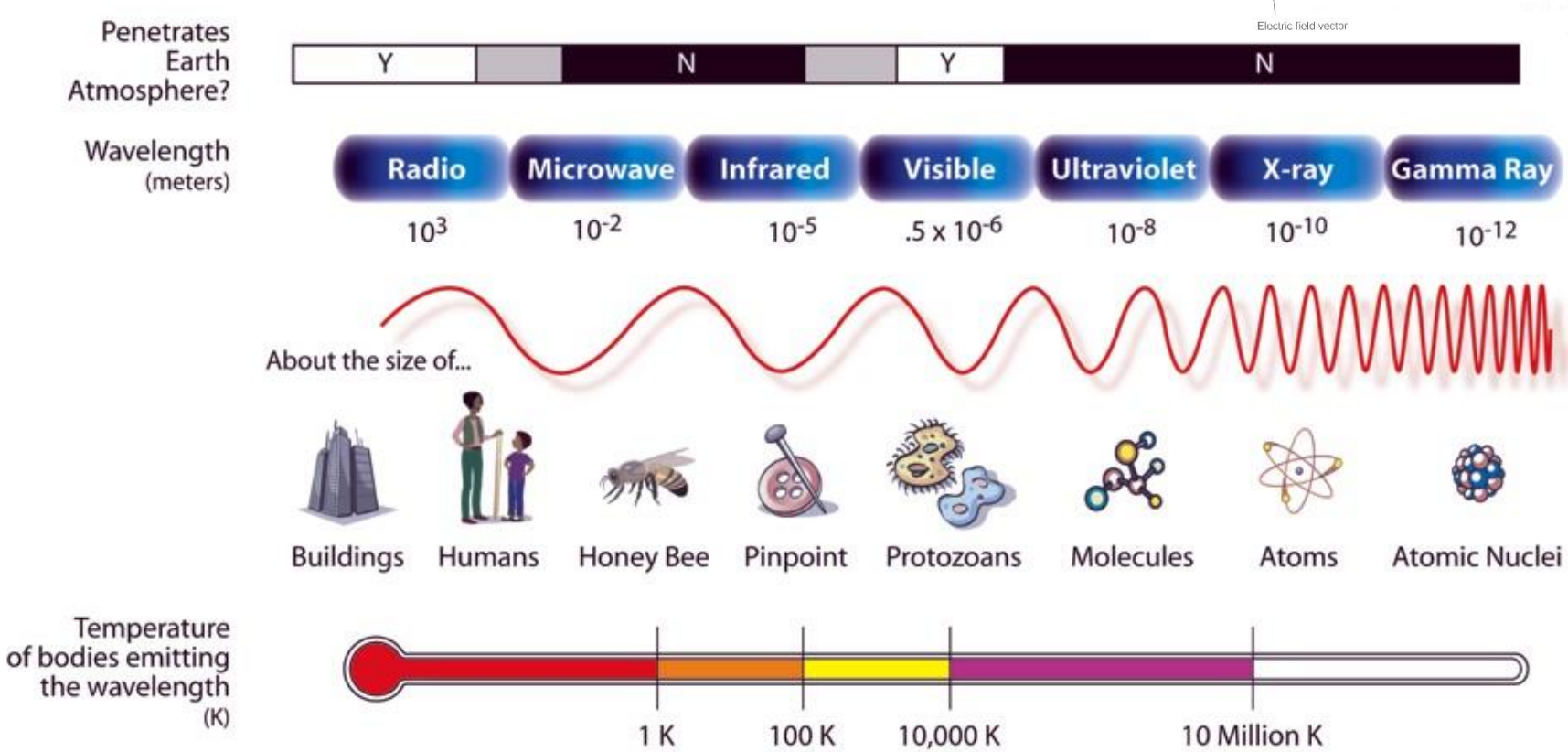
A Very Brief Review of the Electromagnetic Spectrum

- In remote sensing, almost all of the information about a planetary environment is carried by Electromagnetic Radiation
  - a.k.a. “light”





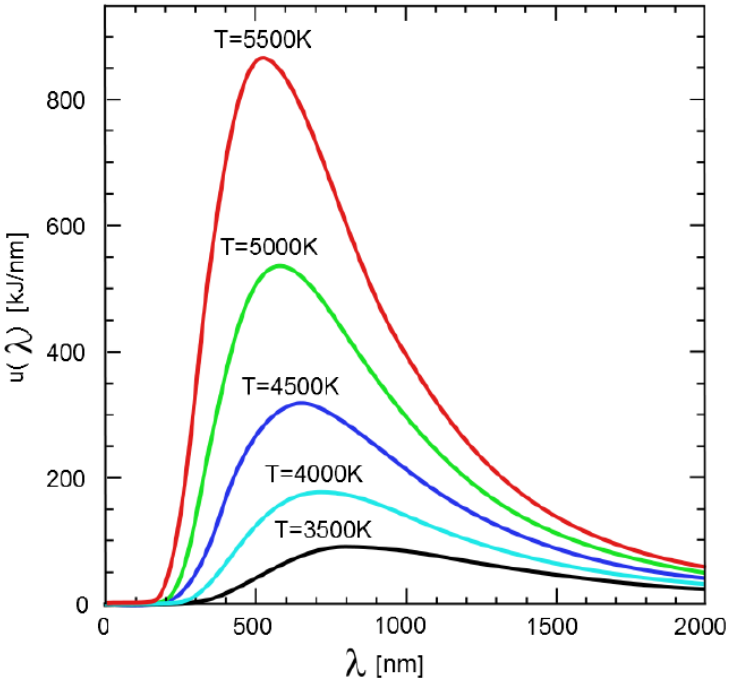
Interaction of Radiation with Planetary Environments

- Electromagnetic radiation can be *absorbed, emitted,* or *scattered* when it interacts with a planetary surface or atmosphere

- Kirchoff''s Law of Thermal Radiation: in thermal equilibrium, the emissivity of a body equals its absorptivity (e.g. a poor reflector is a good emitter)

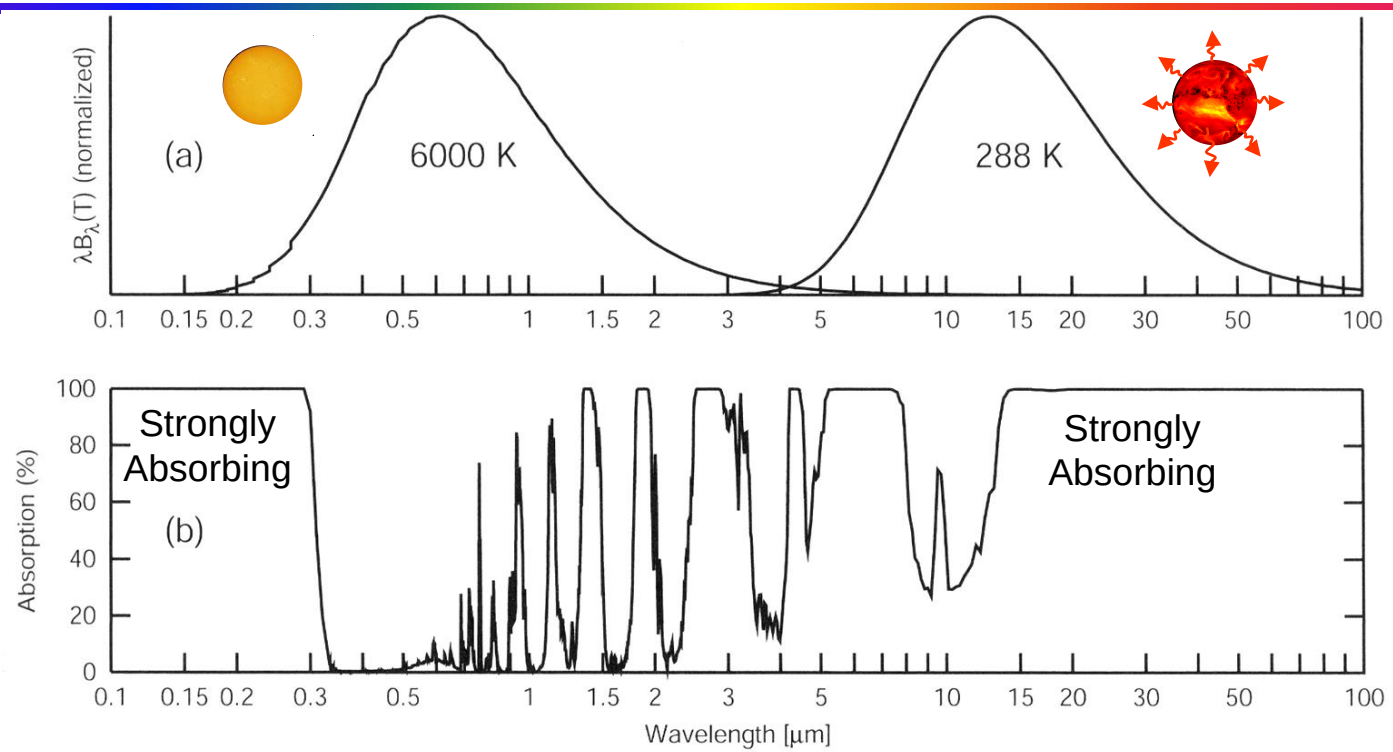
- Wein's Law: The wavelength dependence of emission from a body depends on its temperature

- The wavelength dependence of thermal emission is defined by the Planck function



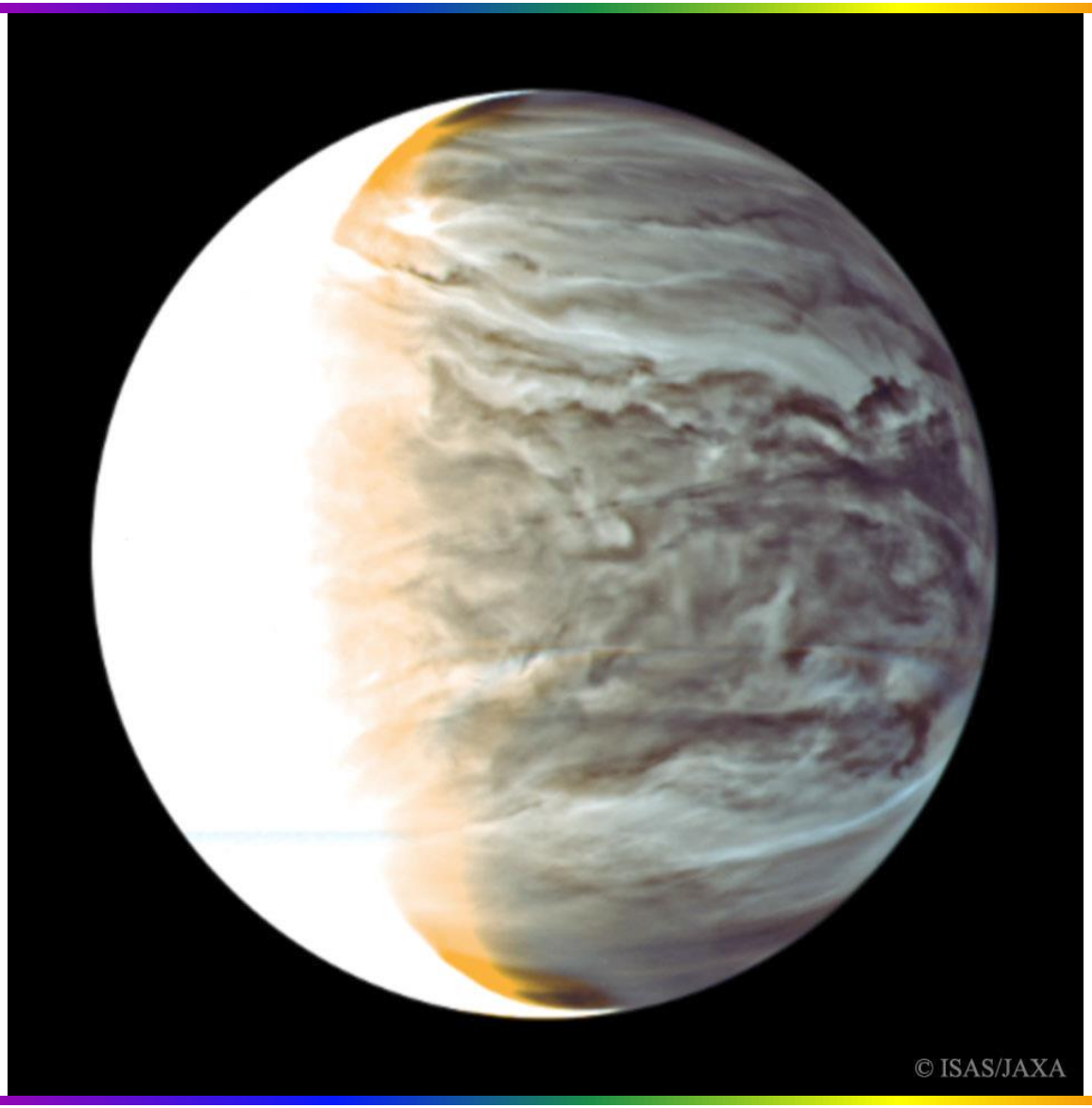
$$B(\lambda,T) = \frac{2hc^2 / \lambda^5}{e^{hc/\lambda kT}-1}$$

Solar and Thermal Radiation



- The sun is hot (5777 K), and therefore emits most of its radiation at short wavelengths (high energies)
- The Earth and (most) other planets substantially cooler, and emit most of their radiation at longer wavelengths (lower energies)
- The Earth's atmosphere absorbs strongly in the UV and thermal IR

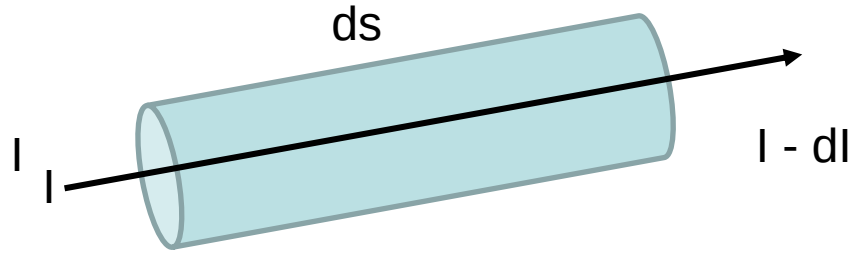
Venus: An Example of Reflection and Thermal Emission



In this near infrared image of Venus taken by the Japanese Akatsuki spacecraft, we see:

- Reflection of sunlight from the bright, sunlit crescent at the left,
- Thermal emission from the hot lower atmosphere that traversers the clouds and escapes to space on night side on the right.

Extinction, Optical Depth, and Transmission



- As light traverses a distance , ds, in a scattering or absorbing medium, it experiences “extinction”
- The amount of extinction is proportional to the “optical depth,” τ, where

$$\tau = \int \sigma (\lambda, s) N(s) ds$$

- Here, σ (λ, s) is the extinction cross section of the molecule or particle, which, in general changes with wavelength, λ , and distance along the path, and the number density of the molecules or particles, N(s).

- The “transmssion” is given by T = exp (-τ)
  - This relationship is often called “Beer’s Law”

Extinction, Absorption, and Scattering

- The total extinction cross section of a gas, aerosol, or cloud particle, σ_{ext}, is given by the sum of the scattering cross section, σ_{scat}, and the absorption cross section, σ_{abs}

- Similarly, the extinction optical depth can be expressed as the sum of a scattering and absorption optical depth,

$$\tau_{ext} = \tau_{abs} + \tau_{sca}$$

- The single scattering albedo is defined as the ratio of the scattering cross section (or optical depth) and the extinction optical depth

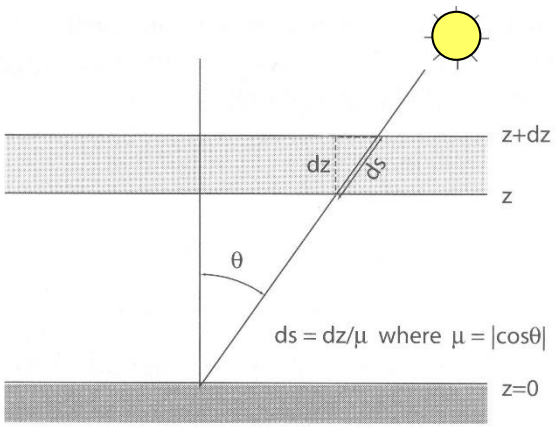
$$\omega = \tau_{scat} / \tau_{ext}$$

Radiance, Flux, and Pathlength

- It is easiest to relate slant paths to vertical paths by considering a horizontally-infinite system
  - Then, the vertical distance, dz, is related to the optical path length

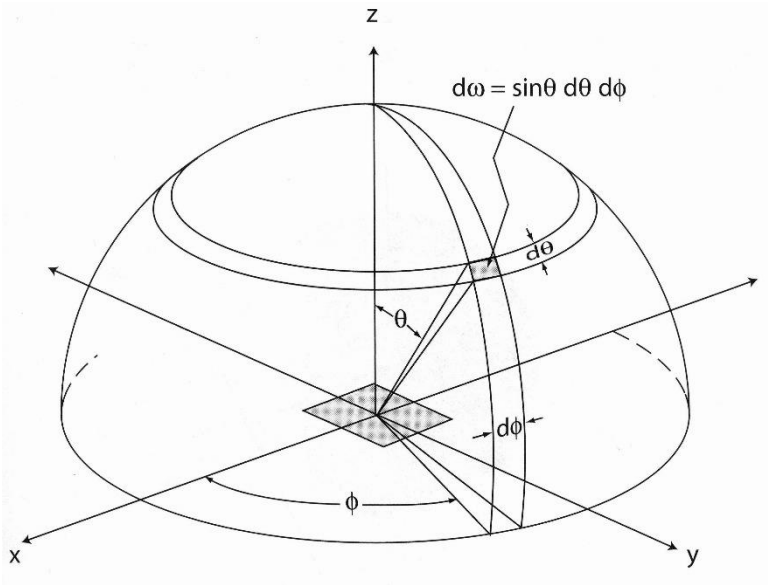
    $$ds = dz / \cos(\theta)$$

    where θ is the zenith angle



- A ***radiance*** or "intensity" specifies the amount of radiation per unit solid angle (e.g. $W/m^2/sr/\mu m$)

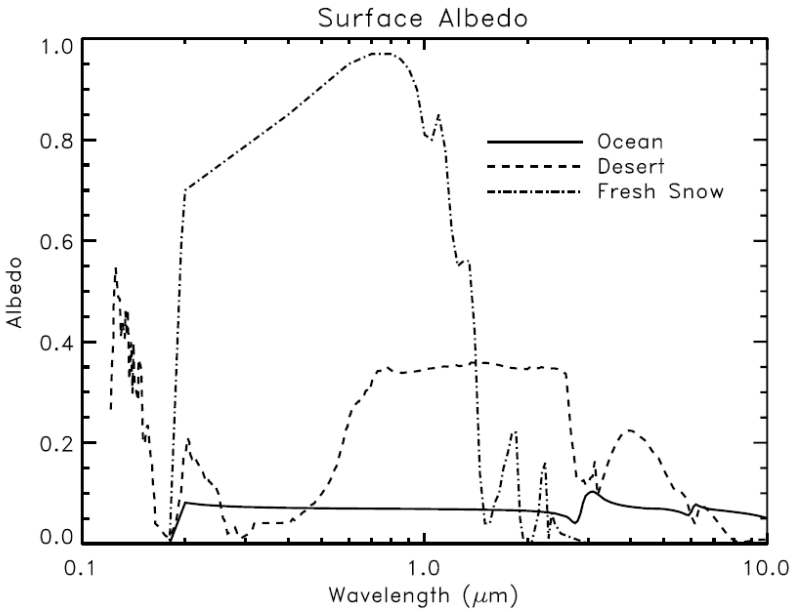
- An ***irradiance*** or "flux" is the total, angle-integrated radiation through a surface

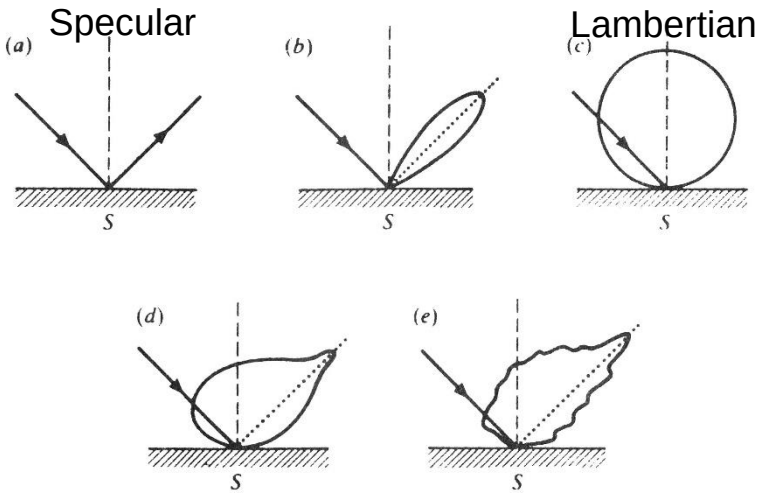


Interaction of Radiation with Planetary Surfaces

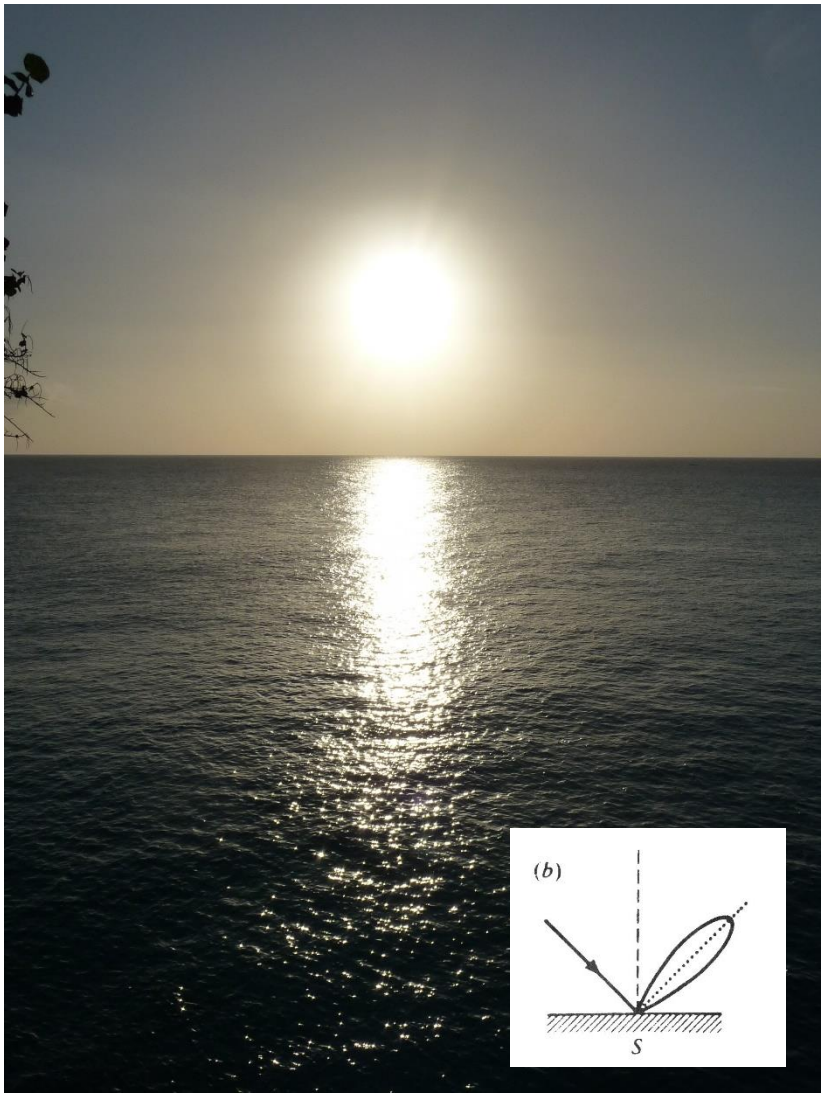
- The reflectance and transmittance of solid and liquid surfaces usually varies slowly with wavelength
  - Some are unusually bland (water)

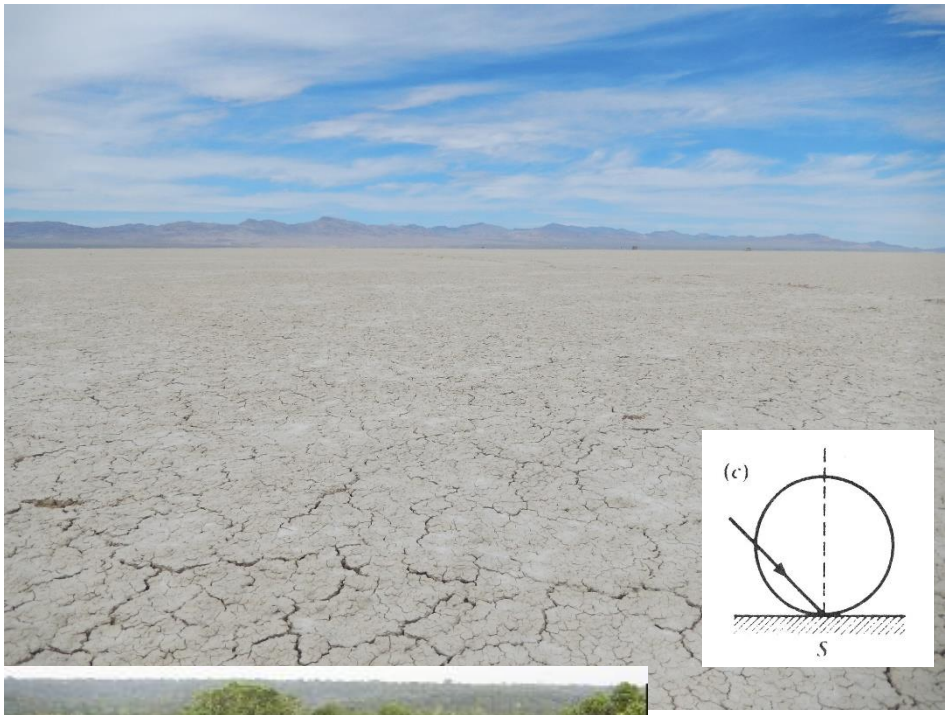
- The angular dependence of the scattering by the surface (the *bidirectional reflection distribution function*) depends primarily on the physical properties of the surface
  - The scattering from flat "specular" surfaces follows the Law of Reflection: $\theta_r = \theta_I$
  - "Rough" or granular surfaces can scatter the incident radiation over a wider range of angles
  - "Lamberian" surfaces scatter the incident radiation equally in all directions

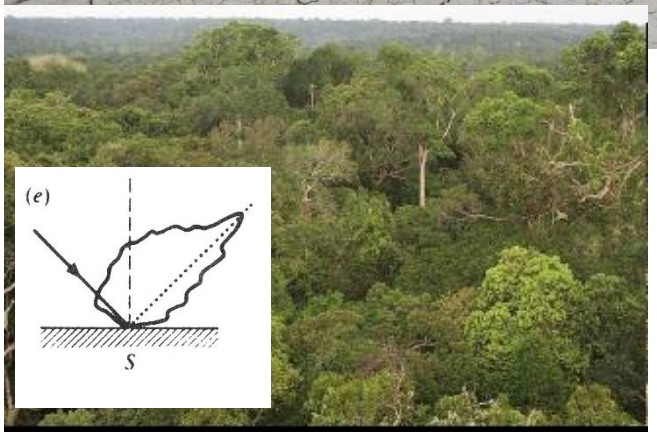




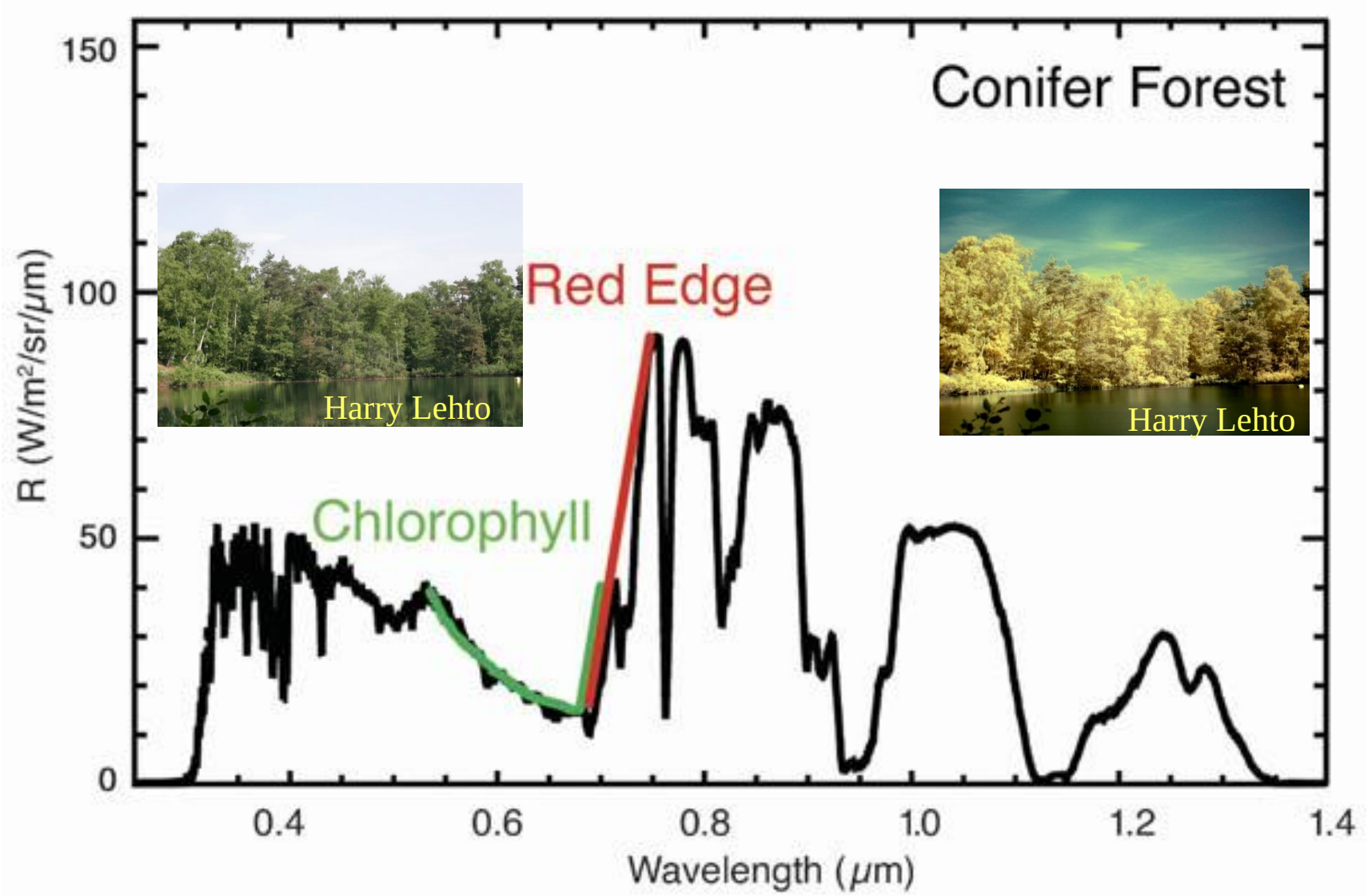
Examples of Surface Scattering





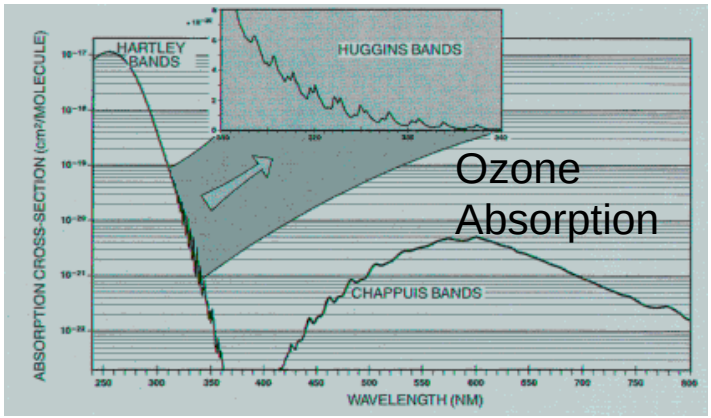


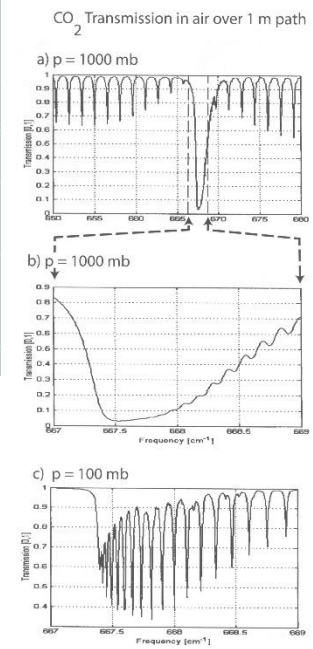
The Photosynthetic Red Edge

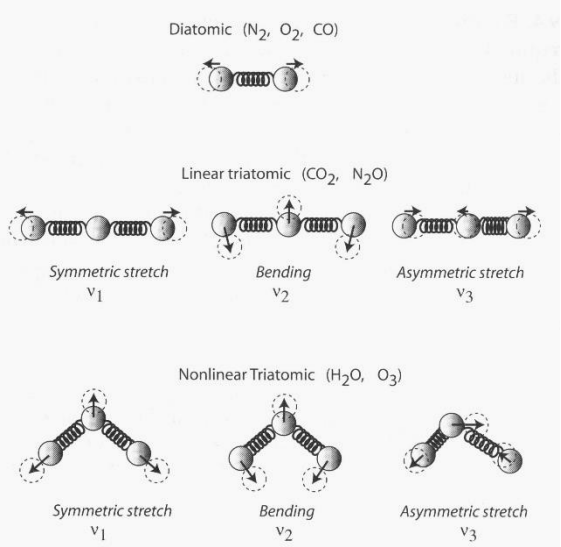


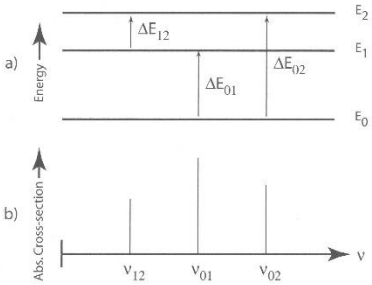
Interaction of Radiation with Planetary Atmospheres

- Atmospheres consist of gases and particles that can absorb, emit, or scatter radiation
- Gases produce
  - Rayleigh scattering at blue and UV wavelengths ($\sigma \propto 1/\lambda^4$)
  - Continuum absorption associated with electronic transitions at UV and visible wavelengths
  - Infrared absorption associated with vibration-rotation transitions at infrared wavelengths
    - Transitions are unique to the elemental composition and molecular structure
    - Provides a distinct spectral *fingerprint* for each gas
  - Some gases produce little IR absorption (N_2, noble gases)



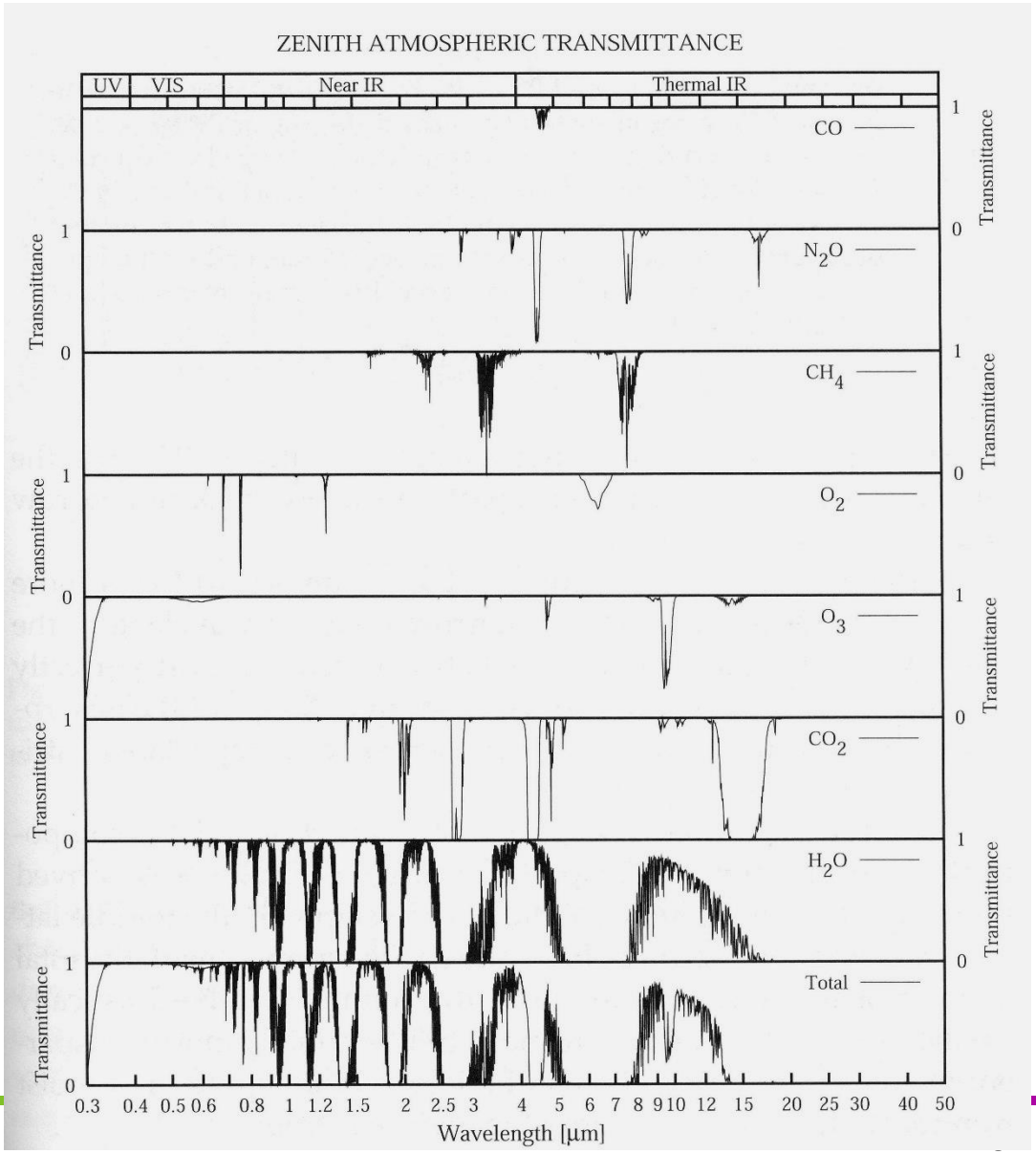




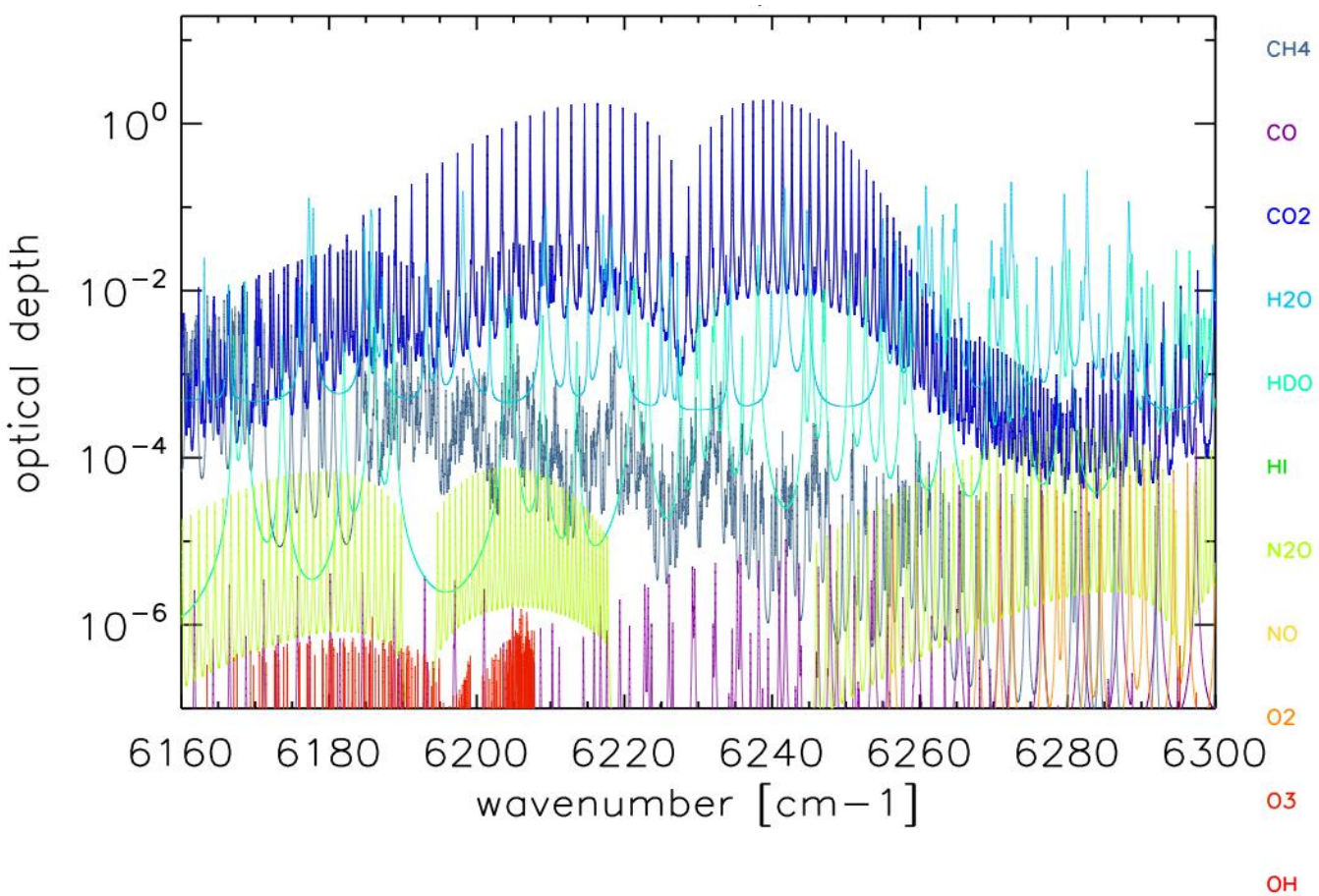


Infrared Gas Transmission in the Earth's Atmosphere

- The primary absorbing gases in the Earth's atmosphere are H_2O, CO_2, O_3, N_2O, CH_4, and CO
  - Water vapor is by far the strongest absorber (and most effective greenhouse gas)
  - The other gases fill in many of the "spectral windows" between the water vapor bands
  - There is relatively little gas absorption at visible wavelengths (0.4 – 0.7 μm)

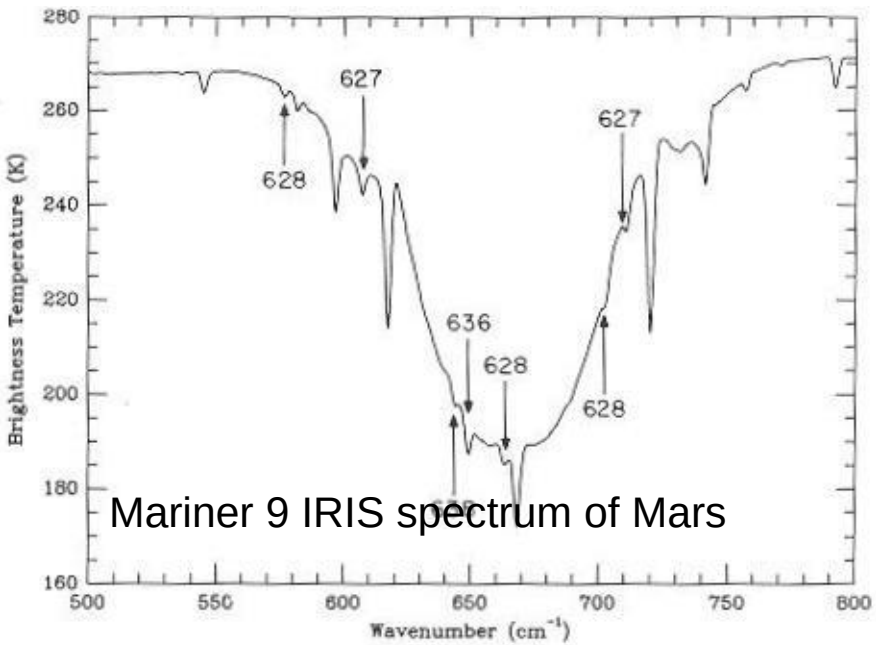


Gas Absorption Cross Sections

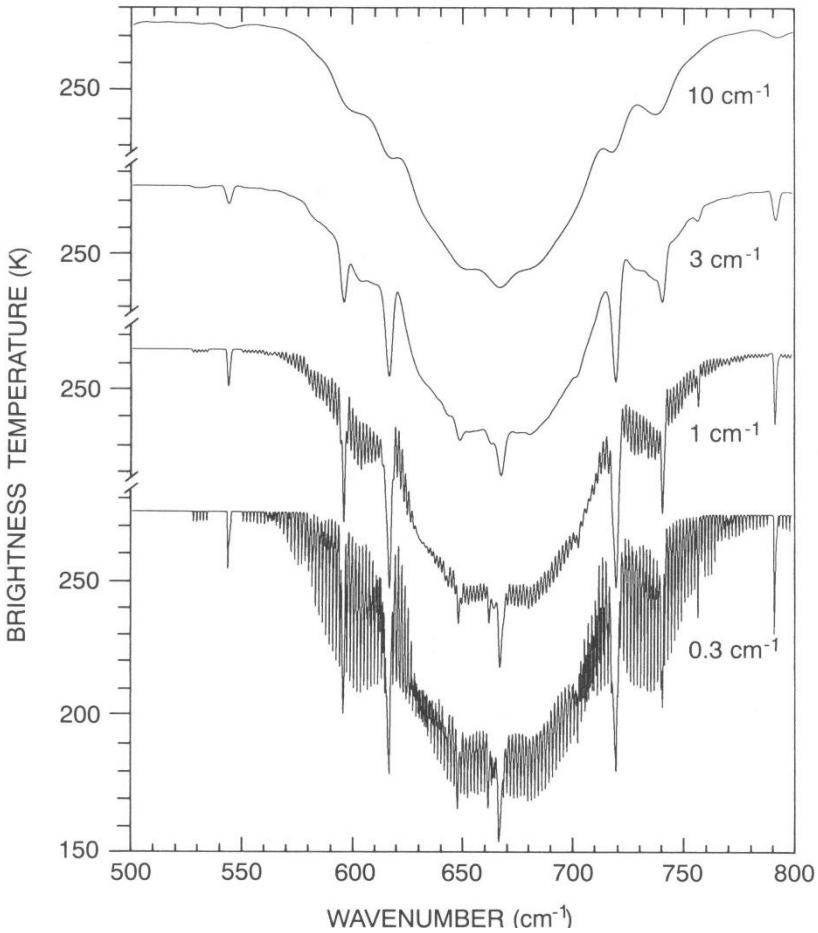


While atmospheric gases have distinct absorption spectra, there is a substantial amount of overlap between their absorption features in most spectral regions.

The CO2 ν_2 Fundamental at 15 μm



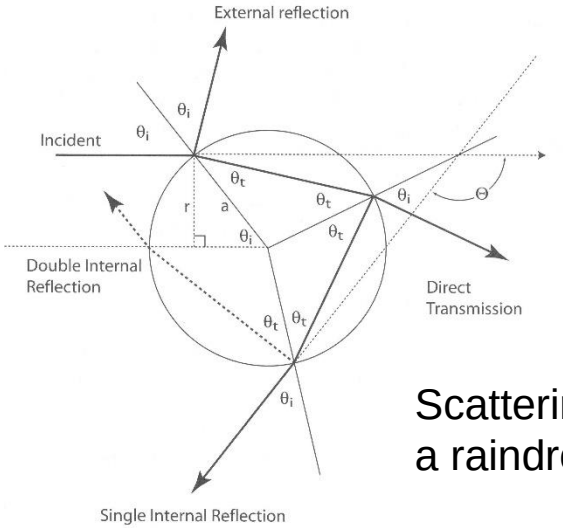
- Radiances are often shown as "Brightness Temperatures". This is the effective black-body temperature of the system



- The CO_2 15 μm (667 cm^{-1}) band is the primary feature used to retrieve the temperature structure of terrestrial planet atmospheres
- With high enough resolution and signal to noise ratios, this band can also yield isotopic ratios

Scattering in Planetary Atmospheres

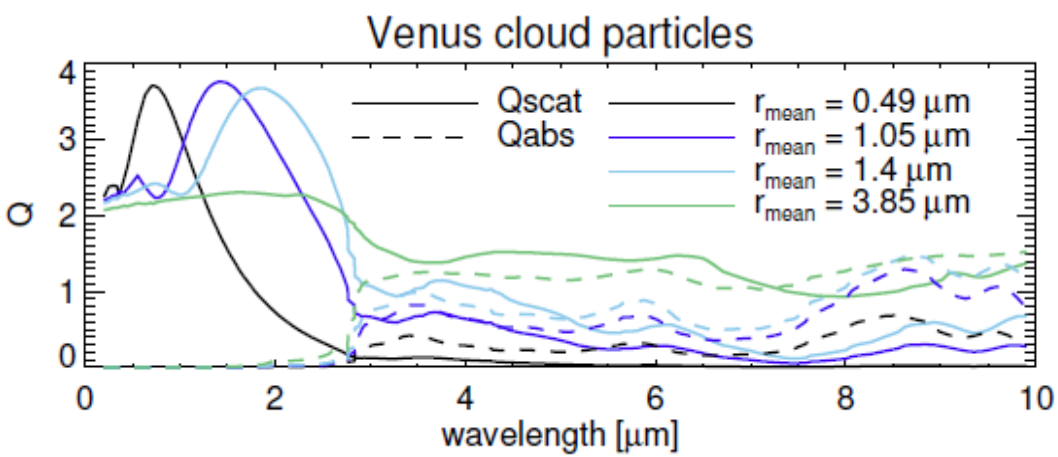
- Atmospheric scattering can be broadly defined as the redirection of radiation out of the original direction of propagation, due to interactions with molecules and airborne particles
  - No energy transformation results, only a change in the spatial distribution of the radiation
  - Reflection, refraction, diffraction etc. are actually all just forms of scattering
- Airborne particles (cloud droplets, aerosols) also absorb and emit radiation
- The total scattering + absorption cross section of a particle is referred to as the “extinction” cross section

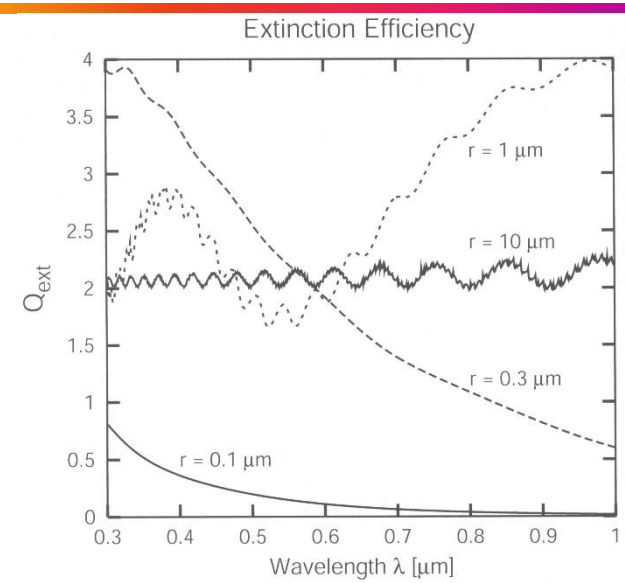


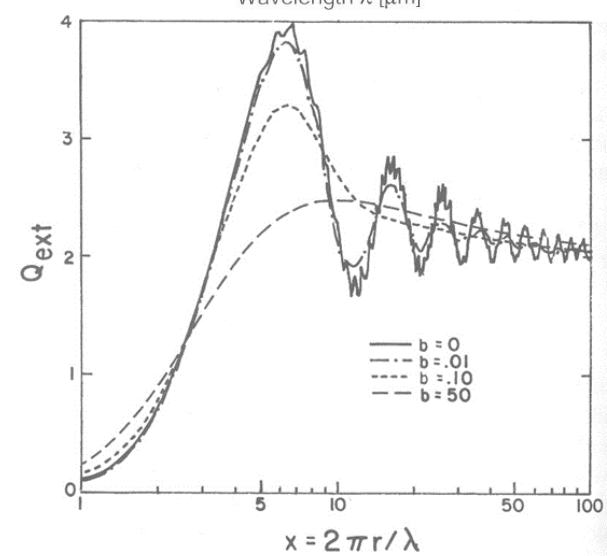
Scattering by a raindrop

Single Scattering by Gases and Airborne Particles

- The extinction cross-section depends on the composition and the particle's size (when compared to the wavelength)
  - Particles are most efficient at scattering radiation with a wavelength comparable to their wavelength
  - The particle composition affects the real and imaginary index of refraction, which affects the particle absorption and scattering efficiencies

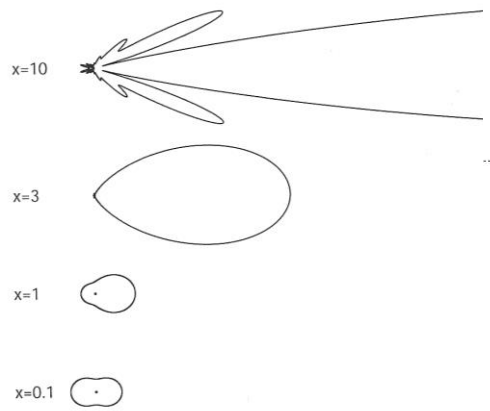


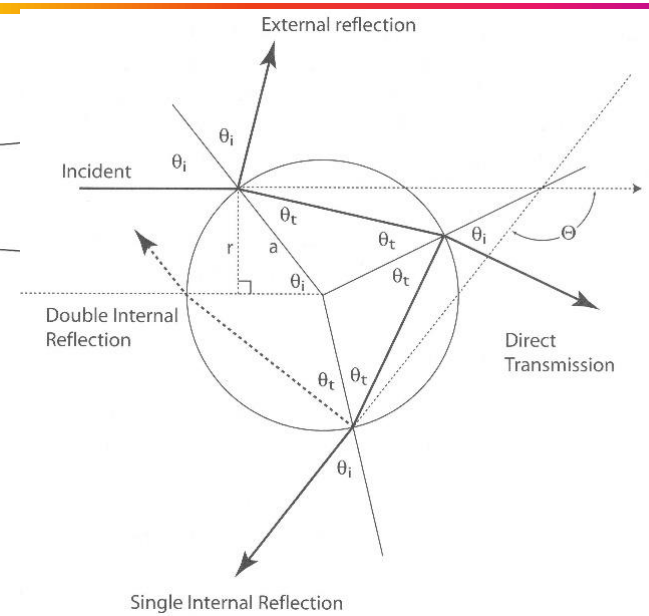


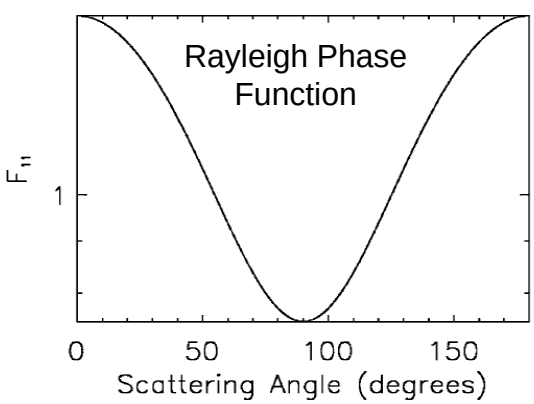


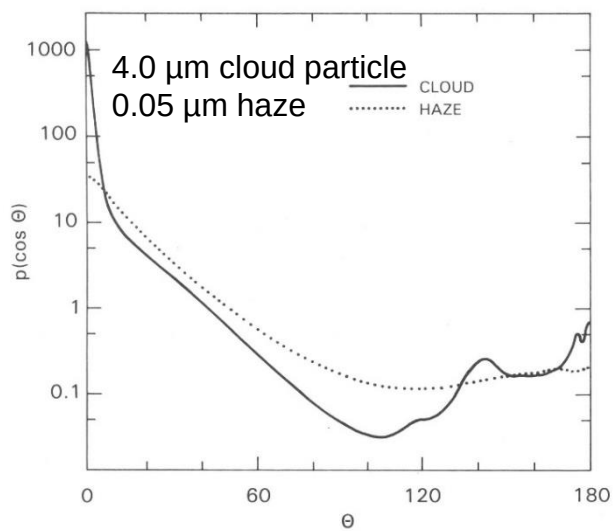
The Scattering Phase Function

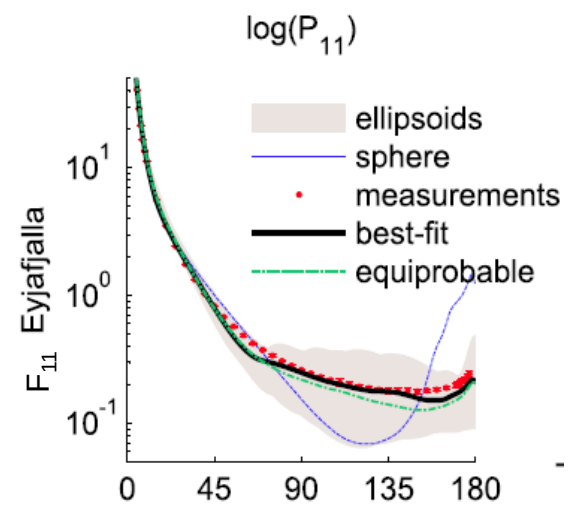
- The angle dependence of the scattering depends on particle size and shape
  - Gases and small particles scatter in both the backward and forward directions
  - Large particles are predominately forward scattering.









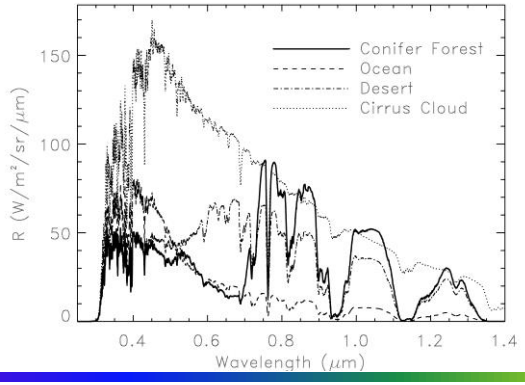


Putting it All Together: Radiative Transfer in Scattering/Absorbing/Emitting Atmospheres

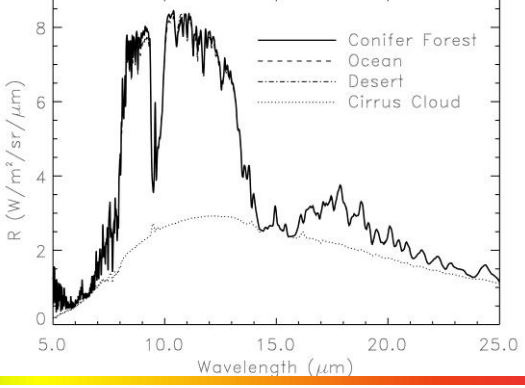
- Efficient, accurate methods for simulating the absorption, emission, and scattering of solar and thermal radiation are essential for:
  - Analysis of remote sensing observations
  - Radiative heating and cooling rate calculations for climate models

- These problems are intrinsically challenging when fluxes or radiances are needed over broad spectral regions because
  - Gas absorption cross sections change rapidly with wavelength
  - Methods for estimating radiances and fluxes in the presence of multiple scattering must be applied to spectral regions that are sufficiently narrow that the optical properties are essentially constant (monochromatic)

Reflected Solar Radiation



Emitted Thermal Radiation



Solution Techniques

- Discrete Ordinates
  - Treat RTE as BVP - boundary conditions at TOA and surface
  - continuity at interface between layers
- Doubling-Adding
  - Compute reflection and transmission properties for very thin sub-layer assuming single scattering
  - Add two such identical sub-layers by computing successive reflections back and forth between the layers (doubling)
  - Repeat process until layer properties computed
  - Add contiguous layers using same technique (adding)
  - Repeat process until all layers treated
  - Start with reflection properties of surface and build up
- Successive Orders of Scattering
  - Reflection and transmission computed individually for photons scattered once, twice, thrice …
  - Stokes parameters obtained as sum over all orders of scattering
  - $(n-1)^{th}$ order of scattering is the source for the nth order of scattering
- Monte Carlo Methods (backward and forward)

Exact Methods for Solving Broad-band Radiative Transfer Problems

- The most accurate solutions to the equation of transfer in scattering, absorbing, emitting atmospheres can be obtained full “full-physics” methods that
  - Employ a spectral grid that is fine enough to resolve the spectral structure of all (gas, aerosol, surface) optical properties, and their variations along the optical path
  - Perform a (vector or scalar) multi-stream, multiple scattering calculation at each spectral grind point

- The primary problem with this approach is its computational expense, especially for broad-band (bolometric) calculations
  - 10^6 to 10^7 spectral grid points are needed to fully resolve the spectral structure throughout the solar and thermal spectral range
  - 4 to 8 vertical grid points are needed per scale height to accurately resolve the pressure and temperature dependent changes in the gas absorption cross-sections and the vertical structure of clouds and aerosols
  - 8 to 256 “streams” are needed to resolve the angle dependence of the radiation field

More Efficient Methods

- A variety of fast approximate methods have been developed to increase the efficiency of radiance and flux calculations for
  - Climate modeling applications, where level-dependent bolometric fluxes are needed at large numbers of spatial grid points, and these fluxes must be recomputed often as the atmospheric and surface properties evolve
  - Remote sensing applications where high resolution radiances and radiance Jacobians ($\partial r_i / \partial x_j$, where r is the radiance at wavelength, i, and x is a an optical property at level, j) are needed for a large number of soundings

- These methods typically increase their efficiency by
  - Reducing the number spectral grid points (band models, etc.)
  - Reducing the angular resolution (2-stream methods)
  - Reducing the vertical resolution (single layer atmospheres)
  - Neglecting all but the dominant radiative process (neglecting multiple scatting within gas absorption bands, considering only the first or 2nd orders of scattering, etc.

- All of these methods can reduce the accuracy and information content of the simulated radiances an fluxes

Band Models

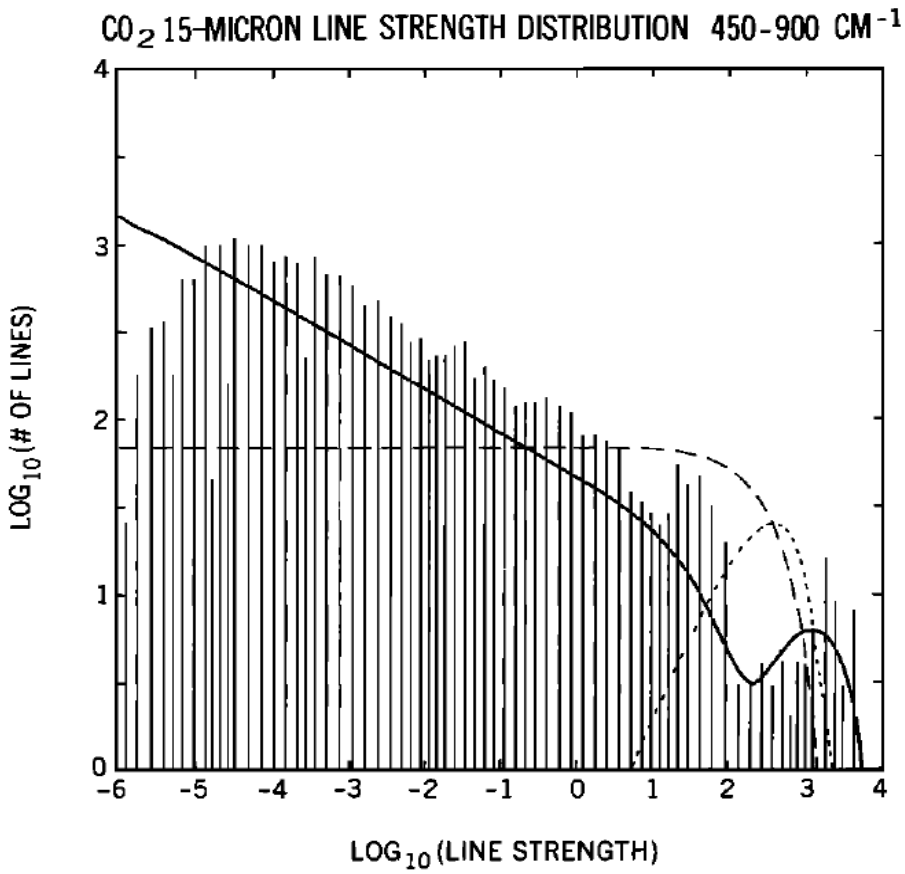
- "**Band models**" use approximate methods to account for the spectrally-dependent structure of gas absorption bands, reducing the number of spectral points where the equation of transfer must be evaluated
  - Most efficient when fluxes are needed at relatively low spectral resolution over a broad spectral range, where the Transmission Function is:

$$\mathcal{T}(\Delta\nu,u) = \Delta\nu^{-1} \int_{\nu 1}^{\nu 2} \exp\{-\tau(\nu,u)\}\, d\nu$$

where ν is wavenumber, u is absorber amount, and τ is the monochromatic optical depth along the path, $\tau = \int \sigma(\nu, s)\, N(s)\, ds = \int \sigma(\nu, u)\, du$.

- Band models can be classified into two broad categories:
  - "Physical band models" employ a simplified physical model of a gas absorption band, that yields an analytic functional form for the absorption or transmission over a finite spectral region
    - Goody and Malkmus Random Models, Quasi-Random Model, Elsasser Model, etc.
    - Generally not applicable to scattering atmospheres
  - "Empirical band models" are typically analytic functions, whose coefficients are fit to the measurements or simulations of the broad-band absorption or transmission

Physical Band Models



Histogram of actual line strengths within the CO_2 15 micron band, compared to the line strength distributions assumed in the Goody (dotted line), Malkmus (long-dash line) and an analytic distribution (Crisp et al. 1986).

Physical band models, such as the Elsasser (1942), Goody (1952), and Malkmus (1967) make assumptions about the positions and intensities of spectral lines within a band that allow the broadband transmission to be expressed as a simple, analytic, "transmission function."

These models are rigorously valid in the "weak line" and "strong line" limits, and work reasonably well (errors < 10%) for a wide range of conditions, but

- Typically work only work where Lorentzian line shapes are valid,
- Cannot be incorporated with most multiple scattering models.

Exponential Sum Methods

- For homogeneous, isothermal optical paths, the broadband transmittance in finite spectral range, $\Delta\nu$, can be approximated by a finite sum of exponentials:

$$\Im \cong \sum_{i=1}^{N} a_i \exp(-k_i u)$$

- The quantity, u, is the absorber amount along the optical path, k_i can be interpreted as a gas absorption coefficient, and the weights, a_i can be interpreted as the fraction of the spectral range characterized by that absorption coefficient.

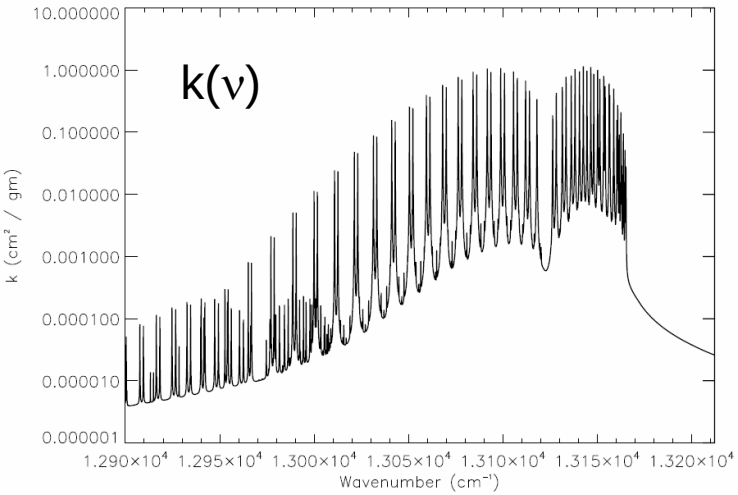
- Instead of dividing the spectrum up into monochromatic spectral segments, exponential sum methods can be interpreted as dividing the range of absorption coefficients within a spectral range.
  - This transformation can be "exact" for homogenous, isothermal optical paths (i.e. k_i is constant along the path) as $N \rightarrow n_{mono}$
  - It can yield accurate results for $N << n_{mono}$, where each weight, a_i, is interpreted as the spectral range over which $k_i = k_i \pm \delta k_i$

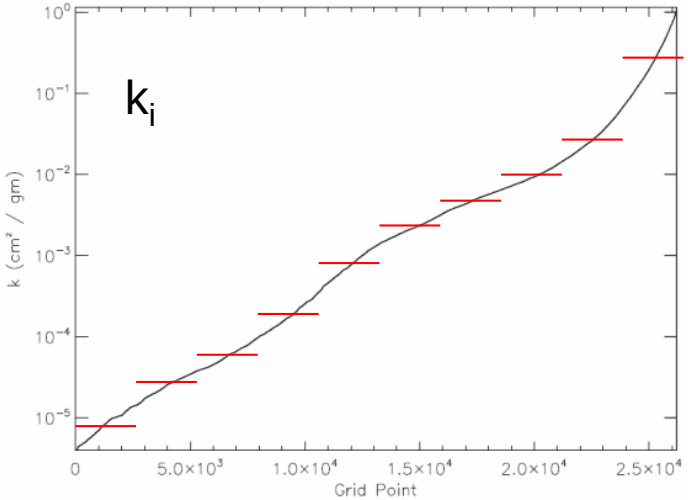
How do Exponential Sums Reduce the Computational Expense?

- The spectrum of gas absorption coefficients for the O_2 A-Band spectrum (top right) is defined by 26000 spectral grid points.

- If we sort the spectrum in order of increasing absorption coefficient (bottom right), we produce a much smoother distribution (bottom right).

- This distribution can be described accurately with a smaller number of discrete terms.
  - Here, we have roughly approximated the distribution by 10 absorption coefficients.





Advantages and Disadvantages of Exponential Sum Methods

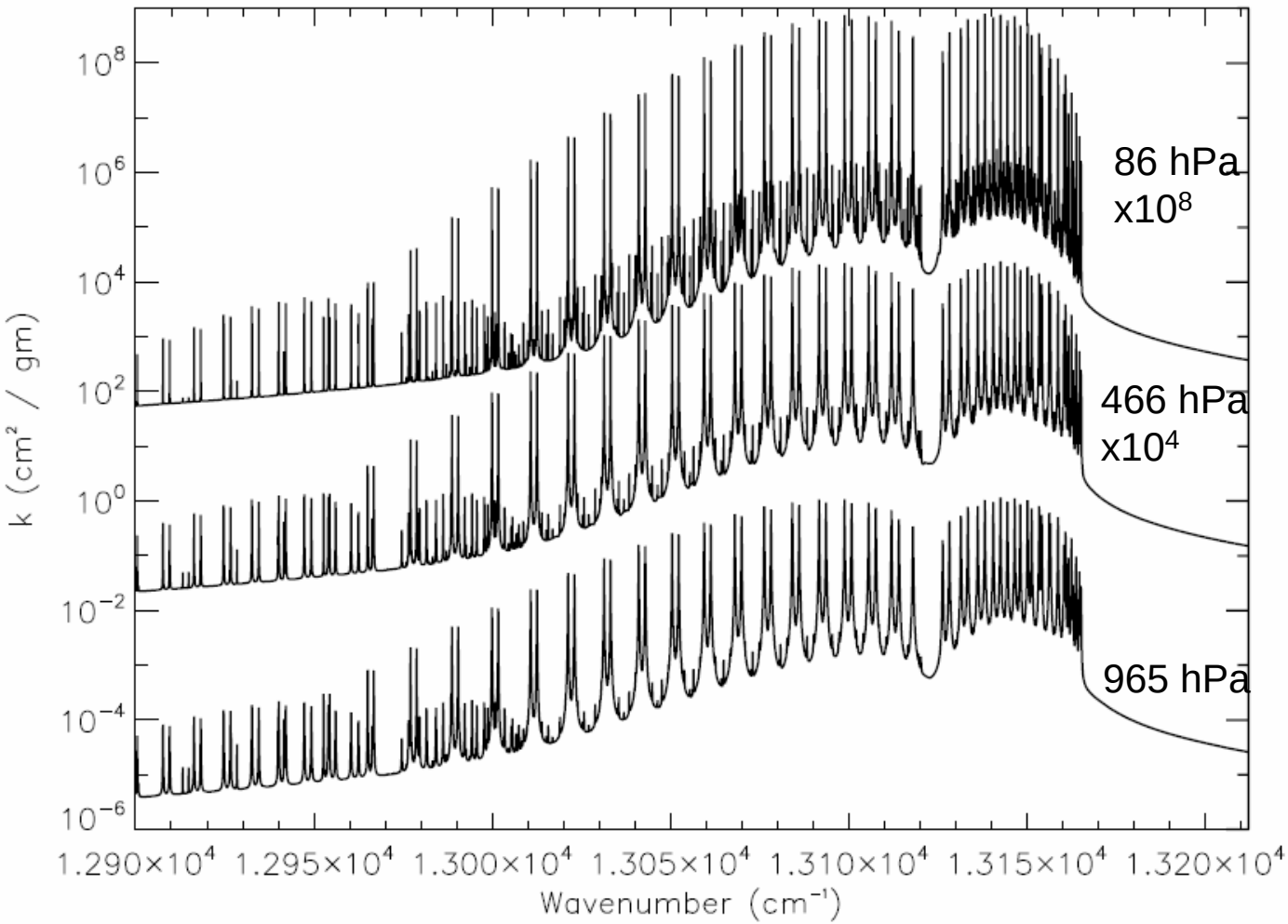
- Exponential sum methods were popularized in the 1970's and 1980's, when
  - Lacis and Hansen[1] (1974) developed a very fast exponential sum model for use in GCM's and
  - Wiscombe and Evans[2] (1977) published a numerically efficient, accurate method for producing the fits (Exponential sum fitting of radiative transmission functions. J. Comp. Phys. 24, 416-444, 1977.)

- If we consider each term in the exponential sum as quasi-monochromatic,
  - The gas optical depth associated with each absorption coefficient, k_i is approximated by: $\tau_{gas(i)} = k_i\, u$
  - This optical depth can be added to the airborne particle optical depth, and a single monochromatic multiple scatting calculation could be performed for each term
  - The computed radiance or fluxes for each term can then be weighted by a_i and summed to yield the broad-band radiances or fluxes within the spectral band

- The primary disadvantage is that this method was that there was no rigorous way to extend it to inhomogeneous, non-isothermal atmospheric optical paths, where the absorption coefficients, k_i, change over the optical path

1) Lacis, A.A., and J.E. Hansen, 1974: A parameterization for the absorption of solar radiation in the Earth's atmosphere. *J. Atmos. Sci.*, **31**, 118-133, doi:10.1175/1520-0469(1974)031<0118:APFTAO>2.0.CO;2.
2) Wiscombe W., and J. Evans (1977). Exponential-sum fitting of radiative transmission functions J. Comp. Phys.,, 24, 416-444

Oxygen Absorption Cross Sections vs Pressure

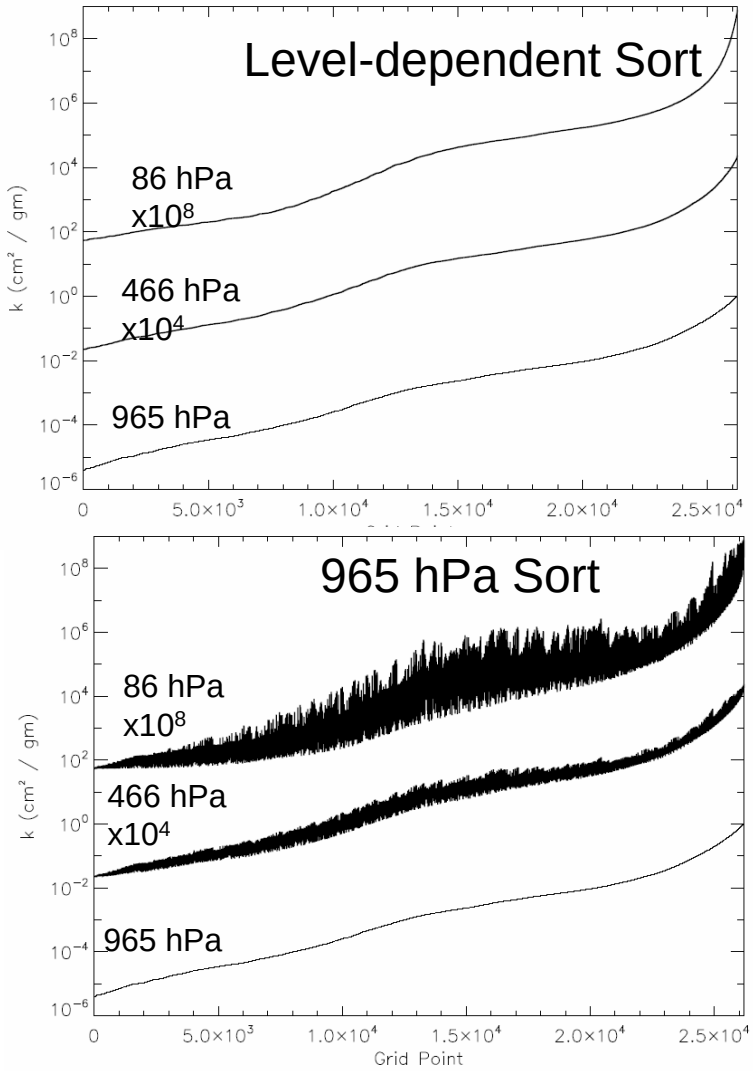


O2 A-band absorption cross sections are shown at 965 (bottom), 466 (middle) and 86 hPa (top). The spectra have been offset by 10^4 for clarity. The basic spectral structure is preserved, but the spectral contrast increases with decreasing pressure.

Absorption Coefficient Sorting in Inhomogeneous Atmospheres

- In principle, level-dependent absorption coefficients, $k_i(p)$, can be derived by simply sorting the monochromatic absorption coefficients at each level of the atmosphere (top right).

- Unfortunately, gas absorption coefficients do not remain "spectrally correlated" along inhomogeneous, atmospheric optical paths
  - A different sorting is needed at each level.
  - If the same sorting is used at all levels to preserve the relationship between k_i and $k(\nu)$, a substantial amount of structure is introduced in the absorption coefficients at some levels
  - This structure undermines the primary advantage of exponential sum methods.



Correlated – k Methods

- While gas absorption coefficients are not completely spectrally correlated along realistic, inhomogeneous, non-isothermal atmospheric optical paths, many (and sometimes most) spectral regions remain well correlated
- To exploit this spectral redundancy, and preserve the principal assets of the exponential sum methods, several groups tried to find reliable ways to extend this method to vertically inhomogeneous optical paths.
- Lacis and Oinas (1991) relied on the success of physical band models for simulating the broadband transmission in inhomogeneous optical paths
  - They found that they could derive absorption coefficients (k – coefficients) and weights directly from the Malkmus band model, by taking its Laplace Transform
  - These k-distributions "implicitly preserved the monochromatic structure of the atmosphere at different pressure levels" .
  - They also found that this approach provided a systematic way to include the overlapping absorption by multiple gases in a given spectral interval.
  - They named this method the "Correlated k Distribution" (CKD) Method
- While the CKD method was adequate for many applications, it was no more accurate than the Malkmus band model, on which was based.

Spectral Mapping Methods for High Resolution Spectra: SMART

- A primary feature of the Spectral Mapping approach is the preservation of the mapping between the spectral grid and the k-coefficients.

- Given the concept of a “spectral map” and wavelength dependent (monochromatic) optical properties of a planetary surface and atmosphere over a finite spectral range, it should be possible to:
  - Define a spectral grid that completely resolves the wavelength-dependent optical properties of the surface and all gases, clouds, and aerosols that contribute to the radiation field throughout the atmospheric column;
  - Identify spectral grid points that have similar optical properties along the entire surface/atmosphere optical path;
  - Map these grid points into a smaller number of quasi-monochromatic bins;
  - Use a multi-stream multiple scattering algorithm calculate the angle-dependent radiances for each bin; and
  - Map these radiances back to their original spectral grid points to create a high-resolution spectrum.

- This approach was first implemented in a model called the Spectral Mapping Atmospheric Radiative Transfer (**SMART**) model.

Spectral Binning in SMART

- Each ***Spectral Bin*** is defined by a mean value and a range of acceptable values at each atmospheric level and at the surface:
  - Mean Values: $x_b = [\delta\tau_b(z), \omega_b(z), g_b(z), a_b]$
  - Range: $\Delta\delta\tau_b(z), \Delta\omega_b(z), \Delta g_b(z), \Delta a_b$
    - Set by the user at run time

- A monochromatic grid point is included in a bin if it satisfies the following criteria at all levels:

  Differential Optical depth: $\delta\tau(z) = \delta\tau_b(z) \pm \Delta\delta\tau_b(z)$

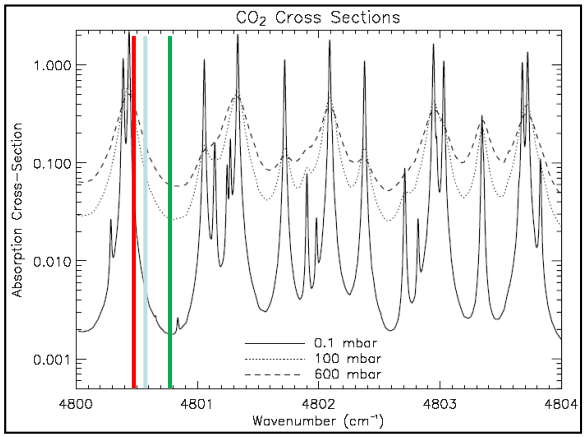
  Single Scattering Albedo: $\omega(z) = \omega_b(z\ )\ \Delta\omega_b(z)$

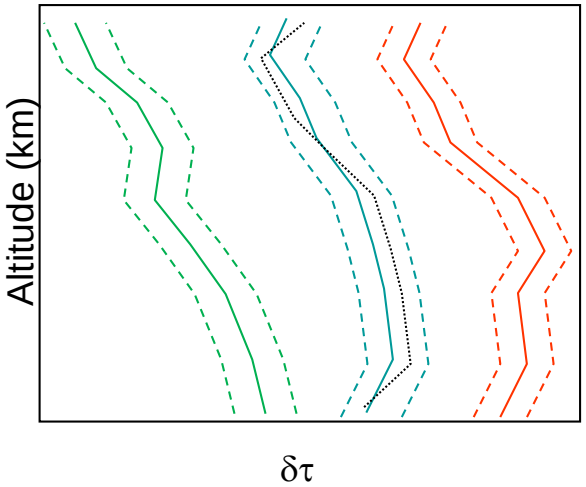
  Asymmetry Parameter: $g(z) = g_b(z) \pm \Delta g_b(z)$

  Surface Albedo $\Delta a = a_b \pm \Delta a_b$

- If the sounding fits in a bin, the bin number is recorded on the "*Spectral Map*"

- If a sounding does not fit in an existing bin, a new bin is defined and initialized with by its optical properties



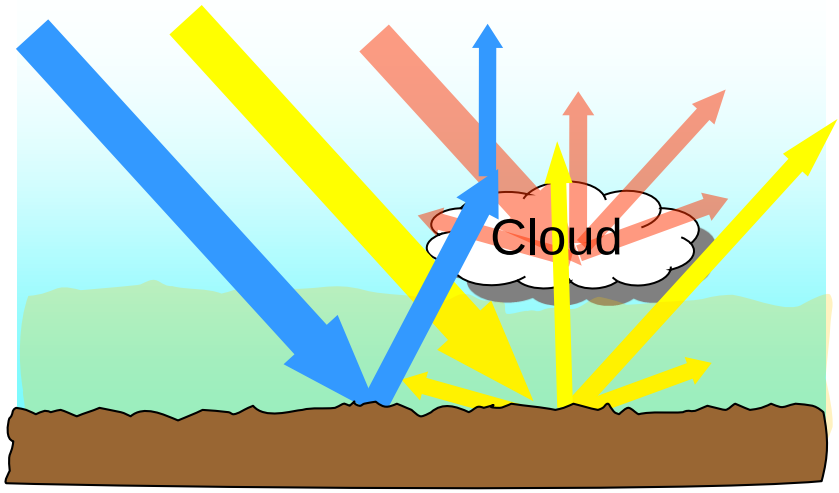


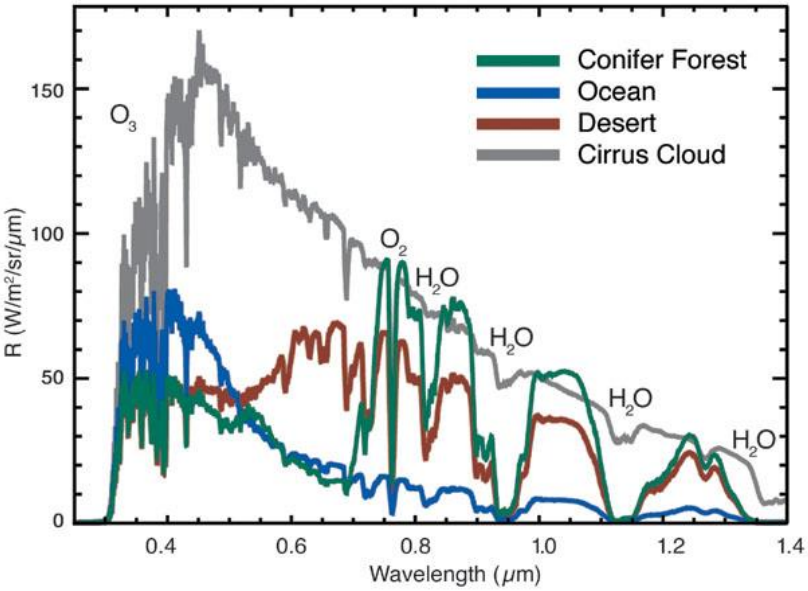
Cartoon showing differential optical depths, δτ(z) for 3 bins (blue, green, red).

Applications: Remote Sensing with Reflected Solar Radiation

Solar spectra yield information about:

- Optical properties of the reflecting surface (cloud deck/ground)
  - Albedo, “surface” cover
- Atmospheric pressure of “surface”
  - Need a well-mixed gas with a known spectrum is needed (e.g. O_2 or CO_2)
- Detection and quantification of column abundance of key trace gases - UV/VIS/near-IR
  - H_2O, O_2, O_3, N_2O, CH_4, NH_3 …
- Limitations
  - Little information about temperatures of surface or atmosphere
  - Clouds preclude full-column or surface measurements





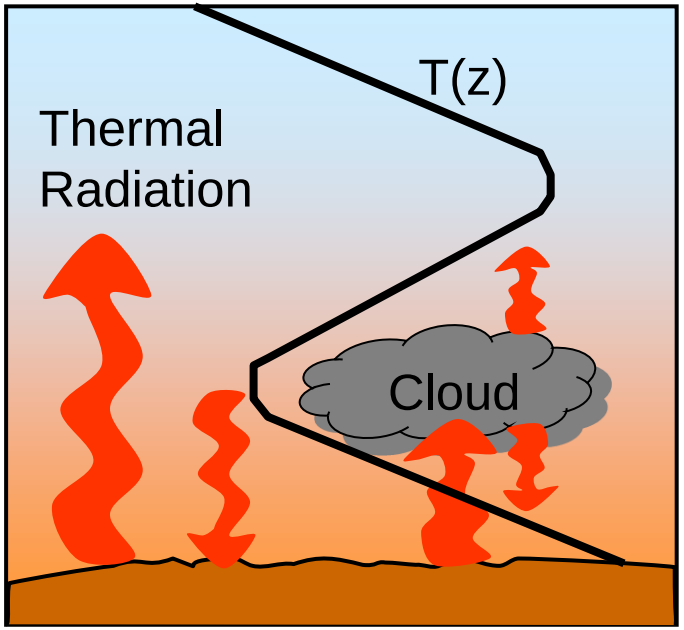
Applications: Remote Sensing Using Thermal IR Spectra

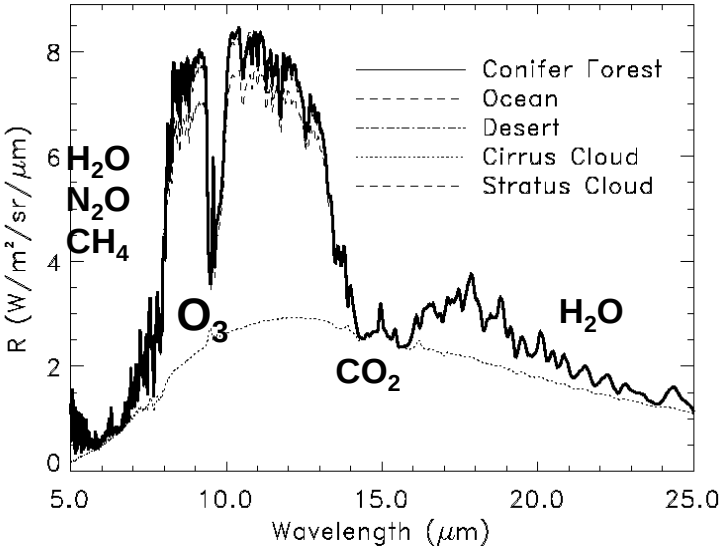
Thermal IR spectra can yield information about

- Temperature of emitting surface
  - Window regions
- Atmospheric thermal structure
  - **Well-mixed gas: CO_2 15 μm band**
- Vertical distribution of temperatures and trace gases above emitting surface
  - **H_2O, O_3, CH_4, N_2O**

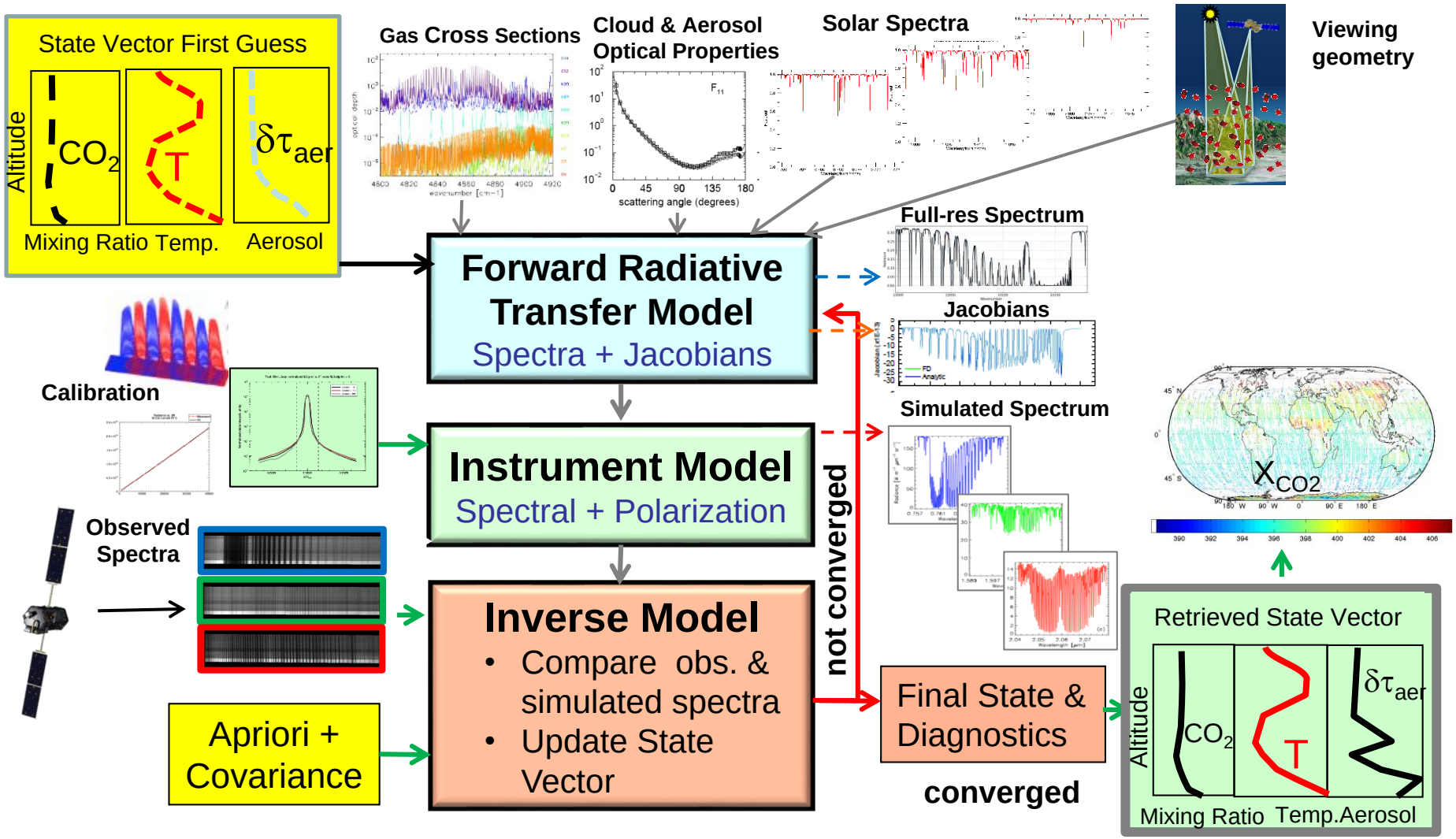
Limitations

- Atmospheric temperature information is essential for retrieving trace gas amounts
  - requires a well-mixed gas with a known spectrum
  - Limited information on constituents near the surface – surface/atmosphere temperature gradient needed
- Thermal IR provides limited constraints on planetary surface composition

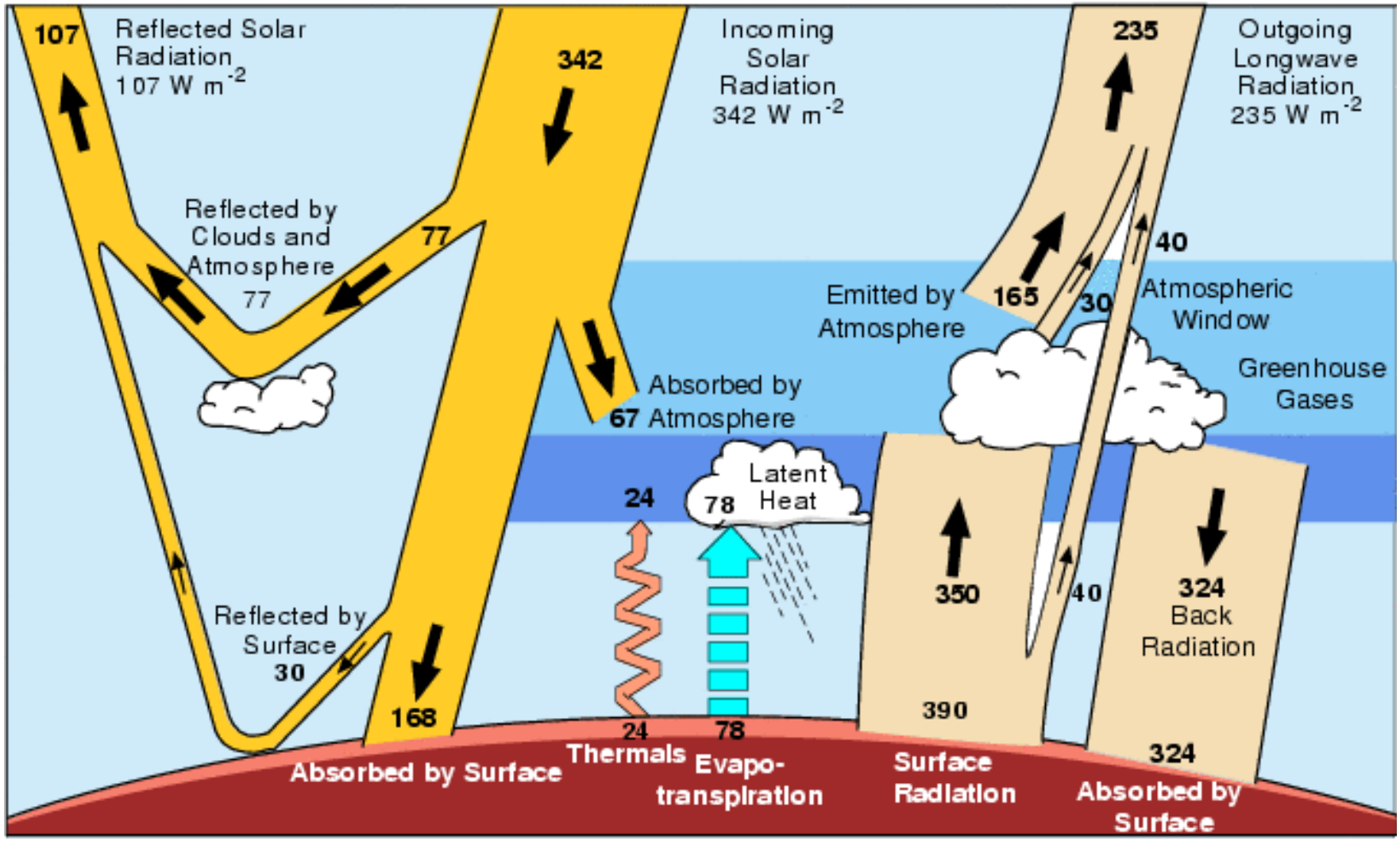




The OCO-2 XCO2 Retrieval Algorithm



Applications: Calculating the Global Energy Balance



Kiehl and Trenberth 1997

Brief Summary of the Territory Covered

We took a quick tour through:

- The solar and thermal components of the electromagnetic spectrum
- Interactions of solar and thermal radiation with the surface and atmosphere
  - Surface reflectance and bi-directional reflection distribution function
  - Absorption and scattering by atmospheric gases
- Methods for calculating the solar and thermal radiative transfer in scattering, absorbing, emitting planetary atmospheres
  - Exact methods
  - Approximate methods
- A few applications