Spatial Data Compression via Adaptive Spatial Dispersion Clustering

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Outline

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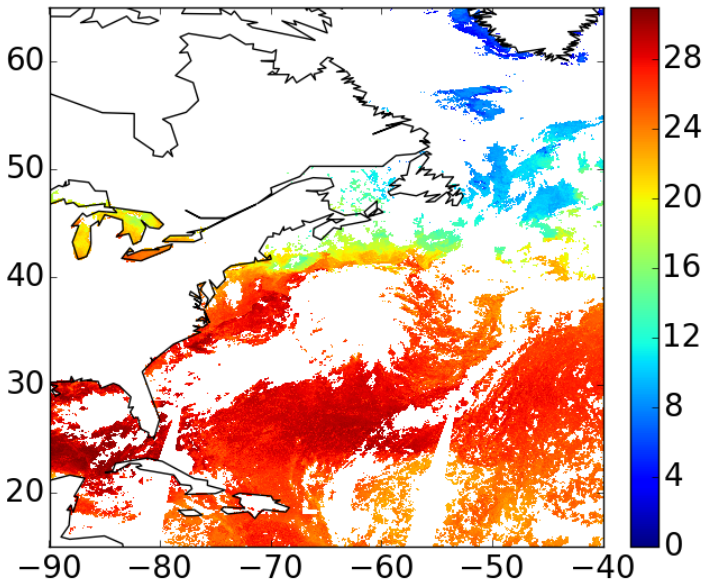
- Discussion

Motivation

Remote sensing data are a vast, largely untapped resource.

- Broad range of applications from climate science to fisheries to disaster management
- Massive but sparse
- Challenging for spatial estimation

Current spatial modeling approaches mainly focus on dimension reduction in the model.



Sea-surface temperature from the MODIS-Aqua instrument, daytime on 07/28/2015. Data are 1km resolution. There are about 2.7 million data points in this region.

Motivation

Existing methodologies might not be sufficient (e.g. computational costs and memory limitations).

Idea:

- Make the data smaller by compressing them.
- Perform spatial inference on compressed data.

Adaptive Spatial Dispersion Clustering (ASDC)

Aggregate data into spatial clusters. Requirements:

- produce spatially contiguous clusters
- preserve spatial covariance structure
- minimize information loss for prediction

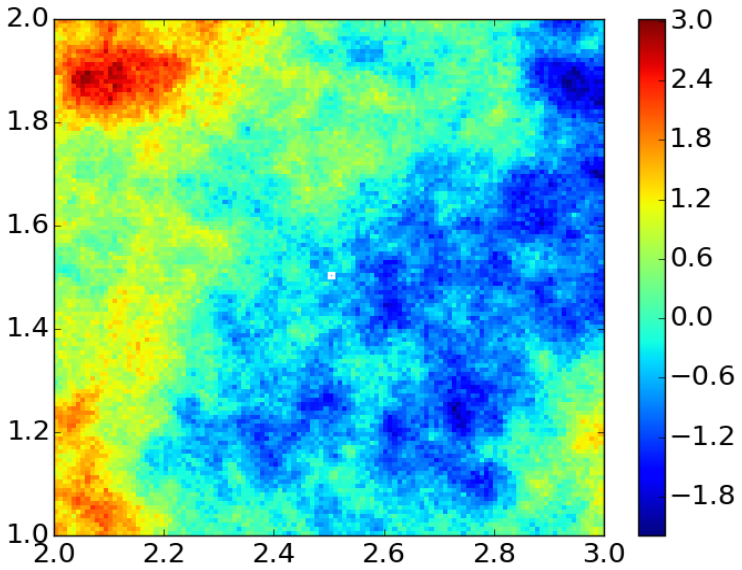
ASDC combines a spatial statistical framework with spectral clustering to meet these requirements.

Distance dispersion functions

Data: $Z(\mathbf{v}) = Y(\mathbf{v}) + \epsilon(\mathbf{v})$,

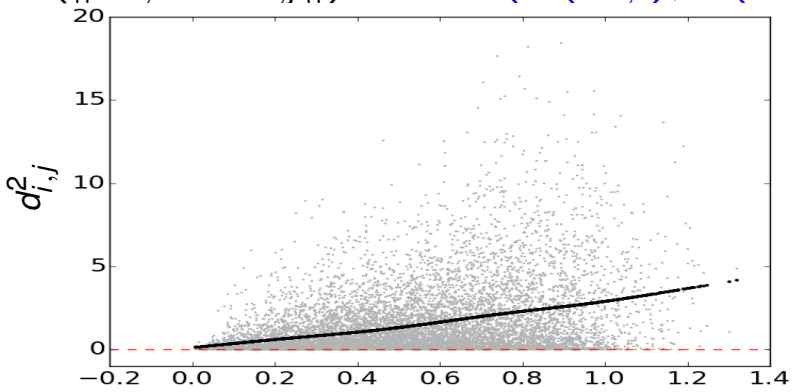
prediction location: $\mathbf{v}_0 \in \mathcal{D}$,

all other locations: $\mathbf{v}_e \in \mathcal{D}_e = \mathcal{D} \setminus \{\mathbf{v}_0\}$.

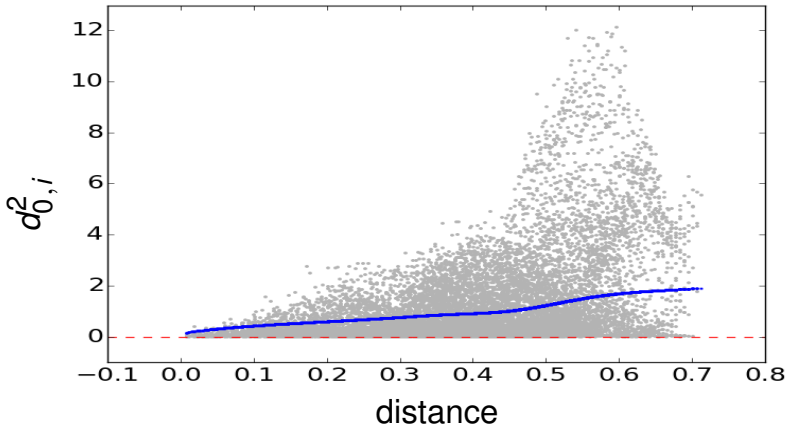


Synthetic spatial domain $\mathcal{D} \subset \mathbb{R}^2$ measurements $Z(\mathbf{v})$, $\mathbf{v} \in \mathcal{D}$.

$f_e(\|\mathbf{v}_{e,i} - \mathbf{v}_{e,j}\|) \Leftrightarrow \mathrm{cov}(Z(\mathbf{v}_{e,i}), Z(\mathbf{v}_{e,j}))$



$f_0(\|\mathbf{v}_0 - \mathbf{v}_{e,i}\|) \Leftrightarrow \mathrm{cov}(Z(\mathbf{v}_0), Z(\mathbf{v}_{e,i}))$



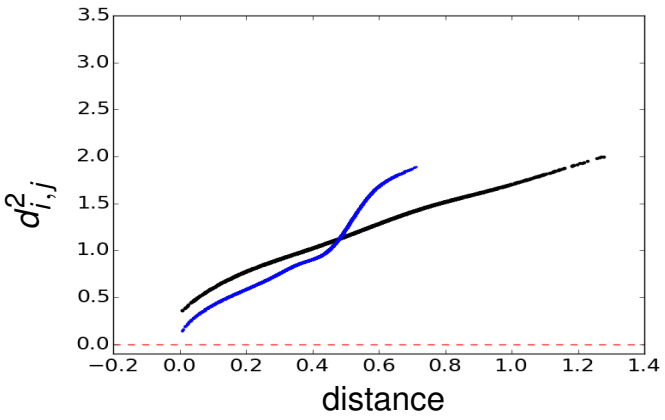
Distance dispersion matrix

Fit f_e and f_0 to $d_{i,j}^2 = |Z(\mathbf{v}_i) - Z(\mathbf{v}_j)|^2$,

$$\gamma_e^{i,j} = f_e(\|\mathbf{v}_{e,i} - \mathbf{v}_{e,j}\|),$$

$$\gamma_0^i = f_0(\|\mathbf{v}_0 - \mathbf{v}_{e,i}\|).$$

f_e and f_0 are fitted by splines



	$\mathbf{v}_0$	$\mathbf{v}_{e,1}$	$\mathbf{v}_{e,2}$	$\cdots$	$\mathbf{v}_{e,N}$	row vector
$\mathbf{v}_0$	0	γ_0^1	γ_0^2	$\cdots$	γ_0^N	$\tilde{\gamma}_0$
$\mathbf{v}_{e,1}$	γ_0^1	0	$\gamma_e^{1,2}$	$\cdots$	$\gamma_e^{1,N}$	$\tilde{\gamma}_e^1$
$\mathbf{v}_{e,2}$	γ_0^2	$\gamma_e^{2,1}$	0	$\cdots$	$\gamma_e^{2,N}$	$\tilde{\gamma}_e^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$\mathbf{v}_{e,N}$	γ_0^N	$\gamma_e^{N,1}$	$\gamma_e^{N,2}$	$\cdots$	0	$\tilde{\gamma}_e^N$

H: dissimilarity matrix/table

Spectral clustering

1. Construct a similarity matrix **W** from **H**:

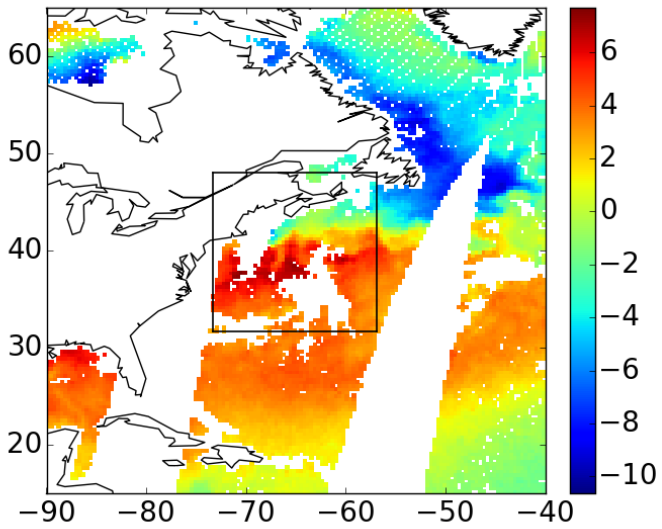
$$w_{i,j} = \exp\left(-\frac{\|\tilde{\gamma}_e^i - \tilde{\gamma}_e^j\|^2}{2\sqrt{\gamma_0^i}\cdot\sqrt{\gamma_0^j}}\right)$$

2. Apply spectral clustering [1] to **W**.

3. Obtain $N \times K$ compression matrix $[\mathbb{Q}]_{ij}$, where ij-th element is 1 if i-th location $\in \Gamma_j$, $j = 1, \ldots, K$.

4. The j-th spatial cluster is $\Gamma_j = \bigcup_{k=1}^{N} \{\mathbf{v}_k : [\mathbb{Q}]_{kj} = 1\}$.

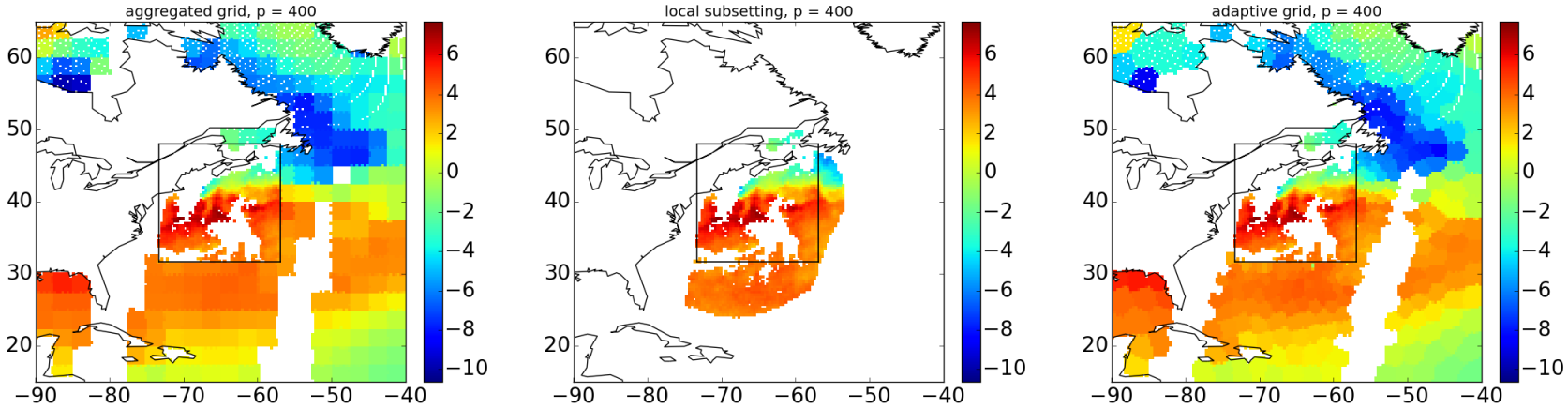
Example: Sea-surface Temperature



Sea-surface temperature from the AMSR-2 instrument on GCOM-W satellite, daytime on 07/28/2015. Data are 25km resolution. There are about 117,000 data points in this region.

- Predict SST in a region of interest (in black square) by kriging compressed data.
- Compare performance to kriging on a proximate subset ("local kriging" [2, 3]) and on binned data.
- Performance benchmark is kriged prediction using the full dataset.

Comparison of methods



binning subsetting ASDC

- A set of M prediction locations $\mathbf{V}_0 = \{\mathbf{v}_{0,1}, \ldots, \mathbf{v}_{0,M}\}$ in $\mathcal{D}_0$ (region of interest).
- Differences in performance are due to differences in compression of $\mathcal{D}_e$ (exterior region).

Step 1: Spatial compression

- Compress the data in $\mathcal{D}_e$ using ASDC, subsetting and binning.
- Obtain compressed datasets $\mathbf{\Psi}^c$, $c = \{\text{Ada}, \text{Sub}, \text{Bin}\}$:

$$\mathbf{\Psi}^c = (\mathbf{Z}_0, \psi^c(\Gamma_1), \ldots, \psi^c(\Gamma_K))^T$$

$$\psi^c(\Gamma_j) = \frac{1}{n_j} \sum_{i=1}^{n} Z(\mathbf{v}_i) 1(\mathbf{v}_i \in \Gamma_j), \quad n_j > 0,$$

is a cluster centroid and n_j is cluster size.

Step 2: Spatial prediction

- Estimate the covariance function parameters from all the available data.
- Perform kriging on full data to predict the region of interest $\mathbf{V}_0$:

$$\hat{Y}(\mathbf{V}_0) = \mathbf{A}^T\mathbf{Z}, \quad \text{where } \mathbf{A} \text{ is a matrix of kriging coefficients.}$$

- Perform kriging on compressed data $\mathbf{\Psi}^c$:

$$\hat{Y}^{\text{Ada}}(\mathbf{V}_0) = \mathbf{B}_{\text{Ada}}^T \mathbf{\Psi}^{\text{Ada}}$$
$$\hat{Y}^{\text{Sub}}(\mathbf{V}_0) = \mathbf{B}_{\text{Sub}}^T \mathbf{\Psi}^{\text{Sub}}$$
$$\hat{Y}^{\text{Bin}}(\mathbf{V}_0) = \mathbf{B}_{\text{Bin}}^T \mathbf{\Psi}^{\text{Bin}}.$$

Performance metrics and results

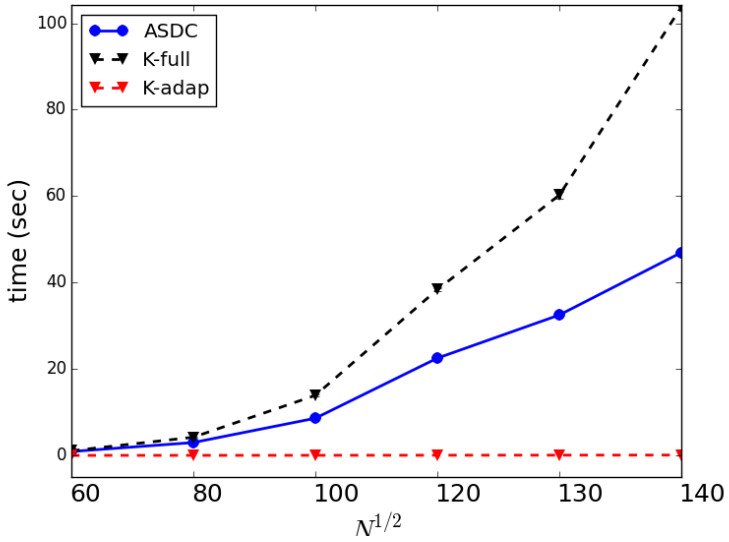
Root mean squared error (RMSE) and mean prediction error ratio (MPER):

$$\mathrm{RMSE}_c = \left[\frac{1}{M} \sum_{i=1}^{M} (\hat{Y}(\mathbf{v}_{0,i}) - \hat{Y}^c(\mathbf{v}_{0,i}))^2 \right]^{1/2}, \qquad \mathrm{MPER}_c = \frac{1}{M} \sum_{i=1}^{M} \frac{\mathrm{var}(\hat{Y}(\mathbf{v}_{0,i}))}{\mathrm{var}(\hat{Y}^c(\mathbf{v}_{0,i}))}.$$

Compressed as	RMSE			MPER		
% of original size	ASDC	Subset	Binned	ASDC	Subset	Binned
1%	0.078	0.208	0.130	1.002	1.029	1.014
2%	0.074	0.177	0.099	0.995	1.015	1.004
4%	0.063	0.106	0.085	0.995	1.005	0.999

Discussion

- Here we demonstrated good performance on a “small" data set against standard competitors.
- Key idea is to preserve both covariance structure in the exterior and between the interior and exterior.
- Simulation studies suggest that computational performance improves with data set size.
- For remote sensing, it will be necessary to combine ASDC with dimension reduction approaches.
- Other potential applications for this methodology include medical imaging, network data, health records, and more.



ASDC computation time vs data size.

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Acknowledgements

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