Statistical ARQ Link Analysis and Planning for Dynamic Links

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***Abstract*—In [1] and [2], we discussed automatic Repeat-reQuest (ARQ) link analysis and planning in terms of effective data rate, effective throughput, latency, and frame-error-rate (FER), under the standard assumption that the signal-to-noise ratio (SNR) remains the same throughout the ARQ communication session. In [3], we argued that the concept of constant SNR might not be valid when considering events over a long time horizon, as many link parameters are inherently statistical. This is particularly true for long-haul ARQ links because the channel SNR changes during subsequent re-transmissions of un-received or non-decodable frames. As shown in [3], this inaccurate assumption of constant SNR might be non-consequential for static links such as S-band and X-band, but can lead to large discrepancies in the analysis and planning of the more dynamic communication links such as Ka-band and optical communication frequencies.**

In this paper, using similar techniques developed in [3], we incorporate the effect of changing SNR, or link uncertainty, into the analysis of ARQ links. SNR is no longer considered as a fixed value, but a random variable whose long-term statistics can be characterized with a probability distribution function. We consider two limiting cases:

1. **"Fast-varying" SNR: when SNRs in subsequent re-transmissions of a code-block can assume different values, and they are independent. One example is the deep space link when the ARQ acknowledgement time is much larger than the coherency time of the channel. For communications between Earth's ground stations and spacecraft at Mars, the round trip light time is 20–40 minutes, and this is much more than the typical atmospheric coherency time of Ka-band.**

2. **"Slow-varying" SNR: when SNR values in subsequent re-transmissions of a code-block remain the same. One example is the proximity link between a low-Mars-orbit orbiter and a surface asset at Mars. In this case, the ARQ acknowledgement time is of the order of milliseconds and we can assume identical channel environment in subsequent re-transmissions.**

We expect the ARQ behavior of real-world dynamic channels would fall in between the "fast-varying" and "slow-varying" cases, thus providing interesting insights on the ARQ data return performance and latency performance.

We illustrate the aforementioned analysis using the NASA (1024, ½) low-density-parity check (LDPC) code.

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1. INTRODUCTION

The fundamental concept of Automatic Repeat-reQuest (ARQ) protocol is that when data are corrupted during transmission, messages can be re-sent multiple times until they are received and acknowledged. We assume that all data transmissions use error-correction coding for channel error-correction and/or error-detection. Much work has been done in the performance analysis of ARQ protocols. Throughput and latency analyses can be found in early papers [4][5] under the assumption that code-block errors occur independently. To analyze wireless communication channels that are characterized by fast fading and bursty errors, recent literature introduces channel models that assume an error process that is not random and is modeled as a Markovian process [6][7][8][9].

In this paper we consider the case when the ARQ system has no limit on the number of re-transmissions. The case of truncated ARQ will be discussed in a subsequent paper. The ARQ link is "error-free" in the sense that a data frame will eventually be successfully delivered (at the first transmission or a subsequent re-transmission). However, the penalties for ARQ link are (i) increased latency for re-transmission and (ii) reduced link efficiency (measured in higher power or lower data rate) to accommodate the re-transmitted data frames.

Thus, the key metrics to measure the quality of the "error-free" ARQ link are:

1. Transmission latency in some statistical sense (e.g., maximum latency, mean latency, etc.)

978-1-4673-7676-1/16/$31.00

2. Effective data rate R_{eff} in terms of the net data throughput, discounting the portion of the bandwidth that accommodates re-transmissions.

The concept of effective data rate is also applicable to "send-once" links[1]. Assuming the smallest data unit to be a frame, and denoting P_{blc} as the Frame-Error-Rate (FER), the effective data rate R_{eff} in terms of the amount of reliable data available on the received side can be measured as

$$R_{eff} = R_b(1 - P_{bk}) \tag{1}$$

where R_b is the raw data rate. Note that in this interpretation of effective data rate for the "sent-once" link, R_{eff} includes the portion of the data frames that are successfully received. The corrupted data frames are lost and are thus non-recoverable.

In many prior ARQ studies and system designs, including the recent ones [1][2] by the main author of this paper, the assumption is that the SNR is fixed throughout the ARQ communication session. This assumption can be reasonable for the following cases:

1. When the ARQ turnaround time is much shorter than the channel coherency time.

2. When the links are relatively static, like the S-band and X-band links, when the SNR is not expected to change significantly during a communication session.

In deep space communications, the ARQ turnaround time includes the round-trip light time that can be tens of minutes long, plus the data processing time at the receiving end. Also, the future deep space links are migrating toward the higher frequency links like the Ka-band and optical communication links that are susceptible to non-Gaussian and non-linear effects such as turbulence, scintillation, antenna pointing, jitter, etc. All these point to the fact that the assumption of a constant SNR in the analysis and design of an ARQ system might not be valid for deep space communications.

In this paper, using similar techniques developed in [3], we incorporate the effect of changing SNR, or link uncertainty, in the analysis of ARQ links. SNR is no longer considered as a fixed value, but a random variable whose long-term statistics can be characterized with a probability distribution function. We consider two limiting cases:

1. "Fast-varying" SNR: when SNRs in subsequent re-transmissions of a code-block can assume different values, and they are independent. One example is the deep space link when the ARQ acknowledgement time is much larger than the coherency time of the channel. For communications between Earth's ground stations and spacecraft at Mars, the round trip light time is 20–40 minutes, and this is much more than the typical atmospheric coherency time of Ka-band.

2. "Slow-varying" SNR: when SNR values in subsequent re-transmissions of a code-block remain the same. One example is the proximity link between a low-Mars-orbit orbiter and a surface asset at Mars. In this case, the ARQ acknowledgement time is of the order of milliseconds and we can assume identical channel environment in subsequent re-transmissions.

We expect the ARQ behavior of real-world dynamic channels would fall in between the "fast-varying" and "slow-varying" cases, thus providing interesting insights on ARQ data return performance and latency performance.

We illustrate the aforementioned analysis using the NASA (1024, ½) low-density-parity check (LDPC) code [13].

The rest of the paper is organized as follows: Section 2 provides a summary of prior results in the area of ARQ link analysis and statistical characterization of SNR uncertainty. Section 3 introduces new statistical ARQ link analysis techniques for the "fast-varying" and "slow-varying" ARQ channels and illustrates the analysis approaches using the NASA (1024, ½) low-density-parity check (LDPC) code. Section 4 discusses the impact of varying SNR on ARQ latency. Section 5 provides concluding remarks.

2. SUMMARY OF PRIOR RESULTS

In this section, we summarize key results in ARQ link analysis in [1] and [2], and in statistical link margin analysis in [3]. These results form the basis for the derivations of the statistical ARQ link analysis techniques, which we will discuss in detail in Section 3.

2.1 Prior Results in ARQ Link Analysis

In [1] and [2], we discussed automatic Repeat-reQuest (ARQ) link analysis and planning in terms of effective data rate, effective throughput, latency, and frame-error-rate (FER), under the standard assumption that the signal-to-noise ratio (SNR) remains the same throughout the ARQ communication session. The analytic expressions for ARQ link performance and latency are given below.

ARQ Link Performance—For the Selective Repeat ARQ protocol[2] and for a lossless acknowledgement channel, the effective data rate R_{eff} as derived in [1] is

$$R_{eff} = R_b(1 - P_{bk}) \tag{2}$$

Note that this expression is the same as the effective data rate R_{eff} of the "send-once" link in Equation (1), where R_b

[1] Links when messages are only sent once. When there are uncorrectable errors, the messages are lost.

[2] Selective Repeat ARQ protocol sends one code-block per re-transmission.

and P_{bk} are as previously defined. The difference is that the un-decodable data are lost in the case of "sent-once" link (Equation (1)), whereas the erroneous data are eventually recovered in the case of ARQ link (Equation (2)).

Let's consider the general case of a constant non-zero acknowledgement channel frame error rate P_{ack}, and the use of the Go-Back-N protocol[3]; then, the effective data rate R_{eff} is given by

$$R_{eff} = R_b \left(1 + \frac{N\left(1-(1-P_{bk})(1-P_{ack})\right)}{(1-P_{bk})(1-P_{ack})} \right)^{-1} \quad (3)$$

Now we express P_{bk} in terms of $f\left(\frac{E_b}{N}\right)$, where $f(.)$ denotes the frame error rate performance curve used in the return link, and $\frac{E_b}{N_0}$ is the information bit signal-to-noise ratio.[4] Also, $\frac{E_b}{N_0} = \frac{P_D}{N_0} - 10Log_{10}R_b$, where $\frac{P_D}{N_0}$ denotes the total data channel power signal-to-noise ratio (in dB), and can be computed from standard link analysis. Assuming R_b is a tunable parameter, we can express $\boldsymbol{R_{eff}}$ as a function of raw data rate R_b as follows:

$$R_{eff} = R_b \left(1 + \frac{N\left(1-\left(1-f\left(\frac{P_D}{N_0} - 10Log_{10}R_b\right)\right)(1-P_{ack})\right)}{\left(1-f\left(\frac{P_D}{N_0} - 10Log_{10}R_b\right)\right)(1-P_{ack})} \right)^{-1} \quad (4)$$

If we define effective signal-to-noise ratio $\left(\frac{E_b}{N_0}\right)_{eff}$ to be the energy per *reliable* information bit-to-noise spectral density, one can express $\left(\frac{E_b}{N_0}\right)_{eff}$ as a function of raw signal-to-noise ratio $\left(\frac{E_b}{N_0}\right)$ as follows:

[3] Typically, $N \geq 2$. Also, $N = 1$ corresponds to the Selective Repeat protocol.

[4] $\frac{E_b}{N_0}$ is expressed in dB, not taking into account re-transmission energy required for reliable communications.

$$\left(\frac{E_b}{N_0}\right)_{eff} = \frac{E_b}{N_0} + 10Log_{10}\left(1 + \frac{N\left(1-\left(1-f\left(\frac{E_b}{N_0}\right)\right)(1-P_{ack})\right)}{\left(1-f\left(\frac{E_b}{N_0}\right)\right)(1-P_{ack})} \right) \quad \text{(in dB)} \quad (5)$$

Note that $\left(\frac{E_b}{N_0}\right)_{eff} \geq \left(\frac{E_b}{N_0}\right)$, and there exists a minimum $\left(\frac{E_b}{N_0}\right)_{eff}$ that achieves lossless communication within the range of $\frac{E_b}{N_0}$.

ARQ Latency—For latency, we assume that when either or both of the code-block and acknowledgement messages are in error, the transmitter would wait for a predetermined time T_{out} before re-transmitting the code-block. For a well-designed ARQ system, $T_{out} \geq 2T_c + \Delta_R$, where T_c denotes the one-way-light-time and Δ_R denotes the receiver processing time to determine if the code-block is correctly decoded and to send an acknowledgement. There can be different ways to respond to missing acknowledgement messages and to those that are received and not decodable, resulting in different latency respond times to re-transmit. To simplify the problem, we assume that the transmitter always re-transmits after time T_{out} if it does not receive an acknowledgement message, or if it receives an un-decodable acknowledgement message. The code-block transmission timeline, the acknowledgement message receiving timeline, and the processing latencies are shown in Figure 1.

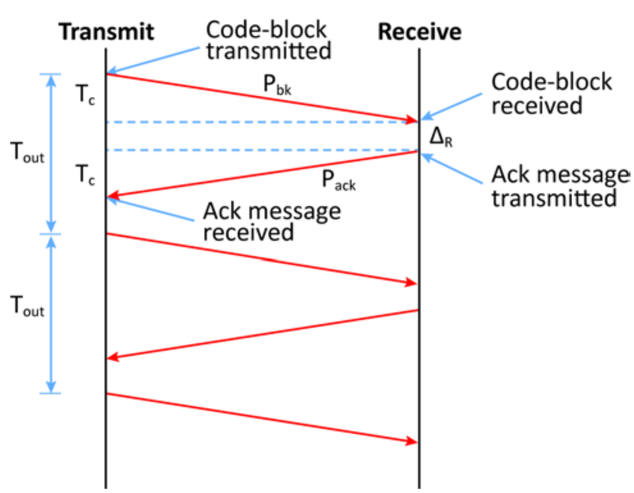

Figure 1. ARQ Transmission and Receiving Timeline

Also assuming that P_{bk} and P_{ack} do not change during the ARQ communication session, the latency of an ARQ link follows the discrete geometric distribution

$$\text{Prob}\left[\text{latency} = T_c + iT_{out}\right] = \theta(1-\theta)^{1} \quad \text{for } i = 0,1,2,\ldots, \quad (6)$$

where $\theta = (1-P_{bk})(1-P_{ack})$ is the probability that the code-block is successfully sent and acknowledged. The mean latency as observed by the receiver is computed to be $T_c + T_{out}\dfrac{1-\theta}{\theta}$.[5] Thus in an average sense, the additional latency cost of an ARQ link compared to a "send-once" link is $T_{out}\dfrac{1-\theta}{\theta}$.

2.2 Prior Results in Statistical Link Margin Analysis

In [3] we argue that during a communication session, signals can be attenuated by various unpredictable non-ideal operation effects and natural phenomena. Different random noises can also be added to the receiver, where the received SNR at the receiver is in fact a random variable. Taking into account this fluctuation in SNR, the code performance curve, and the error rate requirement, we compute the "true" SNR design point that would meet the error rate requirement. We show that this "true" SNR design point can be a lot larger than the SNR threshold value given by the code performance curve, especially for dynamic links such as Ka-band and optical communication links. We summarize the main results as follows.

Without loss of generality, we denote the distribution that corresponds to the long-term statistics of the received SNR to be $h(x|.)$. We showed in [10][11] that this SNR fluctuation could be modeled as a Gaussian process when there is no dominant component in the link. In this case, the SNR distribution can be expressed as

$$h(x|m;\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-m)^2}{2\sigma^2}} \tag{7}$$

where m is the mean of the link parameters (in dB) and σ is the standard deviation. Also m is the maximum likelihood estimate of the SNR in the Gaussian case, so m is chosen to be the SNR design value that the link analysis is based on.

In the non-Gaussian case when there are one or more dominant components, and when empirical measurements for each of the dominant components exist, we show in [12] that the SNR fluctuation can be modeled as a sum of Gaussian random variables with shifted means. Simplifying the discussion in this paper, we only consider the Gaussian distribution to illustrate the ARQ performance.

For the Gaussian case, by averaging the error rate $f(x)$ over the distribution $h(x|m;\sigma)$, the mean error rate $\bar{e}(x,\sigma)$ for a given SNR design point x is given by

$$\bar{e}(x,\sigma) = \int_{-\infty}^{+\infty} f(y) h(y\,|\,x;\sigma)\,dy \tag{8}$$

3. Statistical ARQ Link Analysis

Due to link uncertainty, concepts like effective signal-to-noise ratio and latency in the ARQ link shown in Section 2.1 should be reevaluated statistically by incorporating the idea of "true" SNR defined in Section 2.2. In this section, statistical ARQ link analysis is separated into two limiting cases: "fast-varying" and "slow-varying" ARQ channels. We will compare the performance of a coded ARQ link system using the low-density parity check (LDPC) (1024, ½) code with the "sent-once" link under a high link-uncertainty condition with $\sigma = 1.5$, which is typical for a Ka-band link [3].[6] The conventional coding performance curve (assuming SNR is constant), and the link-adjusted coding performance curve is shown in Figure 2 and Table 1 below.

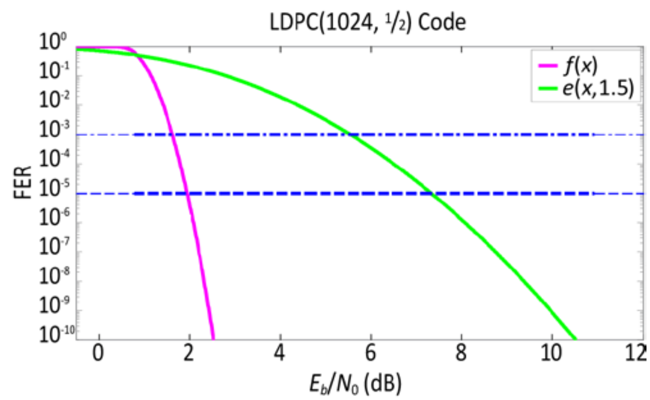

Figure 2. LDPC (1024, ½) Coding Performance

Table 1. Coding Performance for LDPC (1024, ½) Code—FER versus E_b/N_0

Coding Performance Type	FER	E_b/N_0 (dB)
Constant SNR	10^{-3}	1.62
Constant SNR	10^{-5}	1.94
Link Adjusted (σ = 1.5)	10^{-3}	5.54
Link Adjusted (σ = 1.5)	10^{-5}	7.33

3.1 Fast-varying and Slow-varying ARQ Channels

"Fast-varying" alludes to the situation when ARQ acknowledgement time is much longer than the typical atmospheric coherency time of the channel, such as for a link between a ground station on Earth and a spacecraft at Mars. In this case, the round-trip-light-time between Earth and Mars is 20 to 40 minutes long, and the channel

[5] Variance = $T_{out}^2 \dfrac{1-\theta}{\theta^2}$

[6] Though Ka-band links can have non-Gaussian components, we assume Gaussian distribution to simplify the discussion.

conditions between subsequent retransmission are assumed to be independent.

On the other end, "slow-varying" refers to the situation when channel environment changes very little for each subsequent retransmission. For example, for the proximity link between a low-orbit orbiter and a lander at Mars,[7] time between transmission and re-transmission of a code-block is of the order of milliseconds. We assume channel environment remains identical during the transmission and re-transmission of a code-block.

We expect the behavior of real-world dynamic ARQ channels would fall in between the "fast-varying" and "slow-varying" cases.

3.2 Fast-varying ARQ Channel and its Effective SNR

As discussed in Section 2.1, the effective SNR $\left(\frac{E_b}{N_0}\right)_{eff}$ assuming a constant SNR $\frac{E_b}{N_0}$ is given by Equation (5). If $\frac{E_b}{N_0}$ is fast-varying based on the definition given in Section 3.1, the frame error rates $f\left(\frac{E_b}{N}\right)$ during transmission and re-transmission of a code-block are independent of each other. In this paper, we assume $\frac{E_b}{N_0}$ has a Gaussian distribution $h\left(\frac{E_b}{N_0}\middle|m,\sigma\right)$, where m is the designed SNR operation point. Using a similar argument on counting average energy required for reliable transmission of a code-block, the effective SNR $\left(\frac{E_b}{N_0}\right)_{eff\,fast}$ could be shown to be

$$\left(\frac{E_b}{N_0}\right)_{eff\,fast} = \frac{E_b}{N_0} + 10Log_{10}\left(1+\frac{N\left(1-\left(1-\bar{e}\left(\frac{E_b}{N_0},\sigma\right)\right)\left(1-P_{ack}\right)\right)}{\left(1-\bar{e}\left(\frac{E_b}{N_0},\sigma\right)\right)\left(1-P_{ack}\right)}\right) \quad \text{(in dB)} \tag{9}$$

We apply the above analysis to the case of a coded ARQ system using the LDPC (1024, ½) code operating under a dynamic link environment typical of a Ka-band link, with $\sigma = 1.5$. We also assume a lossless acknowledgement link with $P_{ack} = 0$. The effective SNR $\left(\frac{E_b}{N_0}\right)_{eff}$ for the ARQ protocols of Selective Repeat, Go-Back-2, and Go-Back-8, and Go-Back-32, in both deterministic SNR and in fast-varying SNR cases are given in Figure 3.

The lowest effective SNR $\left(\frac{E_b}{N_0}\right)_{eff}$ for the four ARQ protocols considered in both deterministic SNR and "fast-varying" SNR cases are tabulated in Table 2.

Table 2. Lowest Effective SNR for ARQ Protocols for "Fast-varying" Case

Deterministic/ Varying	Protocol Type	Effective SNR (dB)	Raw SNR (dB)
Deterministic	Selective Repeat	1.428	1.32
Deterministic	Go-Back-2	1.5	1.4
Deterministic	Go-Back-8	1.628	1.541
Deterministic	Go-Back-32	1.741	1.66
Fast-Varying	Selective Repeat	3.099	2.09
Fast-Varying	Go-Back-2	3.669	2.78
Fast-Varying	Go-Back-8	4.626	3.963
Fast-Varying	Go-Back-32	5.402	4.84

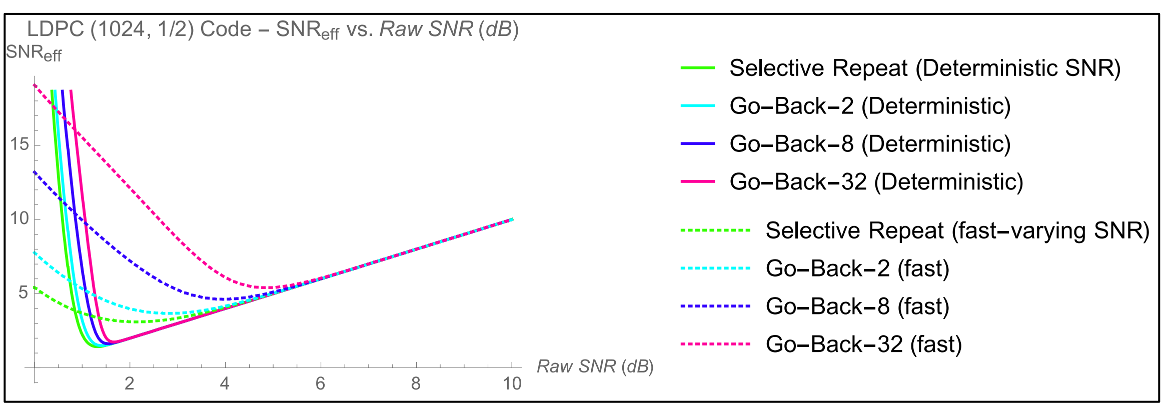

Figure 3. ARQ Link Performance for "Fast-varying" SNR Case

[7] Distance between the orbiter and lander is a few hundred kilometers.

3.3 *Slow-varying ARQ Channel and its Effective SNR*

In the slowing-varying SNR case, we assume $\frac{E_b}{N_0}$ and thus $f\left(\frac{E_b}{N}\right)$ remains constant during transmission and re-transmission of a code-block. As in Section 3.2, we assume $\frac{E_b}{N_0}$ has a Gaussian distribution $h\left(\frac{E_b}{N_0}\middle| m,\sigma\right)$, where m is the designed SNR operation point. In this "slow-varying" SNR case, the effective SNR $\left(\frac{E_b}{N_0}\right)_{eff\,slow}$ is given by

$$\left(\frac{E_b}{N_0}\right)_{eff\,slow} = \frac{E_b}{N_0} + Log_{10}\left(\int_{-\infty}^{+\infty}\left(1+\frac{N\left(1-(1-f(y))(1-P_{ack})\right)}{(1-f(y))(1-P_{ack})}\right)h\left(y\,\middle|\,\frac{E_b}{N_0};\sigma\right)dy\right) \quad \text{(in dB)} \quad (10)$$

We apply the above analysis to the case of a coded ARQ system using the LDPC (1024, ½) code operating under a dynamic link environment typical of a Ka-band link, with $\sigma = 1.5$. We also assume a lossless acknowledgement link with $P_{ack} = 0$. The effective SNR $\left(\frac{E_b}{N_0}\right)_{eff}$ for the ARQ protocols Selective Repeat, Go-Back-2, and Go-Back-8, and Go-Back-32, in both deterministic SNR and in slow-varying SNR cases are given in Figure 4.

The lowest effective SNR $\left(\frac{E_b}{N_0}\right)_{eff}$ for the four ARQ protocols considered in both deterministic SNR and "slow-varying" SNR cases are tabulated in Table 3.

Table 3. Lowest Effective SNR for ARQ Protocols for "Slow-varying" Case

Deterministic/ Varying	Protocol Type	Effective SNR (dB)	Raw SNR (dB)
Deterministic	Selective Repeat	1.428	1.32
Deterministic	Go-Back-2	1.5	1.4
Deterministic	Go-Back-8	1.628	1.541
Deterministic	Go-Back-32	1.741	1.66
Slow-varying	Selective Repeat	3.099	2.09
Slow-varying	Go-Back-2	3.188	2.19
Slow-varying	Go-Back-8	3.362	2.397
Slow-varying	Go-Back-32	3.523	2.52

4. Statistical Latency Analysis for an ARQ System

Using approaches similar to those as in Section 3, we can show that for the "fast-varying" case the mean latency L_{fast} as a function of raw SNR $\frac{E_b}{N_0}$ is given by

$$L_{fast}\left(\frac{E_b}{N_0}\right) = T_c + \frac{1-\hat{\theta}}{\hat{\theta}}T_{out} \quad (11)$$

where $\hat{\theta} = \left(1-\overline{e}\left(\frac{E_b}{N_0},\sigma\right)\right)(1-P_{ack})$, and $\frac{E_b}{N_0}$ is assumed to have a Gaussian distribution as in Equation (9). Again we consider a coded ARQ system using the LDPC (1024, ½) code operating under a dynamic link environment typical of a Ka-band link, with $\sigma = 1.5$. We also assume a lossless acknowledgement link with $P_{ack} = 0$. The mean latency as a function of raw SNR $\frac{E_b}{N_0}$ is given by (see Figure 5)

For the "slow-varying" case, using an approach similar to that as discussed in Section 3, the mean latency L_{slow} as a

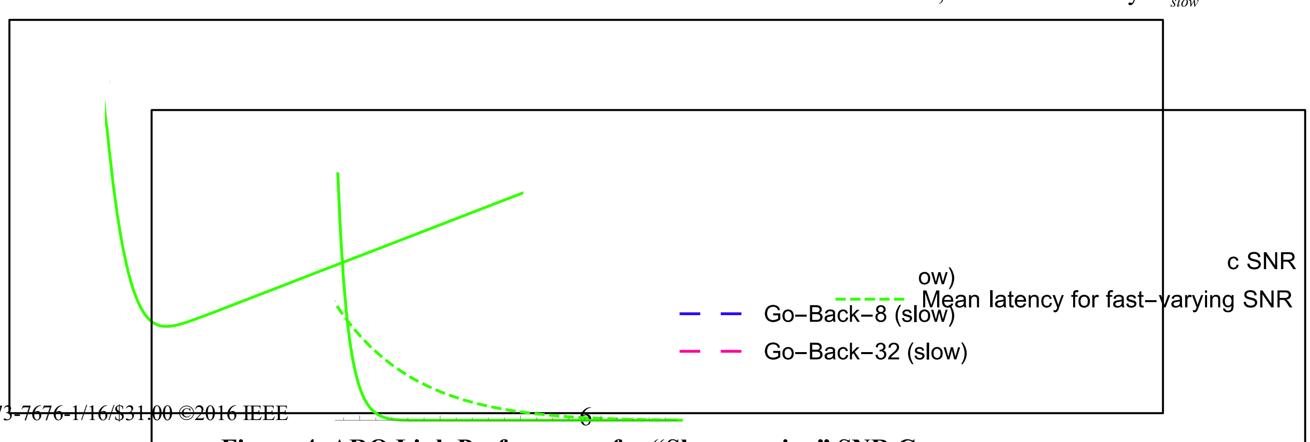

Figure 4. ARQ Link Performance for "Slow-varying" SNR Case

Figure 5. Mean Latency for Coded ARQ System for "Fast-varying" Case

function of raw SNR $\frac{E_b}{N_0}$ is computed to be

$$L_{slow}\left(\frac{E_b}{N_0}\right) = T_o + T_{out} \int_{-\infty}^{+\infty} \frac{1-\left(1-f(y)\right)\left(1-P_{ack}\right)}{\left(1-f(y)\right)\left(1-P_{ack}\right)} h\left(y \mid \frac{E_b}{N_0};\ \sigma\right) dy \tag{12}$$

However, from the above analytical expression, if there exists a finite value $x \geq -\infty$ (negative infinity) such that the monotonous frame error rate $f(x) = 1$, the mean latency can be shown to be infinity (∞). This is consistent with intuition, as in this "slow-varying" case where SNRs are assumed to be the same in the transmission and re-transmissions of a code-block, there is a non-zero probability that the frame error rate $f(x) = 1$, and the ARQ system would keep on transmitting the same code-block indefinitely.

5. Conclusion

In this paper, we incorporate the effect of changing SNR, or link uncertainty, in the analysis of ARQ links. We considered two limiting cases: (a) fast-varying SNR case when SNRs in subsequent re-transmissions of a code-block can assume different values and are independent, and (b) slow-varying SNR case when SNR values in subsequent re-transmissions of a code-block remain the same. We derive analytical expressions of effective SNR and latency for the two cases, and apply the above analysis to the case of a coded ARQ system using the LDPC (1024, ½) code operating under a dynamic link environment typical of a Ka-band link, with $\sigma = 1.5$. We observe the following interesting and insightful characteristics:

1. Using the above example, we illustrate that an ARQ scheme achieves lossless performance with an effective SNR that is significantly lower than a "send-once" link with non-zero error rate. The gain comes in the expense of additional latency. In this case, the "send-once" link achieves FERs of 10^{-3} and 10^{-5} with SNR of 5.54 dB and 7.33 dB, respectively. Whereas an ARQ system using a Selected Repeat protocol achieves lossless communication with an effective SNR of 3.10 dB, for both "fast-varying" and "slow-varying" SNR cases.

2. In the "fast-varying" SNR case that is typical of a deep space channel, different ARQ protocols exhibit significantly difference effective SNR performance. Whereas in the "slow-varying" SNR case that is typical of Mars proximity links, the difference in effective SNR performance is small. This shows that Selective Repeat protocol is suitable for a deep space link, whereas Go-Back-N protocols are suitable for a proximity link.

3. In both "fast-varying" and "slow-varying" SNR cases, the effective SNR changes slowly in the vicinity of the minimum as compared to the constant SNR case. This alleviates the need to estimate SNR accurately for the efficient operation of the ARQ system.

Acknowledgement

The author would like to thank David Morabito for the review of this paper and his helpful comments. The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The research was supported by the NASA's Space Communication and Navigation (SCaN) Program.

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Biography

Dr. Kar-Ming Cheung *is a Principal Engineer and Technical Group Supervisor in the Communication Architectures and Research Section (332) at JPL. His group supports design and specification of future deep-space and near-Earth communication systems and architectures. Kar-Ming Cheung received NASA's Exceptional Service Medal for his work on Galileo's onboard image compression scheme. He has authored or co-authored 30+ journal papers and conference papers in the areas of error-correction coding, data compression, image processing, and telecom system operations. Since 1987 he has been with JPL where he is involved in research, development, production, operation, and management of advanced channel coding, source coding, synchronization, image restoration, and communication analysis schemes. He got his B.S.E.E. degree from the University of Michigan, Ann Arbor, in 1984, and his M.S. and Ph.D. degrees from California Institute of Technology in 1985 and 1987, respectively.*

Thomas Choi is a graduate student at Georgia Institute of Technology studying Aerospace Engineering. He received his B.S. degree in electrical engineering at University of Southern California. His interests include optimization of space communication systems, software defined radio, spacecraft antennas, and small satellite missions.