A Massively Parallel Bayesian Approach to Planetary Protection Trajectory Analysis and Design

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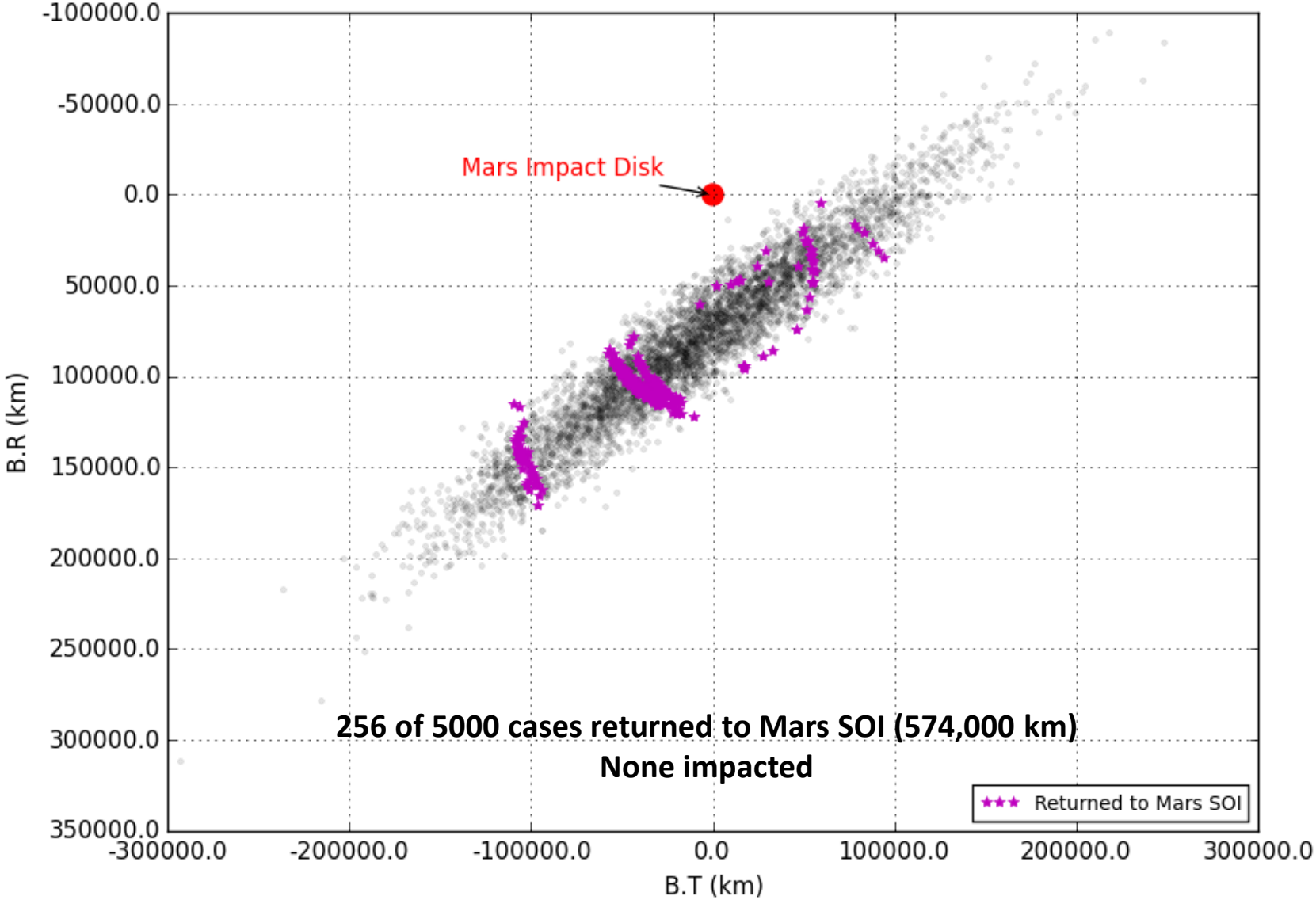
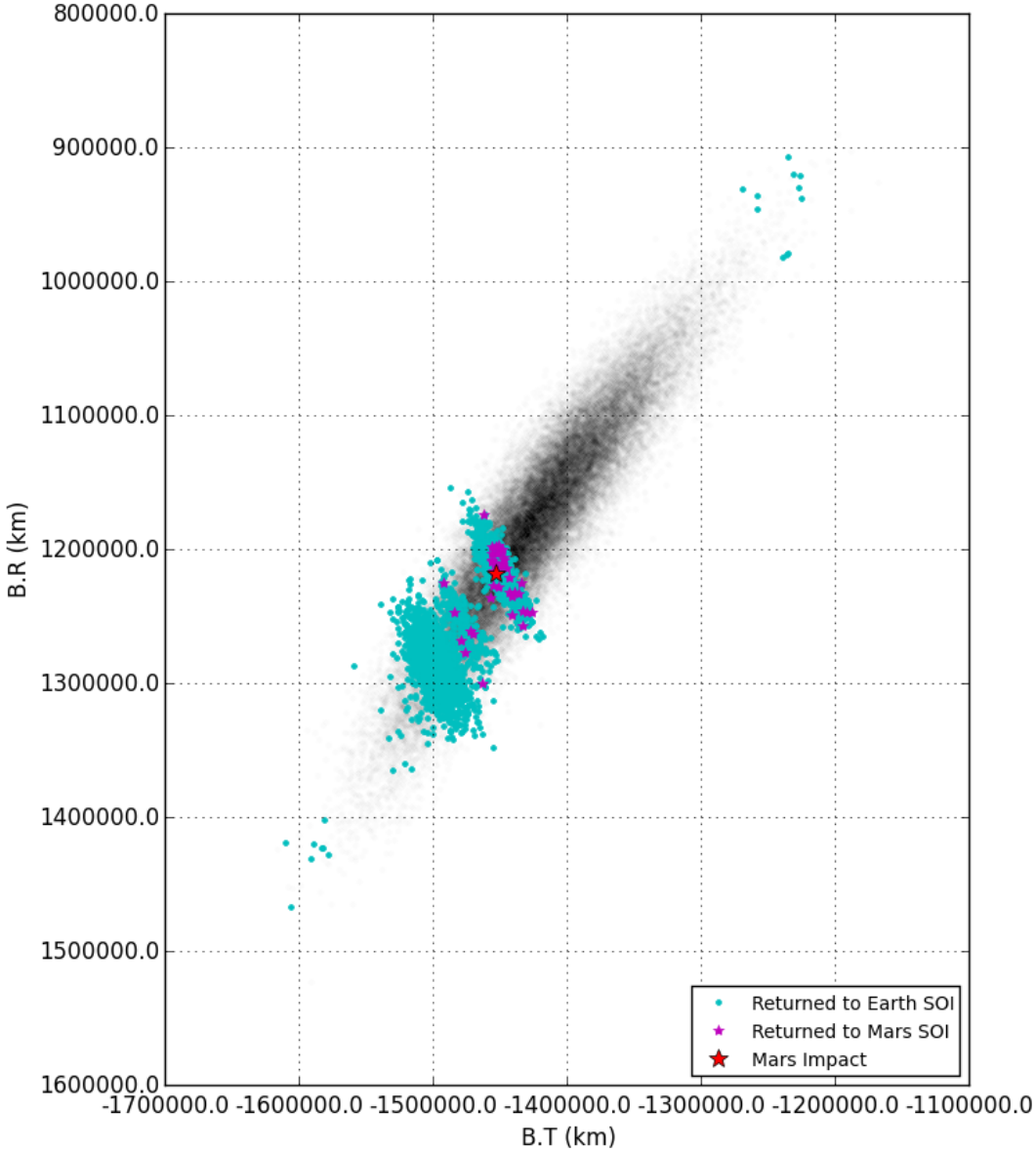
Introduction

- NASA Planetary Protection Office Requirement is that the upper stage have < 10^{-4} probability of impact within 50 years after launch.
  - Applies to the total mission, not just nominal scenario
  - Subject to upper stage reliability.
- Examples throughout use the next Mars lander, InSight
  - "Interior Exploration using Seismic Investigations, Geodesy, and Heat Transport"
  - Scheduled to launch March 2016 on ULA Atlas V 401 with a Centaur upper stage
  - 27 day launch period, compliance is demonstrated for a day if the open, middle, and close of the window is in compliance
- Centaur mission:
  - Separate from InSight
  - Perform the Contamination and Collision Avoidance Maneuver ("CCAM")
  - Dump or burn remaining residual propellants ("Blowdown")

Direct Resonance



Assisted Returns



- 50,000 case Monte Carlo
- Many return to Earth SOI (924,000 km)
- A few thus returned to Mars
- A single case impacted Mars

Statistical Formulation

$$P(I) = \begin{aligned} & P(\bar{s})\left[P(I_0|\bar{s}) + P(\bar{I}_0|\bar{s})P(I_{50}|\bar{I}_0,\bar{s})\right] + \\ & P(s)P(\bar{c})\left[P(I_0|s,\bar{c}) + P(\bar{I}_0|s,\bar{c})P(I_{50}|\bar{I}_0,s,\bar{c})\right] + \\ & P(s)P(c)P(\bar{b})\left[P(I_0|s,c,\bar{b}) + P(\bar{I}_0|s,c,\bar{b})P(I_{50}|\bar{I}_0,s,c,\bar{b})\right] + \\ & P(s)P(c)P(b)\left[P(I_0|s,c,b) + P(\bar{I}_0|s,c,b)P(I_{50}|\bar{I}_0,s,c,b)\right] \end{aligned}$$

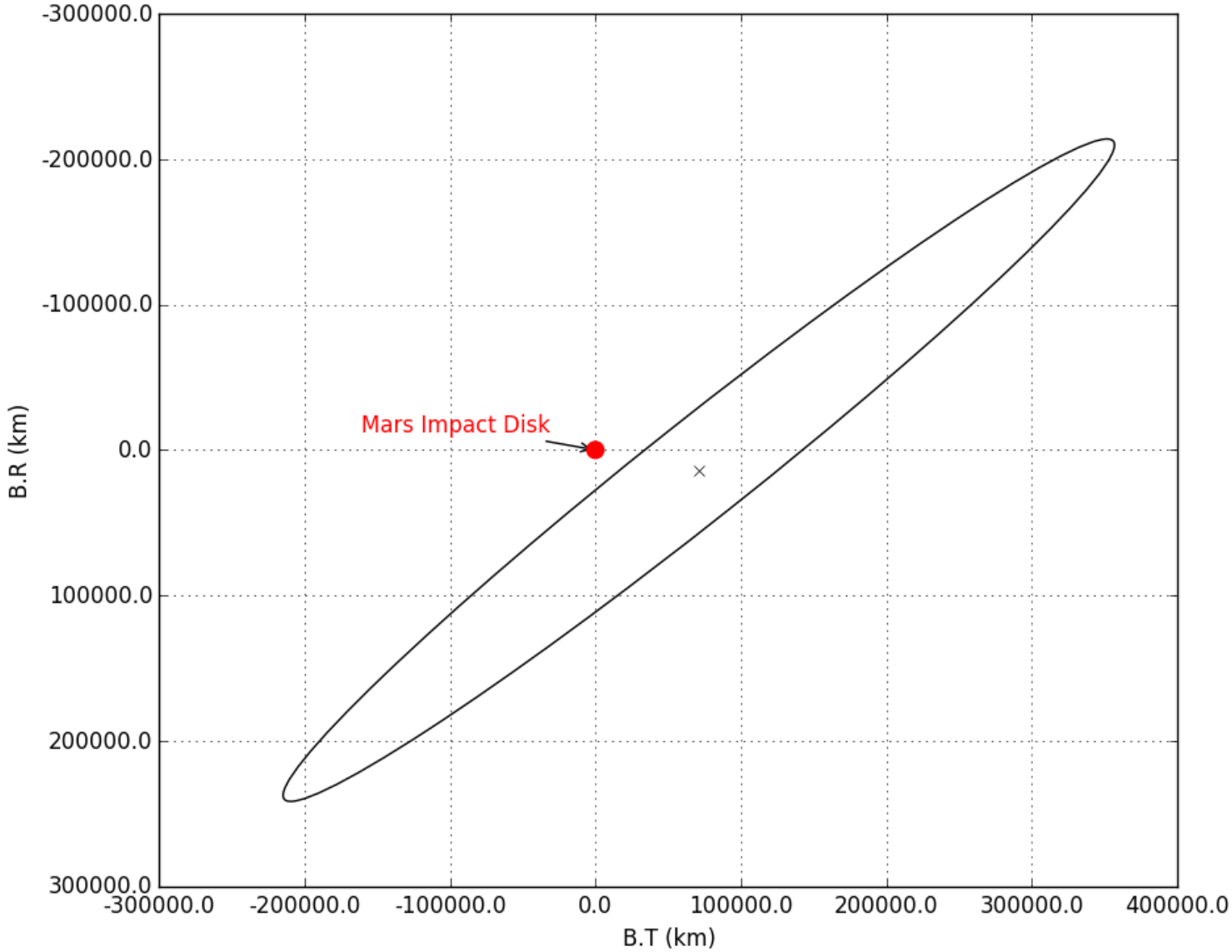
Probabilities that the Centaur achieves each step in its mission

Probabilities of impact on the first encounter

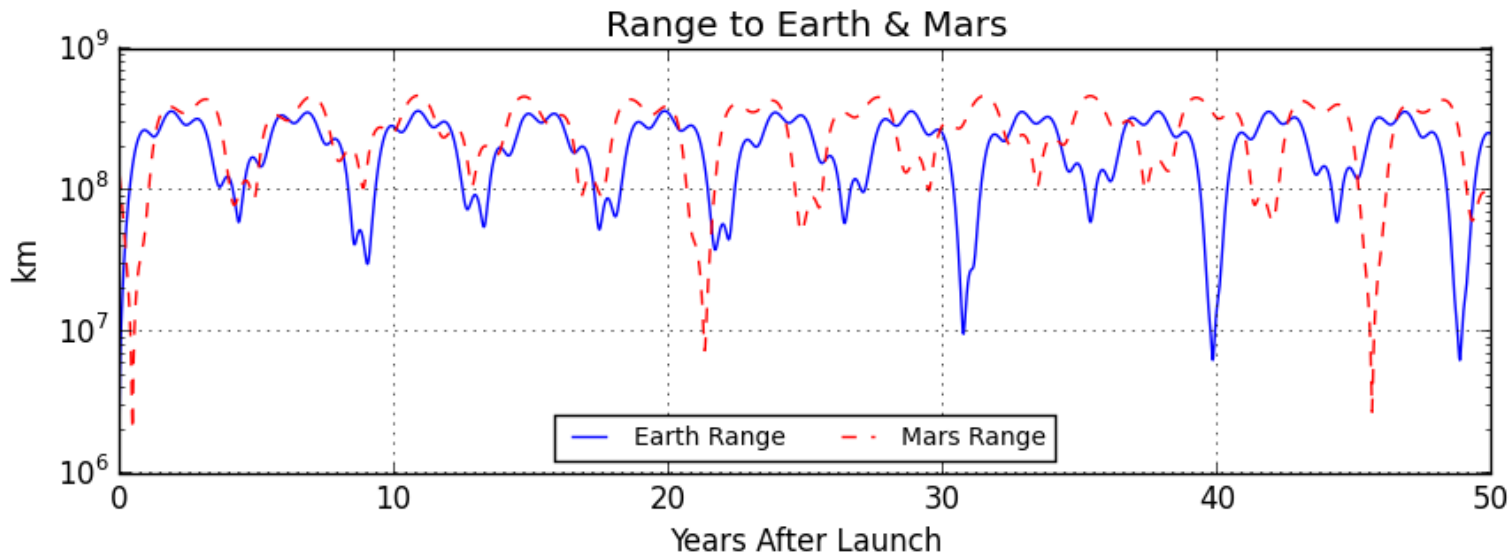
Probabilities of impact after the first encounter

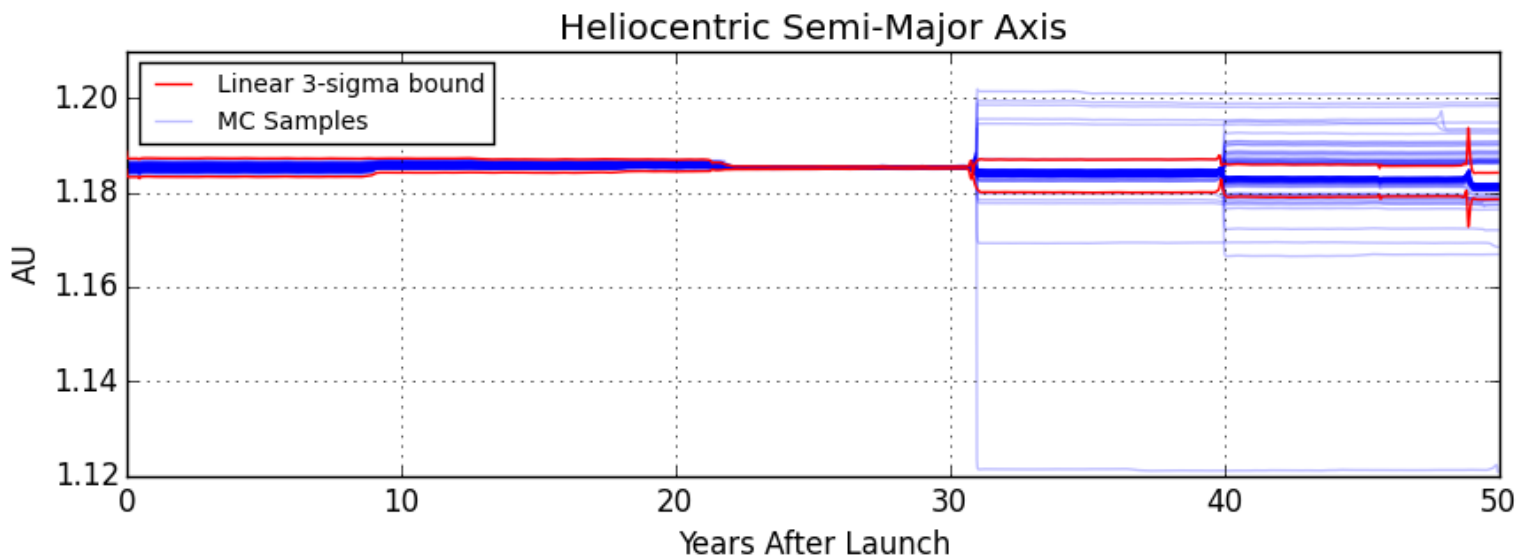
Recall: $P(\bar{e}) = 1 - P(e)$

First-Encounter Impacts



Limits of Linear Mapping





Confidence Intervals

- Normal Approximation (Wald Interval)

$$CI = \left\{ \hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right\}$$

  - If estimate is zero, interval is { 0 , 0 }
  - If estimate is small, lower bound can be negative
  - Requires a relatively large number of recorded events
  - Effectively unbounded number of Monte Carlo runs required

- Wilson Interval

$$CI = \left\{ \frac{n}{n+z^2} \left[\hat{p} + \frac{z^2}{2n} \pm z \sqrt{\frac{4n\hat{p}(1-\hat{p}) + z^2}{4n^2}} \right] \right\}$$

$$CI|_{x=0} = \left\{ 0, \frac{z^2}{n+z^2} \right\}$$

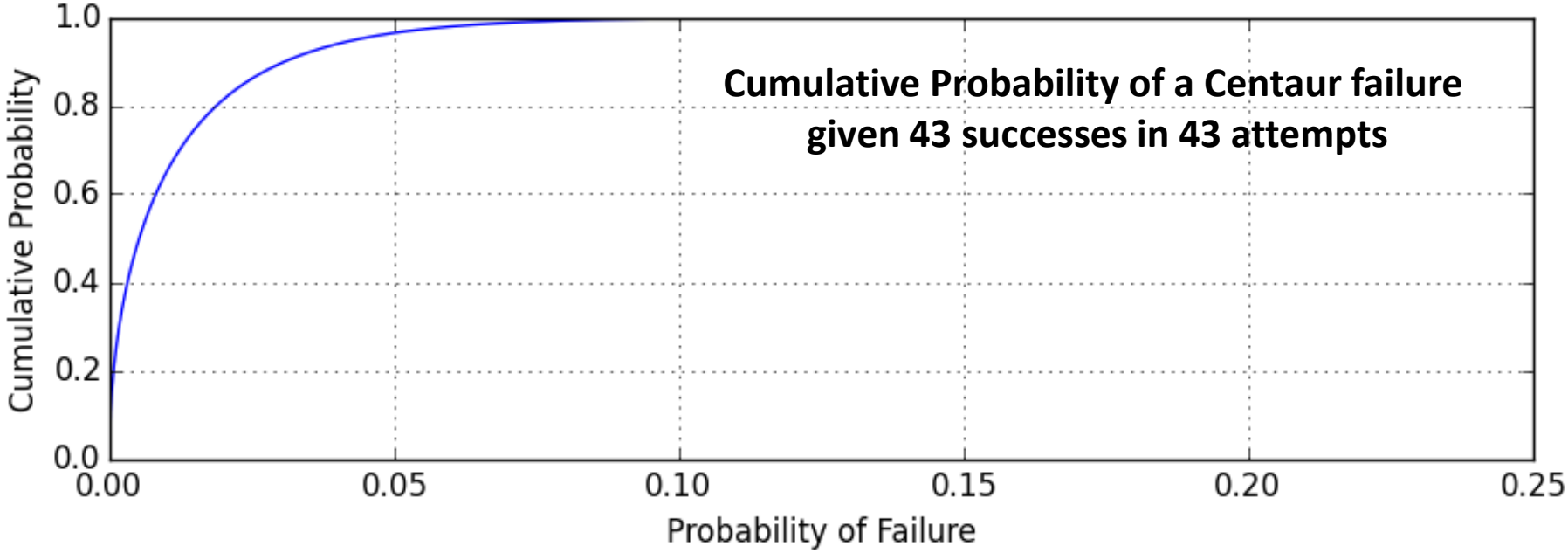
  - Well defined behavior at extreme estimates
  - Known to be conservative

A Brute Force Approach

- Use the Wilson interval for the 50 year probabilities of impact and ignore the Centaur reliability.
  - If each term of the equation < 10^{-4}, then the Centaur reliability is irrelevant
- Number of runs:
  - 85,169 for scenarios likely to have direct resonances
  - 66,352 otherwise
- Requires 10-computer years to complete.

A Bayesian Approach

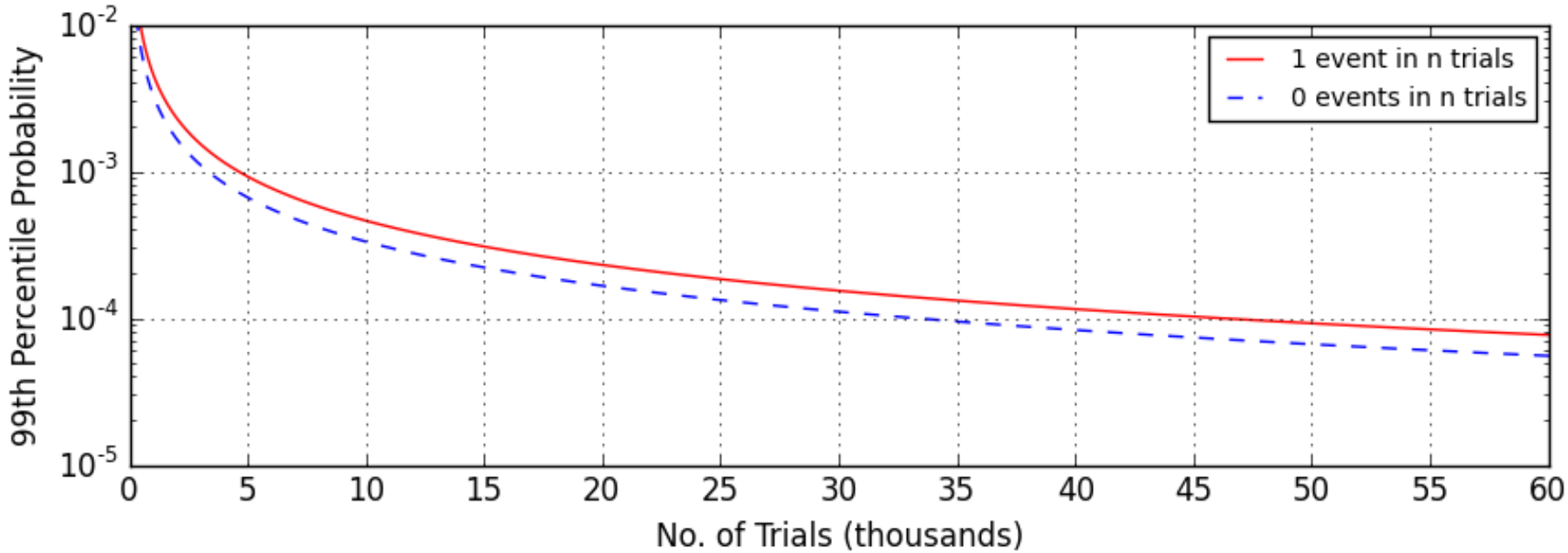
- The Beta distribution: $\beta(\alpha,\beta)$, captures the distribution of the estimate of probability of an event given prior experience
  - α: number of recorded events
  - $\alpha+\beta$: number of trials
  - Increment both by 0.5 if either is zero.



The Method

- Determine the first-encounter probabilities of impact for each of the four scenarios (no-separation, no-CCAM, no-blowdown, and the nominal mission).
- Perform 50-year Monte Carlos for each of the four scenarios and record how many cases impact Mars during the integration.
- Generate one million samples of each of the seven Beta distributions representing the probability of each of the scenarios and the resulting probability of impact.
- Solve the equation one million times with each sample from the seven sets of Step 3 and the results from Step 1.
- Sort the resulting values and determine the 99th percentile value.

Sizing the Monte Carlo



- Assume:
  - Centaur reliability: 0.9
  - First Pass Probability of Impact:
    - 10^{-4} for anomalous scenarios
    - 0 for nominal mission
- Then, to meet 10^{-4} requirement:
  - Anomalous scenario probability of impact $\approx 10^{-3}$
  - Nominal scenario probability of impact $\approx 10^{-4}$

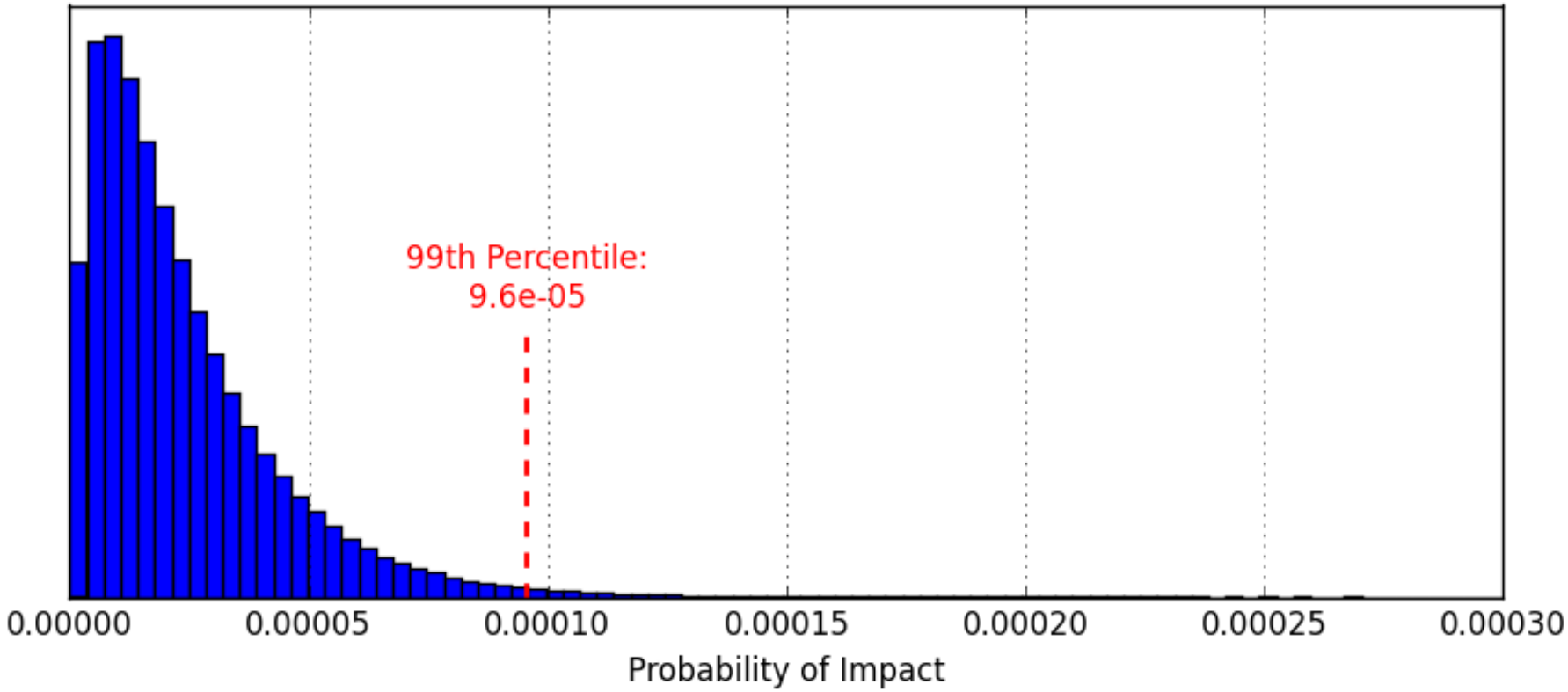
An Example

Example First-Encounter Probabilities of Impact

Scenario	First-Encounter Impact Probability
Failure to separate	9.0×10^{-6}
Failure to perform CCAM	9.3×10^{-6}
Failure to perform the blowdown	1.4×10^{-8}
Nominal Mission	1.3×10^{-300}

Example Results from Monte Carlos

Scenario	Monte Carlo Size	Number of Impacts	Beta Distribution
Failure to separate	5,000	0	β(0.5, 5000.5)
Failure to perform CCAM	5,000	0	β (0.5, 5000.5)
Failure to perform the blowdown	5,000	1	β (1, 4999)
Nominal Mission	50,000	1	β (1, 49999)



Applying Parallel Computing

- For InSight, 8,265,000 Monte Carlos were required to demonstrate compliance using this method (731 computer-days)
- Monte Carlos are embarrassingly parallel
  - Each initial condition is an independent draw
  - No propagation depends on any other
- 776 CPUs available, theoretically reduces runtime to 24 hours
  - Other users on the system
  - Real-world performance ≈80 hours

Conclusion

- Space is big; Mars is small. The probabilities of impact are accordingly tiny.
  - Textbook statistical methods require an unbounded number of Monte Carlo runs
  - Brute force frequentist approach places an upper bound on the problem, but still intractably large.
- Use the demonstrated reliability of the upper stage and a Bayesian approach to reduce the number of propagations required.
- Apply parallel computing to demonstrate compliance in 80 hours instead of two years.