Statistical Ka-Band Link Analysis

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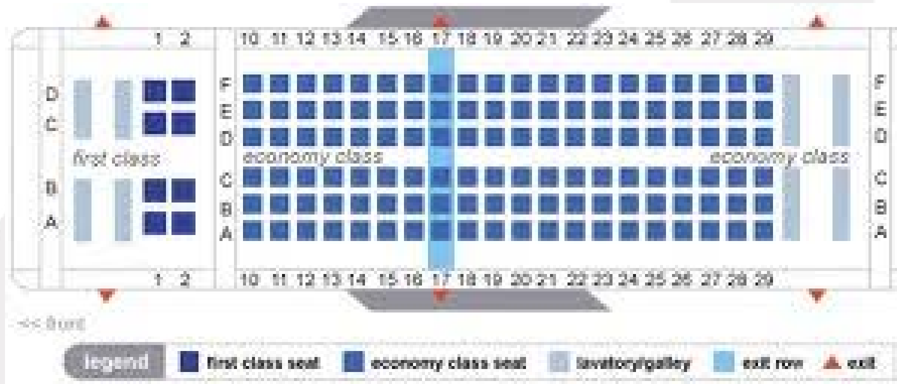
Outline of Talk

- Introduction – Motive of Study
- Review of Link Budget and Statistical Link Analysis
- Weather Availability and Weather Model
- Outline of New Techniques for statistical Ka-band Link Analysis
- Comparison of Current and New Approaches
- Concluding Remarks

Introduction – Motive of Study (1)

- Unlike S- and X- bands, Ka-band link is highly susceptible to weather effects
  - Slow-fading phenomena with long coherence time
  - Range of Ka-band weather loss can be 10+ dB's
  - Non-linear and non-Gaussian
- Accurate statistical link characterization can
  - Avoid overly pessimistic link design – Waste of dB's
  - Enable link margin to be expressed in statistically quantifiable term of tolerable risk (e.g. 5% chance of not closing the link for a 95% weather availability)

Introduction – Motive of Study (2)



- An analogy…
  - Link analysis planning for mission data return is analogous to booking passengers on a flight – once the plane takes off, all empty seats are useless

Review of Link Budget/Analysis (1)

- Link analysis is the tabulation of useful signal power and interfering noise power along the signal propagation path from transmitter to receiver
- Signal and noise terms are mathematical abstractions of the performance behavior
  - Deterministic: Transmission power, antenna gains, etc.
  - Statistical: weather effects, AWGN, axial ratio mismatch

Receivable Signal Hot body noise, cosmic noise, etc. Equipment noise (AWGN)

$$y_k = \alpha(x_k + n_k) + n_k$$

Noise is additive

Attenuation is multiplicative

$$R_b \leq \frac{\frac{G}{T} EIRP}{k L_o L_s \left(\frac{E_b}{N_o}\right)_{Th} M}$$

Need good understanding on
- Laws of Physics
- Mathematics Principles
- CONOPS

Review of Link Budget/Analysis (2)

- By “summing” up the link terms (in dB), one can come up with an overall signal-to-noise ratio (SNR) estimate to compare with some SNR thresholds to derive the supportable data rate and margin

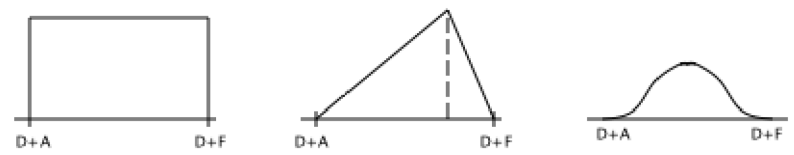
$$\frac{P_r}{N_0}(dB-Hz)=EIRP(dB-W)+G/T(dB-K^{-1})-k(dB-m^2kgs^{-2}K^{-1})-L_s(dB)-L_o(dB)$$

- Problem with link budget (deterministic approach)
  - Use worst-case numbers
  - Margin policy makes no statistical sense. Why 3 dB?

Review of Link Budget/Analysis (3)

- Statistical link analysis treat each link parameter as a random variable, each associated with a design value, a min value, a max value, and a pdf
- This in turn translates into mean and variance, and the resulting SNR is assumed to be Gaussian

Computation of mean and variance of common pdf (from J. Yuen's Book)



Uniform	Triangular	Gaussian
$M = D + (F+A)/2$	$M = D + (F+A)/3$	$M = D + (F+A)/2$
$V = (F-A)^2/12$	$V = (F^2+A^2-AF)/18$	$V = (F-A)^2/36$

D = Design value
M = Mean
V = Variance
A = Adverse tolerance, which has a negative value for all link parameters except noise spectrum density and noise bandwidth
F = Favorable tolerance, which has a positive value for all link parameters except noise spectrum density and noise bandwidth

For noise spectral density and noise bandwidth the favorable tolerance is a negative number while the adverse tolerance is a positive number.

Review of Link Budget/Analysis (4)

- By Lyapunov's principle, the Gaussian assumption can only be upheld if there is no "dominant term(s)"

The Lyapunov's condition states that for a sequence of independent variables x_i 's such that each x_i has a finite mean m_{x_i} (m_{x_i} can be different from m_{x_j}) and a finite variance $\sigma_{x_i}^2$ ($\sigma_{x_i}^2$ can be different from $\sigma_{x_j}^2$), then the radom variable $z = \sum_i x_i$ converges to a Gaussian distribution $N(m_z, \sigma_z^2)$ as long as the following condition is satisfied:

$$\text{For some } \delta > 0, \lim_{N \to \infty} \frac{1}{\sigma_z^{2+\delta}} \sum_{i=1}^{N} E\left(\left|x_i - m_{x_i}\right|^{2+\delta}\right) = 0$$

- But Ka-band weather effects are dominant term(s)
  - Ka-band weather loss variance can be 10+ times the sum of other variances combined
- Current statistical link analysis use worst-case loss for a given weather availability
- New mathematical techniques is needed to accurately perform statistical Ka-band link analysis

Weather Availability and Weather Models (1)

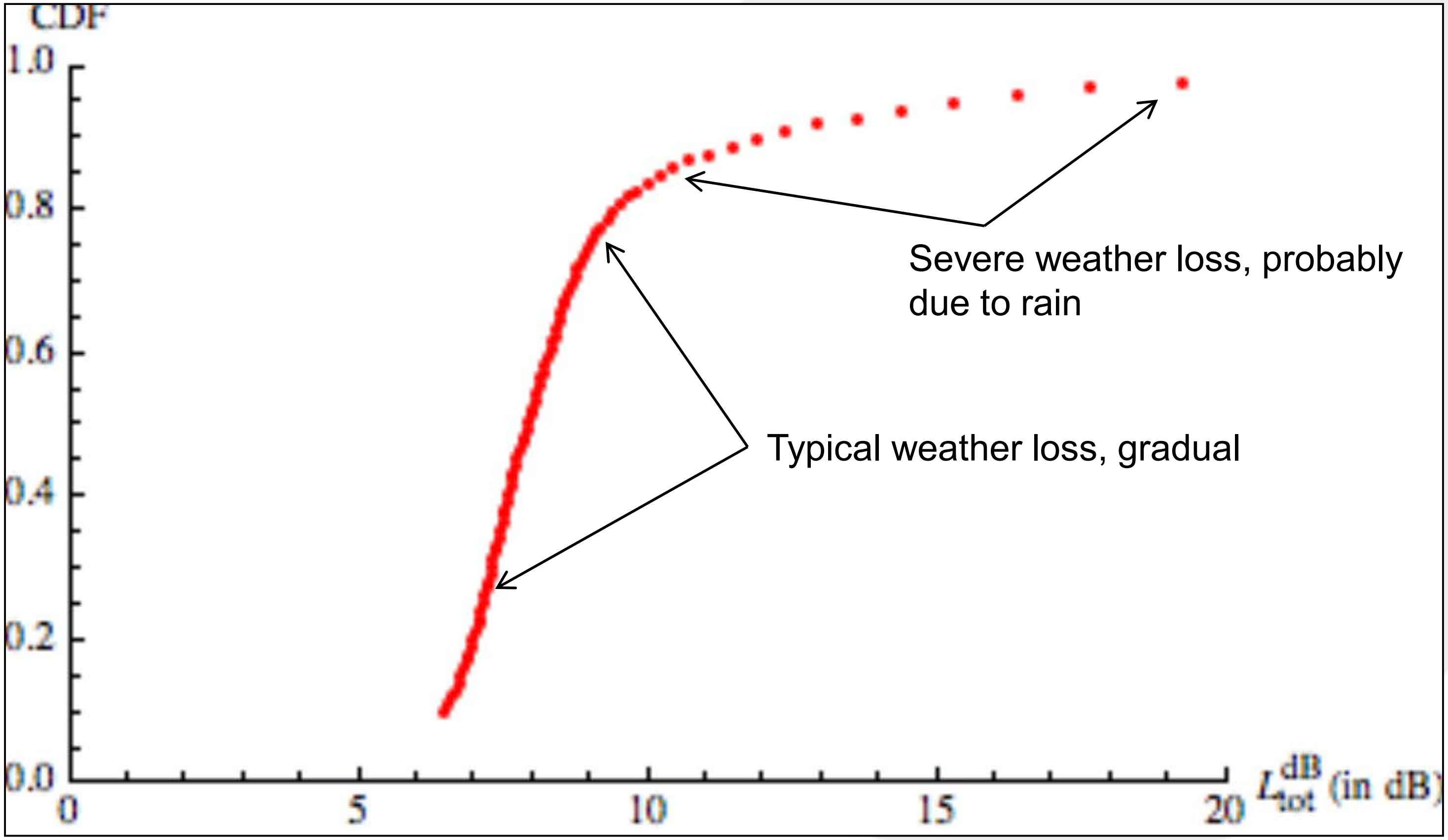
- Weather availability (CD in %) is the long-term statistics of the weather effects that impact the link
  - Ka-band weather effects typically are slow-fading phenomena with long coherence times, affecting a large fraction, if not all of the entire communication session
  - One can regard weather availability as the percentage of communication sessions such that the dB loss due to weather effects is less than a certain value
    - For example, the dB loss due to weather effects for weather availability CD of 90% at 10^{o} elevation angle of the Madrid site is 10 dB or lower (see Figure 2 of paper)

Weather Availability and Weather Models (2)

- Weather effects (esp. Ka-band) include
  - Increase in system noise temperature T_{sys} due to increase in atmospheric noise temperature T_{atm}
  - Atmospheric attenuation L_{atm} resulting from the aggregate of oxygen, water vapor, clouds, and rain, etc.
    - ITU models the weather effects individually, double counting the loss
- Relationship between T_{atm} and L_{atm} can be found in Section III of paper and 810-005
- Decades of measurements of L_{atm} at all DSN sites are available to compute T_{atm} and overall weather loss L_{tot} (dB) for a given elevation angle θ

Weather Availability and Weather Models (3)

- Example of CDF of L_{tot} of Madrid at 10° elevation



Outline of New Techniques for Statistical Ka-band Link Analysis(1)

- Sum of link parameters *S* excluding the weather effects has Gaussian distribution

$$N(s,m,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(s-m)^2}{2\sigma^2}} \qquad F(s,m,\sigma)=\int_{-\infty}^{s}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{(s-m)^2}{2\sigma^2}}ds$$

- From CDF of L_{tot}, compute the discrete probability distribution conditioned on the weather availability CD

$L_D=\{l_{10},l_{11},\cdots l_{CD}\}$ with probability $\{P_{10},P_{11},\cdots P_{CD}\}$, where l_i corresponds to the i-th percentile value of the total weather loss $L_{tot}^{dB}(\theta)$

- Complicated convolution ➔ sum of Gaussian

$$X=S+L_D$$

$$P[X\le x]=\sum_{i=10}^{CD}P_i\times F[x,m+l_i,\sigma]$$

Outline of New Techniques for Statistical Ka-band Link Analysis(2)

- Solving the above expression, one can compute the SNR value (mean + margin) required to guarantee a certain probability of closing the link
  - For example, the DSN downlink margin policy is 2σ (in Normal sense), which corresponds to a probability of 0.97725

Comparison of Current and New Approaches

Madrid

Weather Availability (%)	Current Approach (dB)	New Approach (dB)	Difference (dB)
80	13.53	12.65	0.88
90	16.35	14.66	1.68
95	20.40	17.72	2.69
99	29.80	22.54	7.26

Goldstone

Weather Availability (%)	Current Approach (dB)	New Approach (dB)	Difference (dB)
80	12.88	12.20	0.68
90	14.11	12.91	1.20
95	17.21	14.77	2.44
99	24.92	19.23	5.69

Canberra

Weather Availability (%)	Current Approach (dB)	New Approach (dB)	Difference (dB)
80	13.44	12.64	0.80
90	14.67	13.54	1.13
95	16.43	14.68	1.75
99	23.82	17.79	6.03

Concluding Remarks

- Link Budget (deterministic link parameters) can be overly conservative for Ka-band link analysis
- Current statistical link analysis is better, but is still conservative due to the use of worst-case weather loss for a given weather availability
- Using sound statistical principle the new approach removes the artificial conservatism of the current link analysis approaches and yields a lower and tightly upper bound of the received power required to close the link
  - Computation is simple also

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