

Flyby Error Analysis Based on Contour Plots for the Cassini Tour

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The maneuver cancellation analysis consists of cost contour plots employed by the Cassini maneuver team. The plots are two-dimensional linear representations of a larger six-dimensional solution to a multi-maneuver, multi-encounter mission at Saturn. By using contours plotted with B•R and B•T components, it is possible to view the effects on ΔV for various encounter positions in the B-plane. The plot is used in operations to help determine if the Approach Maneuver (ensuing encounter minus three days) and/or the Cleanup Maneuver (ensuing encounter plus three days) can be cancelled and also is a linear check of an integrated solution.

Nomenclature

OTM	= Orbit Trim Maneuvers
$LAMBIC$	= Linear Analysis of Maneuvers with Bounds and Inequality Constraints
ΔV	= change in velocity
ΔV_{REF}	= change in velocity given in the reference trajectory
OD	= Orbit determination
B	= B-vector or B-plane
R	= R-vector of the B-plane
T	= T-vector of the B-plane
T_n	= nth encounter of Titan
S	= the incoming asymptote
$C3$	= excess velocity or characteristic energy
r_o	= nominal trajectory Apoapsis Maneuver location
Δr	= difference in the Apoapsis Maneuver location for a perturbed trajectory
ΔB_{xy}	= the variations in the B-plane with a perturbed trajectory
$\Delta B \bullet R_y$	= y component of grid points in the B-plane
$\Delta B \bullet T_x$	= x component of grid points in the B-plane
ΔB_{RT}	= Grid variation vector
ΔB_A	= Asymptote change B-plane state vector
$B \bullet R$	= vertical axis of the B-plane
$B \bullet T$	= horizontal axis of the B-plane
$V1$	= the magnitude of the velocity for a Cleanup Maneuver
$V2$	= the velocity magnitude for an Apoapsis Maneuver
i	= leg index
M	= the B-plane mapping
K	= K-matrix—a maneuver capability matrix
$J_{x,y}$	= ΔV calculation of a B-plane grid point for an Approach Maneuver
$J_{x,y,c}$	= ΔV calculation of a B-plane grid point for a Cleanup Maneuver
$G_{x,y}$	= grid point value on the contour plot for an Approach Maneuver
$G_{x,y,c}$	= grid point value on the contour plot for a Cleanup Maneuver
LFT	= Linearized Flight Time

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I. Introduction

The Cassini-Huygens mission to Saturn launched on October 15, 1997 and successfully entered Saturn orbit on July 1, 2004. The orbital phase started after Saturn Orbit Insertion and continued through the prime mission which ended formally on 30-June-2008 with the extended mission commencing after this date. The driver for the Cassini-Huygens orbital segment are the gravity-assist encounters with Titan—Saturn’s largest moon. In between encounters with Titan or any other Saturnian moon, there are typically three opportunities for Orbit Trim Maneuvers (OTMs). These three maneuvers have been named the Cleanup Maneuver—which takes place at approximately three days after an encounter; the Apoapsis Maneuver or Shaping Maneuver—that usually takes place around Apoapsis of the orbit; and the Approach Maneuver—which takes place approximately three days before the next encounter. These maneuvers can be seen in Fig. 1¹. The targets for the maneuvers are $B \cdot R$ and $B \cdot T$ components of the B-plane and Linearized Flight Time (LFT).² A description of the B-plane is found in the appendix.

During the eleven years since launch, there have been many Trajectory Correction Maneuvers, OTMs implemented, and also a number of canceled maneuvers. In order for the Cassini-Huygens Maneuver Team to provide information for project management to determine if cancellation of a maneuver was the appropriate course of action, some tools had to be developed that would quantify what would happen with and without a particular maneuver. Thus, the maneuver cancellation analysis software was developed.³ One piece of this maneuver cancellation analysis software is the ΔV cost contour plot.

The contour plot is a two-dimensional linear representation of a larger six-dimensional solution to the multi-maneuver, multi-encounter operation that takes place on the Cassini-Huygens mission at Saturn. Shown on the plots are ΔV costs for different B-plane encounter positions of the spacecraft with respect to the nominal B-plane encounter aimpoint. There are two versions of the plot—one for the Approach Maneuver and another for the Cleanup Maneuver. These plots are generated by using a computer program named LAMBIC (Linear Analysis of Maneuvers with Bounds and Inequality Constraints), which produces the statistics of ΔV magnitude and delivery accuracy by simulating the execution of a sequence of maneuvers through the use of the Monte Carlo method. Included in LAMBIC is an optimization routine that has been used to develop maneuver optimization strategies and design OTMs. LAMBIC⁴⁻⁵ can also generate ΔV costs corresponding with grid points that represent flyby errors. Data associated with the grid points are used to make the contours, but the plots are not made by the LAMBIC program. The contour plots are produced by MATLAB⁶ with the grid point data from LAMBIC.

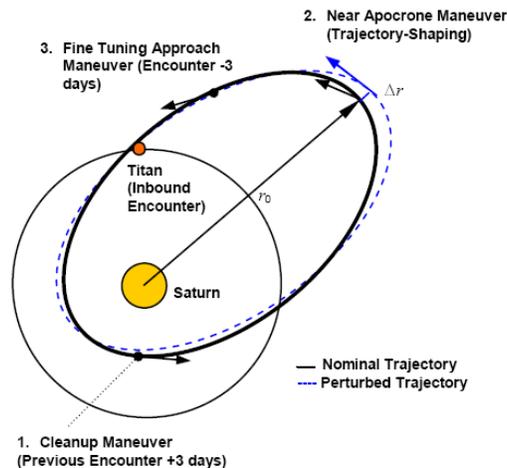


Figure 1. Maneuver Locations of a typical Cassini-Huygens encounter with Titan. An illustration of a Nominal and Perturbed Trajectory is included.

II. Methodology and Development

The contour plots are made by plotting ΔV costs for a grid of B-plane data points, which makes it possible to view the effects on ΔV for various encounter positions in the B-plane, as shown in Fig. 2. Plotting the ΔV contours on the B-plane allows for the maneuver implementation delivery ellipse and current orbit determination (OD) delivery ellipse to be drawn on the plot. (B-plane information can be found in the appendix.) By doing this, statistics are able to be applied to the contour plot thus giving the navigation team probabilities that certain ΔV costs will or will not occur.

Originally, the Approach maneuver contour plot was used as a visual aid to show the sensitivity of flyby errors; if the nominal flyby aimpoint represents a classical quadratic minimum; and if the encounter would be on the flyby altitude boundary (i.e. the boundary of the altitude requirement to prevent the spacecraft from tumbling due to atmospheric drag)—all of which is of limited value for operations. However, after some appropriate modifications were made to account for the change between incoming nominal and perturbed asymptotes, the Approach Maneuver cost contour plot has become an important tool in the decision making process of canceling Approach Maneuvers. The modifications will be further covered in Section III.

It was then theorized that a similar contour plot could be constructed for the Cleanup Maneuvers. The Cleanup Maneuver cost contour was formed by differencing two LAMBIC runs and then mapping that difference back to the encounter just before the Cleanup Maneuver. The first LAMBIC run is with the Cleanup Maneuver included in the simulation while the second LAMBIC run is without the Cleanup Maneuver. The reason for mapping the cost contours to the previous flyby is that there is not a well-defined B-plane target at the next encounter for the Cleanup Maneuver. This is because the Cleanup Maneuver and the Apoapsis Maneuver (the maneuver between the Cleanup and Approach Maneuvers) are designed together and the intermediate target is flexible. (The target of the Cleanup Maneuver is the intermediate target. The intermediate target is shown as $r_o + \Delta r$ in Fig. 1.)

While using these contour plots, another application for these tools was discovered. The Approach maneuver contour plot became useful for biasing the targets of the Approach Maneuvers. As a result, there have been two instances where the B-plane targets of an encounter were changed because a sizable negative ΔV region (a negative ΔV region represents an area of ΔV savings) was discovered on the Approach Maneuver contour plot that was big enough to target. Therefore, the encounter target was changed to take advantage of the ΔV savings. This will be

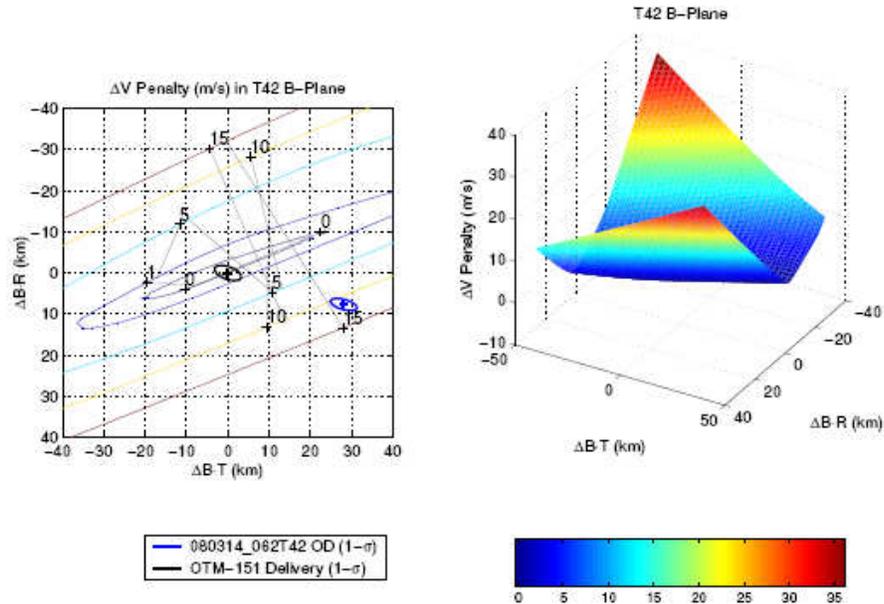


Figure 2. Contour Plot for Titan Encounter 42. *The plot on the left is the contour plot. The plot on the right is the contour surface.*

further discussed in Section IV. In addition, the contour plots help the maneuver analysts determine that the current setup for a particular maneuver is correct and more importantly, it is a linear check of the integrated solution that is nominally processed. These contour plots also take into account future encounters and how the current encounter

target effects the downstream ΔV costs associated with those future encounters. It can also be seen how the future encounters affect the ΔV cost at the next encounter. This is evident in the changes to the contour plots when the length of the optimization chain is changed when LAMBIC is run.

The contour lines are shaped and oriented in a manner that corresponds to the orbital change that takes place due to the flyby of the encounter body. For instance, if the altitude of the flyby is the primary cause of the orbital change, the semi-minor axis of the contours will line up with the B-vector of the encounter. Or stated another way, if the orbital change that takes place because of the encounter is sensitive to the altitude of the encounter, the steepest gradient of the contour lines will line up with the B-vector.

On the Approach Maneuver contour plot in Fig. 2, there are two one-sigma ellipses: the blue ellipse is the current trajectory without a maneuver taking place which is provided to the Maneuver Team by the Orbit Determination Team (OD team); and the black ellipse is the delivery ellipse if the maneuver takes place. Included in the delivery ellipse are the one-sigma maneuver execution errors.⁷ The axes of the contour plot are position differences in $B \cdot R$ and $B \cdot T$ with respect to the nominal aimpoint. So the nominal aimpoint is at point (0,0). The contour values are ΔV differences with respect to the ΔV value for the nominal aimpoint. This is why the zero contour always passes through the origin of the plot (More in-depth information on this topic is found in Section III). The reason why ΔV differences with respect to the nominal aimpoint are used is that the ΔV difference is more consistently accurate than the absolute ΔV values.

III. Theory

A. Approach Maneuver Cancellation Contour Plot

The ΔV costs shown on the contour come from two sources: B-plane grid variations (encounter position variations) and an asymptote correction that is the result of a B-plane offset which stems from the fact that the B-planes are different for a nominal and perturbed trajectory. Figure 3 shows an example of the B-plane used for the

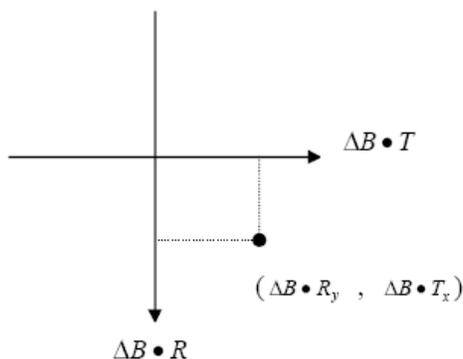


Figure 3. Example B-plane with grid point sample

contour plot with a sample grid point. Each grid point is a summation of the ΔV for multiple maneuvers over multiple encounters. For the Approach Maneuver case, the algorithm that the optimizer uses is the following cost function.

$$J_{x,y} = \sum_{i=0}^3 (\|V1_i\| + \|V2_i\|) \quad (1)$$

$J_{x,y}$ represents the ΔV calculation for a grid point, $V1$ represents the magnitude of the velocity for a Cleanup Maneuver and $V2$ represents the velocity magnitude for an Apoapsis Maneuver. The i index represents a leg number. Usually, four legs or four sets of Cleanup and Apoapsis Maneuvers are used in the optimization process. Now the value at each grid point is subtracted from the cost at the nominal aimpoint. (The nominal aimpoint is at the origin of the contour plot.) So the value of the grid point is computed from:

$$G_{x,y} = J_{x,y} - J_{0,0} \quad (2)$$

$G_{x,y}$ represents the values on the contour plot. Note that this is the cost due to flyby error assuming the ensuing Cleanup Maneuver will be implemented, the Approach Maneuver is assumed to be cancelled, and the encounter takes place at the grid points. The grid variation can be written in the following equation since the only terms that have a value are the $B \cdot R$ and $B \cdot T$ terms.

$$\Delta B_{RT} = \begin{bmatrix} \Delta B \cdot R_y \\ \Delta B \cdot T_x \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

Figure 4 is an illustration of the B-plane offset which goes with Fig. 1, which is an example of a perturbed trajectory (perturbed with respect to the nominal reference trajectory). B_o is the B-vector for the nominal trajectory, S_o is the incoming asymptote for the nominal trajectory, T_n is the n th Titan encounter, T_{n+1} is the $n+1$ th Titan encounter, B is the B-vector for the perturbed trajectory, and S is the incoming asymptote for the perturbed trajectory.

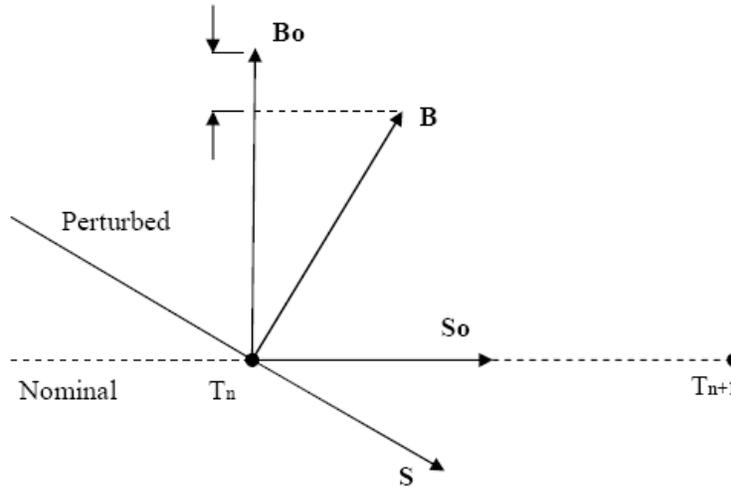


Figure 4. Illustration of B-plane offset. *The offset is due to difference of the perturbed trajectory incoming asymptote and the nominal trajectory incoming asymptote.*

It can be seen in Fig. 4 that there is an offset in the B-plane due to the incoming asymptote being different than the asymptote of the original trajectory. The target at T_n is the same but the “route” to get to the target changed from the one that was planned. This asymptote change can be written in B-plane state as

$$\Delta B_A = \begin{bmatrix} \bar{B} \cdot \bar{R}_o \\ \bar{B} \cdot \bar{T}_o \\ LFT \\ \bar{S} \cdot \bar{R}_o \\ \bar{S} \cdot \bar{T}_o \\ C3 \end{bmatrix} - \begin{bmatrix} \bar{B}_o \cdot \bar{R}_o \\ \bar{B}_o \cdot \bar{T}_o \\ LFT_o \\ \bar{S}_o \cdot \bar{R}_o \\ \bar{S}_o \cdot \bar{T}_o \\ C3_o \end{bmatrix} \quad (4)$$

The LFT term has negligible effect. Therefore, it is set to zero. Also, $\bar{S}_o \bullet \bar{R}_o$ and $\bar{S}_o \bullet \bar{T}_o$ terms are zero by definition. Now ΔB_A and ΔB_{RT} can be brought together in the equation

$$\Delta B_{x,y} = \Delta B_A + \Delta B_{RT}; \quad (5)$$

$$\Delta B_{x,y} = \begin{bmatrix} (\bar{B} - \bar{B}_o) \bullet \bar{R}_o \\ (\bar{B} - \bar{B}_o) \bullet \bar{T}_o \\ \approx 0 \\ \bar{S} \bullet \bar{R}_o \\ \bar{S} \bullet \bar{T}_o \\ \Delta C3 \end{bmatrix} + \begin{bmatrix} \Delta B \bullet Ry \\ \Delta B \bullet Tx \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (6)$$

$\Delta B_{x,y}$ represents the variations in the B-plane (due to the ΔB_{RT} term) using the perturbed trajectory B-plane state. \bar{B} is the perturbed B-vector, \bar{B}_o is the nominal B-vector, \bar{R}_o is the nominal B-plane R-vector, \bar{T}_o is the nominal B-plane T-vector, \bar{S} is the perturbed incoming asymptote, and $\Delta C3$ is the C3 difference. The $\Delta B \bullet T$ component is the x grid point of the B-plane, with the $\Delta B \bullet R$ component the y grid point. So the B-plane variation mapped (M is the B-plane state mapping) from the n th Titan encounter to the $n+1$ th Titan encounter for the perturbed trajectory is

$$M_n^{n+1} * \Delta B_{x,y}. \quad (7)$$

The ΔB_{RT} component is produced from a special setting in LAMBIC. What this setting does is produce ΔV values that correspond to the grid of position errors with respect to the flyby. Now this setting is merged with another feature of LAMBIC which, combined with the aforementioned special setting, has the function of centering the grid of flyby errors at the perturbed flyby conditions represented by the incoming asymptote error estimate. The estimate is computed by using the current trajectory from operations. Thus, the result is a grid of ΔV values in the B-plane that corresponds to the perturbed trajectory, which is much more accurate than using the nominal or reference trajectory B-plane.

The influence of using the current trajectory from operations rather than the nominal reference trajectory can be seen in the difference between Fig. 5 and Fig. 6 which are contour plots of the T32 encounter. Figure 5, shows the case for Approach Maneuver OTM115 without applying the incoming asymptote correction. It shows that the current OD ellipse is close to the 5 m/s contour; the delivery ellipse is directly on top of the zero m/s contour point; and there is no negative ΔV region. Hence, the best maneuver strategy would be targeting to the nominal aimpoint. Figure 6 shows the cost contour for the same maneuver, but this plot takes into account the incoming asymptote change. The OD ellipse is not as close to the 5 m/s contour as in Fig. 5; the delivery ellipse is still on the zero m/s contour but the zero m/s contour is now an oval rather than a point; and there is a substantial negative ΔV region. Now, targeting to the nominal aimpoint doesn't look like the best option. (Note: The project decided to bias the encounter targets for the T32 encounter. There is more discussion on biased targeting in Section IV.) This proves how important the asymptote correction is in accurately displaying the ΔV costs associated with a flyby.

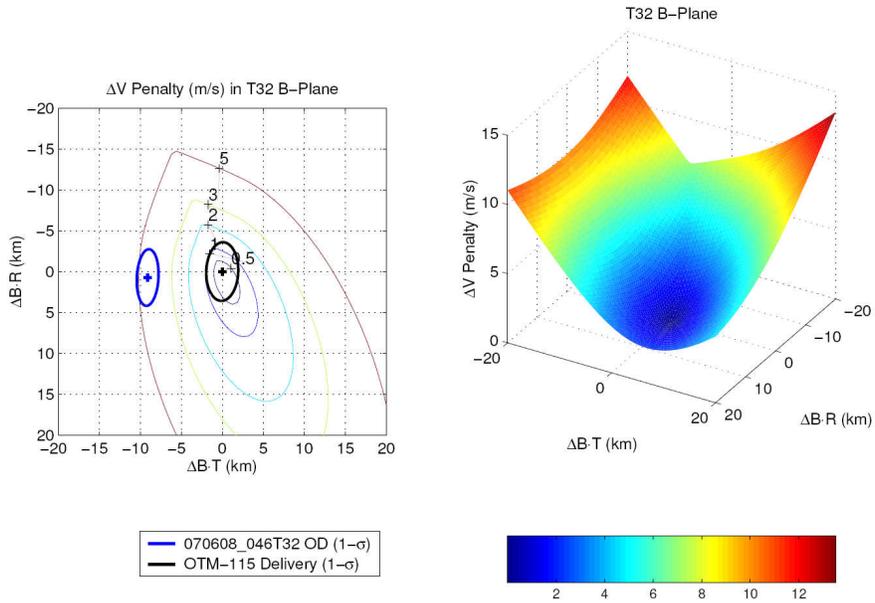


Figure 5. T32 Approach OTM115 Cancellation Cost without incoming asymptote correction.

T32 Approach: OTM-115 Cancellation Cost

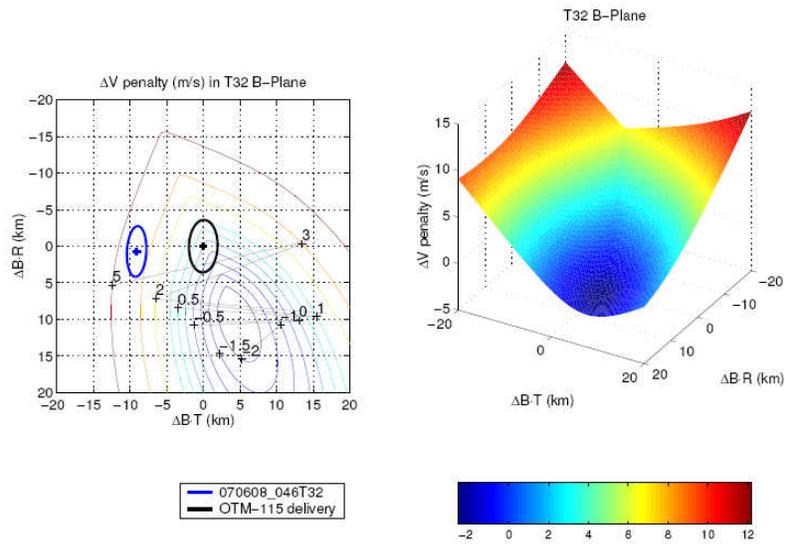


Figure 6. Contour Plot for OTM115 with incoming asymptote correction. Shows the negative ΔV contours before the target bias.

B. Cleanup Maneuver Cancellation Contour Plot

After the Approach Maneuver Contour Plot was assembled, the possibility of cancelling a Cleanup Maneuver emerged so a Cleanup Maneuver Contour Plot was formulated and is now used in the analysis to consider cancellation of a Cleanup Maneuver. There are some differences to how the procedure is performed for the Approach Maneuver case and the Cleanup Maneuver case.

For the Cleanup Maneuver cancellation case, it is assumed that the Cleanup Maneuver is cancelled with the subsequent Apoapsis Maneuver implemented. As mentioned before there are two LAMBIC runs performed to generate the data for the Cleanup Maneuver Contour Plot and the difference between the two LAMBIC runs produce the values of the contour plot. The first run is the nominal run that produces values from Eq. (1). The second run uses a different cost function which is the minimization of

$$J_{x,y,c} = \|V2_0\| + \sum_{i=1}^3 (\|V1_i\| + \|V2_i\|) \quad (8)$$

Where $J_{x,y,c}$ represents the ΔV calculation for a grid point, $V2_0$ represents the upcoming Apoapsis Maneuver ($V1_0$ is cancelled). Usually, four legs or four sets of Cleanup and Apoapsis Maneuvers are used in the optimization process, but for this case there are only three legs in the summation. Let $G_{x,y,c}$ be the difference of the ΔV at a grid point without the Cleanup Maneuver minus the ΔV at a grid point with the Cleanup Maneuver.

$$G_{x,y,c} = J_{x,y,c} - J_{x,y} \quad (9)$$

The B-plane error mapped from the n th Titan encounter to the $n+1$ th Titan encounter for the perturbed trajectory, assuming cancellation of the Cleanup Maneuver, is:

$$M_n^{n+1} * \Delta B_{xy} - K_{6 \times 3} \Delta V_{REF} \quad (10)$$

Remember the variations ΔB_{xy} for the cost contours is with respect to the previous flyby, not the ensuing flyby and M is the B-plane mapping. The $-K_{6 \times 3} \Delta \vec{V}_{REF}$ component of Eq. (10) is included in order to subtract or take out the Cleanup Maneuver which is nominally included with the reference trajectory since it might have a deterministic component (If a cleanup maneuver with a deterministic component is cancelled, its effect needs to be accounted for in the LAMBIC simulation. This is done by adding the negative influence of the maneuver.) The K term is the maneuver capability matrix at the Cleanup Maneuver, and the $\Delta \vec{V}_{REF}$ is the nominal deterministic ΔV planned in the reference trajectory. Multiplying these together gives the B-plane change due to the nominal (reference) Cleanup Maneuver.

Figure 7 is an example of a Cleanup Maneuver Contour Plot where there was a deterministic ΔV component of 1.17 m/s. The plot shows the contours running parallel with a zero contour channel. On the contour, three OD solutions are plotted. Two OD solutions, 071116_053T38 (short) and 071118_053T38 (short), are pre-encounter solutions. The last OD solution, 071120_053T38 (short), is a post-encounter solution which shows where the encounter took place in the B-plane. For this Cleanup Maneuver Contour plot example, using the 071120_053T38 (short) solution shows that the cost to cancel OTM134 is approximately 0.7 m/s. This was in very close agreement with the integrated solution which produced a value of approximately 0.8 m/s for OTM134 cancellation.

‘Post-T37’ OTM-134 Cancellation Cost

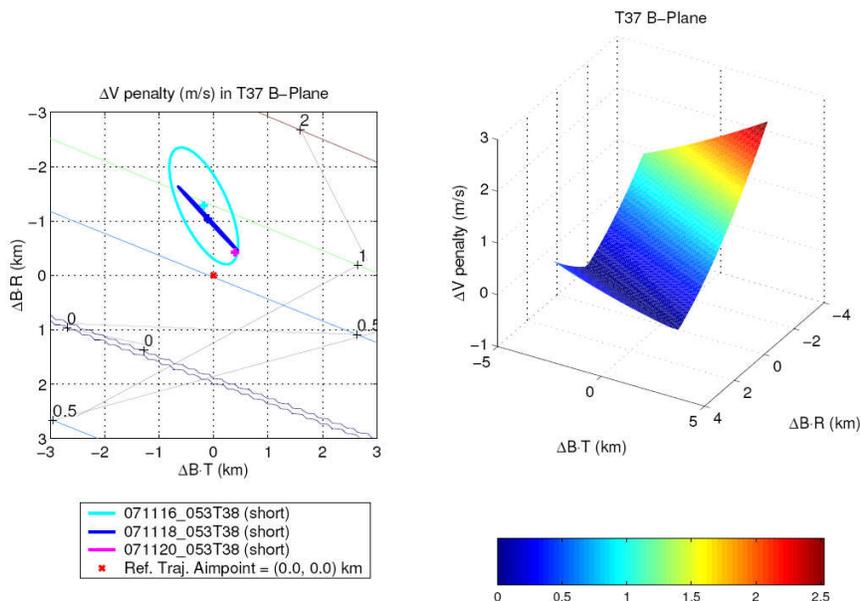


Figure 7. Cleanup Maneuver Cancellation Cost Contour Plot for OTM134. *The (0,0) point of the contour is still the nominal trajectory aimpoint, but the zero m/s cost contour usually does not pass through it. The light blue ellipse is an OD solution before the flyby as well as the blue ellipse. The magenta cross is the actual encounter position—if OTM134 was cancelled there would be a ΔV cost of approximately 0.7 m/s. Notice even if the aimpoint was met precisely, there would be a 0.5 m/s cost to cancel OTM134.*

IV. Biasing Targets⁸

As mentioned previously, the contour plots are used to estimate the cost of cancellation and the one-sigma OD and delivery ellipses can provide some statistics on the cost of cancellation. There is one more use that should be discussed in some detail since it saved ΔV for the Cassini-Huygens Project. This use is biasing the targets of an encounter. Biasing or changing targets are not new. The Galileo project changed targets; however, that mission had more time to adjust the targets. For Cassini-Huygens the time to design a maneuver and make all the appropriate decisions is about three days (given a typical 5-day spacing of OTMs). So the quick time frames do not usually permit retargeting. However, there were some instances where the contour plot aided in target biasing since it was so simple to take the appropriate information from the plot itself. Biasing the encounter targets was done twice (for OTM109 and OTM115 which targeted to T30 and T32 respectively) and saved approximately 4.6 m/s of ΔV . The original contour plot for OTM115 is shown in Fig 6. It can be seen from Fig 6 that there is a sizable negative ΔV area which was mentioned in Section III. It was determined that changing the targets did not have any undesirable downstream effects and the trajectory would be closer to the reference trajectory by targeting the maneuvers to biased aimpoints. By changing the $B \cdot R$ target by 9.4 km and the $B \cdot T$ target by 4.1 km without changing the encounter time would put the center of the delivery ellipse at the absolute minimum of the contour plot. The contour plot then changed and looked like Fig. 8. The decision to bias the target was not just a navigation and ΔV savings issue. There had to be discussion with the science team to make sure that the biased target did not negatively affect the science taking place. However, since there were two benefits (the 4.6 m/s of ΔV savings and the trajectory would be closer to the reference trajectory by implementing the maneuver with the biased targets), the science team agreed to the change.

T32 Approach: OTM-115 Cancellation Cost

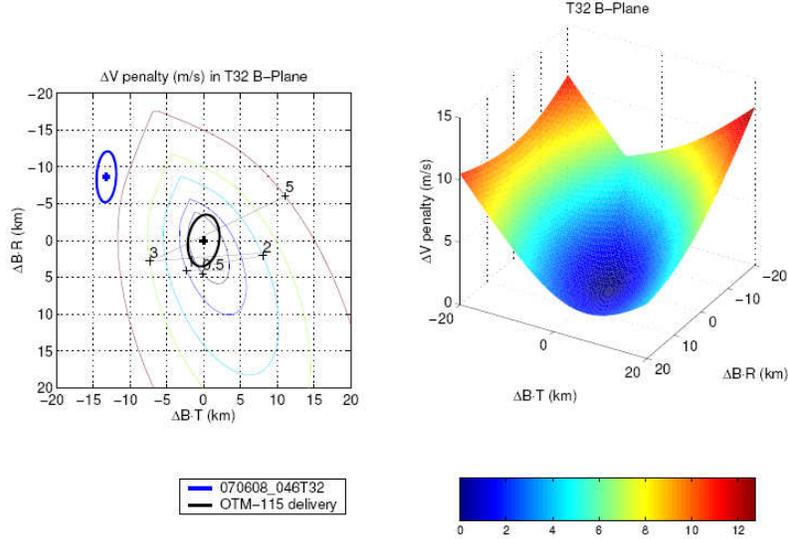


Figure 8. Contour Plot for OTM115. Shows the contours with the biased target.

V. Future Work

The contour plots show very accurate results when dealing with a Titan encounter. However, there is not the same level of accuracy for a contour plot of the smaller and less massive Saturnian satellites such as Enceladus. It is hypothesized that the reason for the inaccuracy is due to the asymptote difference being the dominant variable for ΔV costs in these instances whereas for a Titan encounter the grid points or more specifically where the encounter takes place with respect to Titan is dominant. The asymptote correction explained in Section III is an external one based on operations. For a Titan encounter this is sufficient since the effects of asymptote variations from grid point to grid point are negligible. On the other hand, for an encounter of a less massive body near periapsis, the asymptote can change substantially from one grid point to another and this asymptote change must be generated internally in addition to the correction due to operations. So the grid points, which are based only on two-dimensional variations would expand to five dimensions. The extra three dimensions would deal with the asymptote differences.

Therefore, an augmented or enhanced grid generation could be added to the LAMBIC software to better account for asymptote variation among grid points.

$$K_{6 \times 3} = \begin{bmatrix} A_{3 \times 3} \\ S_{3 \times 3} \end{bmatrix} \quad (11)$$

Assume that $K_{6 \times 3}$ found in Eq. (11) is the K-matrix at the Approach (or prior) Maneuver point, but it is broken into two 3x3 matrices— A and S . To take account the asymptote variation among the grid points multiply the $K_{6 \times 3}$ matrix in the following manner to adjust the asymptotes accordingly

$$K_{6 \times 3} A_{3 \times 3}^{-1} \begin{bmatrix} \Delta B \bullet R \\ \Delta B \bullet T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{3 \times 3} \\ S_{3 \times 3} \end{bmatrix} A_{3 \times 3}^{-1} \begin{bmatrix} \Delta B \bullet R \\ \Delta B \bullet T \\ 0 \end{bmatrix} = \begin{bmatrix} I_{3 \times 3} \\ S_{3 \times 3} A_{3 \times 3}^{-1} \end{bmatrix} \begin{bmatrix} \Delta B \bullet R \\ \Delta B \bullet T \\ 0 \end{bmatrix}. \quad (12)$$

Notice that the $\Delta B \cdot R$ and $\Delta B \cdot T$ remains the same due to the identity matrix (I) and the S-matrix contains the desired asymptote adjustments. An adjustment to the LAMBIC software would be the next course of action with some additional work to be done to verify the algorithm.

VI. Conclusion

In conclusion, the contour plots used by the Cassini-Huygens maneuver team have helped in the maneuver cancellation decision-making process and has become a valuable tool to save ΔV as the opportunity presents itself. Furthermore, the position variations of an encounter without the time of flight component are a significant source to show the ΔV variations of an encounter.

Appendix: The B-plane

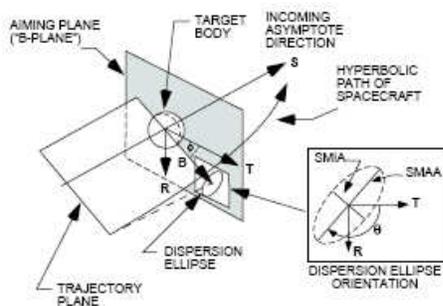


Figure 9. An Example of a B-plane

The B-plane is another name for the Aiming Plane which is a plane that is perpendicular to the incoming asymptote vector of the spacecraft and passes through the target body. The B-vector is the vector from the center of the target body to the location in the B-plane where the path of the spacecraft is projected onto the B-plane. (As shown in Fig. 9.) The B-vector is also known as the impact parameter and is the place of closest approach to the target body if the target body was massless. The B-vector position can be plotted by using the two principle axes of the B-plane— R and T . The T axis is usually aligned with either the ecliptic or body equatorial plane of the target body and the R axis is the third vector that completes the right handed system with the S and T axes. Now the $B \cdot R$ and $B \cdot T$ components would provide the position of the encounter in the B-plane. The inset on Fig. 9 shows the $1-\sigma$

dispersion ellipse that develops around the impact parameter point. This ellipse has a semi-major axis dimension (SMAA), semi-minor axis dimension (SMIA), and an orientation value θ , which is the angle between the T axis and the semi-major axis of the ellipse in the clockwise direction. The S component of the dispersion is a time of flight dispersion or distance dispersion along the S axis.

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