Wavefront Controls for a Large Submillimeter-Wave Observatory

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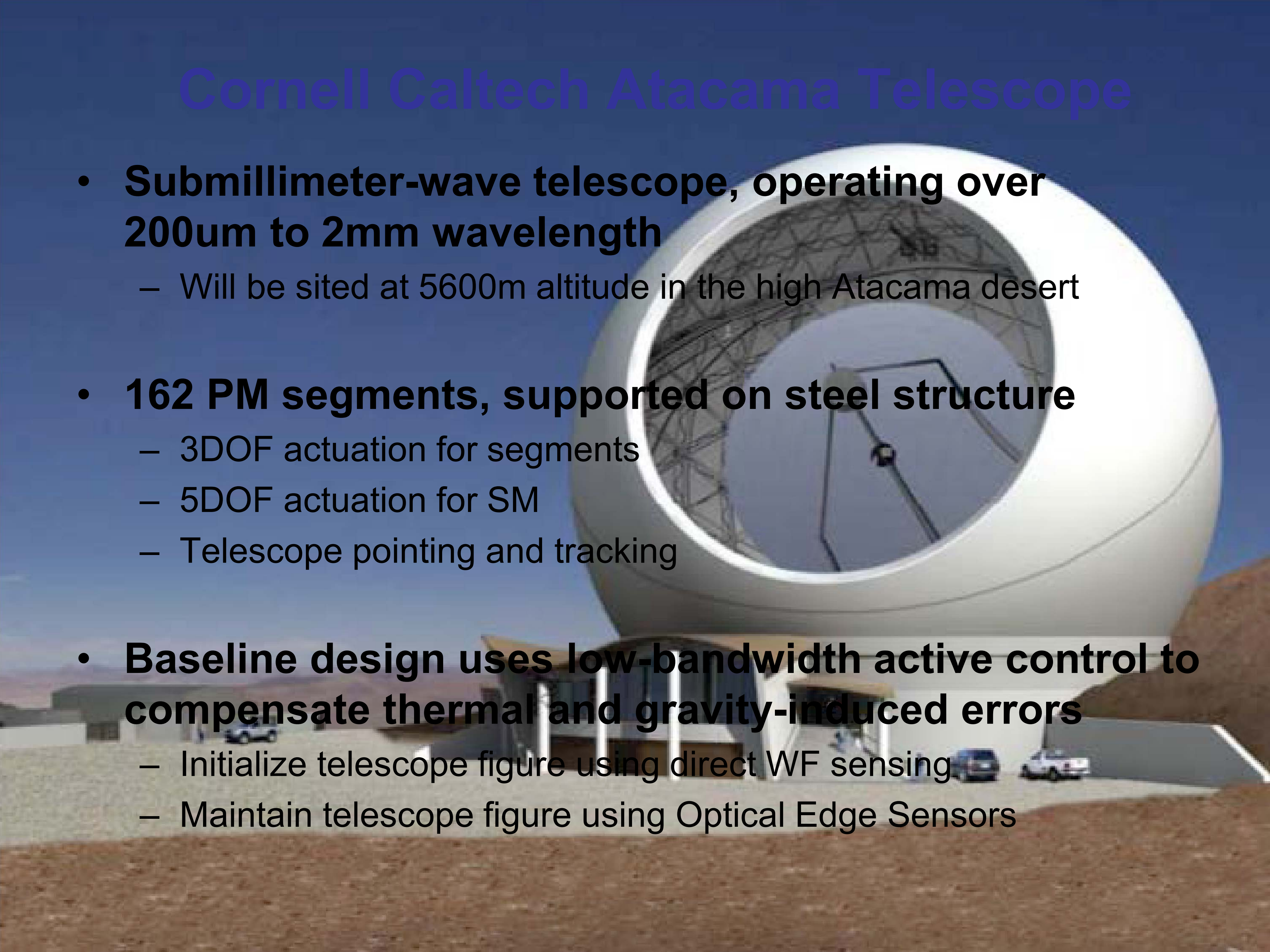
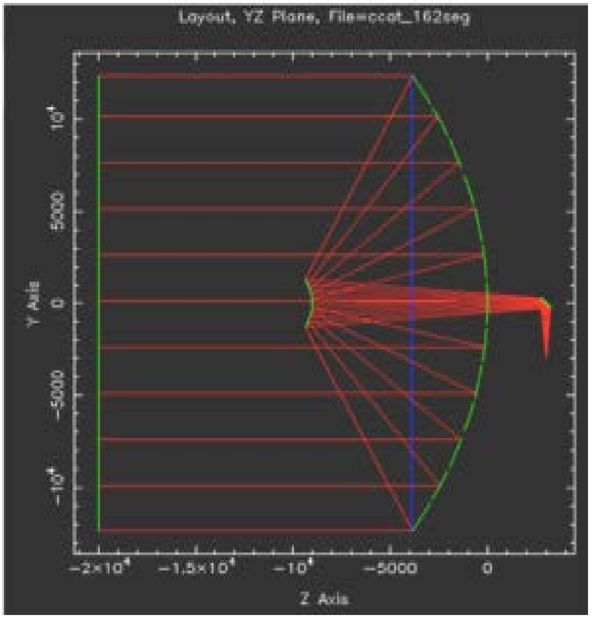
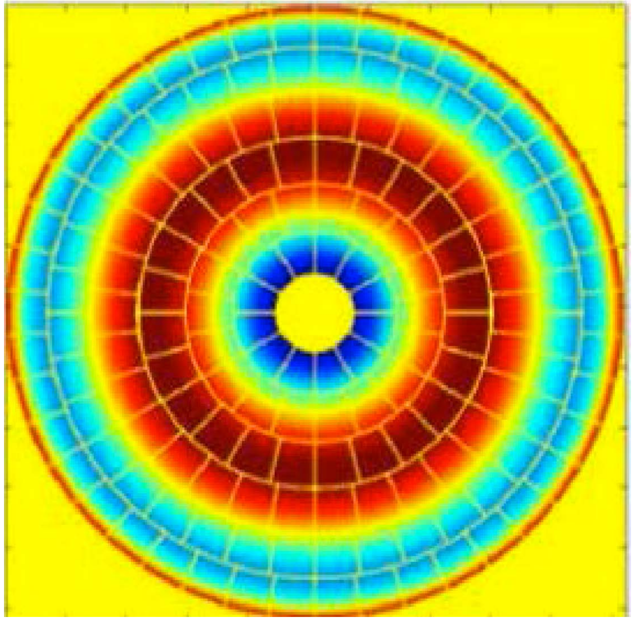
Cornell Caltech Atacama Telescope

- **Submillimeter-wave telescope, operating over 200um to 2mm wavelength**
  - Will be sited at 5600m altitude in the high Atacama desert
- **162 PM segments, supported on steel structure**
  - 3DOF actuation for segments
  - 5DOF actuation for SM
  - Telescope pointing and tracking
- **Baseline design uses low-bandwidth active control to compensate thermal and gravity-induced errors**
  - Initialize telescope figure using direct WF sensing
  - Maintain telescope figure using Optical Edge Sensors



CCAT Controls Model

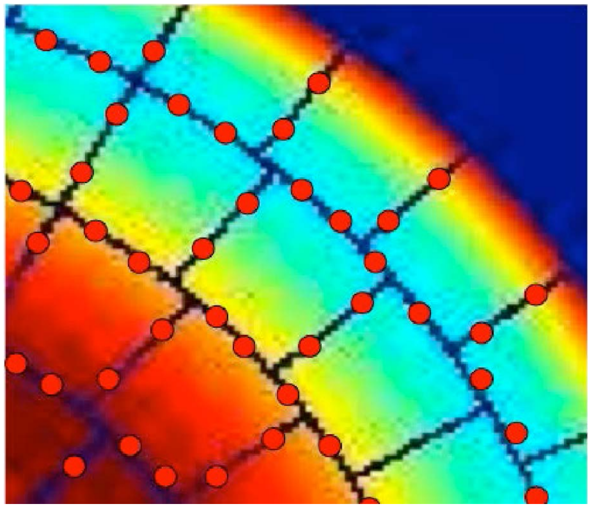


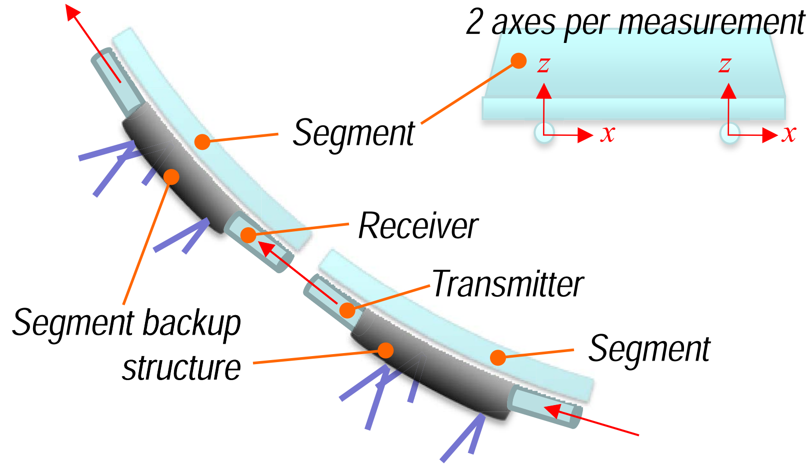


- **CCAT Integrated Model is under development, in parallel with design activities**
  - Optics: MACOS ray-trace and Fourier physical optics model computes WF and pointing from deformed structures
  - Structures: NASTRAN FEA model currently used for quasi-static deformations from thermal and gravity
  - Controls and simulation: MATLAB

Optical Edge Sensors

- **Optical edge sensors**
  - *Transmitter* sends collimated beam across gap
  - *Receiver* uses CCD to measure beam displacement in x and z
  - Measurements are sensitive to dihedral angle, relative piston and lateral translation
  - Noise < 0.1 micron RMS
  - Sample rate > 1 Hz





- **Two sensors per segment edge on average**
  - 624 sensors, for 1248 total measurements of 972 DOFs
- **Measurements provide observability of most modes**
  - Global tilt mode is not observed
  - A few other low-order modes are poorly sensed, so care is required in the control

Linearized Model

- **The optical state x_i: perturbations to the nominal position/orientation/figure of each optic**
  - Subscript i indicates time step

$$x_{Opticj} = \left[\delta_x \delta_y \delta_z \theta_x \theta_y \theta_z\right]^T \qquad x = \begin{bmatrix} x_{Optic1} \\ \vdots \\ x_{Opticn} \end{bmatrix}$$

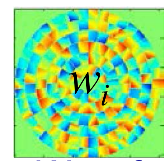
x is the CCAT state vector: 6 rigid-body DOFs per optic; plus deformation states

- **State x_i changes in time, driven by process noise ξ_i, quasi-static modes T_i, actuations u_i**

$$x_i = x_{i-1} + \frac{\partial x}{\partial u} u_i + \frac{\partial x}{\partial T} T_i + \xi_i$$

State transition equation governs the evolution of the state with time

- **Wavefront output w_i**
  - Wavefront w_i is affected by state changes x_i and actuation u_i



$$w_i = \frac{\partial w}{\partial x} x_i + w_0$$

Wavefront is a function of the state

- **Optical edge sensor measurements l_i**
  - Edge sensor measurements l_i are affected by state changes x_i and noise δl_i

$$l_i = \frac{\partial l}{\partial x} x_i + l_0 + \delta l_i$$

Optical edge sensor measurement equation defines how state is seen by the measurements

Control Approaches

- **Conventional approach is to control so that edge sensor measurements are driven to null**
  – This approach nulls errors in the controlled DOFs only

- ***Real objective is to minimize WF error*, while keeping control effort in balance**
  – This can be done if WF is measured directly
  – It can also be accomplished using an "*Optical State Estimator*" to estimate WF from edge sensor measurements

- **WF-based control...**
  – *Compensates optical effects of all errors* by actuation of the controlled DOFs

$$u_i = -pinv\left(\frac{\partial l}{\partial x}\frac{\partial x}{\partial u}, ptol\right) l_i$$

ptol chosen to suppress poorly-observed global modes

$$\min_u \; J = w^T w + c_u u^T u$$

WF control cost function

$$u_i = -G w_i$$

$$G = \left[c_u I + \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T \frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right]^{-1} \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T$$

WF control law and control gain matrix

$$u_i = -G\overline{w}_i$$

WF control can be used with WF measurement w or WF estimate $\overline{w}$

Optical State Estimator

- **Kalman filter recursively estimates full state $\hat{x}_i$ from optical edge sensor measurement l_i , previous control u_{i-1}, and prior estimates $\hat{x}_{i-1}$, $\hat{x}_{i-2}$, $\hat{x}_{i-3}$, ...**
  - Balances measurement noise against error in prior estimates to produce optimal estimate

- **Estimated state $\hat{x}_i$ used to estimate WF $\overline{w}_i$ for WF control**

- **Full-state controller feeds back $\overline{w}_i$ to minimize WFE**
  - Observes 6DOF per segment, minus unobservable global tilt modes
  - Additional term c_u weights control effort, allows damping of response to avoid exciting structure and reduce noise sensitivity

$$\hat{x}_i = \left(I - K_i \frac{\partial l}{\partial x} \right) \hat{x}_{i-1} + \left(I - K_i \frac{\partial l}{\partial x} \right) \frac{\partial x}{\partial u} u_{i-1} + K_i l_i$$

State estimate combines prior estimates, measurements, and known actuations modes

$$K_i = \mathrm{cov}\left(\hat{x}_i - x_i\right) \left(\frac{\partial l}{\partial x} \right)^T \mathrm{cov}\left(\frac{\partial l}{\partial x} \frac{\partial x}{\partial u} u_{i-1}^T + l_i \right)^{-1}$$

Kalman gain weights contributions of data sources by their expected error (can be fixed or time-varying)

$$\overline{w}_i = \frac{\partial w}{\partial x} \overline{x}_i$$

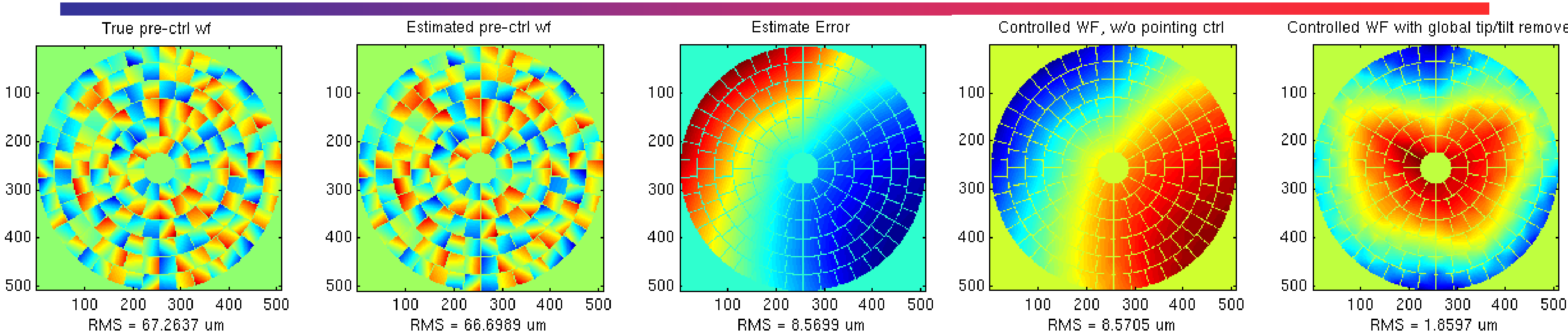
Estimated WF

$$u_i = -G \overline{w}_i = -G \frac{\partial w}{\partial x} \hat{x}_i$$

WF control using estimated WF

$$G = \left[c_u I + \left(\frac{\partial w}{\partial x} \frac{\partial x}{\partial u} \right)^T \frac{\partial w}{\partial x} \frac{\partial x}{\partial u} \right]^{-1} \left(\frac{\partial w}{\partial x} \frac{\partial x}{\partial u} \right)^T$$

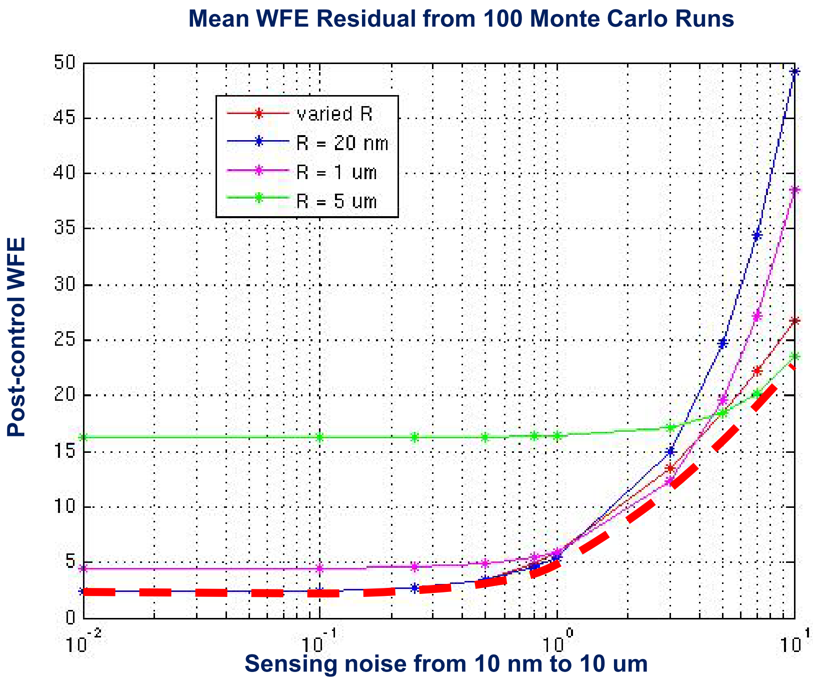
Single-Step Control Example



Initial error std dev = 50 um&urad
Process noise std dev = 0.25 um&urad
Sensing noise std dev = 1 um

- **Simulation driven by random initial errors**
  - $\sigma_x = 50\ \mu m\ \&\ 50\ \mu rad$, no sensing noise
- **Estimator effective in estimating the non-tilt WF errors**
  - Tilt not observable from edge sensors, but does not affect image quality
- **Control effective in removing WF errors**
  - Residual of < 2 um WFE shows residual poorly-controlled global modes
  - Ultimately limited by non-controllable residuals from decenter and twist

WF Control Monte Carlo Example



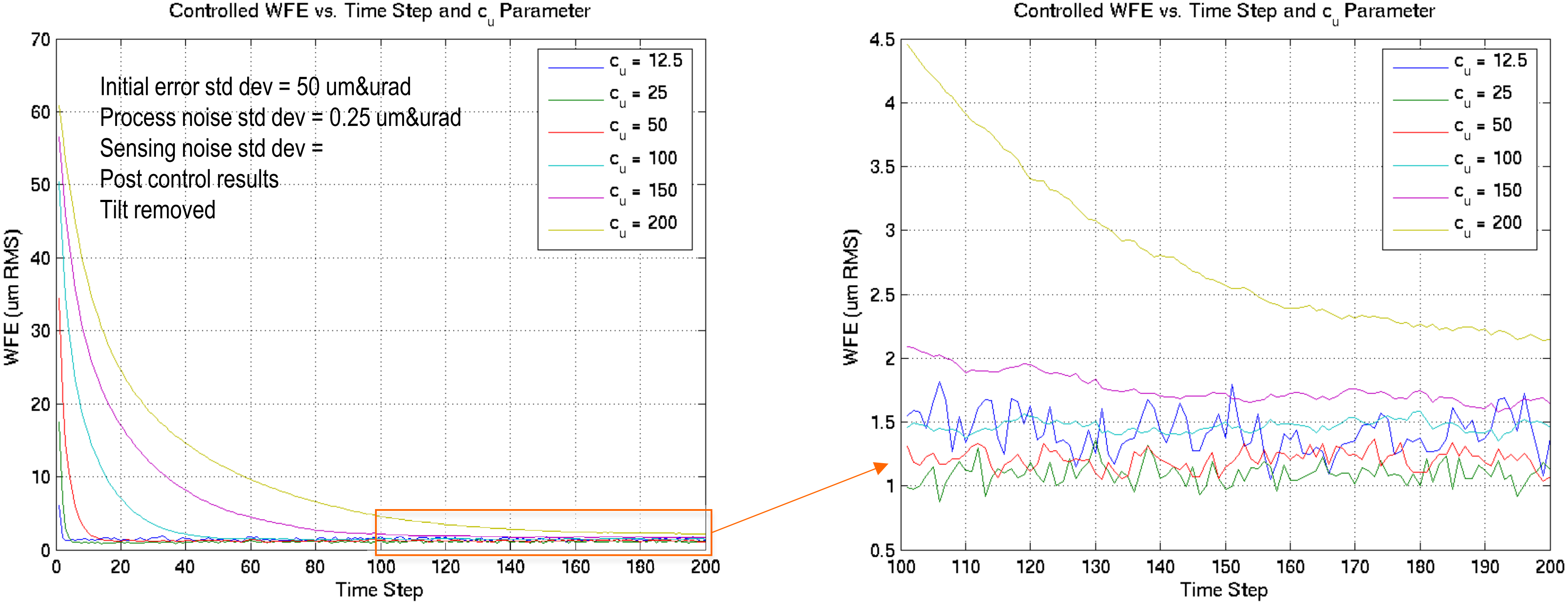
Initial error std dev = 50 um & urad
Single-step control
Sensing noise std dev varies as shown
No control damping ($c_u = 0$)

$$K_i = \mathrm{cov}(\hat{x}_i - x_i)\left(\frac{\partial l}{\partial x}\right)^T \mathrm{cov}\left(\frac{\partial l}{\partial x}\frac{\partial x}{\partial u}u_{i-1}^T + l_i\right)^{-1}$$

Kalman gain weights contributions of data sources by their expected error (can be fixed or time-varying)

- **Good performance is seen for a wide range of sensor noise**
- **Control performance benefits from Kalman filter tuning**
  - Choose values of $R = \mathrm{cov}(l_i)$ for computation of the Kalman gain that are consistent with actual sensor noise

Time Sequence Example



- **Quasi-static simulation provides a test of the controls**
  - Time sequence generated using quasi-static model driven by process noise
    - No dynamics included: not a prediction of actual telescope dynamics (that will come later)
  - Controls are effective in achieving desired WFE in the presence of noise
- **Control behavior can be modulated using c_u parameter**
  - Slow control response to below the modal frequencies of the structure
  - Reduce noise response

Next Steps

- **Evaluate optical edge sensor performance in full range of expected disturbances**
  - Realistic thermal and gravity deformations
  - Dynamics
- **Include initialization step using shearing interferometry**

Conclusion

- **Optical edge sensor-base wavefront control appears to meet the needs of CCAT**