The Gravitational-Wave Discovery Space of Pulsar Timing Arrays

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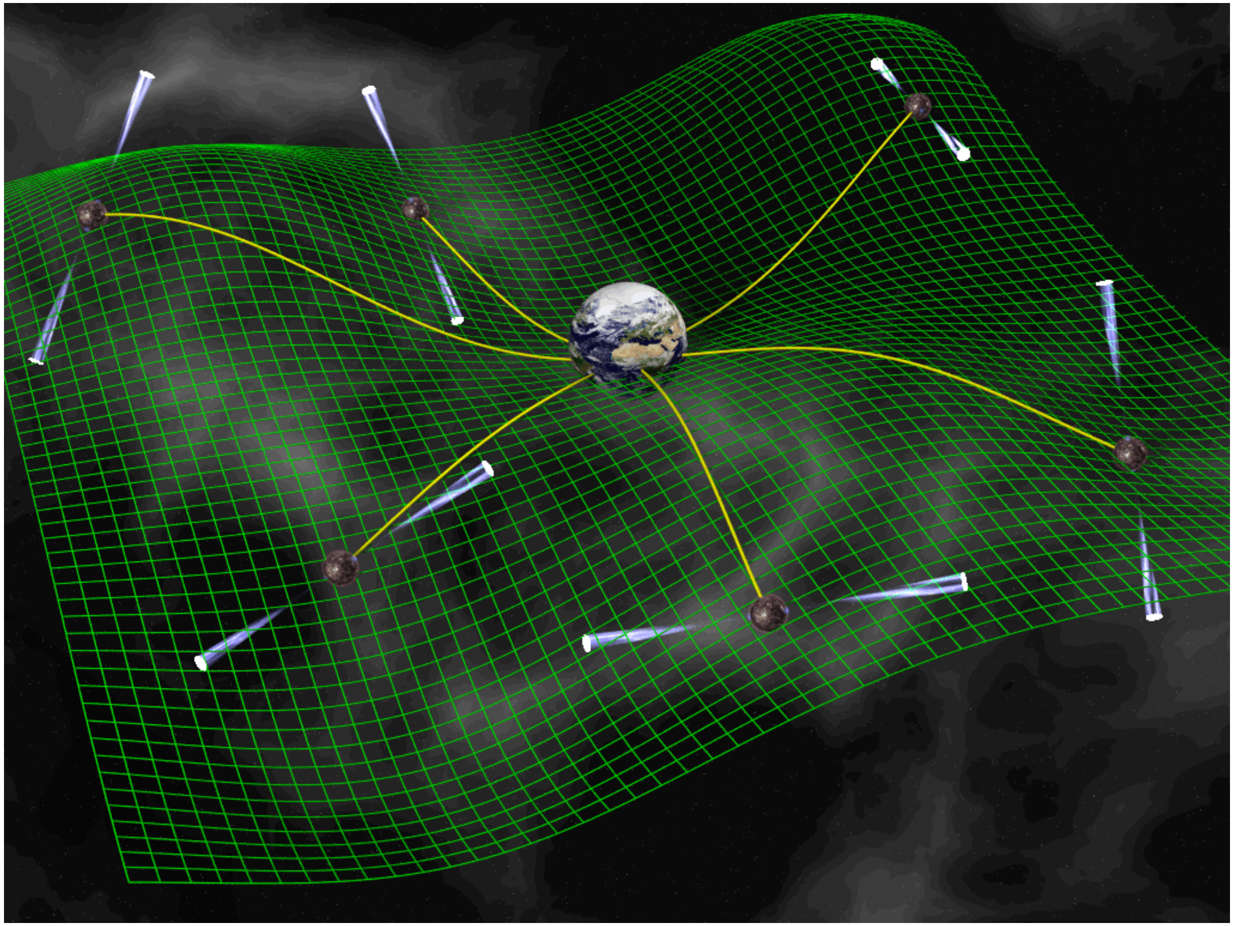


Fig 1: Artist's conception of pulsar timing array (David Champion)

Motivations

- DSN (JPL's Deep Space Network) includes 3 very large radio dishes: ~70m, similar to Parkes. Main job is to communicate with our fleet of satellites, but all typically have ~1 hr/day when not booked. Wahlid Majid currently building state-of-art backend for pulsar searches. Maybe use them for short-cadence ms pulsar observing?

- More generally, the current PTA observing strategies (~once per 2 weeks) and data analyses are focused on SMBHs and stochastic background. Are we missing something?

- Zimmerman&Thorne(1980), "The gravitational waves that bathe the Earth: upper limits based on theorists' cherished beliefs" Timely to re-visit?

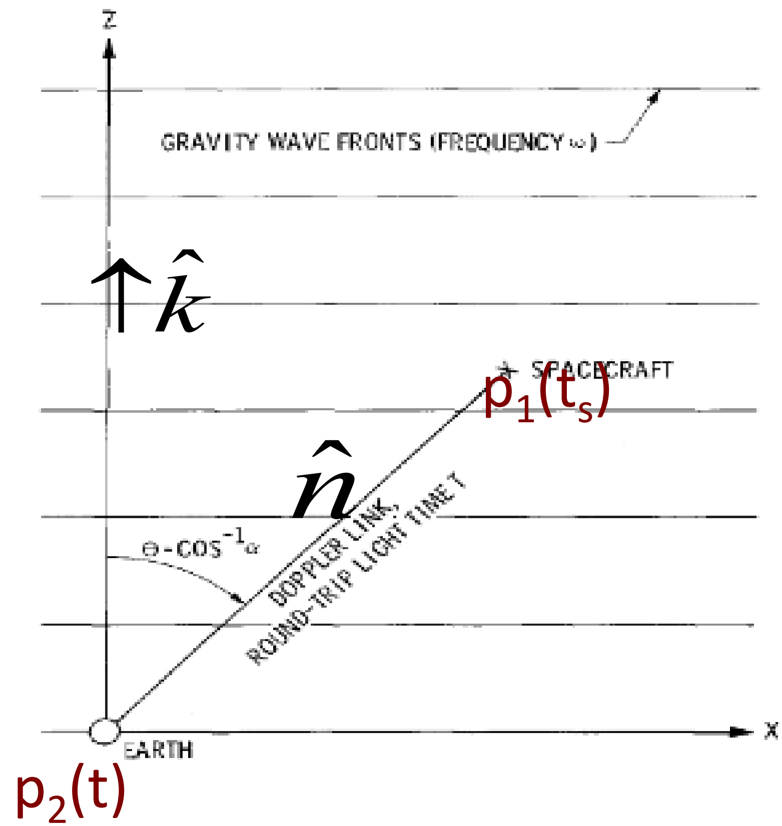
Outline of Rest of Talk

- Review of PTAs –physics, astronomy, some history
- Discovery potential from "direct part" of GWs, both low and high z
- Discovery potential from "memory part" of GWs, both low and high z
- Summary/conclusions

Early papers

- Sazhin (1978)
- Detweiler (1979)
- Estabrook & Wahlquist (1975)
- Hellings & Downs (1983)

From Estabrook&Wahlquist (1975)



$$\frac{\Delta\nu}{\nu} = \frac{1}{2}\frac{\hat{n}^i_{12}(t)\hat{n}^j_{12}(t)\left[h^{\rm TT}_{ij}(p^s_2(t),t) - h^{\rm TT}_{ij}(p^s_1(t_s),t_s)\right]}{1-\hat{n}^m_{12}(t)k_m}$$

Back-of-envelope estimate of detectability

$$\bar{\delta t}_{\rm GW} \equiv \sqrt{\langle \delta t^2_{\rm GW} \rangle} = \frac{1}{4\sqrt{3}\pi}\frac{h}{f} \simeq \frac{1}{20}\frac{h}{f}. \qquad h = \sqrt{h_+^2 + h_\times^2}$$

$$\mathrm{SNR}^2 = \frac{\langle \delta t^2_{\rm GW} \rangle}{\delta t^2_{\rm rms}} (\#\, pulsars)(\#\, obs/pulsar)$$

Plug in fiducial numbers:

$$\delta t_{rms} = 0.1\,\mu s$$

$$f = 10^{-8}\, Hz$$

$$\#\, pulsars = 40$$

$$\#\, obs/pulsar = 200$$

$$\Rightarrow$$

SNR= 7 requires

h = 1.e-15

GW strength of SMBH binary

Example: $10^9 + 10^9 \, M_{sun}$ binary

$f = 10^{-8} \, Hz$ (lifetime ~5e4 yrs) $D = 300 \, Mpc$

$$\Rightarrow h = 10^{-15}$$

Stochastic Background Limits from a PTA

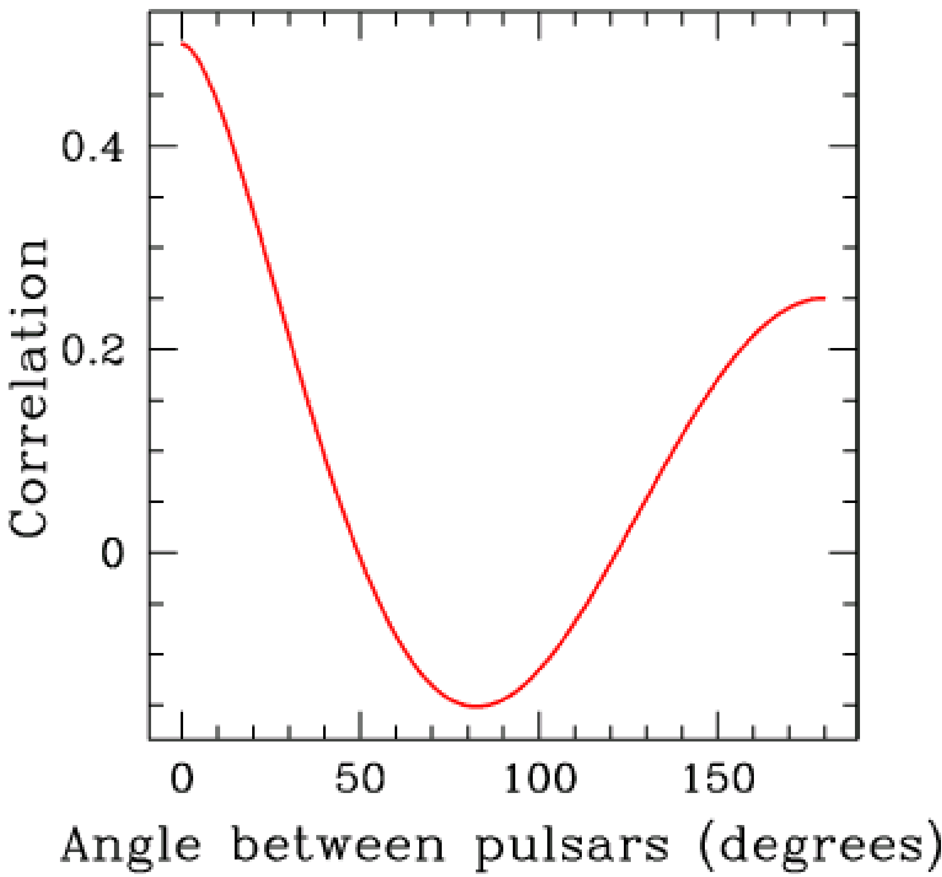
Expected correlation of residuals for pairs of pulsars versus angular separation on sky.

- "Pulsar" terms uncorrelated.
- "Earth" terms correlated.

$$C^{(ab)}_{y,ij} = E\{y_a(t_i)y_b(t_j)\}$$

$$= C_y(t_i - t_j)\zeta(\theta_{ab})$$

- Clock errors monopole
- Ephemeris errors dipole
- GWs quadrupole



(Hellings & Downs 1983) **[JPL]**

Types of Pulsars

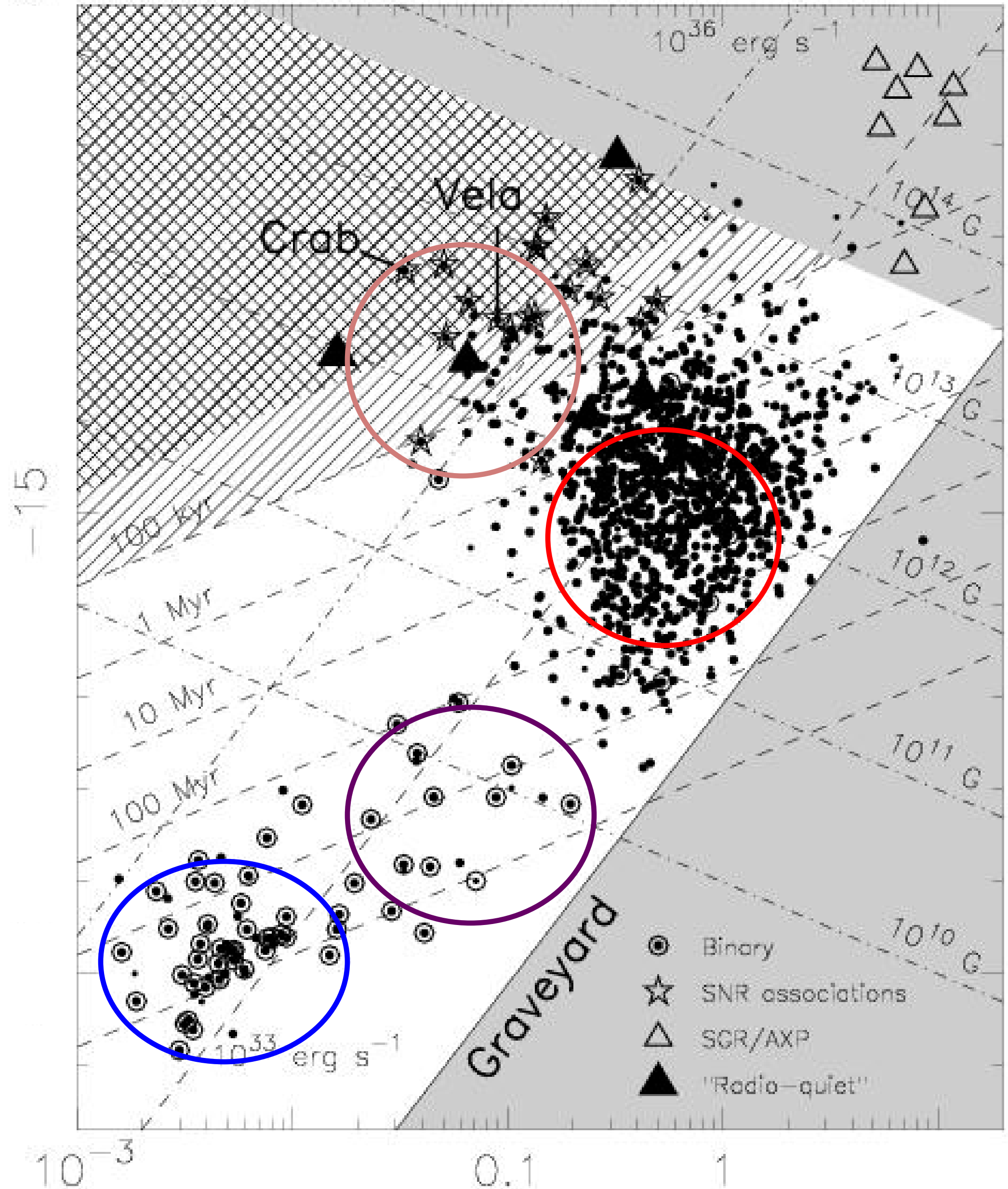
Over 2000 known radio pulsars in the Galaxy.

Young – Energetic, with significant spin-down noise and glitches

Normal – More stable, slower spin-down

Recycled pulsars with high-mass companions - Most in binaries, very stable rotators

Recycled pulsars with low-mass companions – Most in binaries, **extremely** stable rotators ➔ **Millisecond Pulsars**

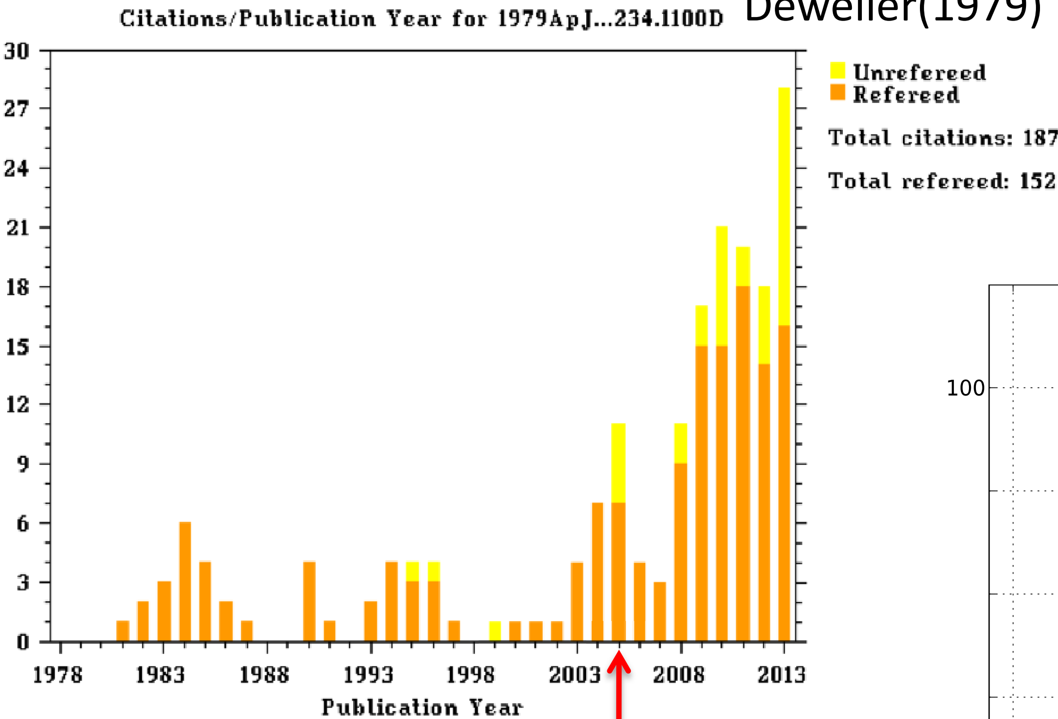


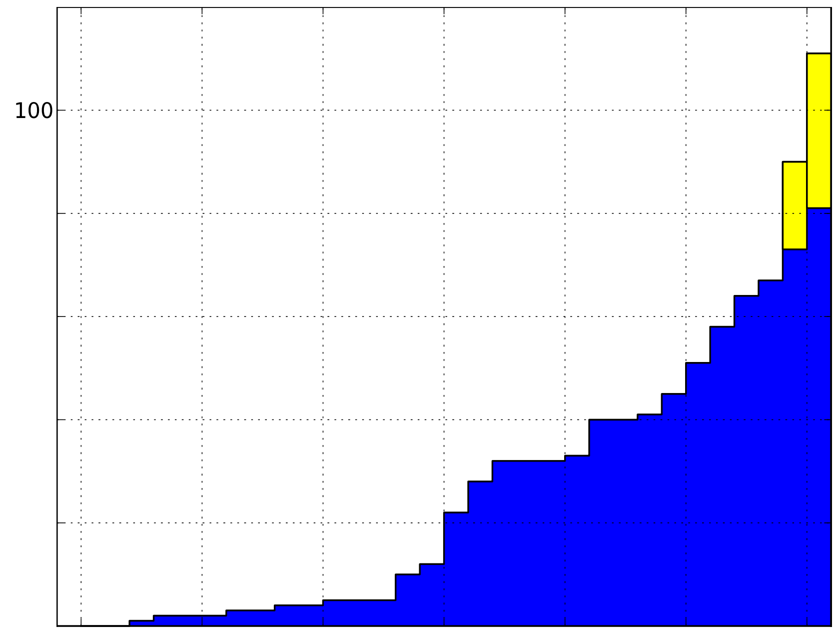
-thanks to Lazio, Burke-Spolaor & Cordes for slide

From Lorimer & Kramer 2005, *Handbook of Pulsar Astronomy*

Growth of PTA Activity, by citations

Deweiler(1979)





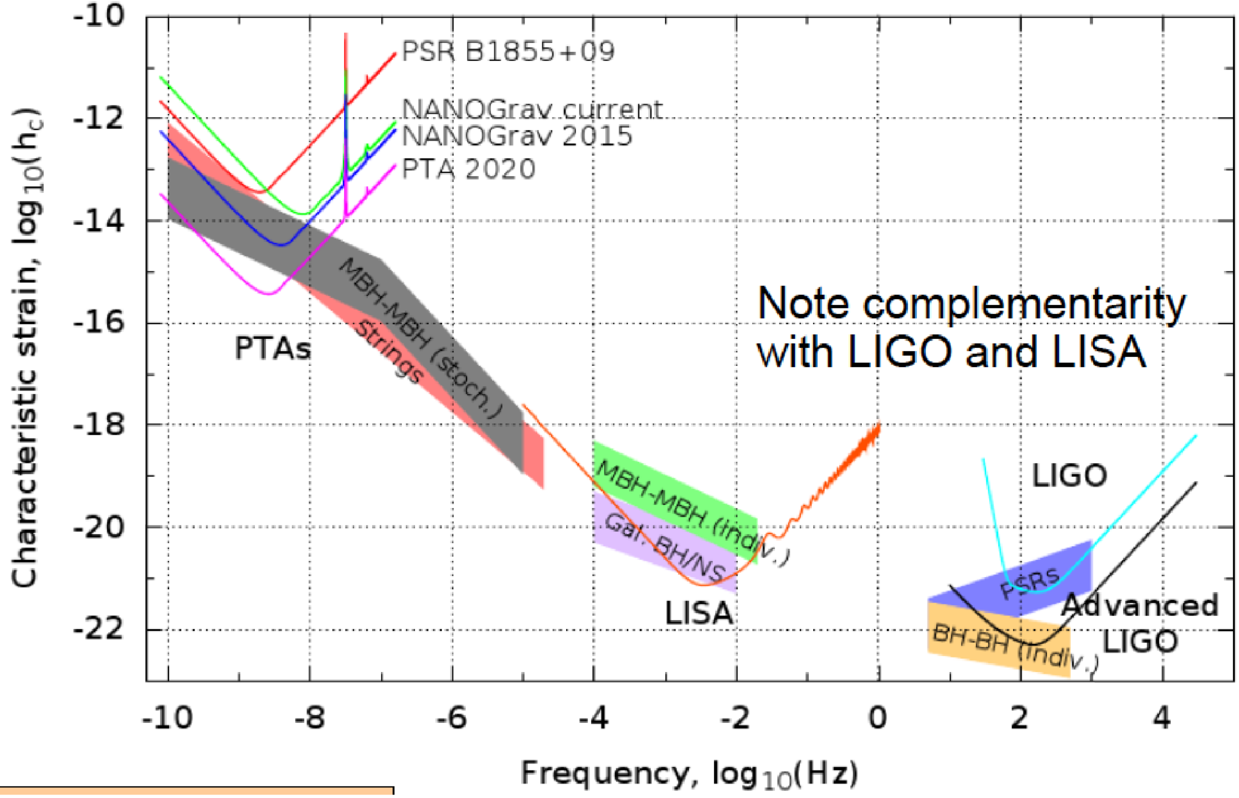
Jenet et al. 2005, ApJ 625, L123, “Detecting the Stochastic Gravitational Wave Background using Pulsar Timing” [124 citations]

International Pulsar Timing Array

PSR B	PSR J	P (ms)	P_b (d)	S1400 (mJy)	Array	PPTA (μs)	EPTA (μs)	NANOGrav (μs)
-	J0030+0451	4.87	-	0.6	EPTA, NANOGrav	-	0.54	0.31
-	J0218+4232	2.32	2.03	0.9	NANOGrav	-	-	4.81
-	J0437−4715	5.76	5.74	142.0	PPTA	0.03	-	-
-	J0613−0200	3.06	1.20	1.4	PPTA, EPTA, NANOGrav	0.71	0.45	0.50
	J0621+1002	28.85	8.32	1.9	EPTA	-	9.58	-
-	J0711−6830	5.49	-	1.6	PPTA	1.32	-	-
-	J0751+1807	3.48	0.3	3.2	EPTA	-	0.78	-
-	J0900−3144	11.1	18.7	3.8	EPTA	-	1.55	-
-	J1012+5307	5.26	0.60	3.0	EPTA, NANOGrav	-	0.32	0.61
-	J1022+1001	16.45	7.81	3.0	PPTA, EPTA	0.37	0.48	-
-	J1024−0719	5.16	-	0.7	PPTA, EPTA	0.43	0.25	-
-	J1045−4509	7.47	4.08	3.0	PPTA	2.68	-	-
-	J1455−3330	7.99	76.17	1.2	EPTA, NANOGrav	-	3.83	1.60
-	J1600−3053	3.60	14.35	3.2	EPTA, PPTA	0.32	0.23	-
-	J1603−7202	14.84	6.31	3.0	PPTA	0.70	-	-
-	J1640+2224	3.16	175.46	2.0	EPTA, NANOGrav	-	0.45	0.19
-	J1643−1224	4.62	147.02	4.8	PPTA, EPTA, NANOGrav	0.57	0.56	0.53
-	J1713+0747	4.57	67.83	8.0	PPTA, EPTA, NANOGrav	0.15	0.07	0.04
-	J1730−2304	8.12	-	4.0	PPTA, EPTA	0.83	1.01	-
-	J1732−5049	5.31	5.26	-	PPTA	1.74	-	-
-	J1738+0333	5.85	0.35	-	NANOGrav	-	-	0.24
-	J1741+1351	3.75	16.34	-	NANOGrav	-	-	0.19
-	J1744−1134	4.08	-	3.0	PPTA, EPTA, NANOGrav	0.21	0.14	0.14
-	J1751−2857	3.91	110.7	0.06	EPTA	-	0.90	-
B1821−24	J1824−2452	3.05	-	0.2	PPTA, EPTA	0.39	0.24	-
-	J1853+1303	4.09	115.65	0.4	NANOGrav	-	-	0.17
B1855+09	J1857+0943	5.37	12.33	5.0	PPTA, EPTA, NANOGrav	0.82	0.44	0.25
-	J1909−3744	2.95	1.53	3.0	PPTA, EPTA, NANOGrav	0.19	0.04	0.15
-	J1910+1256	4.98	58.47	0.5	EPTA, NANOGrav	-	0.99	0.17
-	J1918−0642	7.65	10.91	-	EPTA, NANOGrav	-	0.87	1.08
B1937+21	J1939+2134	1.56	-	10.0	PPTA, EPTA, NANOGrav	0.11	0.02	0.03
B1953+29	J1955+2908	6.13	117.35	1.1	NANOGrav	-	-	0.18
-	J2019+2425	3.94	76.51	-	NANOGrav	-	-	0.66
-	J2124−3358	4.93	-	1.6	PPTA	1.52	-	-
-	J2129−5721	3.73	6.63	1.4	PPTA	0.87	-	-
-	J2145−0750	16.05	6.84	8.0	PPTA, EPTA, NANOGrav	0.86	0.40	1.37
-	J2317+1439	3.44	2.46	4.0	NANOGrav	-	0.81	0.25

Limit from Correlation Analysis

$h_c < 7 \times 10^{-15}$ $(f=1\ yr^{-1})$



Demorest et al. 2012, submitted

thanks to J. Cordes for slide

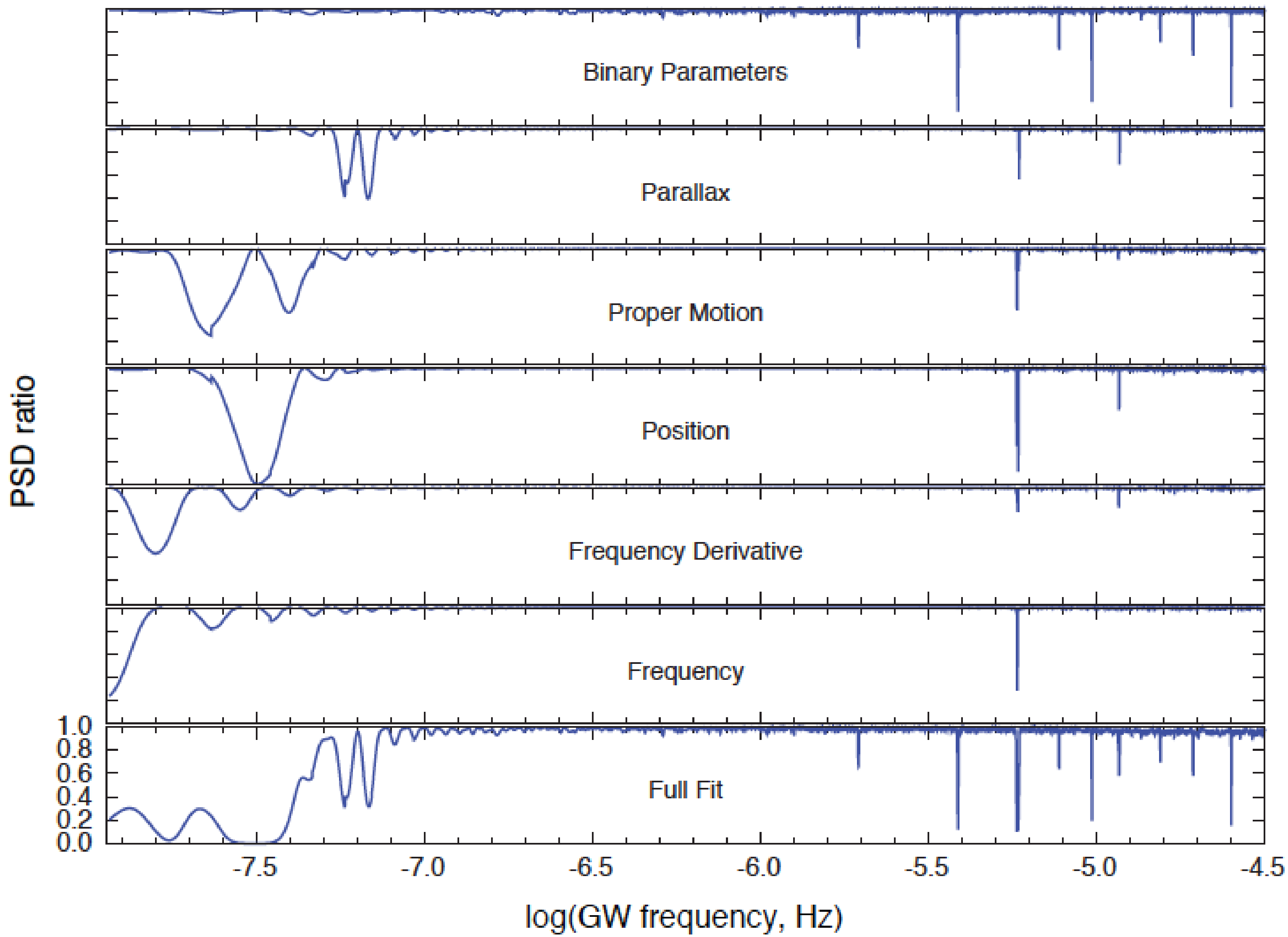


FIG. 1: GW power absorbed by fitting for various pulsar parameters as a function of GW frequency, for pulsar J0613−0200. "PSD ratio" refers to the pre-fit power spectral density value for the given frequency, divided by its post-fit value. All simulated GWs were sinusoids at the given GW frequency. For each panel, only the indicated parameters were used for fitting, while the other parameters were held fixed at the values given in [8]. At high frequencies, only narrow features are evident (mostly due to fitting of the pulsar's binary motions), but low-frequency GW signals are significantly absorbed by standard fitting parameters.

Current constraints on $\Omega_{\mathrm{GW}}(f)$

- From Big Bang Nucleosynthesis; applies to GWs produced before z =10^{10} :

$$\Omega_{\mathrm{GW}} < 1.5 \times 10^{-5}$$

- Current constraint from pulsar timing:

$$\Omega_{\mathrm{GW}} < 1.3 \times 10^{-9} \text{ at } f = 2.8 \times 10^{-9}\,\mathrm{Hz}$$

- Current constraint from LIGO:

$$\Omega_{\mathrm{GW}}(f \sim 100\,\mathrm{Hz}) < 6.9 \times 10^{-6}$$

- From SDSS large-scale structure, combined with Planck, WMAP and H_0:

$$\Omega_{\mathrm{GW}} \lesssim 6 \times 10^{-3}$$

Discovery Space: Constraints

Case 1: z < few. We'll assume:

- $\Omega_{GW} \leq 0.01$
- Copernican Principle: we do not occupy some preferred location in universe
- Can treat space as approximately Euclidean
- No extreme beaming of GWs

In paper we relax assumptions 2 and 4 to allow for fact that we reside in a Galaxy, and to show that even fairly extreme beaming has only modest effect on results.

How to maximize SNR at fixed Ω_{GW} ($z <$ few)

- $h_{\max}^2 f^2 d_{near}^2 T_{sig} \propto \Delta E$
- $\Omega_{GW}\,\rho_0 \propto \Delta E\, \dot{N}\, \tau_0$
- $d_{near}^{-3} \propto \dot{N} \max\{T_{sig}, T_{obs}\}$
- $SNR_{near}^2 \propto h_{\max}^2 f^{-2} \min\{T_{sig}, T_{obs}\}$

with

$$\dot{N} = \frac{\#\,events}{vol - time}$$

$$\tau_0 \approx 10^{10}\, yr$$

$$\rho_0 \approx 0.1/\tau_0^2$$

$$\Rightarrow\ SNR_{near}^2 \simeq 2\times 10^{-4}\frac{\Omega_{\rm GW}}{f^4\tau_0^3}\,\frac{MpT_{\rm obs}}{\delta t_{\rm rms}^2}d_{\rm near}.$$

w/ M = # pulsars
p = obs cadence (e.g., ~1/wk)

So most favorable case has d_{near} as large as possible, which for consistency is $d_{near} \approx \tau_0$
Roughly speaking, SNR is maximized when there is 1 burst per Hubble-volume per T_{obs} .

Zimmerman&Thorne(1980)

Max SNR from direct GWs, z< few

$$\max\{\mathrm{SNR}\} \lesssim 10\left(\frac{f}{10^{-7}\,\mathrm{Hz}}\right)^{-2}\left[\frac{\Omega_{\mathrm{GW}}}{10^{-5}}\right]^{1/2} \times \mathrm{obs.}$$
$$\lesssim 0.03\left(\frac{f}{10^{-5}\,\mathrm{Hz}}\right)^{-2}\left[\frac{\Omega_{\mathrm{GW}}}{10^{-2}}\right]^{1/2} \times \mathrm{obs.}, \tag{5.8}$$

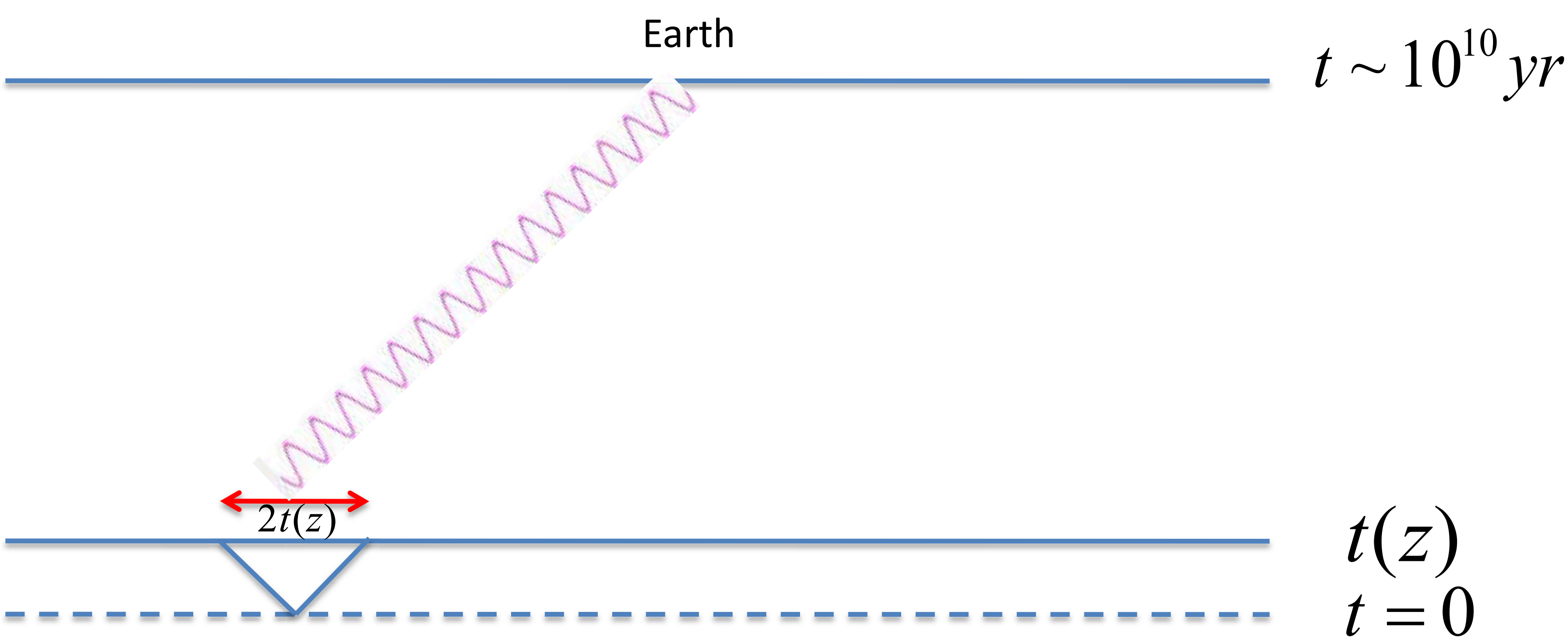
where

$$\mathrm{obs.} = \left[\frac{\delta t_{\mathrm{rms}}}{10^{-7}\,\mathrm{s}}\right]^{-1}\left[\frac{M\,p\,T_{\mathrm{obs}}}{10^{4}}\right]^{1/2}. \tag{5.9}$$

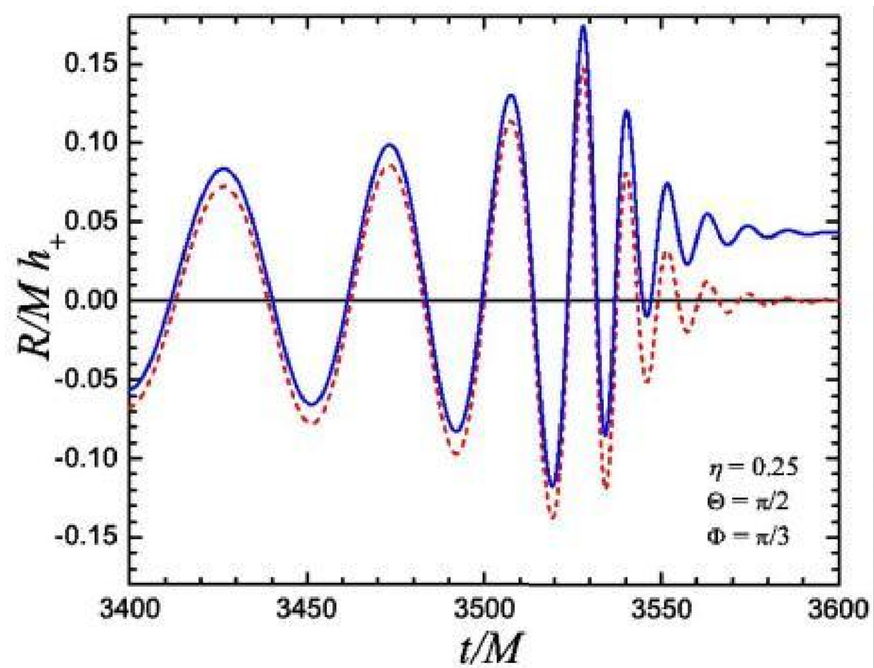
For sources at redshift z > few, $rhs \rightarrow \dfrac{rhs}{(1+z)^{1/2}}$

Discovery Space Constraints: $z \sim 10^1 - 10^{20}$

- Copernican Principle: we do not occupy some preferred location
- $\Omega_{GW} \le 10^{-5}$
- (naïve) Causality: $\Delta E_{burst} < \rho(z)\, t^3(z)$



Christodoulou memory effect: a primer



To understand this, let's start with quadrupole formula:

$$h_{ij}^{TT} = \frac{2}{r}\ddot{I}_{ij}^{TT}$$

$$I^{ij} = \sum_A M_A x_A^i x_A^j \;\Rightarrow\; \ddot{I}^{ij} = \sum_A M_A(\ddot{x}_A^i x_A^j + x_A^i \ddot{x}_A^j + 2\dot{x}_A^i \dot{x}_A^j)$$

Memory term

Accounting for relativistic motion:

$$\Delta h_{jk}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{r\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1 - v_A \cos\theta_A} \right]^{\mathrm{TT}}$$

Christodoulou memory (cont'd)

Christodoulou (1991) showed that you could get memory effect in pure GR (no matter). Thorne (1992) showed that Christodoulou's result was just an extension of older result, with replacement:

$$\frac{M_A}{\sqrt{1-v_A^2}} \rightarrow E_{graviton} \qquad \left|v_A\right| = 1$$

Useful estimate: $h_{\rm mem} \sim \dfrac{\alpha}{\sqrt{6}} \dfrac{\Delta E_{\rm GW}}{d}$

where α is an asymmetry factor that is ≈ 1 for an inspiraling binary.

Christodoulou memory and PTAs

Effect on each pulsar is like a "glitch" : $\Delta\nu / \nu \sim h_{mem}$

which implies a residual arrival time that grows linearly:

$$\delta t_{\rm GW} \sim \theta(t - t_0) h_{\rm mem} t,$$

Of course, in any single pulsar the GW memory effect is indistinguishable for an pulsar glitch, but point is that all pulsars in array seem to glitch at same time (w/in error bars) and with consistent amplitudes. Including loss of significance due to fitting out part of signal with $\Delta\nu$ and $\Delta\dot{\nu}$, van Haasteren and Levin (2010) find:

$$\mathrm{SNR}_{\rm mem} \sim \frac{1}{20} \frac{h_{\rm mem} T_{\rm obs}}{\delta t_{\rm rms}} (M p T_{\rm obs})^{1/2}$$

Effect of redshift on h_{mem}

z<1 result: $h_{mem} \approx \frac{\alpha}{\sqrt{6}} \Delta E / D$

Standard translation to z>1: $M_i \to M_i(1+z)$, $D \to D_L$

implies z >1 result: $h_{mem} \approx \frac{\alpha}{\sqrt{6}} \frac{\Delta E(z)\,(1+z)}{D_L}$

For very high z: $D_L \approx 3\tau_0(1+z)$.

$$\Rightarrow h_{mem} \approx \frac{\alpha}{3\sqrt{6}} \frac{\Delta E_{local}}{\tau_0}$$ independent of z !

So unlike energy, h_{mem} does not redshift away. The universe remembers GW bursts!

What is max SNR_{mem}?

What goes in:

$$\Delta E_0 = \Delta E_z/(1+z)$$

Define $B \equiv \dfrac{\#\ bursts}{Hubble\ Vol\ @z}$

Hubble volumes at z
within current Hubble volume

$$\sim [t_0/t(z)]^3$$

Energetics:

$$\Delta E_0\, B\, [t_0/t(z)]^3 \lesssim \frac{1}{10}\Omega_{\rm GW}\tau_0$$

$$(observed\ rate\ now)\ T_{obs} \geq 1$$

$$\Rightarrow 4\pi(B/\tau_0)[t_0/t(z)]^3\, T_{obs} \geq 1$$

$$h_{\rm mem} \simeq \frac{\alpha}{8}\frac{\Delta E_0\,(1+z)}{\tau_0}$$

$$\mathrm{SNR}_{\rm mem} \sim \frac{1}{20}\frac{h_{\rm mem}\, T_{\rm obs}}{\delta t_{\rm rms}}(M p\, T_{\rm obs})^{1/2}$$

What is max SNR_{mem}?

What comes out:

$$\max\{\mathrm{SNR}_{\mathrm{mem}}\} \simeq \frac{\alpha}{125}(1+z)\frac{\Omega_{\mathrm{GW}}}{\tau_0}T_{\mathrm{obs}}^2\frac{(M\,p\,T_{\mathrm{obs}})^{1/2}}{\delta t_{\mathrm{rms}}}$$

$$\simeq 270\,\alpha\left[\frac{1+z}{10^7}\right]\left[\frac{\Omega_{\mathrm{GW}}}{10^{-10}}\right]\left[\frac{T_{\mathrm{obs}}}{10^8\,\mathrm{s}}\right]^2\times \mathrm{obs.}$$

where

$$\mathrm{obs.} = \left[\frac{\delta t_{\mathrm{rms}}}{10^{-7}\,\mathrm{s}}\right]^{-1}\left[\frac{M\,p\,T_{\mathrm{obs}}}{10^4}\right]^{1/2}.$$

This is a factor ~(1+z) larger than
max SNR-mem for sources at z <1 !!

Summary/Conclusions

- GWs with 10^(-7} < f < 10^{-4.5} Hz would ***not*** be fitted out by small changes to other pulsar parameters.

- However, PTA detection of (direct) GWs with
  f > 10^{-5} Hz requires very lucky coincidence or violation of some fundamental assumptions.

- If some early universe process dumped a lot of energy into GWs, the most detectable signature of that process could easily be their memory effect on PTA timing residuals. We recommend that PTA data analysis efforts include this possible discovery space.