Estimation of Gravitation Parameters of Saturnian Moons Using Cassini Attitude Control Flight Data

Samantha C. Krening[1]
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91109

A major science objective of the Cassini mission is to study Saturnian satellites. The gravitational properties of each Saturnian moon is of interest not only to scientists but also to attitude control engineers. When the Cassini spacecraft flies close to a moon, a gravity gradient torque is exerted on the spacecraft due to the mass of the moon. The gravity gradient torque will alter the spin rates of the reaction wheels (RWA). The change of each reaction wheel's spin rate might lead to overspeed issues or operating the wheel bearings in an undesirable boundary lubrication condition. Hence, it is imperative to understand how the gravity gradient torque caused by a moon will affect the reaction wheels in order to protect the health of the hardware. The attitude control telemetry from low-altitude flybys of Saturn's moons can be used to estimate the gravitational parameter of the moon or the distance between the centers of mass of Cassini and the moon. Flight data from several low-altitude flybys of three Saturnian moons, Dione, Rhea, and Enceladus, were used to estimate the gravitational parameters of these moons. Results are compared with values given in the literature.

Nomenclature

$[BW]$	=	direction cosine matrix from the reaction wheel frame to the Cassini body frame
$[I]$	=	Cassini spacecraft inertia tensor
I_x, I_y, I_z	=	diagonal components of principal axis inertia
L_G	=	gravity gradient torque
$\vec{L}$	=	external torque acting on Cassini
$\vec{R}_c$	=	position vector from the center of mass of the moon to the center of mass of Cassini
R_c	=	magnitude of the position vector $\vec{R}_c$
R_{cx},R_{cy}, R_{cz}	=	components of the $\vec{R}_c$ position vector expressed in the principal axis coordinates
RWA	=	reaction wheel assembly
μ	=	gravitational parameter of the moon
$\vec{\omega}_{B/N}$	=	angular velocity of the Cassini spacecraft with respect to the inertial frame
${}^{B}\vec{\omega}_{B/N}$	=	angular velocity of the Cassini spacecraft with respect to the inertial frame in B frame components

[1] Attitude Control Engineer, Guidance & Control Section, Jet Propulsion Laboratory. 4800 Oak Grove Dr., M/S 230-104, Pasadena, CA 91109.

I. Introduction

THE Cassini Spacecraft was launched on October 15, 1997. Cassini used multiple gravity assists, including two at Venus, one at Earth, and another at Jupiter in order to reach an orbit around Saturn on July 1, 2004. One of the goals of the Cassini mission is to discover and study more of Saturn's satellites. Before the Cassini mission, there were 18 known moons orbiting Saturn. Currently, over 53 moons have been discovered. Some of the most basic information that must be known about each moon is its mass, size, and orbit. For many of these moons, the only data available is from the Cassini mission. This paper discusses a method to estimate the mass of some of Saturn's moons based on attitude control flight data. Other papers have been published that estimate the mass of many of Saturn's moons using satellite astrometry, radiometric tracking, spacecraft imaging, and Doppler tracking.[1,2] But it is beneficial to verify results using different instruments, measurements, and calculations for many reasons. If multiple teams use different data and achieve similar results, it decreases the likelihood that incorrect results are being produced due to issues such as defective instruments and inaccurate processing methods.

Cassini has two types of attitude control systems onboard: thrusters and reaction wheel assemblies (RWAs). During normal operations and science observations, the reaction wheels are generally used for attitude control since the pointing is much smoother and more accurate than using thrusters. Four reaction wheels are mounted onboard Cassini, but three are used at any given time to control the spacecraft's attitude. All three reaction wheels that are currently being used have completed over three billion revolutions during the mission, so the Cassini attitude control engineers monitor the health of the reaction wheels very closely to ensure the hardware lasts until the final mission stage in 2017. Attitude estimation onboard the spacecraft uses star trackers and gyroscopes.

It could be very helpful in future robotic space exploration missions to be able to calculate the gravity gradient torque acting on a spacecraft from the reaction wheel telemetry. Especially if the environment is unknown, if something goes wrong on a spacecraft the telemetry could be analyzed to determine if it is possible the spacecraft had an unexpected flyby with a moon, similar to an incident with Pioneer 11.

Pioneer 11 discovered a new moon by almost running into it. A few minutes after Pioneer 11 crossed Saturn's ring plane on September 1, 1979, the radiation detectors and magnetometer recorded huge disturbances.[3] It was later determined that Pioneer 11 had passed very close to a moon. Epimetheus and Janus are approximately the same size and occupy the same orbit around Saturn – before Pioneer 11, only one of the moons had been discovered. If a similar event were to happen in the future on a spacecraft using reaction wheels, the method in this paper could be used to estimate the mass of the newly-discovered moon. Or, if some other observation gave an estimate of the moon's mass, this method could be used to determine the distance between the center of mass of the moon and the spacecraft. From an operational perspective, if a spacecraft went into safe mode, the reaction wheel telemetry could be analyzed to determine whether an unexpected flyby was a possible cause for fault protection to be triggered. The sudden departure in the reaction wheel speed telemetry from what was expected would give the attitude control engineers an immediate insight that an unexpected external torque was acting on the spacecraft.

II. Theory

The two main analytical theories covered in this paper are gravity gradient torque and the angular momentum of a spacecraft on reaction wheel control. These theories are closely related because the total external torque on a system equals the change of angular momentum. The gravity gradient torque acting on a spacecraft during a close flyby is one external torque acting on the system. Once the change of the system angular momentum is calculated from the reaction wheel and angular velocity telemetry, the total external torque acting on Cassini is known. Knowledge of Cassini's surrounding environment enables us to estimate how much of the external torque acting on Cassini is due to the gravity gradient. This is why the flybys studied are either of moons without atmospheres or flybys in which Cassini did not go through a plume of the moon.

A. Gravity Gradient Torque

When a spacecraft is flying near a moon or planet, the gravitational force exerted on the closer part of the spacecraft is slightly different than the gravitational force on the farther part of the rigid spacecraft. This difference in the gravitational force causes a torque to be applied to the spacecraft, which is called a gravity gradient torque. When the Cassini spacecraft has close flybys of Saturn's moons while using the reaction wheels for attitude control, the gravitational torque causes a change in angular momentum that can be seen in the reaction wheel flight data. The time histories of the reaction wheel speeds could be used to estimate the gravity gradient torque imparted on the spacecraft, and the gravitational parameters of the moon determined accordingly.

The gravity gradient torque exerted on a spacecraft can be expressed by Eq. (1), in which $\vec{L}_G$ is the gravity gradient torque vector, μ is the gravitational parameter, $\hat{R}_c$ is the unit direction vector from the moon's center of mass to Cassini's center of mass, and $[I]$ is the spacecraft inertia tensor.[4]

$$\vec{L}_G = \frac{3\mu}{R_c^3}\hat{R}_c \times [I]\hat{R}_c \tag{1}$$

The inertia matrix in a body-fixed frame that is not the principal axis frame is expressed by a symmetric 3x3 matrix.

$$^{B}[I] = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{2}$$

The direction cosine matrix that rotates the body-fixed coordinate frame to the principal axis coordinates can be calculated by using the eigenvectors of the body inertia matrix. Each row of the direction cosine matrix is an eigenvector of the body inertia matrix. In principal axis coordinates, the inertia tensor is the diagonal matrix shown in Eq. (3) in which the principal axis inertias I_x, I_y, and I_z are the eigenvalues of the body inertia tensor $^{B}[I]$.

$$^{P}[I] = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{3}$$

Let $^{P}\hat{R}_c$ be the unit direction vector from the center of mass of the moon to Cassini's center of mass expressed in principal axis coordinates.

$$^{P}\hat{R}_c = \frac{^{P}\vec{R}_c}{\left|^{P}\vec{R}_c\right|} = {}^{P}\begin{bmatrix} r_{cx} \\ r_{cy} \\ r_{cz} \end{bmatrix} \tag{4}$$

If the gravity gradient torque equation is expressed in principal axis coordinates, it can be expressed in the simpler form of Eq. (5).

$$\begin{aligned} L_{Gx} &= \frac{3\mu}{R_c^3}\left(I_z - I_y\right)r_{cy}r_{cz} \\ L_{Gy} &= \frac{3\mu}{R_c^3}\left(I_x - I_z\right)r_{cx}r_{cz} \\ L_{Gz} &= \frac{3\mu}{R_c^3}\left(I_y - I_x\right)r_{cx}r_{cy} \end{aligned} \tag{5}$$

The gravity gradient torque equation can be rearranged to solve for the gravitational parameter of the moon, μ, as seen in Eq. (6).

$$\mu = \frac{R_c^5\left(\left|\vec{L}_G\right|^2\right)}{3\vec{L}_G^T\left(\vec{R}_c \times [I]\vec{R}_c\right)} \tag{6}$$

The gravity gradient torque equation can also be rearranged to estimate the position vector between the center of mass of the moon and Cassini's center of mass. The unit direction vector $^{P}\hat{R}_c$ equals the unit direction vector from the moon's center of mass to Cassini's center of mass in the spacecraft's principal axis frame.

$$^{P}\hat{R}_c = {}^{P}\begin{bmatrix} r_{cx} \\ r_{cy} \\ r_{cz} \end{bmatrix} = \frac{\begin{bmatrix} a & b & c \end{bmatrix}^T}{\left|\begin{bmatrix} a & b & c \end{bmatrix}^T\right|} \tag{7}$$

The ratio of each component of the radial vector from the moon's center of mass to Cassini's center of mass can be calculated from Eq. (8).

$$\frac{r_{cy}}{r_{cx}} = \frac{L_{Gx}(I_x - I_z)}{L_{Gy}(I_z - I_y)} = \frac{b}{a}$$
$$\frac{r_{cz}}{r_{cy}} = \frac{L_{Gy}(I_y - I_x)}{L_{Gz}(I_x - I_z)} = \frac{c}{b} \tag{8}$$
$$\frac{r_{cx}}{r_{cz}} = \frac{L_{Gz}(I_z - I_y)}{L_{Gx}(I_y - I_x)} = \frac{a}{c}$$

This method assumes it is known which side of the spacecraft the moon is on. More specifically, if it is known whether one of the position vector components, such as r_{cy}, is positive or negative, then the unit direction vector and magnitude of the position vector from the moon's center of mass to Cassini's center of mass can be calculated. To calculate the direction vector from a moon's center of mass to Cassini's center of mass, initially set the value 'b' equal to ± 1.

$$b = sgn(r_{cy}) * 1 \tag{9}$$

Once 'b' is known, 'a' and 'c' can be calculated. The vector ${}^{P}\hat{R}_c$ can then be formulated using Eq. (7).

$$a = b\frac{L_{Gy}(I_z - I_y)}{L_{Gx}(I_x - I_z)} \tag{10}$$
$$c = b\frac{L_{Gy}(I_y - I_x)}{L_{Gz}(I_x - I_z)} \tag{11}$$

The unit direction vector can be used to calculate the distance between the center of mass of the moon and the center of mass of Cassini using Eq. (12).

$$\left|{}^{P}\vec{R}_c\right| = \left(\frac{3\mu\vec{L}_G^T(\hat{R}_c \times [I]\hat{R}_c)}{\left|\vec{L}_G\right|^2}\right)^{\frac{1}{3}} \tag{12}$$

The gravitational parameter and the distance between the moon and spacecraft cannot be estimated simultaneously. If the position vector from the moon to the spacecraft is known from other sources such as navigational data and orbit determination solutions, the gravitational parameter can be estimated using Eq. (6). If the gravitational parameter is assumed to be known from science measurements, the distance between the spacecraft and the moon can be estimated using Eq. (12).

B. Cassini Angular Momentum on Reaction Wheel Control

When the Cassini spacecraft is on reaction wheel control, the thrusters are not being used, so the total angular momentum of Cassini equals the angular momentum of the spacecraft plus the angular momentum of the reaction wheels.

$$\vec{H} = \vec{H}_{sc} + \vec{H}_{rw} \tag{13}$$

The spacecraft angular momentum equals the product of the inertia tensor and the spacecraft angular velocity.

$$\vec{H}_{sc} = [I]\vec{\omega}_{B/N} \tag{14}$$

The angular momentum of each reaction wheel equals the reaction wheel's inertia in the spin direction multiplied by the wheel speed. It is assumed that the reaction wheel spin directions are orthogonal.

$$ {}^{W}\vec{H}_{rw} = \begin{bmatrix} J_{s1}\Omega_1 \\ J_{s2}\Omega_2 \\ J_{s4}\Omega_4 \end{bmatrix} \quad (15) $$

The total spacecraft angular momentum expressed in body frame components is calculated using Eqn. (16).

$$ {}^{B}\vec{H} = {}^{B}[I]\,{}^{B}\vec{\omega}_{B/N} + [BW]\,{}^{W}\vec{H}_{rw} \quad (16) $$

The external torque acting on the spacecraft equals the inertial derivative of the angular momentum.

$$ \dot{\vec{H}} = \vec{L} \quad (17) $$

During the numerical processing, the angular momentum was calculated in the Cassini spacecraft body reference frame. The angular momentum vector was rotated from the spacecraft body frame to the J2000 inertial reference frame. The inertial external torque was calculated as the derivative of the inertial angular momentum.

III. Data Analyses

Data from the Cassini spacecraft are downlinked to the Earth using the Deep Space Network antennas, including the 70-meter antennas in California, Spain, and Australia. The telemetry required for the calculations in this paper are the time histories of the Cassini spacecraft angular velocity vector and the reaction wheel speeds. The equations for the total angular momentum of the spacecraft and the external torque are sensitive to noise and noise amplification, so the raw telemetry must be processed to reduce noise effects and obtain more-accurate results. Two data analysis methods were used to separately calculate the gravitational parameter of a moon during a given flyby. The first method fits polynomials to the telemetry, and the second uses lowpass filters on the data.

A. Processing Telemetry Method 1: Polynomial Fit

The first method of noise reduction used was to fit the raw reaction wheel speed and body angular velocity data to polynomials. This is beneficial because a smooth polynomial will reduce noise issues. Also, the telemetry are not necessarily recorded at equal time intervals, but this is not an issue since the data are fitted to a polynomial and resampled at equal time steps. There are a few issues with this method, such as the polynomial may not be a very accurate representation of the data and the order of the polynomial must be chosen on a case-by-case basis, which makes automated processing not realistic.

The *polyfit* and *polyval* MATLAB® functions were used to create the polynomial approximations of the data in a least squares sense; the order of the polynomial was chosen on a case-by-case basis, but ranged from 3rd to 10th order polynomials. The polynomial approximations of the data were used to calculate the angular momentum and external torque acting on Cassini during the flybys, which allowed the gravity gradient torque, gravitational parameter (GM), and the distance between the moon and Cassini to be calculated using the equations given in Section II.

B. Processing Telemetry Method 2: Filtering

The second method of noise reduction used was to filter the telemetry. The first step of the filtering method was to interpolate the data at a set of equally-spaced times. Next, a zero-phase, lowpass, 5th order, Butterworth filter was applied to the interpolated reaction wheel speeds and body angular velocity components. Using the filtered wheel speeds and angular velocity, the angular momentum, external torque, and gravitational parameter were calculated using the equations given in Section II. The filtering method does not need to be modified on a case-by-case basis. This allows the numerical processing to be automated, which is one reason why the filtering method is preferable to polynomial fits.

C. Gravity Gradient Model

In order to ensure the gravity gradient torque calculated using the raw telemetry is accurate, a gravity gradient model was built for each flyby using Eq. (1). The gravity gradient model uses the SPICE toolkit createdby the NAIF team at JPL to return the position vector from the center of mass of the moon to the center of mass of Cassini during the flyby. This position vector uses the statistical orbit determination solution for the moon as well as the navigator's reconstructed position vector of Cassini based on downlinked telemetry.

IV. Results

A. Dione

On December 12, 2011, Cassini flew by Dione at an altitude of approximately 99 km (see Fig. 1).

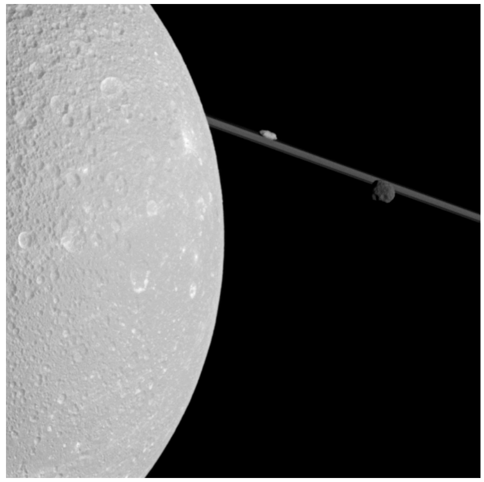

Figure 1. Image taken by Cassini during the December 2011 Dione flyby[5].

The distance from the center of mass of Dione to the Cassini's center of mass throughout the closest approach is shown in Fig. 2. The distance information is from the reconstructed position of the spacecraft and the planetary ephemeris of Dione[6]. The gray, verticle line in the middle of the figure shows the time of closest approach.

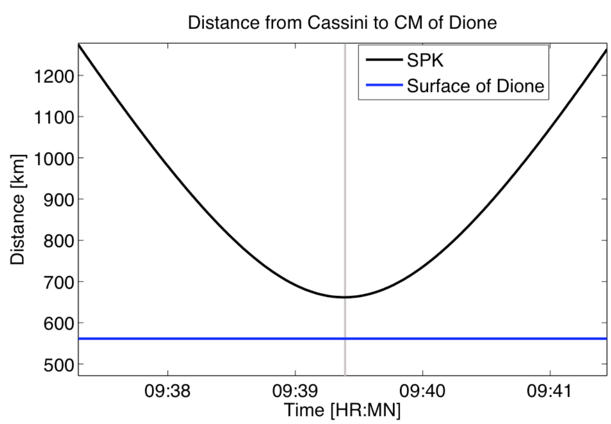

Figure 2. Distance from Dione's center of mass to Cassini's center of mass during the 2011 flyby.

The gravity gradient torque caused a change in the reaction wheel speeds near closest approach. Fig. 3 shows each reaction wheel's raw telemetry along with its polynomial fit and filtered results. The vertical, gray line is the time of closest approach.

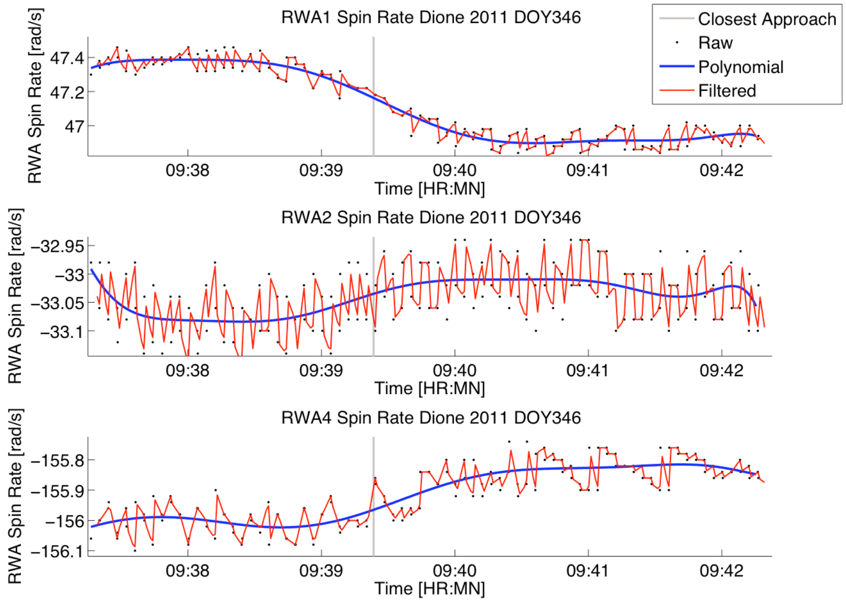

Figure 3. Raw, Polynomial, and Filtered Reaction Wheel Rates.

During the time of closest approach, the Cassini spacecraft was not rotating, and essentially letting the moon pass by its instruments' fields of view. Fig. 4 shows the Cassini body angular velocity telemetry along with the polynomial fit of the data.

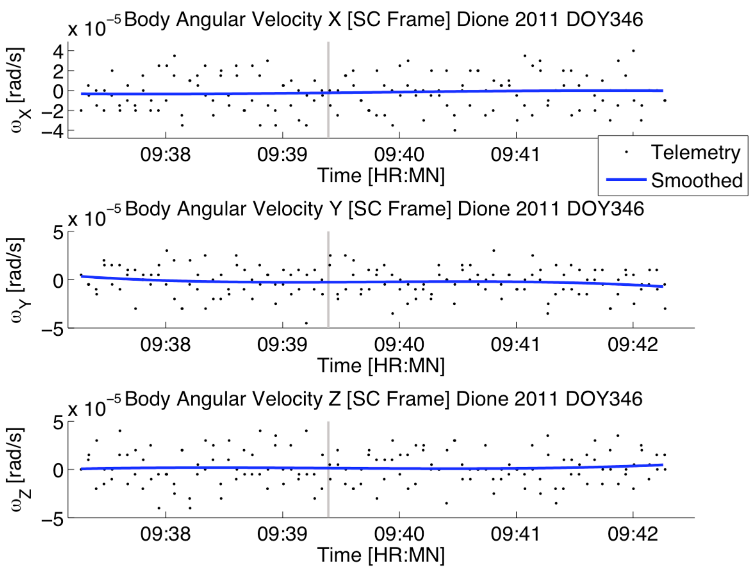

Figure 3. Cassini spacecraft per-axis angular velocity during Dione flyby.

Fig. 5 shows the estimated angular momentum magnitude around the closest approach using the polynomial representations of the data. There is a clear change in the angular momentum magnitude around the closest approach due to the gravity gradient torque exerted on Cassini by Dione.

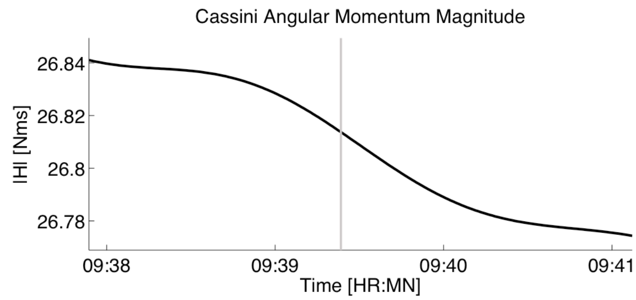

Figure 5. Cassini Angular Momentum Magnitude During Closest Approach.

The derivative of the inertial angular momentum equals the external torque acting on Cassini. Fig. 6 shows the external torque acting on Cassini that was calculated using the polynomial approximations of the telemetry as well as the gravity gradient torque from the theoretical model. The gravity gradient torque calculated from telemetry matches very well with the theoretical results.

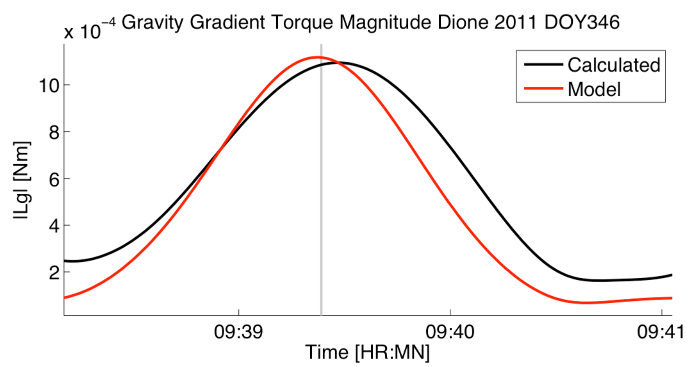

Figure 6. Comparison of Gravity Gradient Torque from Telemetry and Model.

To estimate the distance from Dione's center of mass to Cassini, it was assumed that the gravitational parameter was 73.1221 (km^3s^{-2}), which is the value published by Jacobson[1]. Using Eq. (12), the estimate of the distance from the center of mass of Dione to Cassini's center of mass from telemetry was 650.7 km, while the reconstructed navigation results from the SPK was 661.6 km.

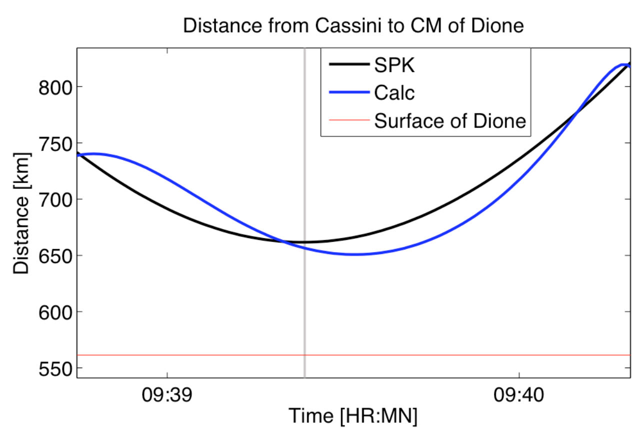

Figure 7. Comparison of Distance between the CMs of Dione and Cassini. SPK is from the navigator reconstruction and JPL ephemeris of Dione. The calculation is from downlinked telemetry.

B. Rhea

The Rhea flyby on March 2, 2010 took the Cassini spacecraft approximately 101.5 km above Rhea's surface. The distance from the center of mass of Rhea to Cassini's center of mass throughout the closest approach is shown in Fig. 8.

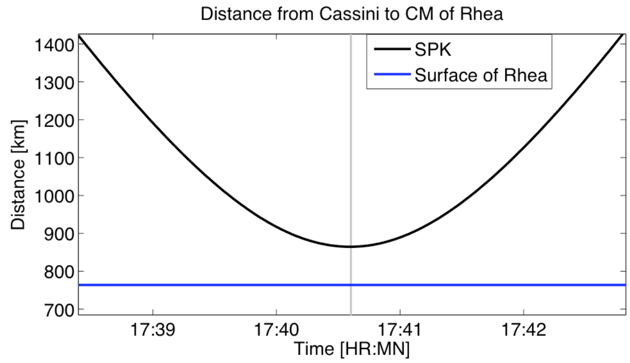

Figure 8. Distance between the CMs of Rhea and Cassini.

Fig. 9 compares the gravity gradient torque during the Rhea flyby from the analytical model and the polynomial calculation from telemetry. The gravity gradient torque model and the polynomial results calculated from telemetry match well nearest the closest approach and have the same profile during the surrounding time.

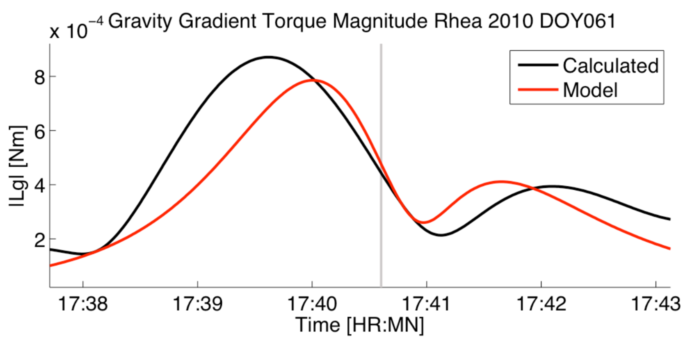

Figure 9. Comparison of Gravity Gradient Torque magnitude calculated from telemetry and the model.

C. Enceladus 12

On November 30, 2010, Cassini flew approximately 45.8 km above the surface of Enceladus (see Fig. 10). The Enceladus 12 flyby was near the north pole so the Cassini spacecraft did not fly through the plume jets.

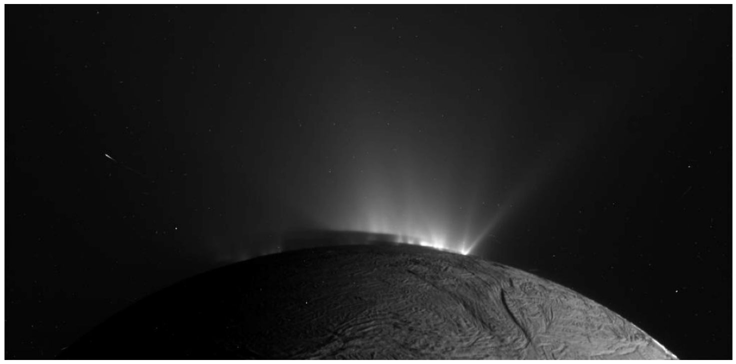

Figure 10. Image of Enceladus taken by Cassini during the Nov. 30, 2010 flyby[5].

The distance from the center of mass of Enceladus to the Cassini's center of mass throughout the closest approach is shown in Fig. 11.

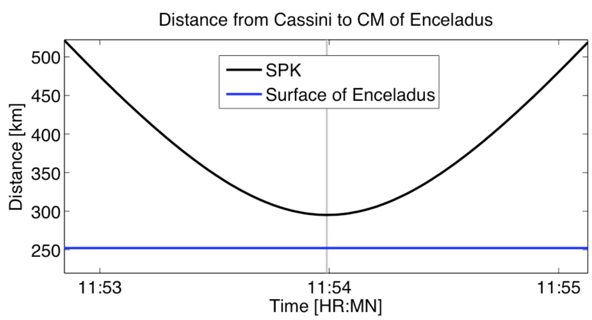

Figure 11. Distance from CMs of Enceladus and Cassini during flyby. The data from the SPK is the reconstructed position of Cassini from the navigators and the JPL ephemeris of Enceladus.

The magnitude of the gravity gradient torque during the Enceladus flyby is shown in Fig. 12. The polynomial approach produced the same profile as the theoretical model for the gravity gradient torque, but the fit for the Enceladus flyby was not as ideal as the Dione flyby. There are many reasons why the results for Enceladus are not as similar to the model as the flybys for Dione and Rhea were. Changing the order of the polynomial fits for the reaction wheel speeds and spacecraft angular velocity components alters the result – the polynomial fits may not have been as ideal for the Enceladus flyby as for Dione and Rhea. Even though Cassini did not fly through the plumes of Enceladus during this flyby, there could have been some atmospheric material that Cassini flew through. The resulting external torque acting on Cassini would cause the results from telemetry to differ from the theoretical gravity gradient model. Additional external torques could have been acting on Cassini during the flyby, including a torque due to solar radiation pressure ($2\text{x}10^{-3}$ Nm), a torque due to the radioisotope thermoelectric generators (RTGs) ($1.83\text{x}10^{-3}$ Nm), and a magnetic torque due to the interaction between the spacecraft's magnetic moment and Saturn's magnetic field ($0.49\text{x}10^{-3}$ Nm)[7]. Another possible external torque is a magnetic torque from Enceladus. Also, the Enceladus results could be less than ideal because Enceladus is a much less massive moon than Dione or Rhea.

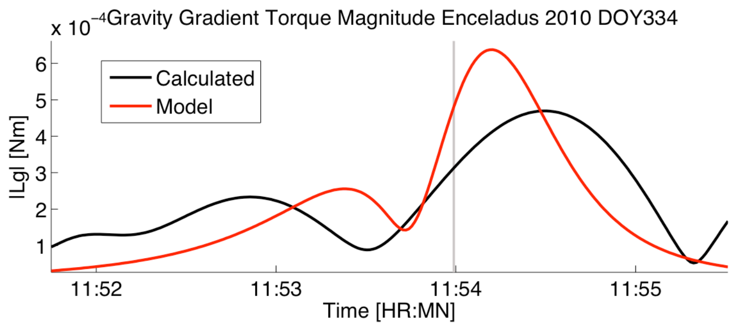

Figure 12. Comparison of the Gravity Gradient Torque magnitude calculated from telemetry and the model.

D. Enceladus 13

On December 21, 2010, Cassini flew approximately 48.4 km above the surface of Enceladus. The distance from the center of mass of Enceladus to the Cassini's center of mass throughout the closest approach is shown in Fig. 13. The Cassini spacecraft did not travel through the plumes of Enceladus during the flyby.

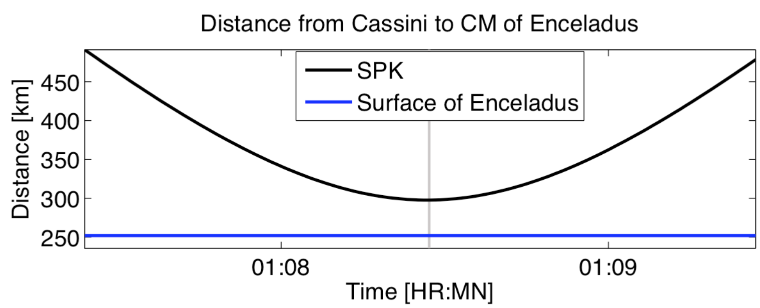

Figure 13. Distance between the CMs of Enceladus and Cassini.

The magnitude of the gravity gradient torque from the model and the calculations from telemetry are shown in Fig. 14. At the time of closest approach, the gravity gradient torque model and the results from processing the telemetry using the polynomial method match very well. The model and polynomial results match less well before and after the closest approach. The less-than-ideal fit between the polynomial results and model on either side of the closest approach could be caused by many reasons, including a poor polynomial fit with the data, unmodeled external torques including atmospheric and magnetic contributions, and the small mass of the moon.

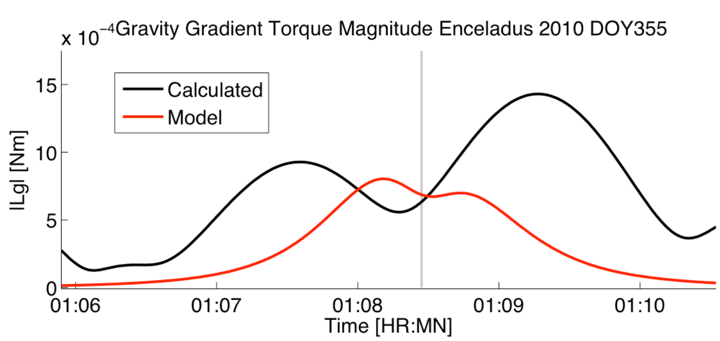

Figure 14. Comparison of Gravity Gradient Torque magnitude calculated from telemetry and the model.

E. Summary

The published values[5] for the gravitational parameters of Rhea, Enceladus, and Dione are given in Table (1) along with the results of the polynomial and filtering methods used in this paper. The gravitational parameter calculated using both the polynomial and filtering methods are close to the published results. The calculations using both methods produce a first-order estimate of each moon's gravitational parameter, but may not be accurate enough to use for high-precision applications.

Table 1. Comparison of the gravitational parameter calculated using the polynomial and filtering methods with the published values. Units of the gravitational parameter μ are given in km^3s^{-2}.

Flybys	Jacobson[1]	Polynomial method μ	Filtering method μ
Rhea	154.5897	144.30	152.19
Dione	73.1221	71.85	73.12
Enceladus 12	6.9495	4.61	9.07
Enceladus 13	6.9495	6.55	7.16

V. Conclusion

During Cassini's close flybys of Saturn's moons, the gravity gradient torque exerted on Cassini can be observed using the reaction wheel speeds. Understanding how the reaction wheel speeds are affected by a gravity gradient torque during a flyby is important to protect the health of the reaction wheels so they successfully operate until the end of the mission. The reaction wheel telemetry can give a first-order estimation and sanity check of the moon's mass or distance from the spacecraft, but may not be accurate enough for high-precision applications. From an operational perspective, filtering the data is preferable to using a polynomial fit because the filtering method can be automated and does not need to be modified on a case-by-case basis.

There are several factors that could improve the results of the polynomial and filtering estimation methods. Better estimates of the spacecraft inertia tensor could improve the results. If the spacecraft recorded and downlinked higher-rate reaction wheel and angular velocity data, the filtering method would be especially improved. Accounting for imperfect mounting of the reaction wheels could improve the results since the reaction wheel spin axes are not perfectly orthogonal. Creating a better model for other external torques, such as torques causes by solar radiation pressure, the atmosphere of each moon, and the onboard radioisotope thermoelectric generators (RTGs), could improve the numerical results. Throughout the mission, the external torque due to the sun and RTGs has been at least an order of magnitude less than the gravity gradient torque acting on the Cassini spacecraft during most close flybys, but creating more-accurate models of these additional external torques could improve the estimates of each moon's gravitational parameter.

Acknowledgments

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Reference to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement by the United States Government or the Jet Propulsion Laboratory, California Institute of Technology The author would like to thank Allan Y. Lee, Tom Burk, Todd Brown, and the Cassini AACS team for their support in this analysis. Allan Y. Lee's and Tom Burk's thorough reviews were very valuable and instrumental to this paper.

References

[1]Jacobson, Robert A., The Orbits of the Major Saturnian Satellites and the Gravity Field of Saturn from Spacecraft and Earth-Based Observations. The Astronomical Journal, 128:492–501, 2004 July.

[2]Jacobson, Robert A., et al, The Gravity Field of the Saturnian System from Satellite Observation and Spacecraft Tracking Data. The Astronomical Journal, 132:2520–2526 2006 December.

[3]Spencer, Henry, "The Perils of Pioneer 11", http://www.astroarts.org/the-perils-of-pioneer-11/. March 18, 2013.

[4]Schaub, Hanspeter, and John L. Junkins, *Analytical Mechanics of Space Systems*, 2nd ed., AIAA Education Series, 2009.

[5]NASA PDS Imaging Node. http://pds-imaging.jpl.nasa.gov/

[6]SPICE Toolkit. http://naif.jpl.nasa.gov/naif/data.html

[7]Lee, A. Y. and Hanover, G., "Cassini Spacecraft Attitude Control System Flight Performance," AIAA 2005-6269, *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, San Francisco, California, August 15-18, 2005.