(Preprint) AAS XX-XXX

THRUST DIRECTION OPTIMIZATION: SATISFYING DAWN'S ATTITUDE AGILITY CONSTRAINTS

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The science objective of NASA's Dawn Discovery mission is to explore the two largest members of the main asteroid belt, the giant asteroid Vesta and the dwarf planet Ceres.[1] Dawn successfully completed its orbital mission at Vesta. The Dawn spacecraft has complex, difficult to quantify, and in some cases severe limitations on its attitude agility.[2] The low-thrust transfers between science orbits at Vesta required very complex time varying thrust directions due to the strong and complex gravity and various science objectives. Traditional thrust design objectives (like minimum ΔV or minimum transfer time) often result in thrust direction time evolutions that can not be accommodated with the attitude control system available on Dawn. This paper presents several new optimal control objectives, collectively called thrust direction optimization that were developed and necessary to successfully navigate Dawn through all orbital transfers at Vesta.

INTRODUCTION

The dawn spacecraft has two long solar arrays which together span 20 meters (see Figure 1.) The solar arrays provide power for the spacecraft including the three ion thrusters. The arrays are sized to allow spacecraft and one ion thruster to operate at heliocentric distances approaching 3 AU at Ceres. The long arrays result in a large moment of inertia along axes orthogonal to the arrays. The attitude of the spacecraft for any given thrust direction is determined by an algorithm called "power

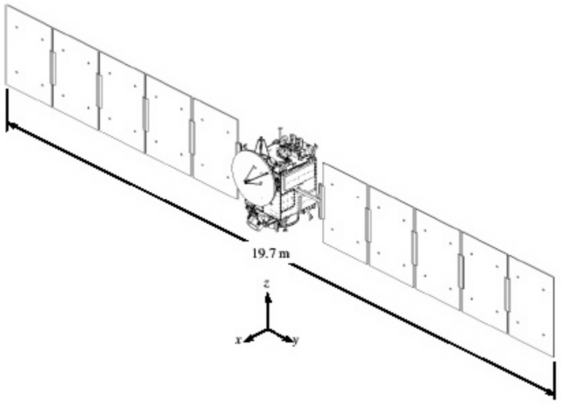

Figure 1. The Dawn spacecraft showing the fully deployed solar array. The ion thrusters are located under (-z direction) the spacecraft in this view.

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steering". The purpose of power steering is to always maximize the power output for ion propulsion. Power steering requires the attitude to be chosen so that the solar array surface is exactly orthogonal to the Sun for any thrust direction. This is accomplished by rotating the spacecraft around the thrust direction (for the -Z axis thruster this amounts a rotation about the z axis) and rotating the arrays relative to the spacecraft about the y axis.

The power steering algorithm has two poles in attitude space that can requite arbitrarily large attitude accelerations and rates. These poles correspond to thrusting in the Sun or anti-Sun direction. If a thrust profile requires thrusting to move near either pole then a rapid, nearly 180 degree flip is required by the solar array. This flip is in an axis associated with the highest moment of inertia of the spacecraft.

Attitude control during ion thrusting is achieved by thrust vector control (TVC) in the two axes perpendicular to the thrust direction and by reaction wheels or hydrazine thrusters in the axis parallel to the thrust direction. TVC is achieved by gimbals on the ion thrusters that have a maximum excursion of about 5 degrees. Using the gimbals to thrust through points outside of the center of the spacecraft mass provides a steering torque. The gentle thrust provided by Dawn's ion thrusters (on the order of 50 [mN]) and the small gimbal excursion ($<$ 5 degrees) provide little torque and hence limited attitude agility. The limited agility combined with the power steering algorithm result in many thrust profiles being unflyable. The agility constraints become particularly limiting when thrusting near the Sun and Anti-Sun directions.

The actual agility limitations are difficult to predict[2] in part because the momentum state of the spacecraft (reaction wheel rates) can not be exactly known or even approximately predicted until a momentum management plan is developed for the thrust profile in question. This is a chicken and egg problem. The agility constraints are therefor not analytic and cannot be directly incorporated into methods to design thrust profiles for orbital transfers. The flyability of any given thrust profile is predicted best by a Monte Carlo analysis of possible momentum management plans and reaction wheel momentum states.[2]

The software used to design all of the maneuvers for the Dawn spacecraft is a tool set called Mystic.[3] The optimization algorithm used in Mystic is the Static Dynamic optimal Control or SDC algorithm.[4] Mystic was originally designed for low-thrust (typically electric propulsion) mission design but has been extended to provide very high fidelity maneuver design for mission operations.

THRUST DIRECTION OPTIMIZATION

Mystic's main function is low-thrust trajectory optimization. The optimization variables include thrust as a function of time, launch or start date, flight time (or equivalently arrival date), up to six initial condition parameters, and the initial mass. State variables that are optimized include position, velocity, and mass of a spacecraft as a function of time. The general problem that Mystic's optimization algorithm solves is

$$J(v(t),w) = \min_{v(t),w} \int_{t_o}^{t_f} F(x(t),v(t),w,t)dt + \sum_{i=1}^{N} H_i(x(t_i),u_i,w,t_i) + G(x(t_N),u_N,w,t_N), \tag{1}$$

subject to a state equation of the form

$$\frac{dx(t)}{dt} = T(x(t),v(t),w,t), \tag{2}$$

and an initial condition of the form

$$x(t_0) = \Gamma(w), \tag{3}$$

where the functions F and G are user selectable, once continuously differentable objective functions; the function T represents the trajectory equations of motion; and the function Γ returns the initial system state. There are three classes of variables: state variables $x(t)$, dynamic control variables $v(t)$, and static control variables w. In the current Mystic implementation, the components of the state vector $x(t)$ are the position, velocity and mass of the spacecraft. The dynamic control vectors $v(t)$ is the the three components of thrust as a function of time. The “static” control paramters w are defined to be the initial spacecraft position, velocity , mass, and the time of flight for each trajectory phase.

The equation used to describe the time evolution of the state T includes ion thruster mass flow, attitude control thruster mass flow, multi-body gravity, gravity harmonics, thrust, and solar radiation pressure. Both linear and non-linear constraints are also allowed in the variables $x(t)$, $v(t)$, and w.

Formulation of Direction Objectives

The objectives J that where initially used to design transfer thrusting at Vesta were maximize final mass (equivalent to minimizing propellant consumption). Direction optimal thrust solutions are defined as the thrust profile that is closest or furthest to a thrust direction target as possible that also meets all constraints and state targets. The direction target can be either inertially fixed for the duration of the transfer or the direction can be a function of time. The direction objective can be applied to part or all of the transfer. Direction objectives can be used in combination with mass objectives to create a pareto optimal front. The general form of an ”attractor” objective is

$$min_{v,w} \int_{t_o}^{t_f} \left[1 - \hat{v}(t) \cdot \hat{D}(x(t), w(t), t)\right]^2 dt, \tag{4}$$

where $\hat{D}$ is unit vector function that provides the target thrust direction. The general form of a ”pole repulsor” objective is

$$min_{v,w} \int_{t_o}^{t_f} \left[1 - (\hat{v}(t) \times \hat{S}(x(t), w(t), t) \cdot (\hat{v}(t) \times \hat{S}(x(t), w(t), t)\right]^n dt, \tag{5}$$

where $\hat{S}$ is unit vector function that provides a pole that is to be avoided in thrust direction space.

An example of a mass optimal versus a direction optimal thrust direction evolution is provided in Figure 2. The mass optimal solution (blue) cannot be flown by Dawn as a result of attitude rates when thrusting near the anti-Sun direction. With the substitution of an attractor direction objective in a fixed direction in the Radial - Transverse - Normal or ”RTN” frame the transfer becomes flyable (green in Figure 2). The RTN frame is defined relative to the location of the Dawn spacecraft and the center of Vesta. The radial axis is along the Vesta - Dawn direction. The transverse direction is the local horizontal direction relative to Dawn above Vesta that is closest to the relative velocity of Dawn with respect to Vesta. The normal direction completes the right handed coordinate system. The RTN frame is non-inertial so a fixed direction in the RTN frame is not inertially fixed. The RTN direction objective is indicated by a black star in Figure 2. Notice that the direction optimal solution moves the thrusting above the anti-Sun direction (lower red line). This reduces the attitude rate and acceleration requirements of the power steering solar array flips.

Direction optimization turned out to be essential to the success of the Vesta mission by allowing flyable and timely maneuver solutions that delivered Dawn to each science orbit.

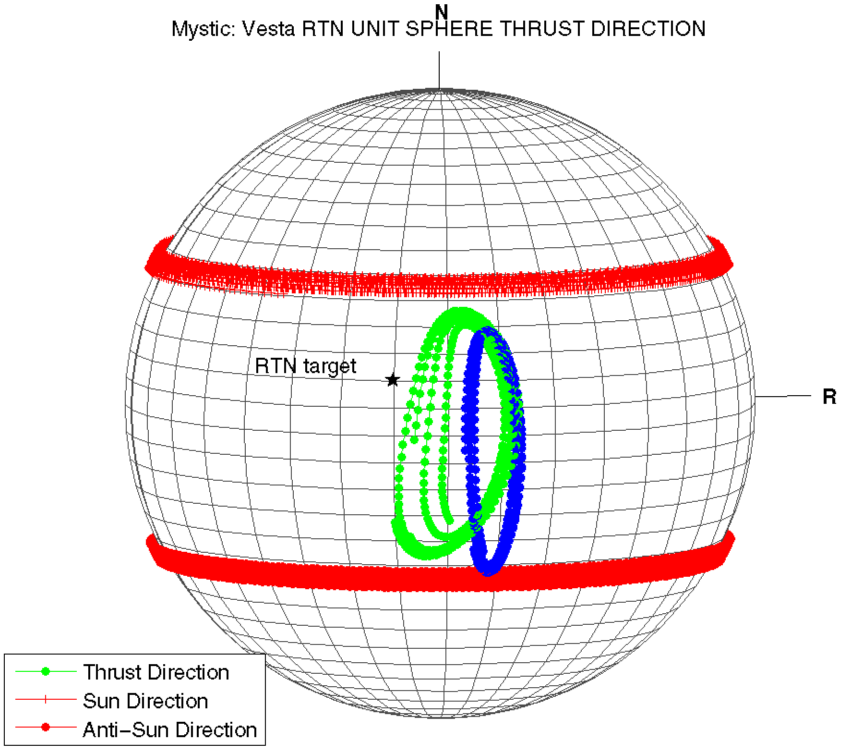

Figure 2. An example of a mass optimal versus a direction optimal thrust direction evolution in the Radial - Transverse - Normal or "RTN" frame. The mass optimal solution is indicated in blue and the direction optimal solution is indicated in green. The Sun and anti-Sun directions are indicated in red. Transverse - Normal frame

ACKNOWLEDGMENT

This research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract to the National Aeronautics and Space Administration.

REFERENCES

[1] M. D. Rayman, T. C. Fraschetti, C. A. Raymond, and C. T. Russell, "Dawn: A mission in the development for exploration of main belt asteroids Vesta and Ceres," *Acta Astronautica*, No. 58, 2006, pp. 605–616.

[2] C. A. V. Brett A. Smith and A. Y. Lee, "Understanding Spacecraft Agility for Orbit Transfers on the Dawn Low-Thrust Mission," *Proceedings of the 35th Annual Guidance and Control Conference*, Breckenridge, Colorado, February 3-8, 2012. Paper AAS2012-097.

[3] G. J. Whiffen, "Mystic: implementation of the Static Dynamic Optimal Control Algorithm for High-Fidelity, Low-Thrust Trajectory Design," *Proceedings of the AIAA/ASS Astrodynamics Specialists Conference*, Keystone, Colorado, 21-24 August 2006. Paper AIAA 2006-6741.

[4] G. J. Whiffen, "Static/Dynamic for Optimizing a Useful Objective," United States Patent, No. 6,496,741, Filed March 1999. Issued December 2002.