Computational Thermo-Physics for Future Space Missions

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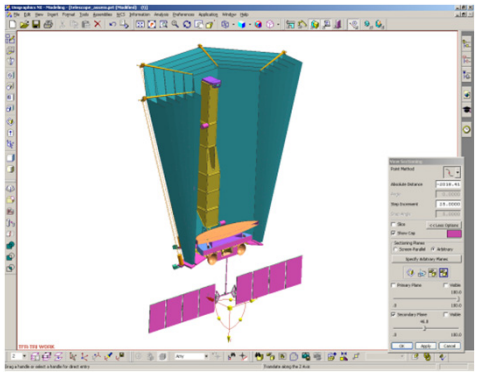
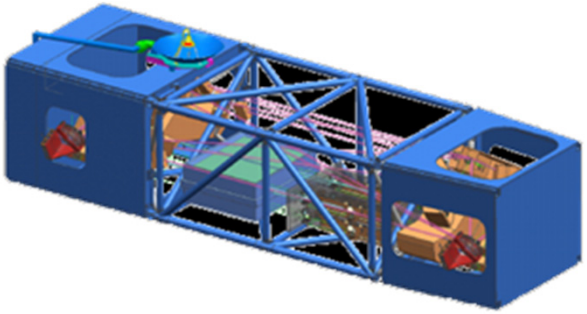
Organization: Jet Propulsion Laboratory

California Institute of Technology

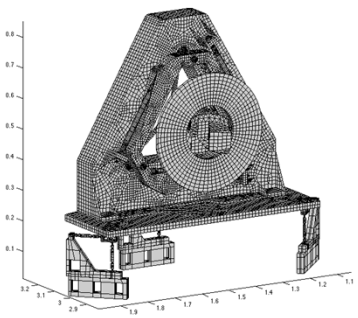
Mission and Instrument Characterization

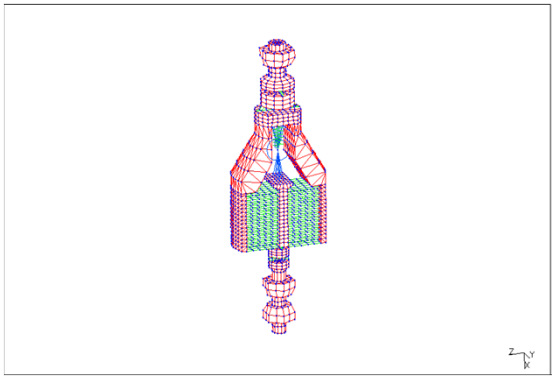




- Extraordinary precision, stability, accuracy levels
- Extreme environments-Mars, Venus, L2
- Limited Full Scale Testing
- Non-Serviceable

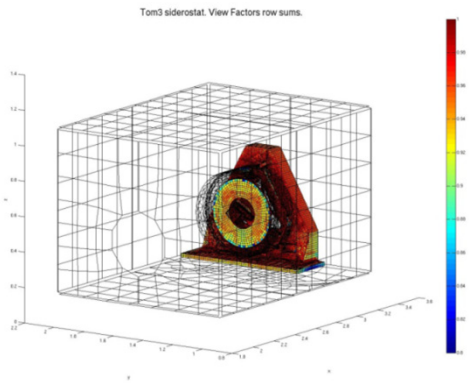
Thermal Analysis Requirements

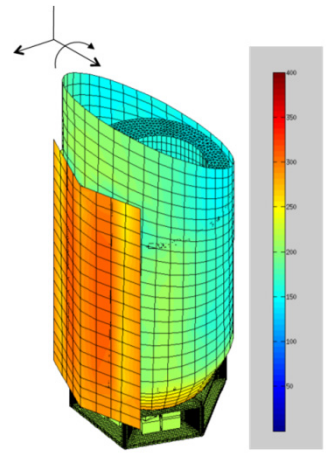




- High fidelity models which exhibit spatial and temporal integrity
- Multidisciplinary interactions
- High performance-algorithms, numerics, multiprocessing
- V & V Process

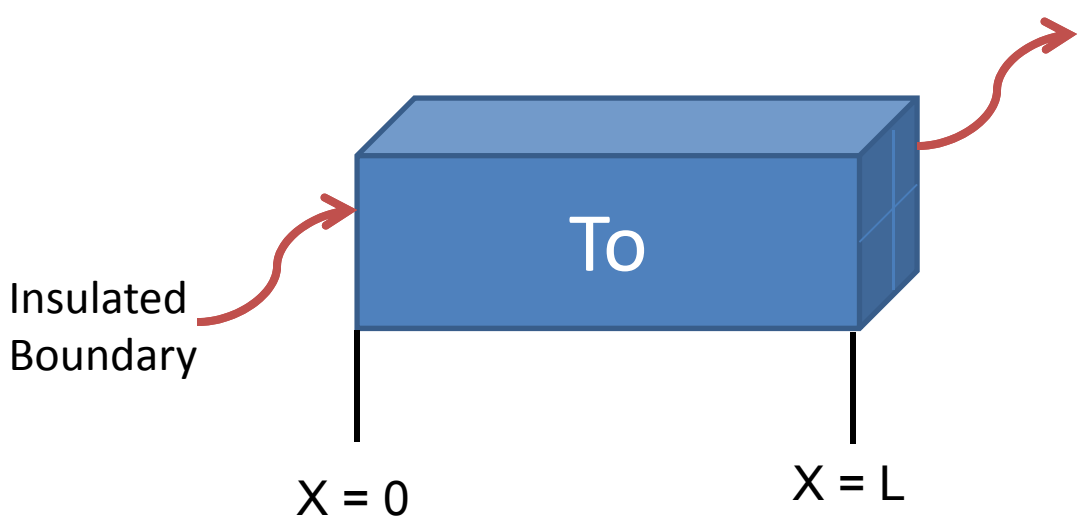
Thermal Physics Requirements





- K(T), Rho*C_p (T)
- $\rho(\lambda,\theta,\phi,T),(\propto+\rho=1.0),T=?$
- Conservation Enforced
- Active/Passive Control
- Contact

Time Accurate Solutions



Find T (x,t) for
x=L, t = 18000.sec
x=L, t = 72000.sec

L = 304.80mm
h = $2.8391 \cdot 10^{-5}$ w/mm^2 ° C
K = $8.6573 \cdot 10^{-4}$ w/mm° C
ρ = $8.0092 \cdot 10^{-8}$ kg/mm^3
C_p =$2.0934 \cdot 10^4$ J/kg°
T_o = 593.33° C
T_∞ = 37.778° C

$$\propto = k / \rho C_\rho$$

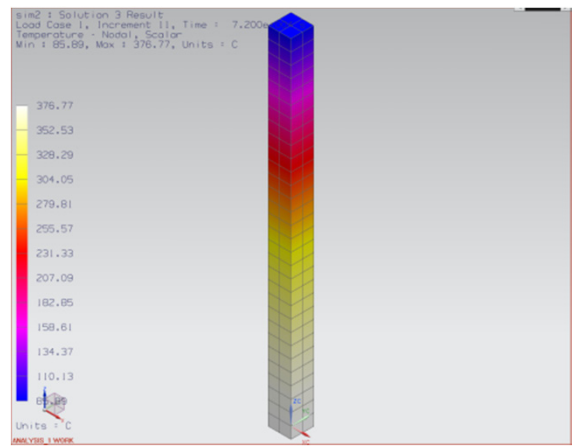
Analytic Solution

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\sigma}\frac{\partial \theta}{2t}$$ in $0 \le x \le L$, $t > 0$

$$\frac{\partial \theta}{\partial x} = 0$$ @ x = 0, t > 0

$$K\frac{\partial \theta}{\partial x} + h\theta = 0$$ @ x =L, t > 0

$$\theta(x,t) = \frac{T(x,t) - T_\infty}{T_0 - T_\infty}$$

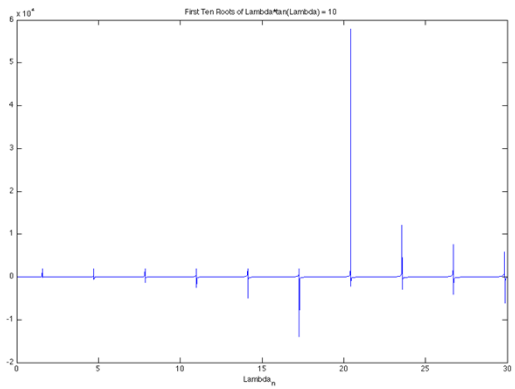


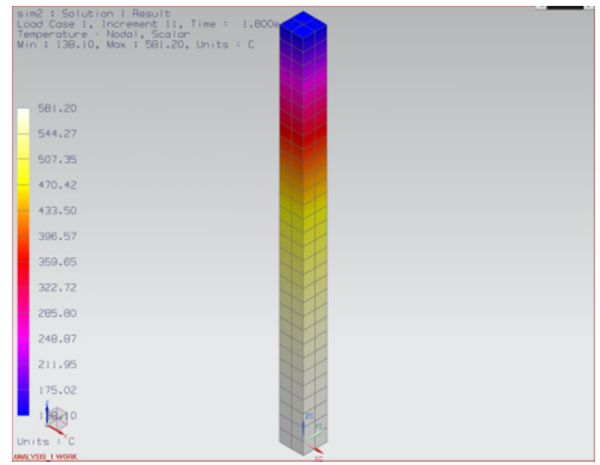
$$\theta(x,t) = \sum_{n=1}^{\infty} \frac{1}{N} e^{-\propto \lambda_n^2 t} \frac{\sin \lambda_n L}{\lambda_n} \cos \lambda_n x$$

Where $$\frac{1}{N} = \frac{2(\lambda_n^2 + H^2)}{L\left(\lambda_n^2 + H^2\right) + H}$$

$$H = h/k$$

and $$(\lambda_n L)\tan(\lambda_n L) = \frac{hL}{K}$$





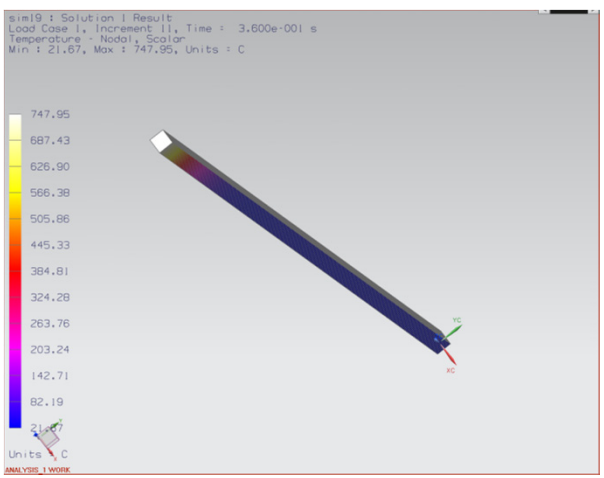
T (L, 18000.sec) = 575.7° C
T (L, 72000.sec) = 347.4° C

	Insulated Boundary		Convection Boundary ° C	
N-Divisions	t = 18000.s	t = 72000.s	t = 18000.s	t = 72000.s
30	581.2	377.5	138.1	86.0
100	575.3	348.4	132.8	81.8
Analytic	575.7	347.4	132.5	81.6

Nonlinear Time Accurate Solutions

Determine T(x=o,t_n)

for t_1 = .0036 sec
t_2 = .036sec
t_3 = .36sec



$$L = 9.5494mm$$

$$T_0 = 294.44°K$$

$$K_0 = 1.7037 \cdot 10^{-3}\ {}^{w}\!/_{mm°K}$$

$$C_{p_0} = 20934\ {}^{J}\!/_{kg°K}$$

$$\beta = .009\ {}^{1}\!/_{°K}$$

$$\rho = 8.0092 \cdot 10^{-6}\ {}^{Kg}\!/_{mm^3}$$

$$\dot{q} = 79.496\ {}^{w}\!/_{mm^2}$$

$$\sigma = 1.0323\ {}^{mm^2}\!/_{\sec}$$

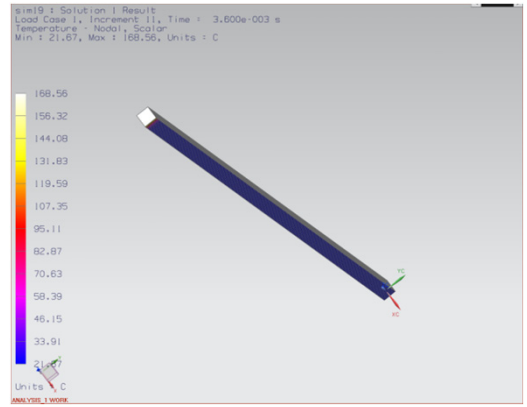
$$K(T) = K_0[1.0 + \beta(T - T_0)]$$

$$C_p(T) = C_{p_o}[1.0 + \beta(T - T_0)]$$

Let: $$F_0 = \frac{\propto}{L^2} \cdot t$$

$$x^2 - x = \frac{x}{L}$$

$$\theta = \frac{T(x,t) - T_0}{T_0}$$

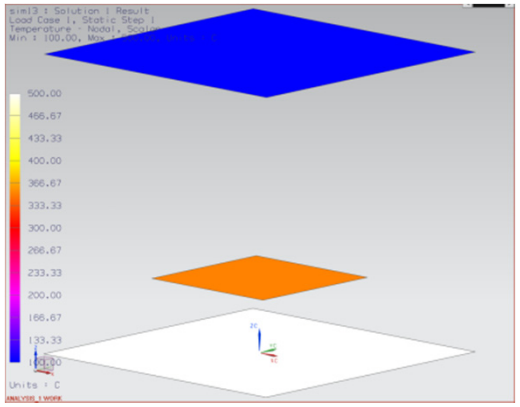


Solution: $$\frac{\beta T_0 \theta^2}{2} + \theta = \frac{\dot{q} \cdot L}{K_0 T_0} \left[\frac{x^2}{2} - x + \frac{1}{3} + F_0 - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi X e^{-F_0 n^2 \pi^2} \right]$$

Results

Time	T(0,t) Analytic ° C	B	F-B	F
.0036 sec	197.3	168.7	168.8	180.9
.36 sec	755.28	747.9	748.0	749.5

Radiation Exchange



Analytic View Factors Diffuse:

$f_{3\text{-}1} = .11562 = f_{1\text{-}3}$

$f_{2a\text{-}1} = .79482$

$f_{2b\text{-}3} = .33691$

$f_{3\text{-}2b} = .08422$

$f_{1\text{-}2a} = .19870$

View Factor Algebra:

$$f_{1\text{-}\infty} = 1.0 - (f_{1\text{-}2a} + f_{1\text{-}3}) = .68568$$

$$f_{2a\text{-}\infty} = 1.0 - f_{2a\text{-}1} = .20518$$

$$f_{2b\text{-}\infty} = 1.0 - f_{2b\text{-}3} = .66309$$

$$f_{3\text{-}\infty} = 1.0 - (f_{3\text{-}2b} + f_{3\text{-}4}) = .79947$$

Network Solution

Nodal Potential: σT^4

Surface Resistance: $\frac{1-\epsilon}{\epsilon A}$

Space Resistance: $\frac{1}{Af}$

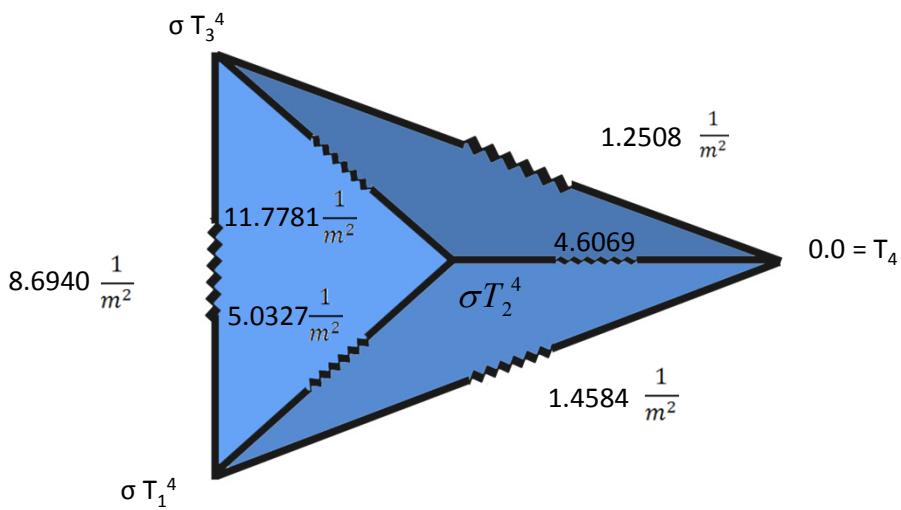
Case – 1

Let $T_3 = 500^\circ$ C (773.15° K)

$T_1 = 100^\circ$ C (373.15° K)

$\epsilon_1 = \epsilon_{2a} = \epsilon_{2b} = \epsilon_3 = 1.0$ (black body)

$T_\infty = T_4 = 0.0^\circ$ K



$$Q_{3-2} = Q_{2-1} + Q_{2-4}$$

$$\frac{\sigma T_3^{\,4} - \sigma T_2^{\,4}}{11.7781} = \frac{\sigma T_2^{\,4} - \sigma T_1^{\,4}}{5.03727} + \frac{\sigma T_2^{\,4}}{4.6069}$$

$$T_2 = 512°K(239.3°C)$$

$$NX7.5(238.7°C)$$

Case – 2

Let $T_3 = 100^\circ$ C

$T_1 = 500^\circ$ C

$\epsilon_1 = \epsilon_{2a} = \epsilon_{2b} = \epsilon_3 = 1.0$

$T_\infty = T_4 = 0.0^\circ$ K

Solution Case – 2

T_2 = 618. ° K (345° C) Analytic Solution

View Factor Calculation Methods

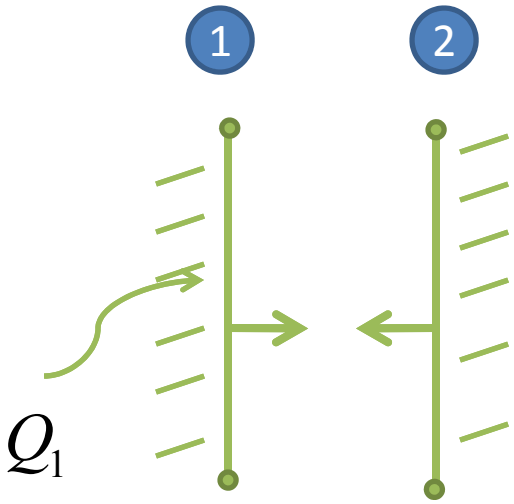
- Hemi cube (low, Medium, high) Yields
  VF_SUM_SURF_1 = 1.0634
  NX7.5 T_2 = 351° C

- Deterministic with Max Div = 5 Yields
  VF_SUM_SURF_1 = .9846
  NX7.5 T_2 = 344.7° C

- Monte Carlo

N = 2000 Rays	T_2 = 342.4°C
N = 10,000 Rays	T_2 = 344.0°C
N = 20,000 Rays	T_2 = 345.1°C
N = 50,000 Rays	T_2 = 344.7°C
N = 100,000 Rays	T_2 = 344.5°C

An Interesting Variation of Simple Radiation Exchange



$A_1 = A_2 = 1.0$

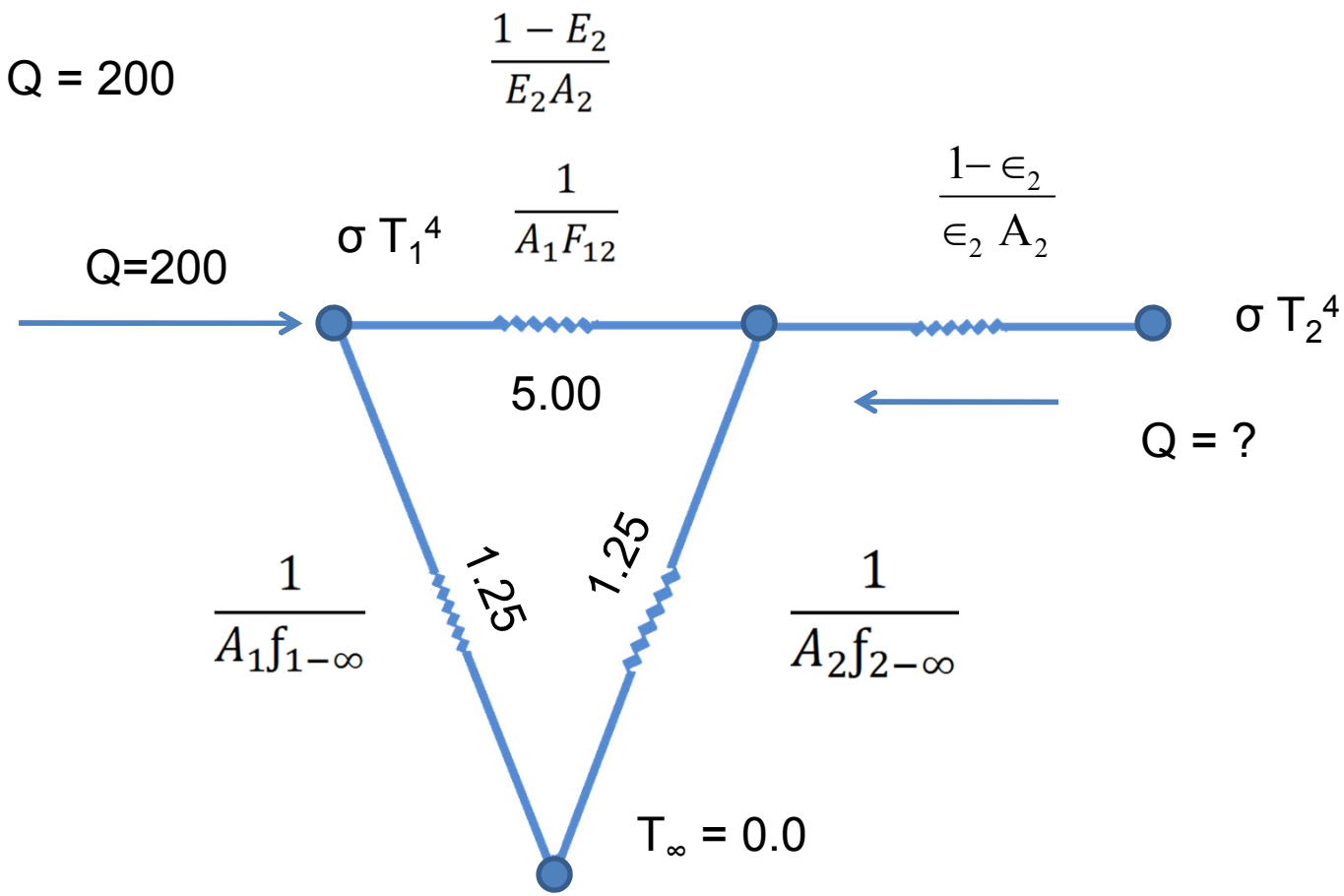
$\epsilon_1 = 1.0$

$\epsilon_2 = .05$

$f_{12} = f_{21} = .20$

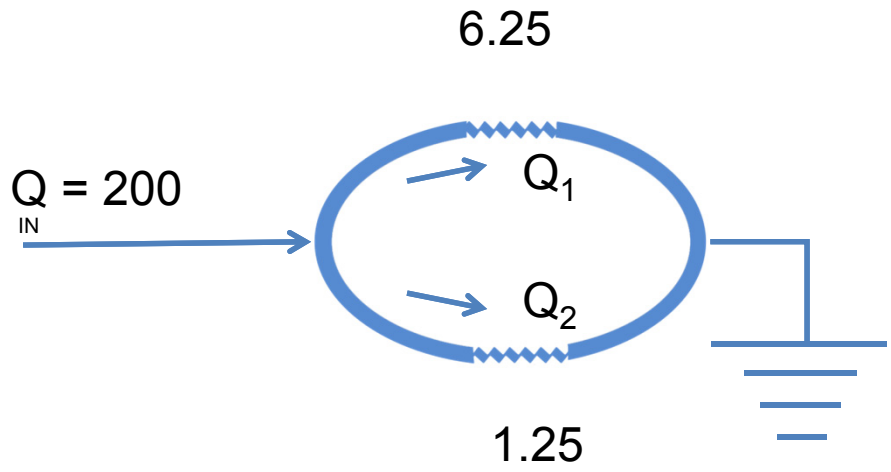
$$Q_1 = 200w$$

$$\propto_1 = 1.0$$



NX7.5

E_2	T_1 °K	T_2 °K
.50	244.95	164.20
.10	243.95	163.26
.001	243.70	162.95

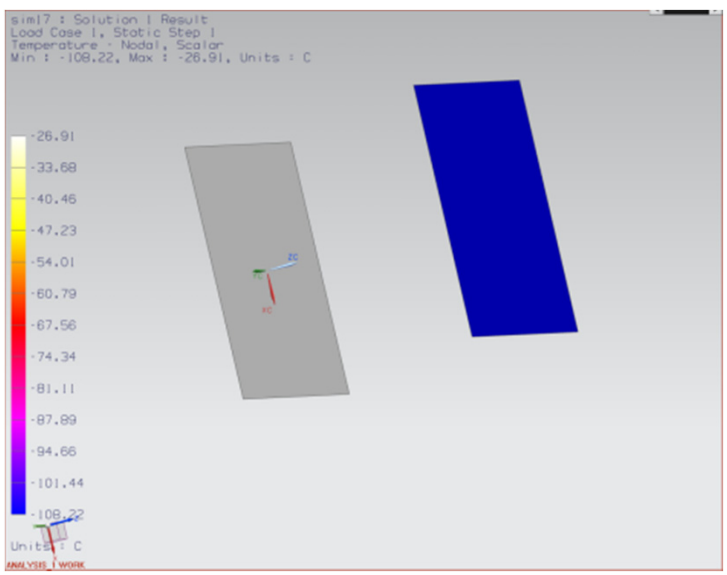


$Q_1 = 33.33w$

$Q_2 = 166.67w$

$T_1 = 246.\ ^\circ\ K$

$T_2 = 165.\ ^\circ\ K$



View Factors

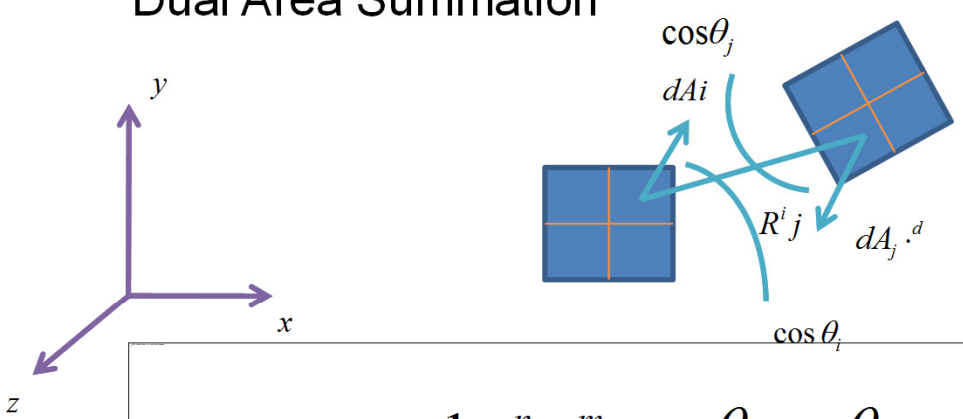
Basic Equation:

$$f_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \ \cos\theta_j}{\pi R_{ij}^2} dA_i dA_j$$

(Assumptions)

Solutions:

Dual Area Summation

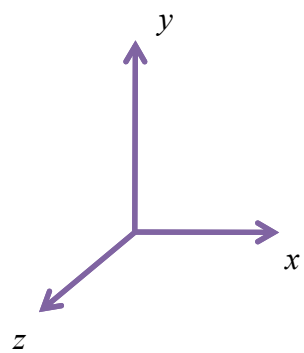


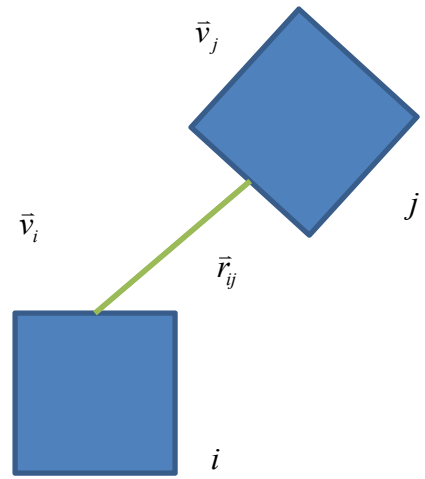
$n = 4$

$m = 4$

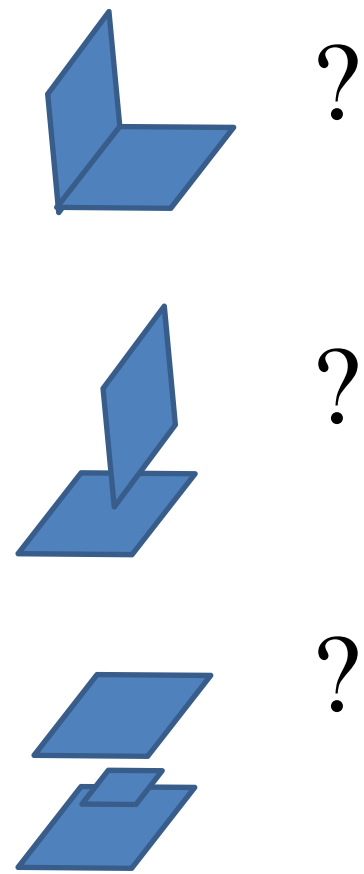
$$f_{ij} = \frac{1}{A_i} \sum_{1}^{n} \sum_{1}^{m} \frac{\cos\theta_i \cos\theta_j}{\pi R^2_{ij}} dA_i dA_j$$

Contour Integration





$$f_{ij} = \frac{1}{2\pi A_i} \sum_{i=1}^{n} \sum_{i=1}^{m} \ell n \left| r_{ij} \right| \left(\vec{v}_i \vec{v}_j \right)$$



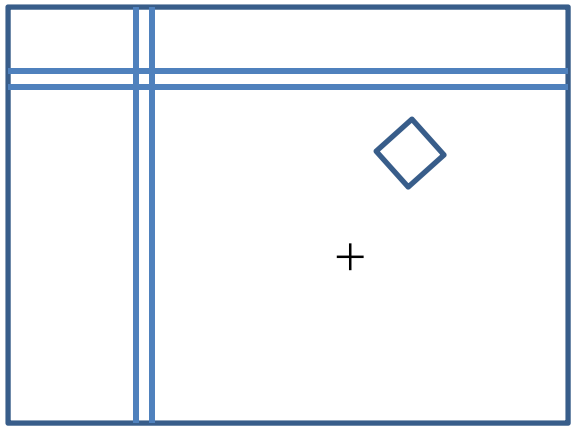
Projection Methods

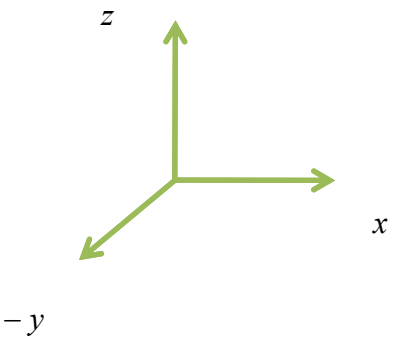
- Nusselt Sphere
- Hemi Cube
- Hemi Plane

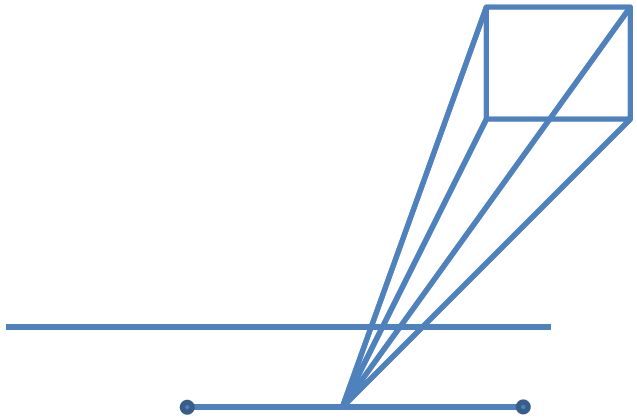
Monte Carlo

$$\in = 1.0$$

Hemi Plane







N-Surface Radiation Exchange

$$[\mathrm{A}_e]\{q_e\}^{in} = [\mathrm{A}_e F]\{q_e\}^{out}$$

$$\{q_e\}^{out} = \sigma \in \{T_e\}^4 + [I - \alpha]\{q_e\}^{in}$$

$$\{Qe\}^{net} = [\mathrm{A}]\left(\{q_e\}^{out} - \{q_e\}^{in}\right)$$

$$\{Qe\}^{net} = -[R_e]\{T_e\}^4$$

$$[R_e] = \sigma\left[[A][\in] - [A][\propto]\left([A] - [AF][I - \propto]\right)^{-1}[AF][\in]\right]$$

Wavelength Dependent Radiation Surface Properties

Cryogenic Detectors, Mirrors, Optical Train

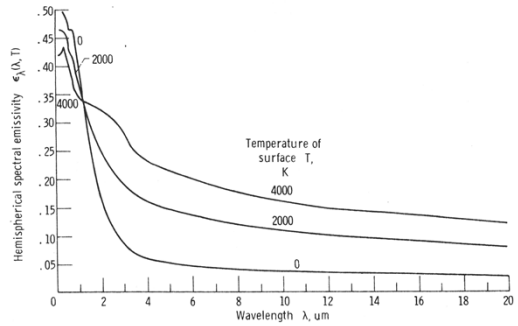


Figure 10-2 Hemispherical spectral emissivity of tungsten.

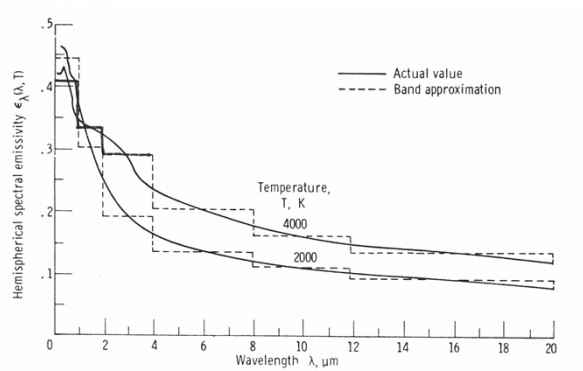


Figure 10-6 Band approximations to hemispherical spectral emissivity of tungsten.

Radiation Exchange Matrix Band by Band

$$\{f_e\} = \{FRAC_{0-\lambda_2 U_e}\}$$

$f_e \rightarrow =$ Fraction of the radiant output of a black body that is contained in the n-th wavelength band where $\Delta\lambda = \lambda_2 - \lambda_1$.

$$FRAC_{0-\lambda U_e} = \frac{15}{\pi^4} \sum_{m=1,2,\cdots} \frac{e^{-mv}}{m^4\{[(mv+3)mv+6]mv+6\}, v \geq 2}$$

$$FRAC_{0-\lambda U_e} = 1\frac{15}{\pi^4} v^3 \left\{ \frac{1}{3} - \frac{v}{8} + \frac{v^2}{60} - \frac{v^4}{272160} - \frac{v^8}{13305600} \right\}, v < 2$$

$$v = \frac{PLANCK2}{\lambda U_e},$$ Where PLANCK2 $= \left. \begin{matrix} 25898 \mu m^\circ R \\ 14388 \mu m^\circ K \end{matrix} \right\} (TYP)$

$$\left[R_e^\lambda\right]_n = \sigma \left[A\varepsilon(\lambda) - A\alpha(\lambda)\left(A - AF\left(I - \alpha(\lambda)\right)\right)^{-1} AF\varepsilon(\lambda) \right]_n$$

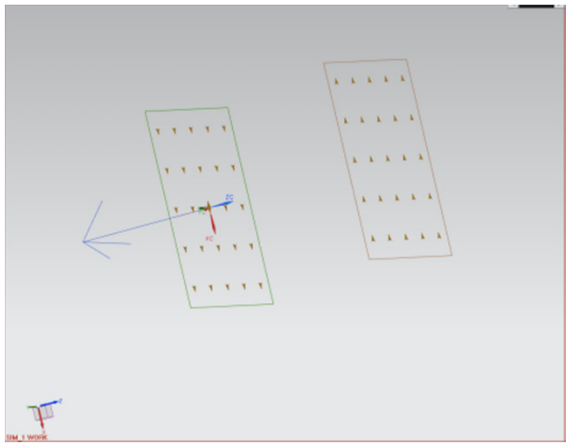
$$\left\{Q_e^{\lambda}\right\}_n = -\left[R_e^{\lambda}\right]_n \left[\ \diagdown\ f_e\ \diagdown\ \right]_n \left\{U_e\right\}^4$$

$$\left\{Q_e^{\lambda}\right\}^{net} = -\left[R_e^{\lambda}\right]^{net} \left\{U_e\right\}^4$$

Where,

$$\left[R_e^{\lambda}\right]^{net} = \sum_{n=1}^{n\,\max} \left[R_e^{\lambda}\right]_n \left[\ \diagdown\ f_e\ \diagdown\ \right]_n$$

Net Reflected Diffuse Loads



$$\{Q\}_{net} = [A]([\propto])([A]-[A][F][I-\propto])^{-1}[A]\{q\}^{IN}$$

$$[\overline{R}] = ([A]-[A][F][I-\propto])^{-1}$$

$$Q_1 = A_2 \propto_1 \overline{R}_{11} A_1 q_1^{IN} + A_1 \propto_1 \overline{R}_{12} A_2$$

$$Q_2 = A_2 \propto_2 \overline{R}_{21} A_1 q_{14}^{IN} + A_2 \propto_2 \overline{R}_{22} A_2$$

Thermal Control Strategy

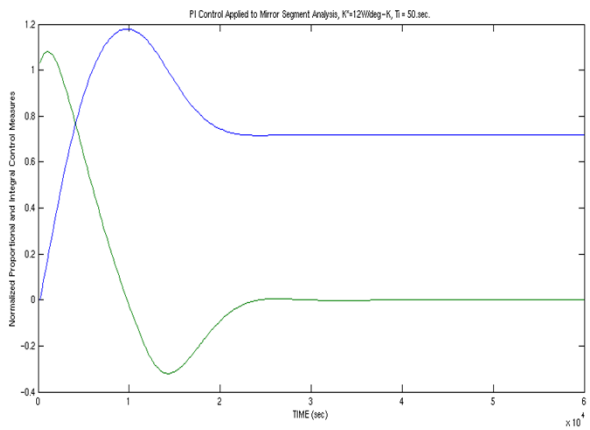
$$F(t) = KF(t) = Kp\left(e(t) + \frac{1}{T_i}\int_0^t e(t)d\tau + T_d \frac{d}{dt} e(t) \right)$$

$$e(t) = Tsp - T_{Ref}$$

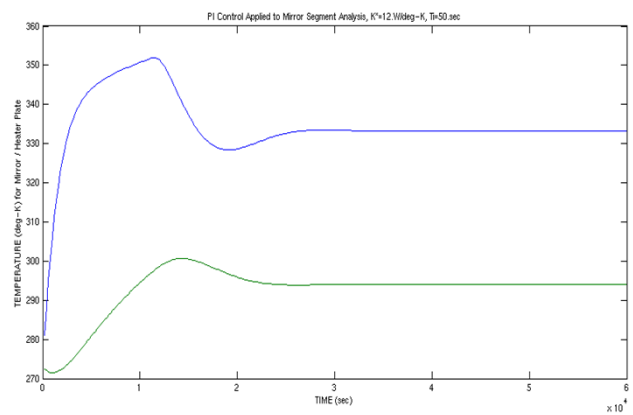
T_i = Integral Time (reset rate)

T_d = Derivate Time

T_{sp} = Set Point Temperature



Green = Proportional control contribution to applied heat load
Blue = Integral control contribution to applied heat load



Green = Mirror temperature
Blue = Backing plate heater temperature

Summary and Conclusions

Mesh fidelity still counts

Time accurate solutions demand attention

Independent V & V is useful

- Best practices
- Defaults
- Limitations
- Efficiencies