



# Hyper-parametric oscillations in a whispering gallery mode fluorite resonator

Anatoliy Savchenkov, Dmitry Strekalov, Makan Mohageg, Vladimir  
Ilchenko, Andrey Matsko and Lute Maleki

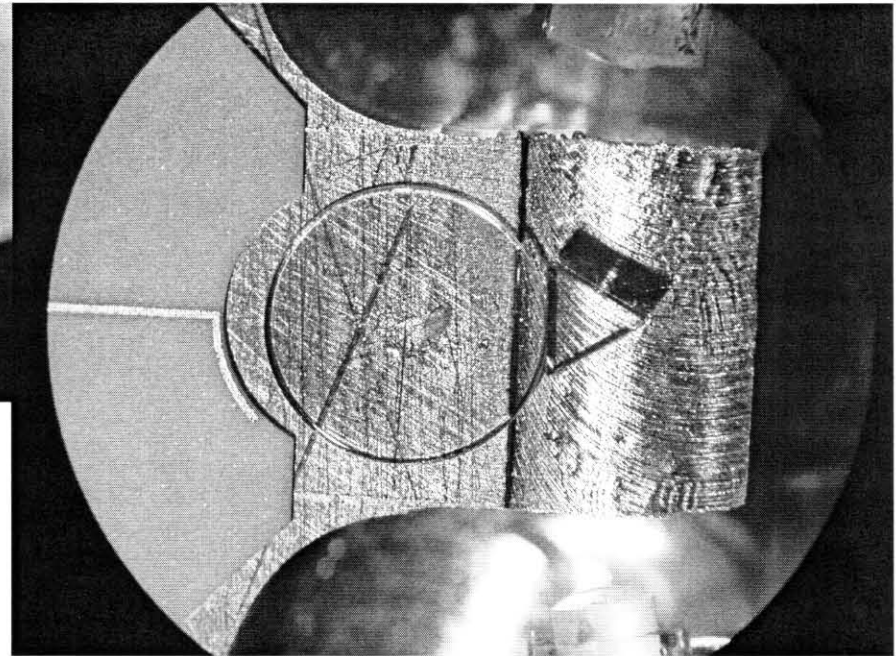
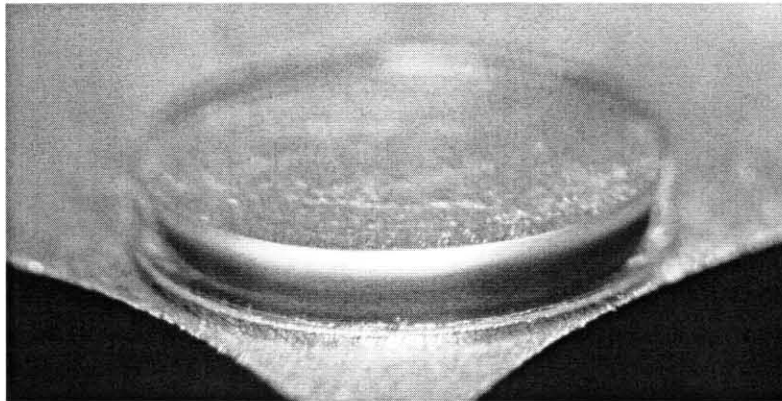
*JPL, Quantum Sciences and Technology group*

**[Dmitry.V.Strekalov@jpl.nasa.gov](mailto:Dmitry.V.Strekalov@jpl.nasa.gov)**

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*Thank\$: NASA, DARPA*

# Whispering Gallery Mode resonators



... Come in various shapes and sizes.

We are mostly interested in the disk resonators. They:

- have cleaner spectrum;
- are ideal for electro-optical applications;
- may have very small mode volume (good for nonlinear optics).

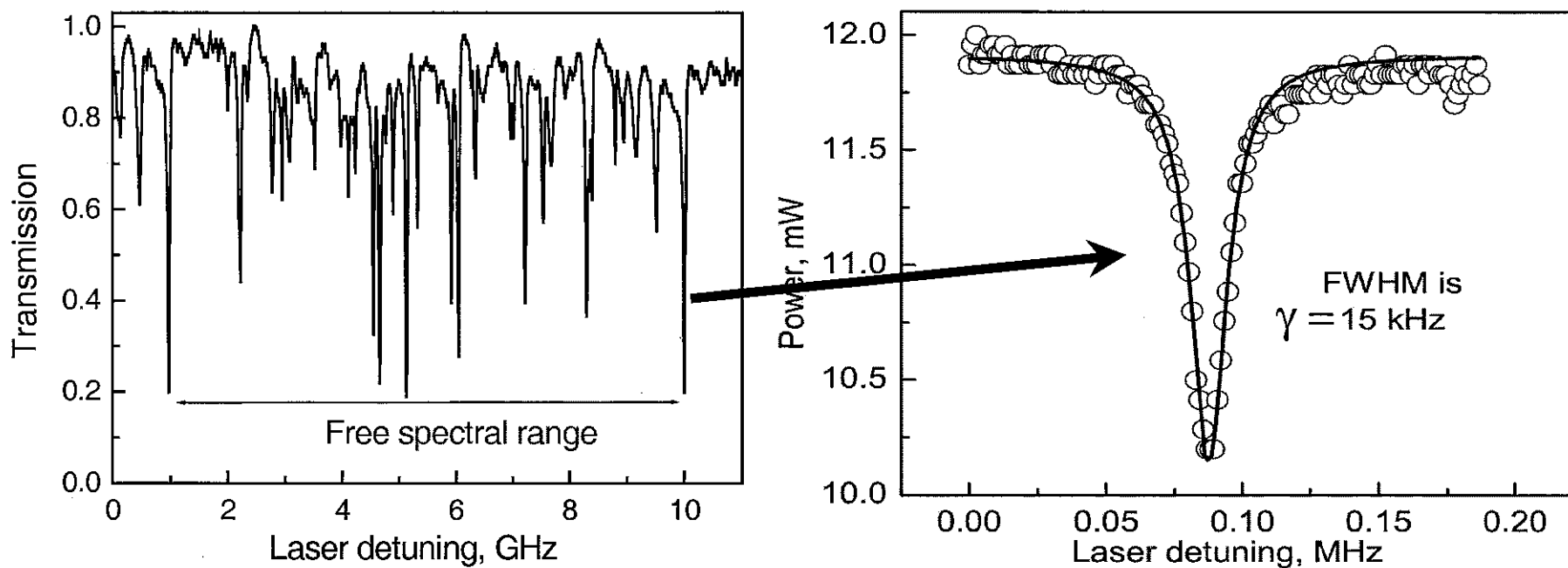
Low microwave power EO modulators

Continuously tunable, narrow-band pass and stop optical filter

[References available]



## Typical WGM spectrum



$$Q = \omega/\gamma > 2 \times 10^{10}$$



## Important parameters

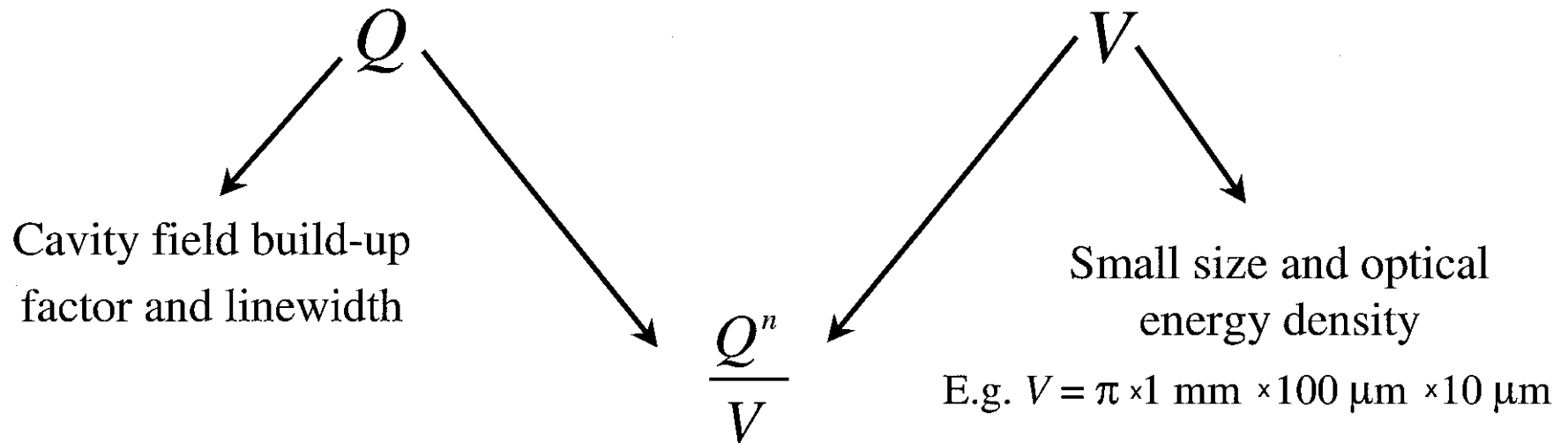


Figure of merit for nonlinear processes:

Purcell's factor:  $n=1$

SRS and FWM:  $n=2$

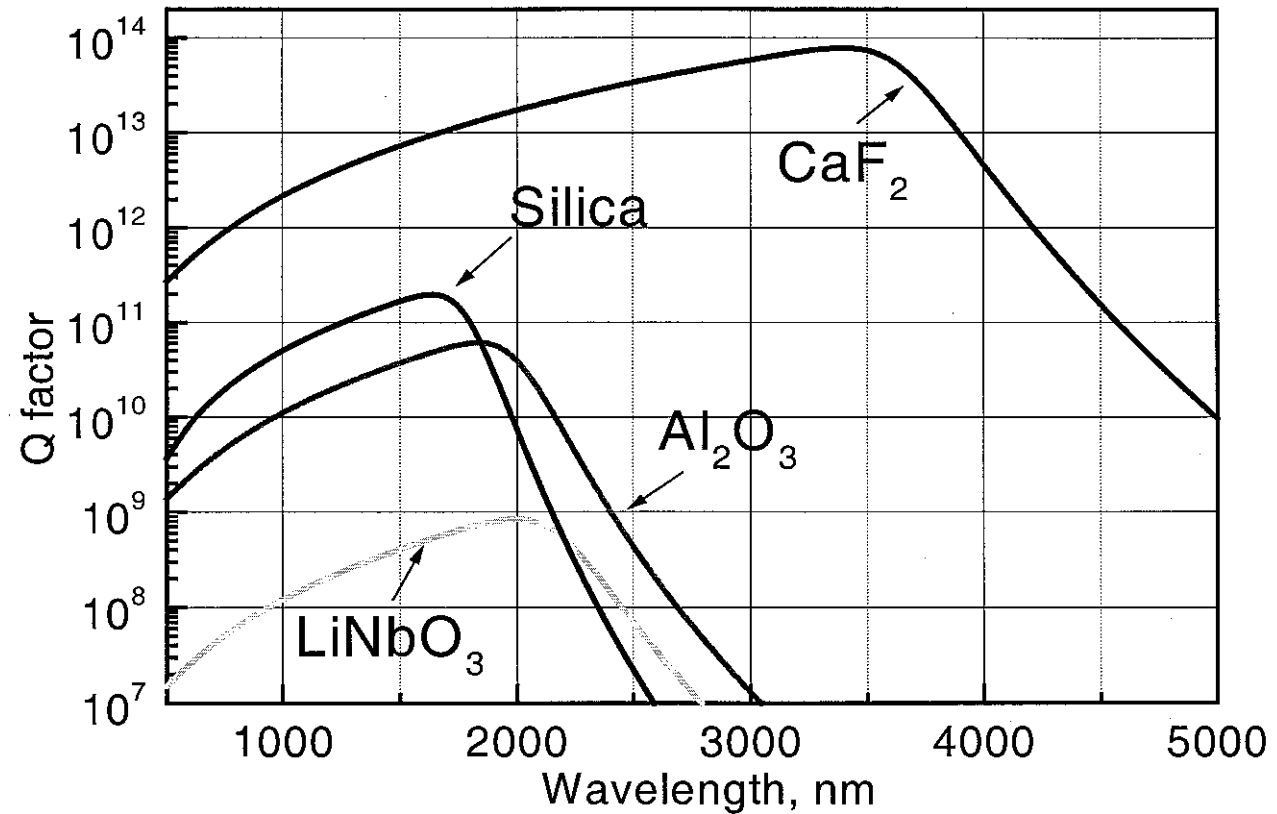
Frequency doubling:  $n=3$

[V.S. Ilchenko et al.,  
JOSA B 20, 1304 (2003)]



For crystalline resonators, linewidth is ultimately determined by the material absorption  $\alpha$ :

$$2\gamma^{-1} = n_0(\alpha c)^{-1}$$
$$\Rightarrow Q = \frac{2\pi n}{\alpha \lambda}$$

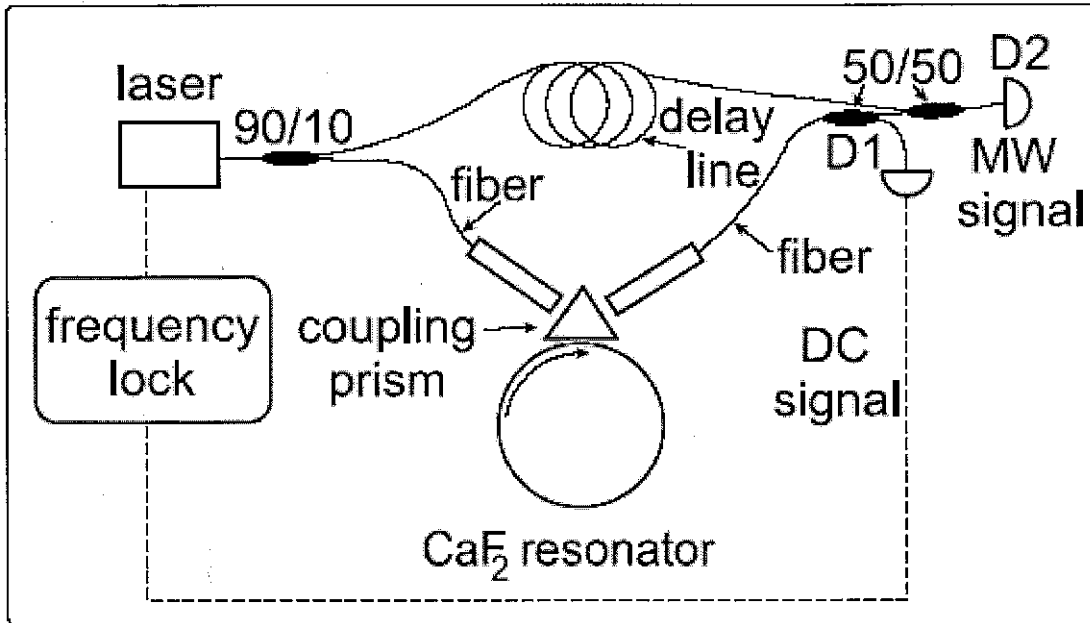


$$\text{For } \alpha \simeq \alpha_{UV} e^{\lambda_{UV}/\lambda} + \alpha_R \lambda^{-4} + \alpha_{IR} e^{-\lambda_{IR}/\lambda}$$

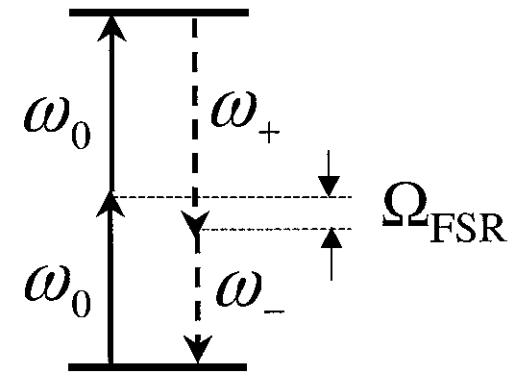
*E.D.Palik, "Handbook on optical constants of solids", Academic, NY, 1998*



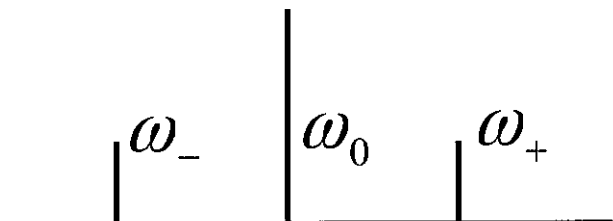
# Hyper-parametric oscillations in fluorite resonators



Transition diagram



Optical spectrum



$$Q = 2 \times 10^{10} \text{ at } \lambda = 1310 \text{ nm}$$

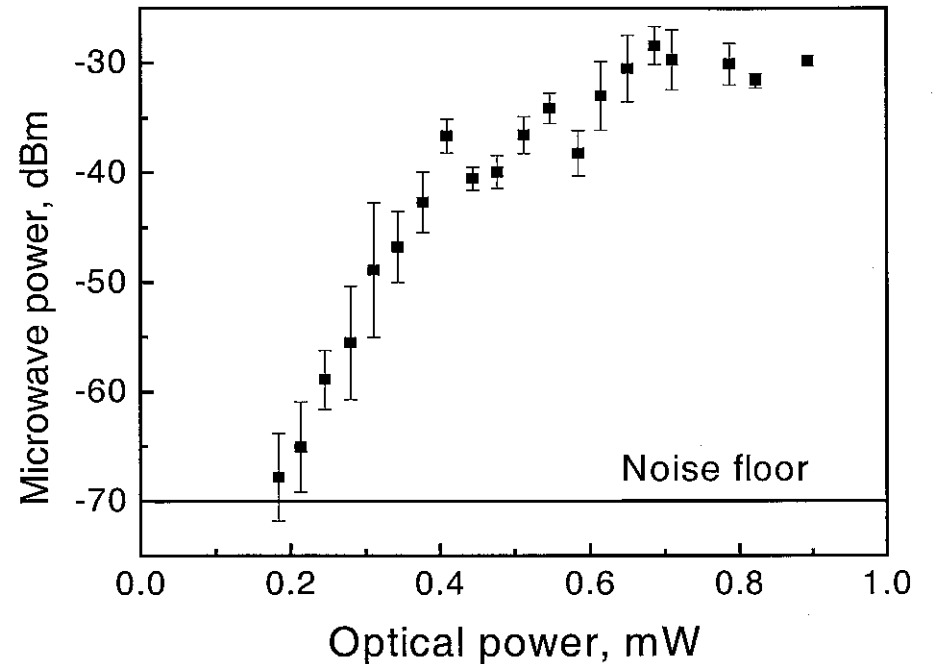
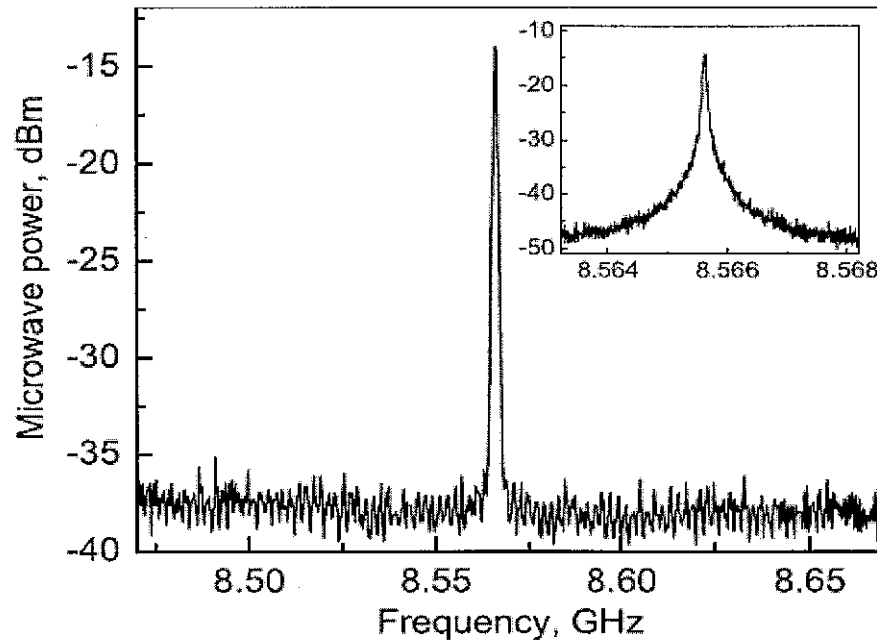
*Selection rules*

*FWM: TE-TE*


*SRS: TE-TM*

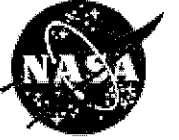


## Microwave beat note observed



 Second-order ( $2\Omega_{\text{FSR}}$ ) beat note is insignificant

 Raman scattering is not observed (expected at  $322 \text{ cm}^{-1}$ )



## Analysis

Kerr Hamiltonian:  $H = H_0 + V$ ,  $H_0 = \hbar\omega_0 a^\dagger a + \hbar\omega_+ b_+^\dagger b_+ + \hbar\omega_- b_-^\dagger b_-$ , where

$$V = -\hbar\frac{g}{2}(a^\dagger a^\dagger a a + b_+^\dagger b_+^\dagger b_+ b_+ + b_-^\dagger b_-^\dagger b_- b_-) - 2\hbar g(b_-^\dagger b_+^\dagger b_+ b_- + a^\dagger b_+^\dagger b_+ a + a^\dagger b_-^\dagger b_- a)$$

Self-phase modulation

$$-\hbar g(b_-^\dagger b_+^\dagger a a + a^\dagger a^\dagger b_+ b_-)$$

Cross-phase modulation

Four-wave mixing

Equations of motion in an open system:

$$\begin{aligned} \dot{a} &= -(i\omega_0 + i\kappa(T) + \gamma_0 + \gamma_{c0})a + ig[a^\dagger a + 2b_+^\dagger b_+ + 2b_-^\dagger b_-]a + 2iga^\dagger b_+ b_- + f_0 + f_{c0}, \\ \dot{b}_+ &= -(i\omega_+ + i\kappa(T) + \gamma_+ + \gamma_{c+})b_+ + ig[2a^\dagger a + b_+^\dagger b_+ + 2b_-^\dagger b_-]b_+ + igb_-^\dagger a a + f_+ + f_{c+} \\ \dot{b}_- &= -(i\omega_- + i\kappa(T) + \gamma_- + \gamma_{c-})b_- + ig[2a^\dagger a + 2b_+^\dagger b_+ + b_-^\dagger b_-]b_- + igb_+^\dagger a a + f_- + f_{c-} \end{aligned}$$

↑  
Temperature tuning
↑  
SPM and CPM
↑  
FWM

Where  $\langle f_{c0} \rangle = \sqrt{\frac{2\gamma_{c0} P_0}{\hbar\omega_0}} e^{-i\omega t}$  and  $g = \omega_0 \frac{n_2}{n_0} \frac{\hbar\omega_0 c}{V n_0}$





## Results of analysis

## *Experiment*

1. Complex dynamics of the system (laser lock helps)



2. Threshold power:  $P_{th} \simeq 1.54 \frac{\pi}{2} \frac{\gamma_0 + \gamma_{c0}}{2\gamma_{c0}} \frac{n_0^2 \mathcal{V}}{n_2 \lambda Q^2}$ ,



3. Phase modulation (when critically coupled)



4. Beat note frequency  $\omega - \tilde{\omega}_- = \tilde{\omega}_+ - \omega = \frac{1}{2}(\omega_+ - \omega_-)$ ,  
is independent on the nonlinear dynamics of the system







5. Phase diffusion is very low:  $D_{min} = \frac{(\gamma_{c0} + \gamma_0)^2}{2} \frac{\hbar\omega}{P_{B\ opt}} =$   
 $\omega_0^2 \frac{\gamma_0 + \gamma_{c0}}{2\gamma_{c0}} \frac{\hbar\omega_0 n_2 \lambda}{4\pi \mathcal{V} n_0^2}$ .

*To be tested*



## Summary and conclusions

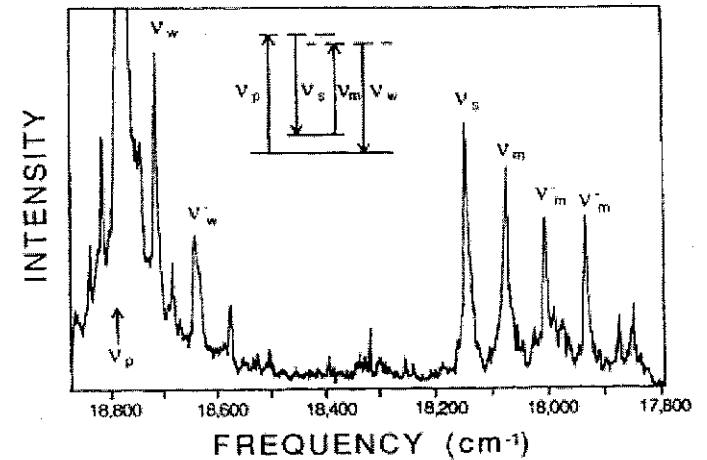
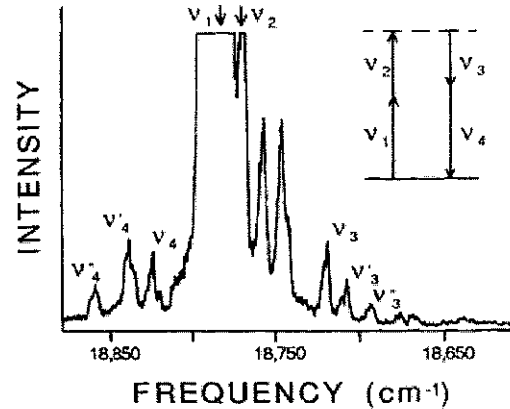
-  Very high Q-factor and small volume allow us to reach the oscillation threshold for the four-wave mixing process in WGM disk resonators with low power DC optical fields.
-  The generated fields can be represented as the sidebands of the phase-modulated carrier (the pump). The sidebands frequency is very stable in spite of complex mode dynamics of the system, which suggests the possibility of its application as a secondary frequency standard.
-  The absence of the SRS in the pump polarization (TE) is a consequence of the selection rules; however its absence in the TM polarization is surprising and is in contradiction with the results for amorphous materials [*S.M. Spillane, T.J. Kippenberg, K.J. Vahala, Nature 415, 621 (2002)*]. This may be due to asymmetry of the Brillouin zone in crystals. Further research is needed.
-  Just like parametric down conversion, hyper-parametric conversion can produce nonclassical (e.g. entangled or squeezed) light. We plan on carrying out the research of quantum optical properties of our system.



## Earlier results on CW four wave mixing in amorphous materials

*H.-B. Lin and A.J. Campillo, Phys. Rev. Lett. 73, 2440 (1994).*

*CS<sub>2</sub> droplets*



*S.M. Spillane, T.J. Kippenberg, K.J. Vahala, Nature 415, 621 (2002).*

*Fused silica microspheres*

