Dynamics of Drag Free Formations in Earth Orbit

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ABSTRACT

In this paper the translational equations of motion of a formation of n spacecraft in Earth orbit, n_f of which are drag-free spacecraft, are derived in a coordinate-free manner using the balance of linear momentum and direct tensor notation. A drag-free spacecraft consists of a spacecraft bus and a proof mass shielded from external disturbances in an internal cavity. By controlling the spacecraft so that the proof mass remains centered in the cavity, the spacecraft follows a purely gravitational orbit. The results described in this paper provide a first step toward coupling drag-free control technology with formation flying in order to mitigate the effect of differential aerodynamic drag on formation flying missions (e.g., Earth imaging applications) in low Earth orbit.

1. INTRODUCTION

The concept of a drag-free spacecraft was introduced by Lange in 1964.[6] The idea is based on placing a proof mass constructed from a gold-platinum alloy within a cavity of a larger spacecraft. As a result, the proof mass is shielded from aerodynamic forces, solar radiation pressure, and other environmental interactions. Protected inside the spacecraft, the proof mass is constrained to follow a purely gravitational trajectory.[2] A control system is then utilized to maneuver the spacecraft about the proof mass so that the proof mass remains centered in the cavity.

In this paper the translational equations of motion of a formation of spacecraft in low Earth orbit (LEO) are developed, where n_f spacecraft are assumed to be drag-free. The eventual goal of this research is to couple formation-flying guidance and control,[9,10] with drag-free technology in order to mitigate the dominant aerodynamic disturbances acting on formations in LEO. For example, the science community has been actively considering using distributed spacecraft for Earth science missions. One specific application is to use a large number of small spacecraft in place of a large deployable antenna to achieve very large sparse for Earth imaging.[3] Compared to their monolithic aperture counterparts, formation flying sparse antennas offer launch and deployment efficiency, and have the advantage of avoiding the structural complexity and pointing issues associated with large, lightweight, antenna dishes in space. However, the success of missions of this type is dependent on the ability to control the formation in the presence of the significant (differential) aerodynamic forces associated with LEO. The dynamic models derived in this paper are a critical step toward determining the feasibility of using drag-free control to mitigate aerodynamic disturbances for formation-flying based Earth imaging in LEO.

In the sequel, the equations of motion of the formation are developed relative to a circular reference orbit. For non-drag-free spacecraft the objective is to determine the equations of motion of the center-of-mass of the spacecraft relative to the reference orbit. For drag-free spacecraft, the objectives are to (1) determine the equations of motion of the center of mass of the spacecraft relative to the reference orbit, and (2) determine the equations of motion of the proof mass relative to the center of mass of the spacecraft. The derivation of the equations of motion given here is based on the use of direct tensor notation. Direct tensor notation leads to physical insight into the structure of formation dynamics with a minimum of notational overhead. Further, direct tensor notation is especially powerful in applications such as formation flying where a large number of reference frames are involved in the dynamic analysis. Further, once a specific set of generalized coordinates has been chosen, the tensorial equations admit a concise matrix form which is amenable to computer simulation.

This paper is organized as follows. First, some preliminary material from rotational kinematics and tensor analysis is reviewed. Next, the geometry of an Earth orbiting formation is described. A derivation of the

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nonlinear equations of motion of a non-drag-free spacecraft about a circular reference orbit is then given. In the final set of motion equations, terms containing quadratic nonlinearities are retained. Next, the nonlinear equations of motion of a drag-free spacecraft are developed. A discussion of the various disturbances acting on the proof mass is also given. In the final section, conclusions and some directions for further research are presented.

2. KINEMATIC PRELIMINARIES

In this section a few basic concepts and notation from kinematics and tensor analysis are reviewed; see[5] and[7] for further discussion.

In the sequel all vectors are *geometric* or *Gibbsian* vectors; i.e., directed line segments in three dimensional Euclidean point space E^3 obeying the parallelogram rule of addition. A vector will be denoted as $\vec{Q}$ and its magnitude as Q. A reference frame is a set of three dextral orthonormal vectors located at an arbitrary point in E^3. A reference frame will be denoted as $\mathcal{F}_A$, $\mathcal{F}_B$, etc.

A fundamental result relating the time rates of change of a Gibbsian vector relative to different rotating reference frames, $\mathcal{F}_A$ and $\mathcal{F}_B$, is the *Transport Theorem*[5]:

$$\overset{A}{\vec{Q}}=\overset{B}{\vec{Q}}+[{}^A\vec{\omega}^B]\vec{Q} \tag{1}$$

where $[{}^A\vec{\omega}^B]$ denotes the skew-symmetric cross product operator $[{}^A\vec{\omega}^B]\vec{Q}={}^A\vec{\omega}^B\times\vec{Q}$, $\vec{Q}$ denotes an arbitrary geometric vector, ${}^A\vec{\omega}^B$ denotes the angular velocity of $\mathcal{F}_B$ in $\mathcal{F}_A$, and

$$\overset{A}{\vec{Q}} \equiv \dot{Q}_1\vec{a}_1+\dot{Q}_2\vec{a}_2+\dot{Q}_3\vec{a}_3 \tag{2}$$

$$\overset{B}{\vec{Q}} \equiv \dot{Q}'_1\vec{b}_1+\dot{Q}'_2\vec{b}_2+\dot{Q}'_3\vec{b}_3 \tag{3}$$

The notation $\overset{A}{\vec{Q}}$ (resp. $\overset{B}{\vec{Q}}$) can be interpreted physically as the rate of change of $\vec{Q}$ as seen by an observer rigidly fixed to $\mathcal{F}_A$ (resp. $\mathcal{F}_B$). As a consequence, if $\vec{Q}$ is a vector fixed in $\mathcal{F}_A$ (resp. $\mathcal{F}_B$) then $\overset{A}{\vec{Q}}=\vec{0}$ (resp. $\overset{B}{\vec{Q}}=\vec{0}$).

Second rank tensors (also called *dyadics*) will also be utilized in the derivation of the equations of motion. In complete analogy with vectors, a second rank tensor $\overleftrightarrow{T}$ is a geometric object that is independent of any observer. For our purposes, we regard a dyadic as a linear operator $\overleftrightarrow{T}$: $E^3\mapsto E^3$; i.e., a dyadic is a linear mapping on the space of geometric vectors. An important class of second rank tensors is defined via the *tensor (dyadic) product*:

$$(\vec{u}\vec{v})\vec{r}=\vec{u}(\vec{v}\cdot\vec{r}) \tag{4}$$

where $\vec{u}$,$\vec{v}$, and $\vec{r}$ denote arbitrary vectors, and $\cdot$ is the standard Euclidean dot product. The tensor product will be used extensively in the derivation of the equations of motion.

3. FORMATION DYNAMICS: TRANSLATIONAL EQUATIONS OF MOTION

3.1. Orbital Geometry

In this section we consider a formation of n spacecraft in Earth orbit. We will assume that n_f spacecraft are drag-free. An inertial frame of reference $\mathcal{F}_N$, is attached to the center of the Earth and described by the unit vectors $\{\vec{n}_1,\ \vec{n}_2,\ \vec{n}_3\}$. The motion of the formation will be described with respect to a circular reference orbit (See Figure 1). The reference orbit defines an orbiting reference frame $\mathcal{F}_O$ as shown in Figure 1. The orbital frame serves as the primary frame to analyze the dynamics of the formation. The unit vector $\vec{o}_1$ points anti-nadir, the unit vector $\vec{o}_3$ points in the direction of the orbit normal, and $\vec{o}_2$ completes the right-handed triad. The location of the origin of the reference orbit, denoted $\vec{R}_o$, is a circular solution to the following two-body problem with Earth as the central body:

$$\overset{NN}{\vec{R}}_o = -\frac{\mu \vec{R}_o}{R_o^3} \tag{5}$$

Here $\mu = 3.986 \times 10^5$ $[\frac{km^3}{s^2}]$ denotes the gravitational parameter of the Earth, and R_o denotes the magnitude of $\vec{R}_o$.

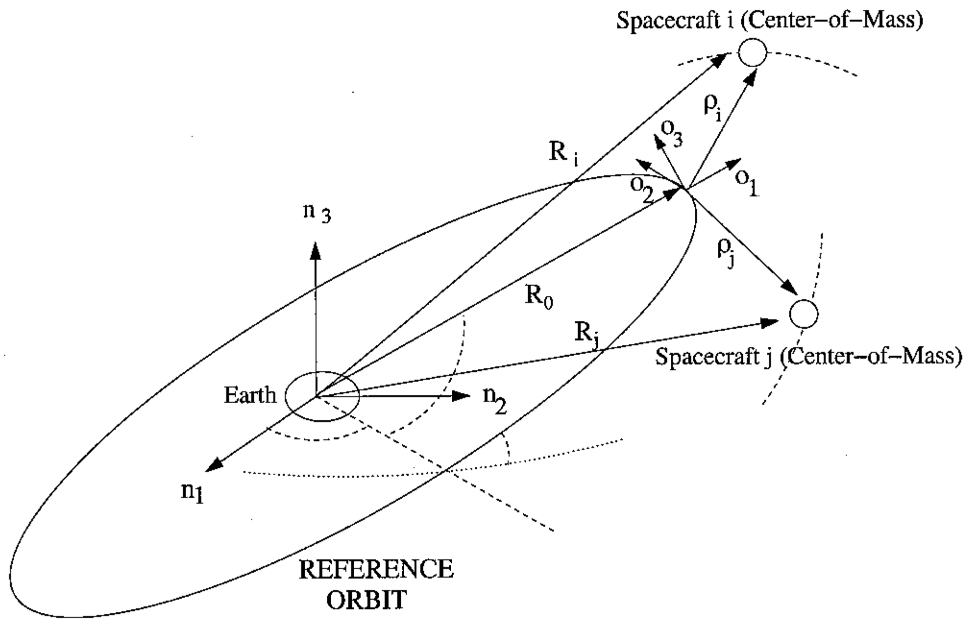

Figure 1. Formation Geometry: Non-Drag-Free Spacecraft

3.2. Formation Equations of Motion: Non-Drag Free Spacecraft

The equations of motion of the non-drag-free spacecraft in the formation are given by

$$\overset{NN}{\vec{R}_i} = -\frac{\mu \vec{R}_i}{R_i^3} + \frac{\vec{F}_i}{m_i} \qquad i = 1, 2, \cdots, n - n_f \tag{6}$$

where $\vec{R}_i$ is the absolute position of the center-of-mass of the i^{th} spacecraft, R_i denotes the magnitude of $\vec{R}_i$, $\vec{F}_i$ is the resultant of all external forces at the center-of-mass of the i^{th} spacecraft, and m_i is the mass of the i^{th} spacecraft. In LEO

$$\vec{F}_i = \vec{F}_{ai} + \vec{F}_{si} + \vec{F}_{oi} + \vec{F}_{pi} + \vec{F}_{ci} \tag{7}$$

where $\vec{F}_{ai}$ denotes the resultant aerodynamic force, $\vec{F}_{si}$ is the resultant solar radiation force, $\vec{F}_{oi}$ is the force due to Earth oblateness effects, $\vec{F}_{pi}$ is the gravitational perturbation due to the Moon and Sun, and $\vec{F}_{ci}$ is the control force due to thrusters. Analytical models of the disturbance forces are well-known and will not be discussed here. The reader should consult[8] for further information.

The position of the center-of-mass of the i^{th} spacecraft relative to the origin of the reference orbit is

$$\vec{\rho}_i = \vec{R}_i - \vec{R}_o \tag{8}$$

Differentiating (8) twice and substituting (5) and (6) yields

$$\overset{NN}{\vec{\rho}_i} + \mu \Delta \vec{a}_g(\vec{R}_i, \vec{R}_o) = \frac{\vec{F}_i}{m_i} \tag{9}$$

where the differential acceleration due to gravity is:

$$\Delta\vec{a}_g(\vec{R}_i, \vec{R}_o) = \frac{\vec{R}_i}{R_i^3} - \frac{\vec{R}_o}{R_o^3} \tag{10}$$

The nonlinear relative equations of motion of the i^{th} non-drag-free spacecraft about the reference orbit are given by (9).

Under the assumption that $\rho_i << R_o$, the expression $\Delta\vec{a}_g(\vec{R}_i, \vec{R}_o)$ can be expanded in a Taylor series about the circular reference orbit. From (8) we find

$$R_i^3 = \left[(\vec{R}_o + \vec{\rho}_i) \cdot (\vec{R}_o + \vec{\rho}_i)\right]^{\frac{3}{2}} \tag{11}$$

$$= R_o^3 \left[1 + \frac{\vec{\rho}_i \cdot \vec{\rho}_i + 2\vec{R}_o \cdot \vec{\rho}_i}{R_o^2}\right]^{\frac{3}{2}} \tag{12}$$

Substituting (12) into (9) yields

$$\overset{NN}{\vec{\rho}_i} + \frac{\mu}{R_o^3}\left[(\vec{R}_o + \vec{\rho}_i)\left[1 + \frac{\vec{\rho}_i \cdot \vec{\rho}_i + 2\vec{R}_o \cdot \vec{\rho}_i}{R_o^2}\right]^{-\frac{3}{2}} - \vec{R}_o\right] = \frac{\vec{F}_i}{m_i} \tag{13}$$

Noting that $\rho_i << R_o$ it follows from the Taylor approximation $(1+x)^q \approx 1 + qx$ that

$$\left[1 + \frac{\vec{\rho}_i \cdot \vec{\rho}_i + 2\vec{R}_o \cdot \vec{\rho}_i}{R_o^2}\right]^{-\frac{3}{2}} \approx 1 - \frac{3}{2}\left[\frac{\vec{\rho}_i \cdot \vec{\rho}_i + 2\vec{R}_o \cdot \vec{\rho}_i}{R_o^2}\right] \tag{14}$$

Substituting (14) into (13) and rearranging yields

$$\overset{NN}{\vec{\rho}_i} + \frac{\mu}{R_o^3}\left[\rho_i - \frac{3}{R_o^2}\left[(\vec{R}_o \cdot \vec{\rho}_i)\vec{R}_o + (\vec{R}_o \cdot \vec{\rho}_i)\vec{\rho}_i\right] - \frac{3}{2R_o^2}\left[(\vec{\rho}_i \cdot \vec{\rho}_i)\vec{R}_o + (\vec{\rho}_i \cdot \vec{\rho}_i)\vec{\rho}_i\right]\right] = \frac{\vec{F}_i}{m_i} \tag{15}$$

The following identities are an immediate consequence of the definition of the tensor product given in equation (4),

$$(\vec{R}_o \cdot \vec{\rho}_i)\vec{R}_o = (\vec{R}_o\vec{R}_o)\vec{\rho}_i \tag{16}$$

$$(\vec{R}_o \cdot \vec{\rho}_i)\vec{\rho}_i = (\vec{\rho}_i\vec{R}_o)\vec{\rho}_i \tag{17}$$

$$(\vec{\rho}_i \cdot \vec{\rho}_i)\vec{R}_o = (\vec{R}_o\vec{\rho}_i)\vec{\rho}_i \tag{18}$$

$$(\vec{\rho}_i \cdot \vec{\rho}_i)\vec{\rho}_i = (\vec{\rho}_i\vec{\rho}_i)\vec{\rho}_i \tag{19}$$

Substituting (16)-(19) into (15) yields the equations of motion of the formation relative to a reference orbit under the assumption $\rho_i << R_o$,

$$\overset{NN}{\vec{\rho}_i} + \frac{\mu}{R_o^3}\overleftrightarrow{\mathcal{G}}\vec{\rho}_i - \frac{3\mu}{2R_o^5}(2\vec{\rho}_i\vec{R}_o + \vec{\rho}_i\vec{R}_o + \vec{\rho}_i\vec{\rho}_i)\vec{\rho}_i = \frac{\vec{F}_i}{m_i} \tag{20}$$

where the *gravity gradient dyadic* is

$$\overleftrightarrow{\mathcal{G}} = \frac{R_o^2 \overleftrightarrow{1} - 3\vec{R}_o\vec{R}_o}{R_o^2} \tag{21}$$

$$= \overleftrightarrow{1} - 3\vec{o}_1\vec{o}_1 \tag{22}$$

and $\overleftrightarrow{1}$ denotes the unit dyadic. Note that we have used the fact that $\vec{o}_1 = \frac{\vec{R}_o}{R_o}$ in the above expression. An alternate form for the gravity gradient dyadic results from applying the identity $\vec{o}_1\vec{o}_1 = \overleftrightarrow{1} + [\vec{o}_1][\vec{o}_1]$ yielding $\overleftrightarrow{\mathcal{G}} = -2\overleftrightarrow{1} - 3[\vec{o}_1][\vec{o}_1]$.

Upon expressing inertial derivatives in terms of orbital frame derivatives via two applications of the transport theorem (1) we find

$$\overset{OO}{\vec{\rho}_i} + 2[\vec{\omega}] \overset{O}{\vec{\rho}_i} + [\vec{\omega}][\vec{\omega}]\vec{\rho}_i + \frac{\mu}{R_o^3}\overset{\leftrightarrow}{\mathcal{G}}\vec{\rho}_i - \frac{3\mu}{2R_o^5}(2\vec{\rho}_i\vec{R}_o + \vec{R}_o\vec{\rho}_i + \vec{\rho}_i\vec{\rho}_i)\vec{\rho}_i = \frac{\vec{F}_i}{m_i} \tag{23}$$

where $\vec{\omega}$ denotes the angular velocity of $\mathcal{F}_O$ in $\mathcal{F}_N$. The magnitude of $\vec{\omega}$ is the mean motion of the circular orbit and is given by $\omega_o^2 = \frac{\mu}{R_o^3}$. The equations of relative motion (23) are the main result of this section

A useful control design model results from neglecting all quadratic coupling terms in (23),

$$\overset{OO}{\vec{\rho}_i} + 2[\vec{\omega}] \overset{O}{\vec{\rho}_i} + [\vec{\omega}][\vec{\omega}]\vec{\rho}_i + \frac{\mu}{R_o^3}\overset{\leftrightarrow}{\mathcal{G}}\vec{\rho}_i = \frac{\vec{F}_i}{m_i} \tag{24}$$

From Figure 1 it follows that the geometric vectors appearing in (24) have the following representations in $\mathcal{F}_O$

$$\begin{aligned}
\vec{R}_o &= R_o\vec{o}_1 && (25)\\
\vec{\omega} &= \omega_o\vec{o}_3 && (26)\\
\vec{\rho}_i &= x_i\vec{o}_1 + y_i\vec{o}_2 + z_i\vec{o}_3 && (27)\\
\overset{O}{\vec{\rho}_i} &= \dot{x}_i\vec{o}_1 + \dot{y}_i\vec{o}_2 + dotz_i\vec{o}_3 && (28)\\
\vec{F}_i &= F_{x_i}\vec{o}_1 + F_{y_i}\vec{o}_2 + F_{z_i}\vec{o}_3 && (29)
\end{aligned}$$

Substituting (25)-(29) into (24) results in

$$\begin{aligned}
\ddot{x}_i - 2\omega_o\dot{y}_i - 3{\omega_o}^2 x_i &= \frac{F_{x_i}}{m_i} && (30)\\
\ddot{y}_i + 2\omega_o\dot{x}_i &= \frac{F_{y_i}}{m_i} && (31)\\
\ddot{z}_i + \omega_o^2 z_i &= \frac{F_{z_i}}{m_i} && (32)
\end{aligned}$$

Under the further assumption that $\vec{F}_i$ only involves control forces, (30)-(32) are called the *Clohessy-Wiltshire-Hill* (CWH) equations.[1]

3.3. Formation Equations of Motion: Drag-Free Spacecraft

In the formation of n spacecraft, we will assume that n_f spacecraft are designated as drag-free. The geometry of a single drag-free spacecraft in Earth orbit is shown in Figure 2. A free-floating proof mass (or ball) B of mass m_B is confined to a cavity in a larger spacecraft of mass m_S. A spacecraft-fixed reference frame $\mathcal{F}_S$ has origin at the center-of-mass of the spacecraft S. By design, the proof mass is shielded from all aerodynamic forces, solar radiation pressure, and other ambient disturbances. Protected from the space environment inside the cavity, the proof mass follows a pure gravitational orbit. The goal of drag-free control is to actuate the spacecraft in such a way that the proof mass B remains centered at S; i.e., the drag-free control system forces the spacecraft to follow the proof mass. In practice, the null point of the proof mass position sensor, denoted C, is not coincident with the spacecraft center-of-mass S. As a result, we will assume that the control system acts to drive $\vec{\rho}_{BC}$, rather than $\vec{\rho}_{BS}$, to zero. Moreover, although it is desirable to have the control null point, the spacecraft center of mass, and the point of zero self-gravity (i.e., the point where the resultant gravitational attraction of the spacecraft on the proof mass vanishes) coincide, this is not possible in practice.[6]

The translational equations of motion of a drag-free spacecraft consist of (1) the translational equations of motion of the proof mass about the nominal reference orbit, and (2) the translational equations of motion of the center-of-mass of the spacecraft relative to the proof-mass. See Figure 2 for a complete description of the system geometry.

The equations of motion of the proof mass are given by

$$\overset{NN}{\vec{R}_B} = -\frac{\mu \vec{R}_B}{R_B^3} + \frac{\vec{F}_B}{m_B} \tag{33}$$

where $\vec{R}_B$ is the absolute position of the proof mass, and $\vec{F}_B$ is the resultant force acting on the proof mass. The resultant force can be decomposed as $\vec{F}_B = \vec{F}_{BS} + \vec{F}_{BNG}$ where $\vec{F}_{BS}$ is the gravitational force on the proof mass due to the mass distribution of the spacecraft, and $\vec{F}_{BNG}$ is the resultant of all non-gravitational forces acting on the proof mass. Although the dominant force acing on the proof mass is the mass attraction of the surrounding satellite on the proof mass $\vec{F}_{BS}$, other disturbance forces are still present. Specifically, $\vec{F}_{BNG}$ may contain forces due to electric fields if a capacitive pickoff is used, radiation pressure if an optical pickoff is used, gas pressure in the cavity, Brownian motion, magnetic field gradients, and thermal gradients.

Noting that $\vec{R}_B = \vec{R}_o + \vec{\rho}_B$ where $\rho_B << R_o$, the translational equations of motion of the proof mass relative to the reference orbit are identical in form to the translational equations developed in the previous section,

$$\overset{OO}{\vec{\rho}_B} + 2[\vec{\omega}]\overset{O}{\vec{\rho}_B} + [\vec{\omega}][\vec{\omega}]\vec{\rho}_B + \frac{\mu}{R_o^3}\overset{\leftrightarrow}{\mathcal{G}}\vec{\rho}_B - \frac{3\mu}{2R_o^5}(2\vec{\rho}_B\vec{R}_o + \vec{\rho}_B\vec{R}_o + \vec{\rho}_B\vec{\rho}_B)\vec{\rho}_B = \frac{\vec{F}_{BS} + \vec{F}_{BNG}}{m_B} \tag{34}$$

where $\vec{\rho}_B$ denotes the position of the center-of-mass of the proof mass relative to the origin of $\mathcal{F}_O$.

We now develop the translational equations of motion of the proof mass B relative to the spacecraft center of mass S. The equations of motion of the spacecraft are

$$\overset{NN}{\vec{R}_S} = -\frac{\mu \vec{R}_S}{R_S^3} + \frac{\vec{F}_S}{m_s} \tag{35}$$

where $\vec{R}_S$ is the absolute position of the center-of-mass of the spacecraft, m_s is the mass of the spacecraft, and $\vec{F}_S$ denotes the resultant force acting on the spacecraft. The resultant force can be decomposed as $\vec{F}_S = \vec{F}_{SB} + \vec{F}_{SNG}$ where $\vec{F}_{SB} = -\vec{F}_{BS}$ is the gravitational force on the spacecraft due to the proof mass, and $\vec{F}_{SNG}$ is the resultant of all non-gravitational forces acting on the spacecraft (e.g., aerodynamic drag, solar radiation pressure).

From Figure 2 we find

$$\overset{NN}{\vec{\rho}_{BS}} = \overset{NN}{\vec{R}_S} - \overset{NN}{\vec{R}_B} \tag{36}$$

Upon substituting (33) and (35) we find

$$\overset{NN}{\vec{\rho}_{BS}} + \mu\left(\frac{\vec{R}_S}{R_S^3} - \frac{\vec{R}_B}{R_B^3}\right) = \left(\frac{\vec{F}_{SNG}}{m_S} - \frac{\vec{F}_{BNG}}{m_B}\right) - \left(\frac{1}{m_S} + \frac{1}{m_B}\right)\vec{F}_{BS} \tag{37}$$

As discussed above, we assume that the position sensor measures the displacement of the proof mass B relative to a null point C, rather than the spacecraft center of mass S. The most common position sensors are based on optical and capacitive measurement techniques.[2] It follows from Figure 2,

$$\vec{\rho}_{BS} = \vec{\rho}_{BC} - \vec{\rho}_{SC} \tag{38}$$

where $\vec{\rho}_{BC}$ is the position of the null point relative to the proof mass, and $\vec{\rho}_{SC}$ is the position of the null point relative to the spacecraft center-of-mass. As the drag-free control system will act to drive $\vec{\rho}_{SC}$ to zero, we will express (37) in terms of $\vec{\rho}_{BC}$. Upon expanding the differential acceleration $\Delta\vec{a}_g(\vec{R}_S, \vec{R}_B)$ in a Taylor series about the circular orbit, and expressing all inertial rates of change in terms of rates of change relative to $\mathcal{F}_S$ via two applications of the transport formula, we obtain

$$\overset{SS}{\vec{\rho}}_{BC} + 2[\vec{\omega}_S]\overset{S}{\vec{\rho}_{BC}} + [\vec{\omega}_S][\vec{\omega}_S]\vec{\rho}_{BC} + \frac{\mu}{R_B^3}\overset{\leftrightarrow}{\mathcal{G}}\vec{\rho}_{BC} - \frac{3\mu}{2R_B^5}(2\vec{\rho}_{BC}\vec{R}_B + \vec{R}_B\vec{\rho}_{BC} + \vec{\rho}_{BC}\vec{\rho}_{BC})\vec{\rho}_{BC} =$$

$$[\vec{\omega}_S][\vec{\omega}_S]\vec{\rho}_{SC} + \frac{\mu}{R_B^3}\overset{\leftrightarrow}{\mathcal{G}}\vec{\rho}_{SC} + \frac{3\mu}{2R_B^5}(2\vec{\rho}_{SC}\vec{R}_B + \vec{R}_B\vec{\rho}_{SC} + \vec{\rho}_{SC}\vec{\rho}_{SC})\vec{\rho}_{SC} + \left(\frac{\vec{F}_{SNG}}{m_S} - \frac{\vec{F}_{BNG}}{m_B}\right) - \left(\frac{1}{m_S} + \frac{1}{m_B}\right)\vec{F}_{BS} \tag{39}$$

where

$$\overleftrightarrow{\mathcal{G}} = \frac{R_B^2 \overleftrightarrow{1} - 3\vec{R}_B\vec{R}_B}{R_B^2} \tag{40}$$

and $\vec{\omega}_S$ is the angular velocity of $\mathcal{F}_S$ relative to $\mathcal{F}_N$. Note that we have assumed that $\vec{\rho}_{SC}$ is fixed in $\mathcal{F}_S$.[6] In summary, the translational equations of motion of the drag-free spacecraft are given by (37) and (39). These equations will serve as the starting point for the synthesis of control logic and sensitivity analysis.

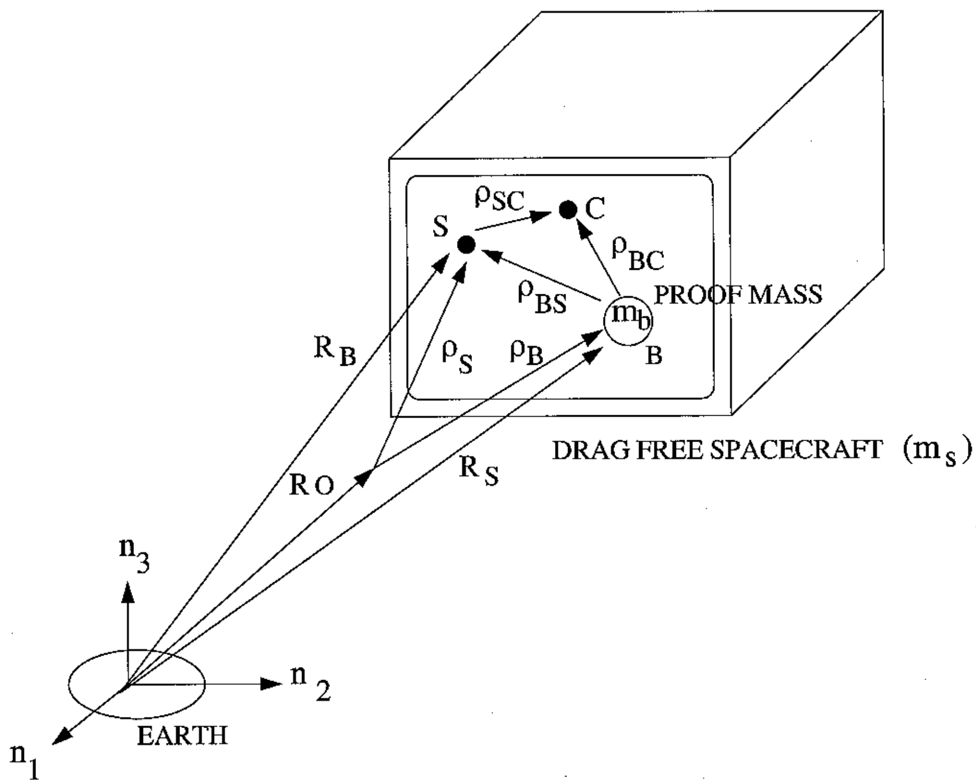

Figure 2. Geometry of a Drag-Free Spacecraft in Earth Orbit

4. CONCLUSIONS

In this paper the translational equations of motion of a formation containing both drag free and non-drag free spacecraft were derived using direct tensor notation. The results presented in this paper are a first step toward coupling drag-free control technology with formation flying in order to mitigate the effect of differential aerodynamic drag on formation flying missions (e.g., Earth imaging applications) in low Earth orbit. For example, the feasibility of creating a virtual drag free instrument (e.g., distributed antenna) via precision formation control will be studied. Specifically, the ability to make the formation behave in a drag-free manner even though only a single spacecraft may be drag free will be investigated.

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