Development of New Modeling and Analysis Tools for Solar Sails

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Existing finite-element-based structural analysis codes are ineffective in treating deployable gossamer space systems, including solar sails that are formed by long space-deployable booms and extremely large thin-film membrane apertures. Recognizing this, the NASA Space transportation Technology Program has initiated and sponsored a focused research effort to develop new and computationally efficient structural modeling and analysis tools for solar sails. The technical approach of this ongoing effort will be described. Two solution methods, the Distributed Transfer Function Method and the Parameter-Variation-Principle method, based on which the technical approach was formatted are also discussed.

Nomenclature

L	=	Length
R	=	Radius
D	=	Diameter
H	=	Thickness
E	=	Modulus of elasticity
M	=	Poisson's ratio
EI	=	Bending stiffness
ε	=	Amplitude of imperfection

I. Introduction

Many of the future space exploration missions planned by NASA will employ solar sails. The typical architecture of these solar sails consists of long space-deployable booms and thin-film membrane apertures. Due to the effects of gravity and air damping, it is difficult and in some cases even impossible to verify in-space performance of a solar sail by ground testing. The development of a solar sail flight system must rely on effective computational capabilities.

Compared to other large lightweight space-deployable structures, a solar sail that uses long booms to deploy and tension its thin-film membrane aperture is structurally unique in two major aspects. Firstly, the booms are very long (up to hundreds of meters in length) and cannot be efficiently treated by existing finite-element modeling (FEM) structural analysis codes. Secondly, the membrane aperture is made of a very thin film material (usually has less than a few microns in thickness) and extremely large (up to hundreds or even thousands of square meters). A thin membrane, by definition, has zero out-of-plane stiffness. It is only when the membrane being subjected to in-plane tension then some level of out-of-plane stiffness (known as differential stiffness) will present. Unless the distribution of its in-plane tension is absolutely uniform, which is impossible to achieve for most, if not all, space deployable solar sail designs, a membrane will wrinkle. Wrinkling of the membrane aperture can significantly affect static and

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dynamic performance of a solar sail. Although the formation of wrinkles and their effects on structural behaviors of large membranes have been studied by a number of researchers[1-4], they remain to be paramount challenges that must be reckoned with in designing and developing all tensioned space membrane structures, including solar sails. At this point, no user-friendly computational capabilities are available to treat the wrinkle issue.

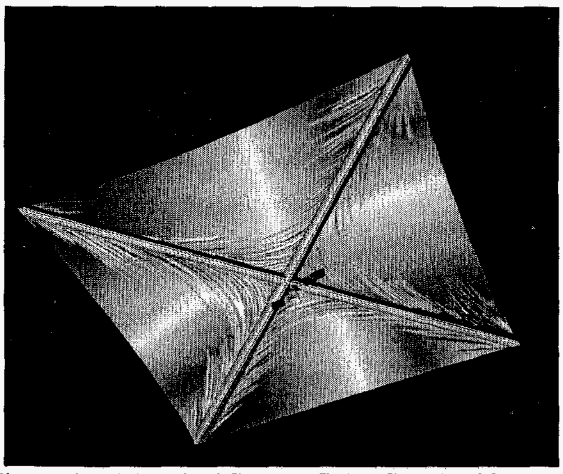


Figure 1. A Typical Square Solar Sail Architecture

This paper will discuss an ongoing effort, which was initiated and sponsored by the NASA Space transportation Technology Program, to develop a new set of analysis modeling and analysis software tools that can potentially meet the above-mentioned challenges. This development effort is based on two innovative solution methodologies, the Distributed Transfer Function Method (DTFM) and the Parameter-Variation-Principle (PVP) method. Results of previously conducted studies have successfully proven that the DTFM is computationally efficient to treat extremely long and slim booms that are with or without material and geometrical imperfections[5-7]. Similar studies have also shown that the PVP method can accurately predict wrinkle formation and patterns in tensioned thin-film membranes of various shapes and boundary conditions[8,9]. Additionally, a discussion will also be included in this paper on the DTFM model synthesis approach for assembling system models to simulate in-space structural performance of solar sails.

II. Technical Approach

The common practice at this time for predicting and analyzing structural and dynamic behavior of large space-deployable membrane structures, including solar sails, is to use commercially available, general-purpose structural analysis codes, such as NASTRAN, ANSYS, and ABAQUS. These finite-element codes were developed specifically for treating traditional structural systems that consist of relatively small and stiff building-block components (i.e., composite and metallic plates, bars, beams, and trusses), and they have been proven to be very effective for these intended uses. However, they are neither efficient nor effective in dealing with large space-deployable membrane structures, such as solar sails, that are extremely large and flimsy, easy to wrinkle, and undergo large-displacement motions during in-space deployment. To advance the state of technology for square solar sails, structural modeling and analysis tools that employ new and innovative solution approaches must be developed that effectively account for the unique structural features and characteristics of the sails.

The authors of this paper and their colleagues have in the last several years successfully developed two innovative solution methods. The first methods, the Distributed Transfer Function Method (DTFM), has been used to successfully treat control and structures problems, including local buckling of booms. The second method, the Parameter-Variation-Principle (PVP) method, was recently developed and applied to analyze membranes with wrinkling and/or slack regions[8,9]. For the ongoing solar sail modeling and analysis tool development, we will build on the obtained results of our previous DTFM and PVP work and extend the application of these two solution methods specifically for long booms, wrinkled membranes, and sail system models. Briefly discussions on these specific DTFM and the PVP applications are given in the following sections.

III. DTFM Modeling of Long Booms

The Distributed Transfer Function Method (DTFM) is an innovative solution technique with unique capabilities for modeling and analysis that can greatly enhance the design and development of large ultra-lightweight space structural systems such as square solar sails. The DTFM was developed in the early 1990s to study control problems of one-dimensional elastic continua in the Laplace domain[10], and over the past few years, has been successfully applied to analyze general structures composed of one-dimensional components.

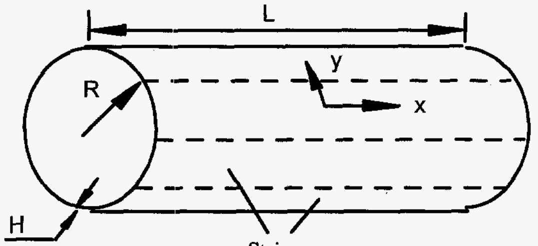
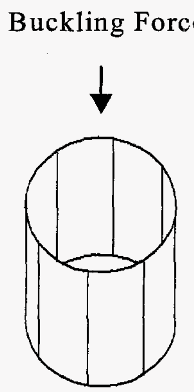
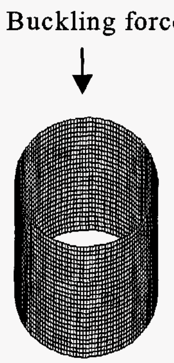


Figure 2. A Shell Divided into Long Strips



Strip Number	Buckling Force
2	381.28
4	381.28
6	381.28
8	381.28



Element Number	Buckling Force
54	1269.6
218	393.3
864	386.8
3456	381.4

Figure 3. DTFM vs. NASTRAN FEM Results

The DTFM is as versatile in modeling multi-body structures of complex configuration as the finite element method (FEM). A major difference between the two modeling techniques, though, is that DTFM uses Distributed Transfer Functions to mathematically represent every component and FEM uses shape functions to represent every element. The combination of higher-level analytical formation and multi-body assembly capability makes the DTFM model synthesis approach unique and powerful in treating large structural systems such as the solar sails. No other existing approaches have such capability. Another advantage of using Distributed Transfer Functions is that it gives exact and closed-form solutions for both displacements and forces of one-dimensional components (semi-analytical solutions for two-dimensional components). Which enables DTFM to model a very long building block of a gossamer structure (e.g. a solar sail boom) with a single component without losing any accuracy. DTFM does not have any numerical approximations and, consequently, does not have any numerical instability. In contrast, FEM uses shape functions, which is a numerical approximation. As a result, numerous elements are required by FEM to model a single boom accurately.

In order to quantitatively illustrate the difference between DTFM and FEM, a previously completed study is presented here as an example. This study is a buckling analysis of a short circular cylindrical shell as shown in Figure 2. In the DTFM, the shell is divided into a number of longitudinal strips. In the strip lateral (y) direction, the shell response is interpolated by polynomials. Along the strip longitudinal (x) direction, the DTFM is applied to obtain closed-form analytical solutions. Figure 3 compares the DTFM and FEM results on the buckling force of this example ($L = D = 100$, $H = 1$, $E = 100$, and $m = 0.3$).

It can be concluded from this example that the DTFM modeling approach delivers a highly accurate semi-analytical solution by using only a very small number of strips while the FEM approach, even with a much larger number of elements, gives only an approximate solution. This example has clearly demonstrated the computationally efficiency and effectiveness of the DTFM for treating boom-type structures.

One obvious advantage of the DTFM over the finite element method is that the DTFM requires far fewer unknowns in order to accurately describe a structure. It should be emphasized that DTFM strip modeling is totally different from the finite strip method[11] in its closed-form analytical solution format and its capability for synthesis analysis of multi-body structures. These features also distinguish the DTFM from other existing analytical methods, such as classical boundary value approach, Rayleigh-Ritz method, Fourier series expansion, and Levy solution scheme[12-14].

In addition to our effort to develop solar sail modeling and analysis tools, NASA Space Transportation Technology Program has also initiated and sponsored a solar sail ground demonstration task. Two 20-meter class ground demonstration sails are currently being designed, fabricated, and assembled for ground testing. One of these sails, developed by AEC-Able, uses coilable multi-bay truss booms. The other sail, developed by L'Garde, uses inflatable thin-film booms. Both of these two sails, will be subjected to a series of performance tests in the ground, including dynamic tests in NASA/GRC's 100-ft vacuum chamber at Plum Brook, Ohio. Since the test results from these 20-meter ground demonstration sails will be used to validate the software tools developed by our ongoing task, we will focus our boom modeling and analysis efforts on these AEC-Able and L'Garde designs.

IV. Buckling of Booms with Initial Geometric and Materials Imperfections

It was shown by a previously conducted research that the structural behaviors of a long boom with initial geometric and material imperfections could be investigated by the DTFM[15]. For instance, in a perturbation analysis, the displacements of a boom in the longitudinal, circumferential and transverse directions can be represented as

$$u_k(x,y) = u_{0,k}(x,y) + \tilde{u}_k(x,y) \tag{1}$$

for $k = 1,2,3$, where $u_{0,k}$ describe the initial geometric imperfections, and $\tilde{u}_k$ the perturbed buckling deformations. In buckling analysis, the perturbed differential equations governing the boom deformation are first cast into a spatial state form, with the coefficients being functions of $u_{0,k}$. The resulting state equation is then solved in closed form by the DTFM.

Depending on the boom configuration, the equations for buckling deformation may take different forms. For instance, for single tubes, the buckling equations are similar to those of shells; and for booms made of isogrid trusses, a set of differential equations can be derived. In either case, the DTFM or DTF synthesis can be applied to determine boom buckling load and deformation.

One obvious advantage of the DTFM over existing methods is that it can conveniently model both global and local boom imperfections, including those caused by packaging and deployment. In FEM, those imperfections may be difficult to model and analyze, even with an extremely large number of elements. This special capability of the DTFM will be extremely useful in design of solar sail booms.

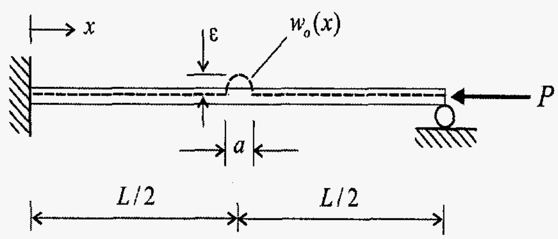


Figure 4. A Column with Localized Imperfection

As an example, consider a column with a localized imperfection $w_0(x)$; see Figure 4. The column can be viewed as a component of an isogrid truss or coil-able boom. Let the column bending stiffness and length be EI = 25 and L = 4, respectively. By using the DTFM, the axial load-deflection curves of the imperfect column at $x = L/2$ are plotted for the imperfection amplitude $\varepsilon = 0.02$, 0.05, 0.1 and a = 0.1; see Figure 5, which indicates that small initial imperfection can cause significant deflection of the column.

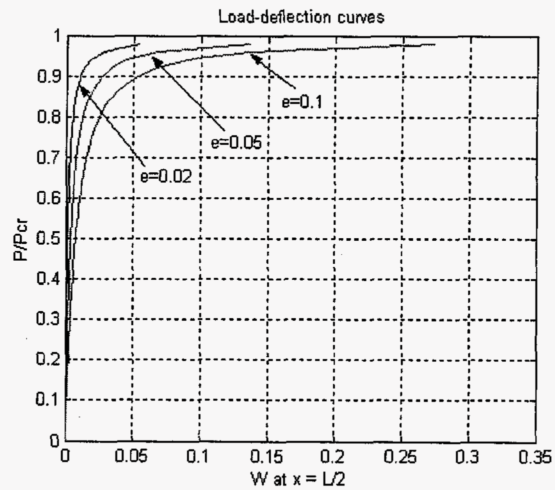


Figure 5. Load-Deflection Curves of An Imperfect Column

In our development of modeling and analysis tools for long booms, we will follow similar perturbation approach to develop analysis processes that focus on studying the effects of geometrical and material imperfections to buckling of long booms. The loading and boundary conditions that represent typical square sail applications of long booms will be considered.

V. PVP Wrinkling Analysis of Membranes

Tensioned thin-film membranes are basic elements of a variety of space structures, including solar sails. Thin-film membranes usually have little compression resistance capability, and hence are easy to wrinkle. Wrinkling eventually leads to out-of-plane deformation of membranes, and variations in in-plane stress distribution.

Membrane analysis usually has three steps. The first step is making a model that does not have out-of-plane stiffness. The second step is pre-tensioning the membrane to get the out-of-plane stiffness, namely differential stiffness. Differential stiffness is a function of the in-plane stress distribution. The third step is applying external loads, such as solar pressure, on the membrane. This step only can be done after the differential stiffness is established. The second step, pre-tensioning a piece of membrane, introduces wrinkled and/or slack regions, which is a local membrane-buckling phenomenon. Slack regions may exist during the analysis process and may not exist when the solar pressure is applied. Using ordinary static analysis to analyze a process that involves buckling introduces numerical instable and divergent.

In order to solve the membrane wrinkling and/or slack problem, an innovative method, namely Parametric Variational Principle (PVP) method, has been developed in the last several years[8,9]. This method is the state-of-the-art for solving membrane pre-tensioning problems. It can predict the formation and distribution of wrinkled and slack regions and will be developed to calculate the out-of-plane deformations. The basic ideas are given as follows:

Let the principal stresses of a membrane be σ_1 and σ_2, $\sigma_1 \geq \sigma_2$. A particle point of the membrane is in one the following three states defined by principal stresses:

(S1) Taut state: $\sigma_1 > 0, \quad \sigma_2 > 0$

(S2) Wrinkled state: $\sigma_1 > 0, \quad \sigma_2 = 0$

(S3) Slack state: $\sigma_1 = 0, \quad \sigma_2 = 0$

In the two-viable-parameter (2-VP) membrane model, material properties in the constitutive law are:

$$\tilde{E} = \frac{E}{1+\lambda_1}, \quad \tilde{\mu} = \mu + \lambda_2 \tag{2}$$

with λ_1 and λ_2 being non-negative viable parameters. The viable parameters can be adjusted such that the conditions of the membrane states (S1, S2 and S3) are satisfied. It can be shown that:

For taut state (S1), $\lambda_1 = 0, \quad \lambda_2 = 0$;

For wrinkled state (S2), $\lambda_1 = 0, \quad \lambda_2 > 0$;

For slack state (S3), $\lambda_1 > 0, \quad \lambda_2 > 0$.

The significance of this is, with proper variation of parameters λ_1 and λ_2, the three states of membranes can be systematically described.

For the 2-VP membrane model, the potential energy functional is then derived and the energy functional contains the viable parameters λ_1 and λ_2:

$$\Pi_\lambda(\{u\}) = \int_\Omega \frac{1}{2}\{\varepsilon\}^T \left[\tilde{D}(\lambda_1,\lambda_2)\right]\{\varepsilon\} h \, d\Omega - \int_{S_\sigma} \{\bar{T}\}^T \{u\} \, dS \tag{3}$$

where $\{u\}$ is the vector of membrane displacements, Π_λ indicates that the energy functional contains the viable parameters λ_1 and λ_2.

The Parametric Variational Principle (PVP) has been developed for the 2-VP membrane model. According to the PVP, the exact displacement solution of the wrinkling problem renders the variation of the potential energy of the membrane equal to zero. This principle lays a foundation for a new numerical wrinkling analysis of membranes.

Based on the 2-VP membrane model and the PVP, a parametric finite element method for numerical wrinkling analysis of membranes of arbitrary shape and boundary conditions has been developed. A membrane is divided into a number of elements. By standard finite element interpolation, the displacements of an element can be expressed as:

$$\left\{u^e(x,y)\right\} = \left[N(x,y)\right]\left\{q^e\right\} \tag{4}$$

where $\left\{q^e\right\}$ represents unknown nodal displacements. Unlike conventional finite element formulation, the stresses $\{\sigma\}$ of the element contains two viable parameters λ_1 and λ_2.

By the PVP described previously,

$$\frac{\partial \Pi_\lambda}{\partial \{q\}} = 0 \tag{5}$$

where $\{q\}$ is the global nodal displacement vector. By Equation (5) and the three membrane states (S1, S2 and S3) defined by the principal stresses, the following two important results are obtained:
(i) The nodal displacement vector $\{q\}$ can be expressed by an explicit function of the viable parameters:

$$\{q\} = g(\{\lambda\}) \tag{6}$$

where $\{\lambda\}$ is the vector of all viable parameters.
(ii) The original wrinkling problem of the membrane is equivalent to the nonlinear complementarity problem (NCP) described by:

$$\Gamma(\{\lambda\}) \cdot \{\lambda\} = 0 \tag{7}$$

This NCP can be solved by a smoothing Newton Method[16].
According to the above discussion, the PVP method for wrinkling analysis takes the following three steps:
Step 1. Solve the NCP for viable parameters $\{\lambda\}$.
Step 2. With $\{\lambda\}$ obtained, compute the membrane displacements by Equations (4) and (6); and
Step 3. With $\{\lambda\}$ and membrane displacements, compute the stress distribution of the membrane, and determine the wrinkled and slack regions.
The break through of the above solution procedure is that the solution of the NCP does not request numerical iterations, and has guaranteed convergence. Thus, the stress distribution of a membrane with wrinkled and slack regions can be accurately and efficiently determined. To test the validity of the PVP analysis approach, a preliminary attempt has been made to study the forming of wrinkles in a tensioned triangular membrane, as shown in Figure 6.

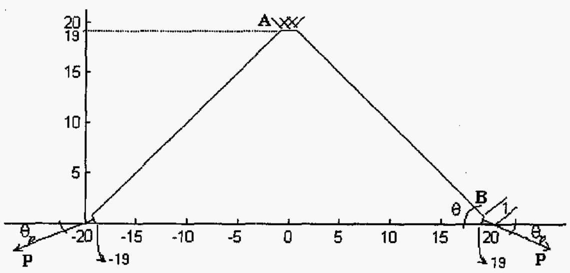


Figure 6. A Tensioned Triangular Membrane

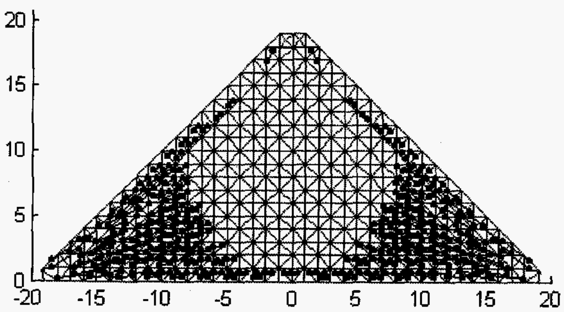


Figure 7. Predicted Wrinkle Pattern

The top corner of the membrane is fixed; the other corners are stretched by force P with angle θ_p to the horizontal axis. The wrinkle pattern of the membrane predicted by the PVP finite element method is shown in Figure 7, where a wrinkled element is marked with a dumbbell-like symbol with the bar orientation and length representing the direction and magnitude of the first principal stress σ_1. As the angle θ_p approaches 90 degrees, a slack region appears near the mid bottom of the membrane, as observed in a preliminary experiment. In this case, conventional stress iteration with standard finite element modeling of the membrane becomes divergent, and fails to deliver any meaningful results. The PVP method, however, can predict the distribution of similar wrinkled and slack regions of the membrane, without any divergence problem in computation.

Solar sail membranes are extremely large and thin with very low in-plane stresses. They are naturally very easy to wrinkle and to have slack regions. All iteration-based methods used to treat solar sail membranes will likely experience numerical instability problems and have difficulties in convergence. The PVP method has been proven computationally efficient for analyzing thin-film membranes without worrying about convergence. Our membrane modeling starts from current start-of-the-art PVP technology. We will finish all equation derivations of the PVP method and develop the analytical process for wrinkle and slack regions as well as their forming processes analyses. This will lead to a new method for predicting the out-of-plane deformation of wrinkled membranes. The key idea of this approach is to introduce the in-plane stresses in the compatible equation of strain components:

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} = \phi(\sigma_x, \sigma_y, \tau_{xy}) \tag{8}$$

where w is the transverse (out-of-plane) displacement of the membrane, and ϕ is a known function of the in-plane stresses $\sigma_x, \sigma_y, \tau_{xy}$. In the literature, equations similar to Equation (8) are sometimes called nonlinear Monge-Ampere Equations, although they have never been used to study the currently considered wrinkled membrane problem. It can be shown that the solution of Equation (8) exists, and is unique[17]. Thus, unlike the conventional analysis methods based on plate/shell models, the problem of numerical singularity can be avoided in the current method. In computing the out-of-plane deformation from Equation (8), knowledge of the in-plane stresses is essential. The in-plane stresses of a wrinkled membrane can be efficiently determined by the 2-VP model and parametric finite element method, which guarantees the feasibility of the proposed method. This research will develop numerical algorithms for solution of the compatibility Equation (8). Stiffness changes introduced by stress changing and membrane deformation will be calculated and superposed to the original stiffness matrices. Dynamic analyses of wrinkled membrane can thus be conducted.

VI. Synthesis and Assembly of Sail System Models

Our experience with the DTFM also has shown that the application of this solution approach is not limited to component-level analysis; it is also capable of modeling and analyzing a system of multiple structural components. This system-level application, referred to as the DTF synthesis[6], takes the following three steps:
(i) *Decomposition:* The structure under consideration is decomposed into a number of components or subsystems. For a solar sail, the major components are thin-film membranes and supporting booms. Also, other components such as beams, plate boxes and rigid bodies can be considered (for modeling the spacecraft attached to a solar sail, for instance).
(ii) *Component Representation:* The response of a component (subsystem) is expressed by its distributed transfer functions in terms of unknown displacement parameters at the nodal points (boundary) of the component. This yields a nodal displacement representation of the response of the component

$$\{q_e\} = [K_e]\{u_e\} \tag{9}$$

where $\{u_e\}$ and $\{q_e\}$ are the nodal displacement and force vectors, and $[K_e]$ is the component dynamic stiffness matrix. The DTFM representation is valid for booms, as well as trusses, plates, beams, and elastic and rigid bodies.
(iii) *Assembly:* The structure is assembled from its components by imposing force balance and displacement compatibility at the interconnecting nodal points of the components, leading to a global dynamic equilibrium equation

$$\{q\} = [K_g]\{u\} \tag{10}$$

where $\{u\}$ and $\{q\}$ are the global displacement and force vectors, and $[K_g]$ is the global stiffness matrix assembled from the component stiffness matrices $[K_e]$. Solution of the global equation, Equation (10), gives the static/dynamic response of the system at every point.

This combination of analytical solution format and multi-body assembly capability makes the DTFM unique and powerful in analysis of solar sails. No other existing approaches have such capability. Using nodal displacements, the DTFM synthesis can efficiently deal with component stiffness matrices obtained by other techniques such as finite element methods. This feature enables the DTFM synthesis to obtain a complete model of solar sails by integrating the boom model that is derived based on the DTFM, with a membrane model that is derived by using the PVP.

With its transfer function formulation, the DTFM is naturally and readily applicable to modeling and analysis of flexible space structures with integrated feedback controllers and embedded smart material layers. Control-structure interactions can be easily studied with the DTFM. Unlike FEM models, the DTFM formulation leads to closed-loop governing equations of much smaller order. Transient (time-domain) response can be obtained in closed form by a newly developed inverse Laplace transform technique.

Following the above-described process, a solar sail system can be decomposed into its major structural components, i.e., booms, membranes, and spacecraft bus and appendages, for component-level modeling and analyses. The component models can also be later assembled into a sail system model for system-level analyses. Figure 8 shows this decomposition/assembly process.

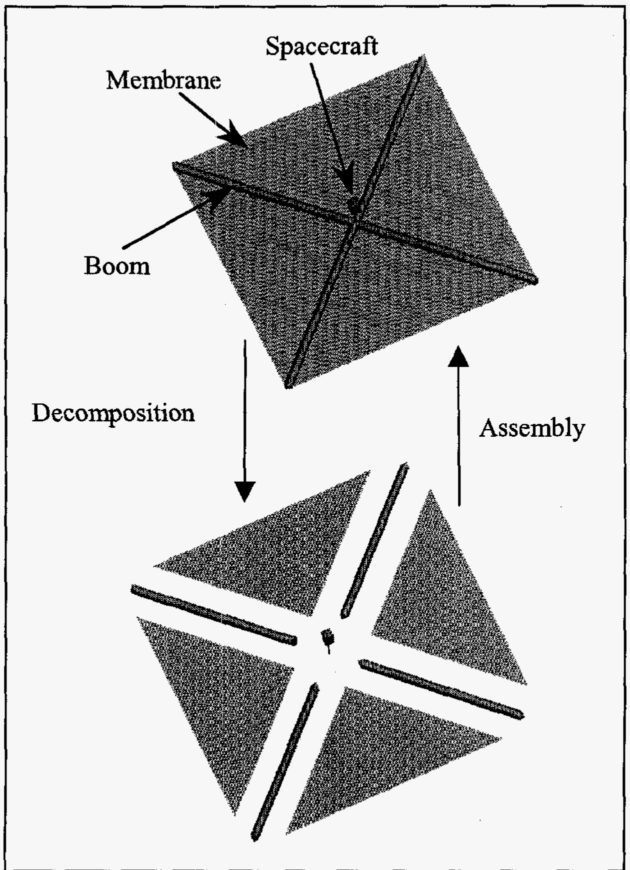


Figure 8 DTFM Synthesis of A Square Sail

On the other hand, the traditional finite element method discretizes individual components of the solar sail into small elements of near square and/or triangular shapes. This usually requires a very large number of elements for each component model. In contrast, the Distributed Transfer Function only requires decomposition of components. For example, the square sail shown in Figure 8 will be decomposed into four pieces of membrane components, four boom components, and a spacecraft as illustrated. That is, only nine "super" components are used by DTFM to model a solar sail to yield results that are equally or more accurate than that given by the FEM analysis. Therefore, DTFM synthesis is particularly suitable for assembling solar sail system models.

VII. Concluding Remarks

Effective treatment of some of the structural issues of large sails formed by long space-deployable booms and extremely large thin film membrane apertures. These include localized geometrical and material imperfections, formation and effects of wrinkles, deployment simulation, and nonlinear dynamics, are beyond the current state-of-the art of existing FEM-based computational capabilities. A focused research effort was initiated to overcome the identified shortcomings of existing FEM analysis codes in modeling and analyzing this particular type of space structures. This development effort is based on two proven solution methods, namely the DTFM and the PVP method. Successful development of these modeling and analysis tools will greatly enhance our capability to predict and analytically verify on-orbit performance of large square solar sails and other similar gossamer structural systems. The developed software will be made readily available to NASA engineers to help enable the implementation of several planned near-term solar sail missions. Additionally, many other NASA missions that have long deployable booms and tensioned thin-film membranes incorporated in their flight systems will also benefit from these new modeling, analysis, and synthesis capabilities.

We recognize that no space missions, including solar sail missions, will be flown based on analytical prediction of on-orbit performance alone. We envision that the modeling and analysis tools developed by our ongoing effort will be used to complement ground testing of sail components (i.e., booms and membrane aperture) and/or subscale models of a complete sail system. These tools can be used to guide test planning, to assist the interpretation and analysis of test data, and to help demonstrate scalability of the test models. They can also be used to evaluate and mitigate the effects on test results of gravity and air damping (if the test is not conducted in vacuum due to facility limitations). This combined test and analysis development approach is essential to advancing the flight readiness level of solar sail technology.

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