Validating Finite Element Models of Assembled Shell Structures

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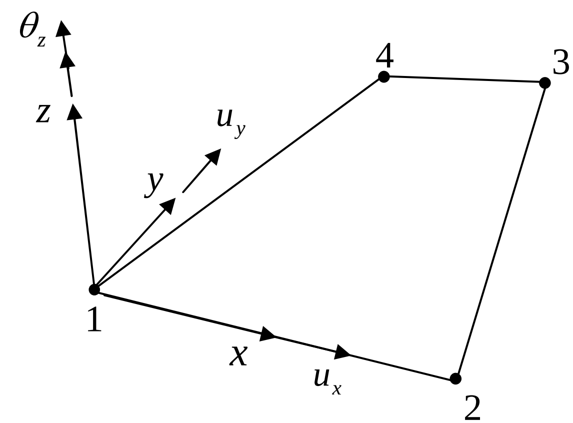
Outline

- **Problems with membrane rotations in assembled shell models**
- **Penalty stiffness for membrane rotations**
- **Physical stiffness for membrane rotations using shell elements with 6 dof per node**
- **Connections avoiding rotations**
- **Conclusions**

Problems with Membrane Rotations in Shells

- Mindlin shells have no stiffness in the membrane rotation θ_z normal to the shell plane
- The rank deficiency can be automatically removed, for example using AUTOSPC in MSC.Nastran
- The membrane rotation may be loaded either by accident or intentionally, for example, through stiffeners, spot welds, rigid elements, concentrated masses, etc.
- False load transfers or spurious modes may occur

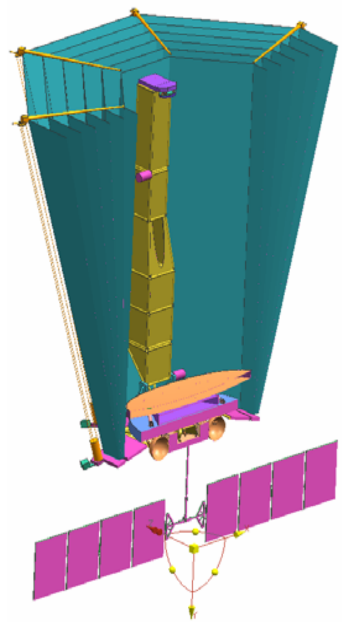


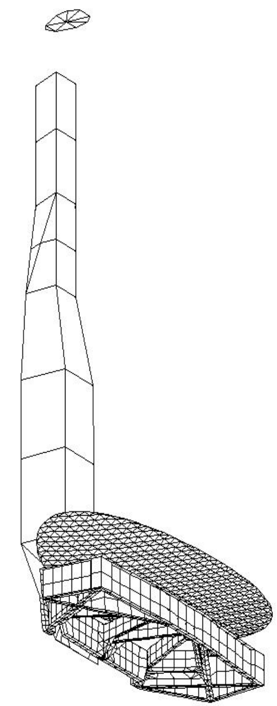
Typical Shell Model in Aerospace

Terrestrial Planet Finder Chronograph

- **Elements**
  QUAD4, TRIA3,
  few HEXAs and PENTAs

- **Connectors**
  CELAS, CBUSH,
  RBE2,
  RBE3

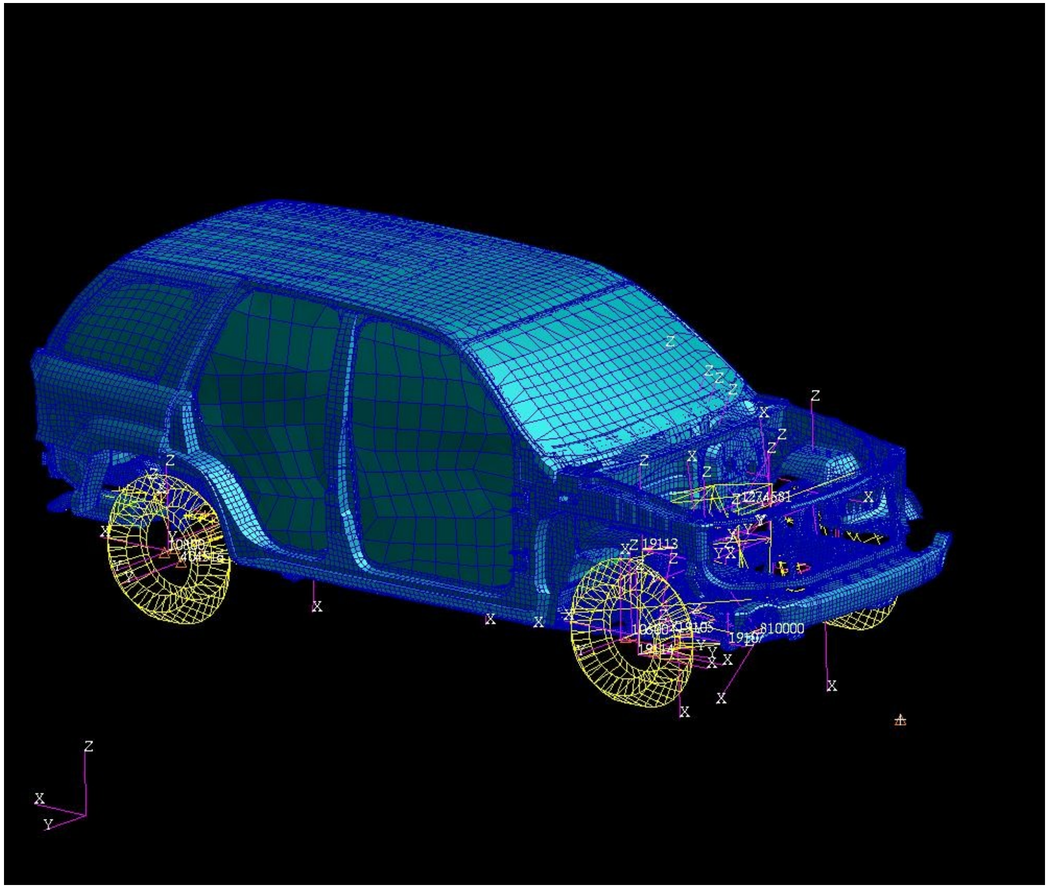




Typical Shell Model in Automotive

BIW with acoustic cavity (courtesy of GM) 1.5 Mill. dof

312,142	GRID
260,348	CQUAD4
17,332	CTRIA3
10,812	CHEXA8
664	CPENTA6
1,880	CELAS2
397	CELAS1
390	CBEAM
14,336	RBE2
17	RBE3
2,310	MPC



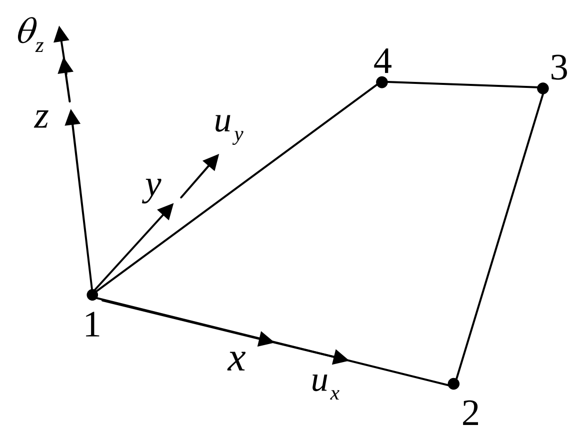
Penalty Stiffness for Membrane Rotations

A penalty term is added to the energy functional

$$\Pi_p = 10^{-6}\ \mathrm{K6ROT}\ \frac{1}{2} G \int_A (\Theta_z - \Omega_z)^2 t\, dA$$

with

$$\Omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$



K6ROT is a user parameter in MSC.Nastran,
default value is K6ROT= 100.

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Good Value of the Penalty Stiffness

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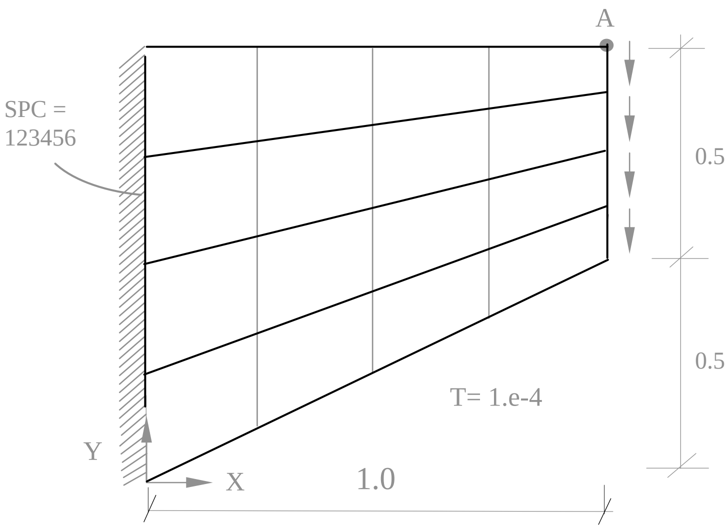
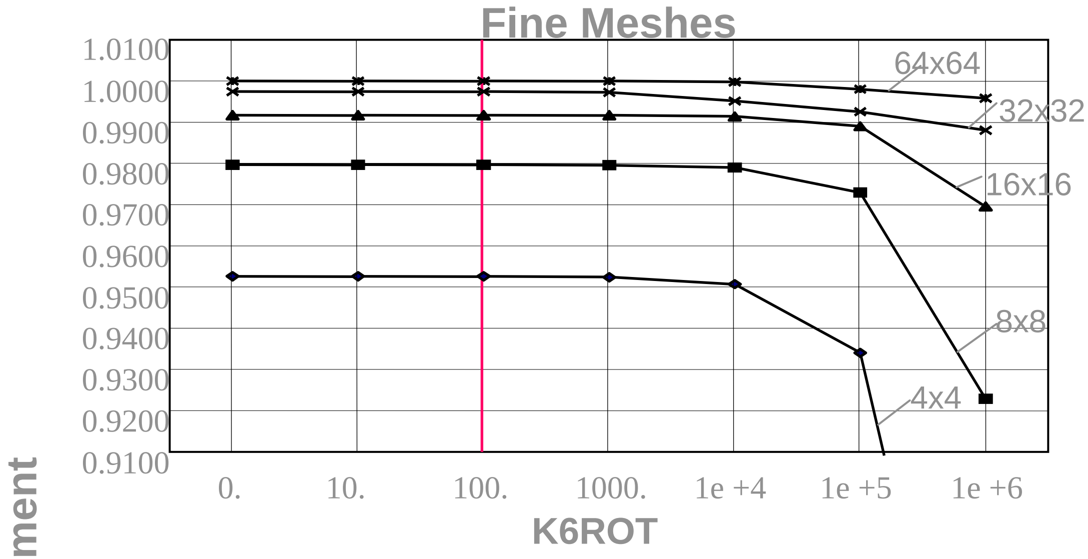
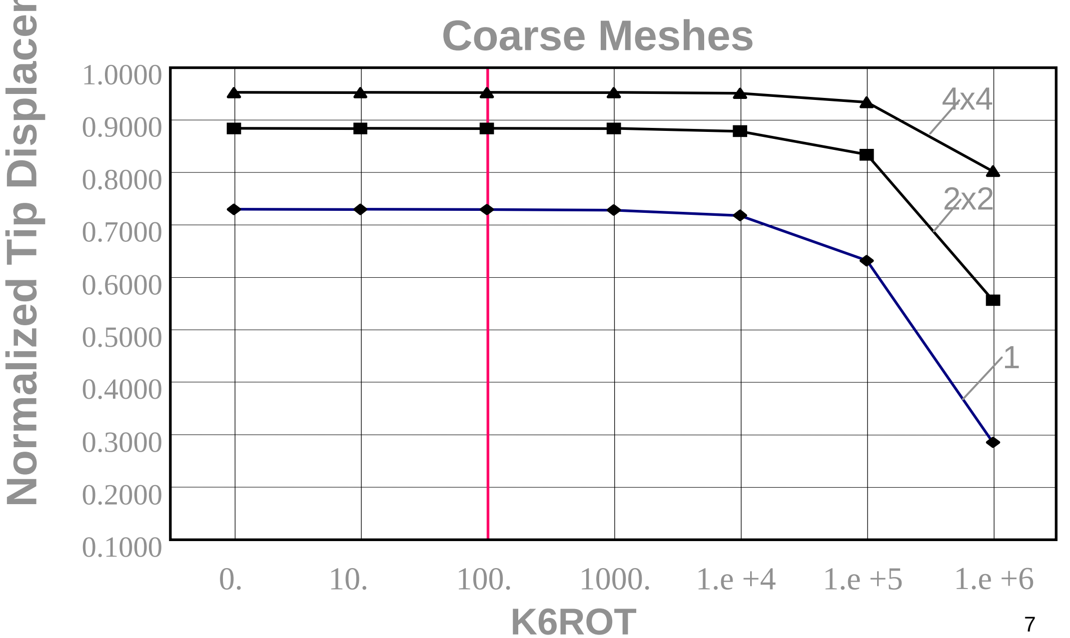
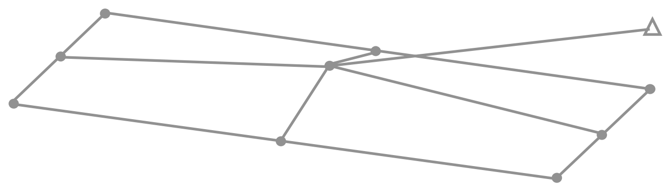






Flagpole Example

K6ROT	No. of Zero Modes	First Nonzero Mode	Comment
0.	7	2.4671e+6	spurious mode, bad first
100.	6	1.2745e+6	no spurious, good first

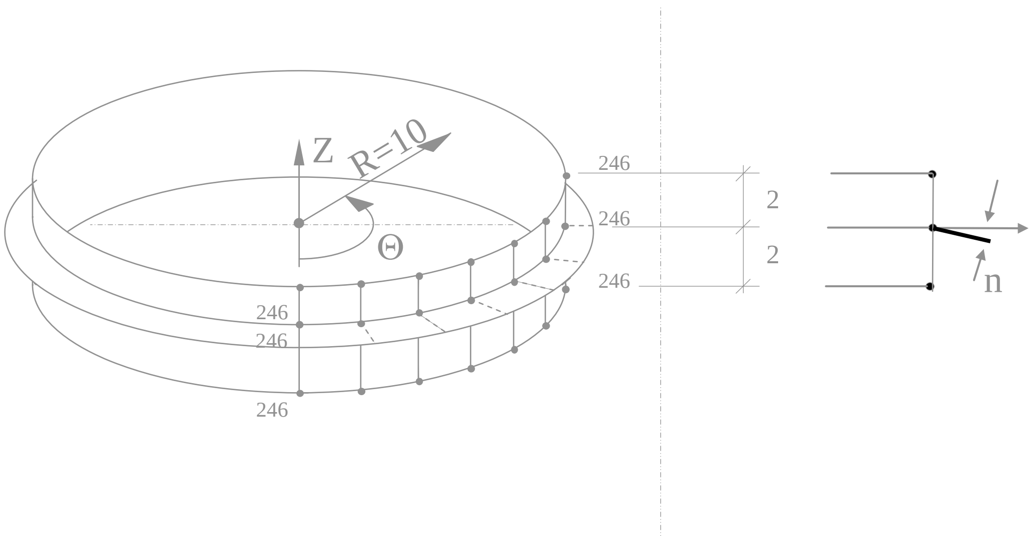


- **Intentional loading of the drilling degree-of-freedom in shells may be discredited as bad modeling practice.**

Effect of Penalty Stiffness – Ring with Stiffener

- **Unintentional loading of the drilling degree-of-freedom occurs in some cases, for example, in stiffened shells modeled with beams and offsets.**

Cylindrical Ring with Stiffener



Ring Modeled with CQUAD4
Stiffener modeled with CBARS and offset
No I2 specified on PBAR
Offset direction and normal direction differ slightly

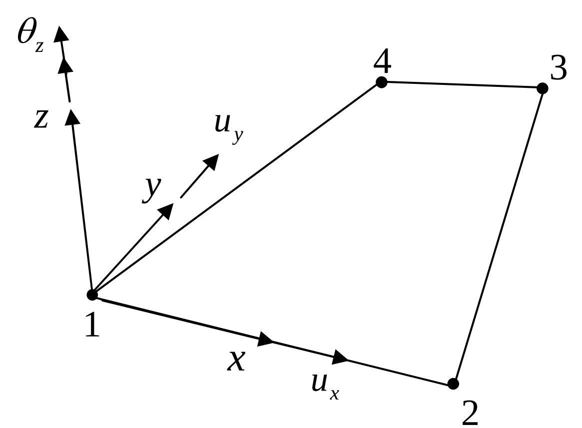
Normal Modes

K6ROT	Offset Disturbance	Comments
0.	$\varepsilon \le 1.8E-4$	1 rigid body mode No spurious modes AUTOSPC catches singularity 2nd mode 9.667
0.	$\varepsilon \ge 1.9E-4$	1 rigid body mode 5 spurious modes 7th mode 9.734
100.	$\varepsilon \ge 1.9E-4$	1 rigid body mode No spurious modes 2nd mode 9.655

Shells with 6 Degrees of Freedom per Node

The membrane displacements and the membrane rotations are coupled with quadratic shape functions, for example, the displacement at the midpoint of edge 1-2 is

$$\boldsymbol{u}_m = \frac{1}{2}(\boldsymbol{u}_1 + \boldsymbol{u}_2) + \frac{1}{8} l_1 (\Theta_{z2} - \Theta_{z1}) \boldsymbol{n}_1$$



The following term is added to the energy functional

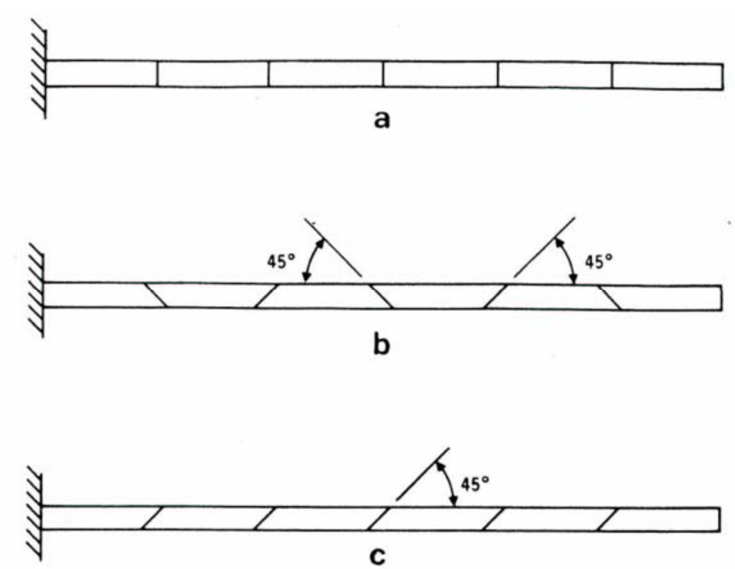
$$\Pi_\Theta = \frac{1}{2} G \int_A (\Theta_z - \Omega_z)^2 t \, dA$$

$$\Omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

Comparison of QUAD4 and QUADR

- **QUADR is superior to QUAD4 for in plane shear and in plane bending**

Straight Beam



Rect

Trap

Skew

Shear load at tip

Tip deflection (normalized with theoretical value)

Version	Rect	Trap	Skew
QUAD4	0.9929	0.0515	0.6323
QUADR	0.9926	0.9613	0.9491

Comparison of QUAD4 and QUADR (cont.)

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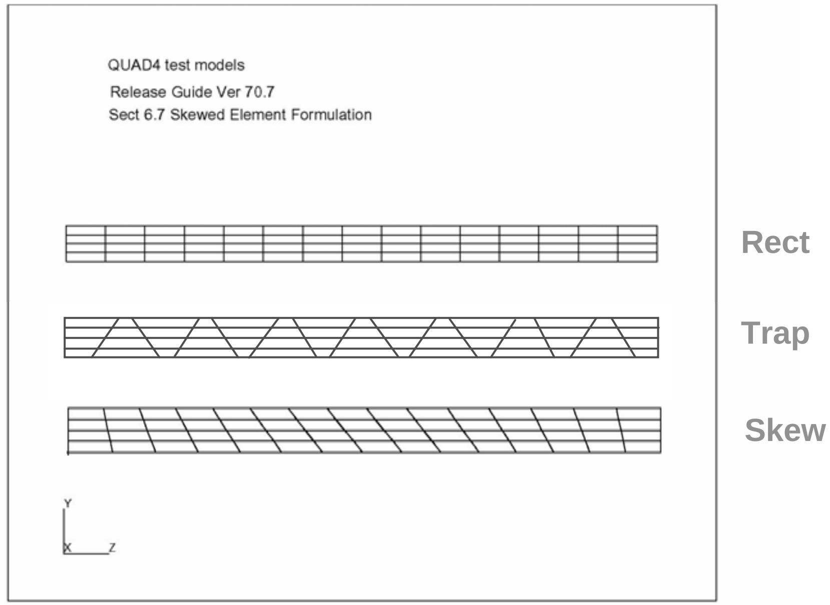
Normal Modes of a Straight Beam



In Plane Bending Frequency

Element	Rect	Trap	Skew
QUAD4	9.39	15.89	9.41
QUADR	9.39	9.42	9.39

Modeling of Connectors

Bolts or spot welds for nearly congruent meshes

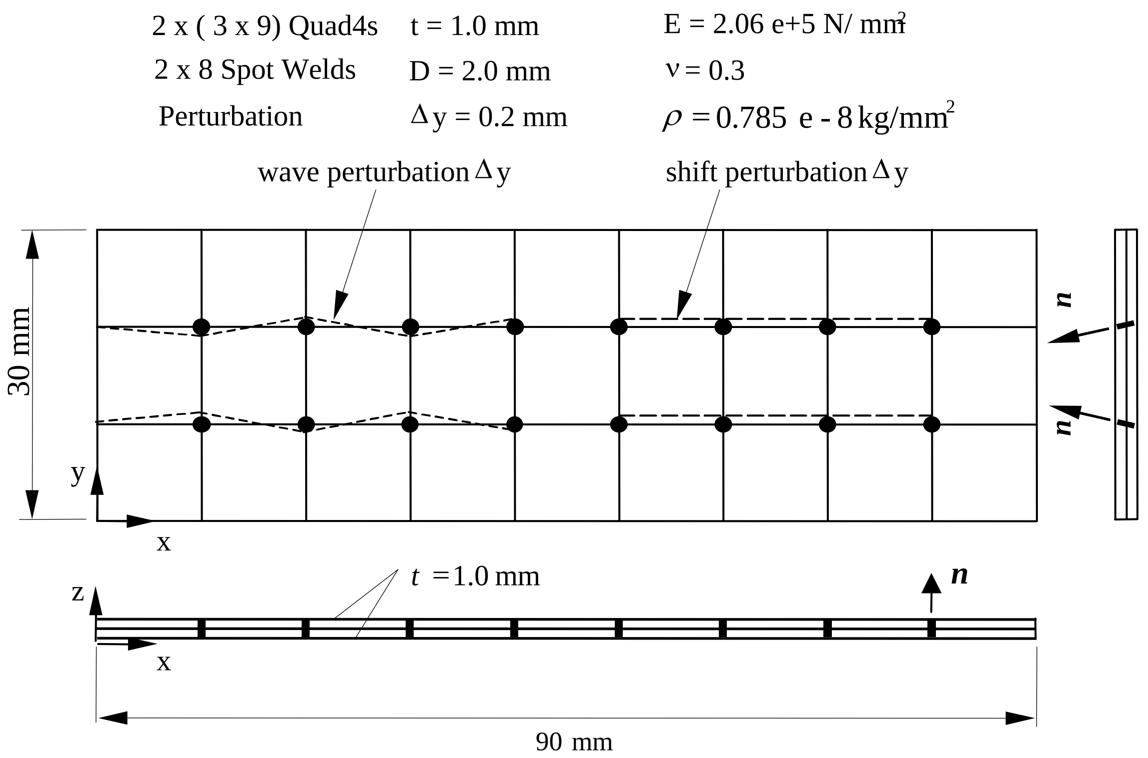


Figure 6. Two Plates Connected with 16 Spot Welds

Example demonstrates how global results change when shell rotations are coupled to connectors

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Modeling of Connectors (cont.)

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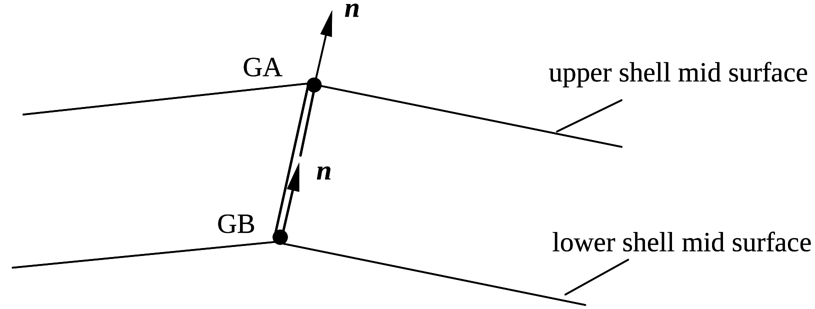
- **Comparing two modeling options**
  - RBAR with no additional constraints
  - Connector with constraints
    - Membrane rotations of the shell are not coupled to the drilling rotations of the bar
  - The results from the perfectly aligned grids are the baseline
  - RBAR modeling causes errors up to 12% in the first eigenfrequencies



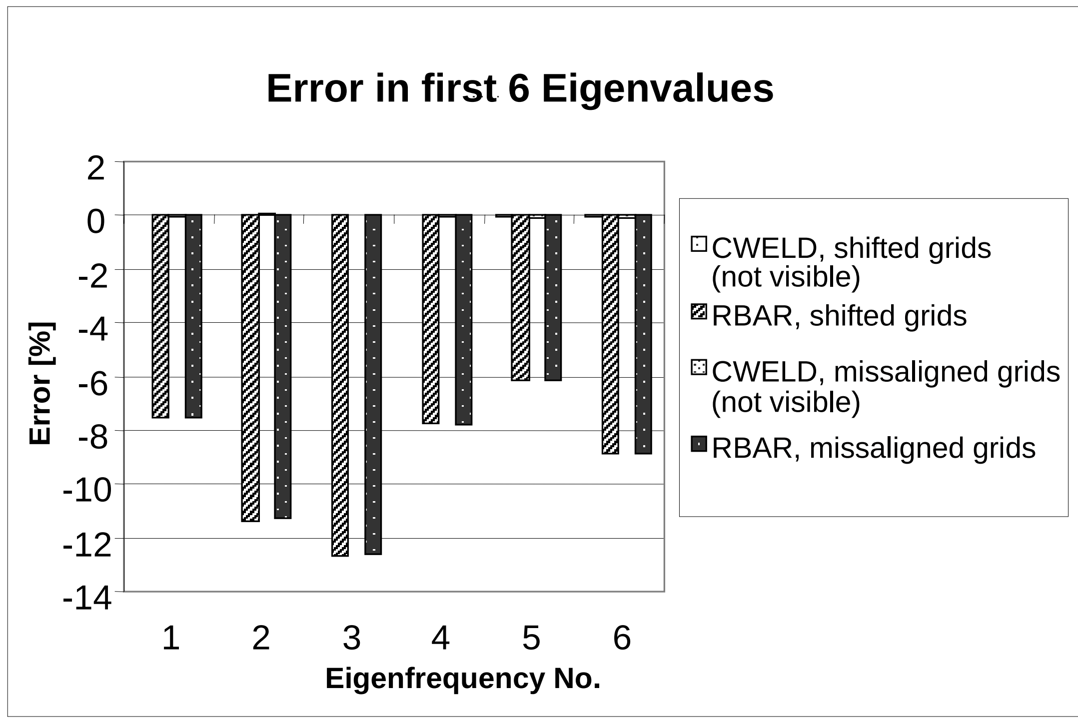


Figure 7. Errors in the first 6 Eigenfrequencies

Connections Avoiding Rotations

Interpolation constraints for translations

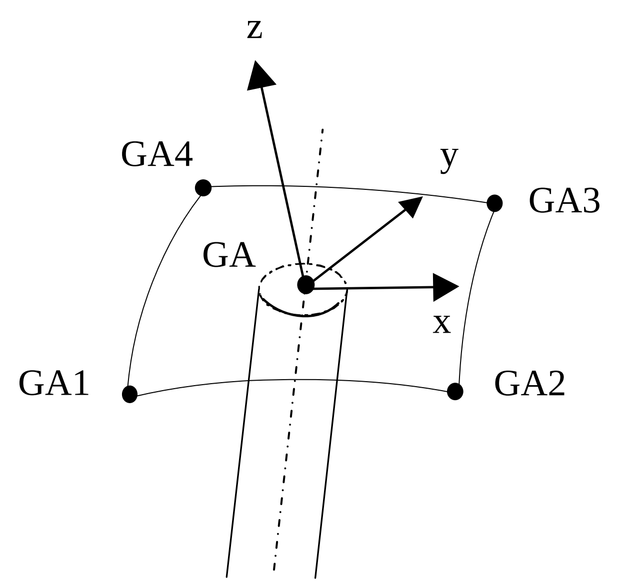
$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_A = \sum N_I(\xi_A, \eta_A) \cdot \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_I$$

Kirchhoff constraints for rotations

$$\theta_x^A = \frac{\partial w}{\partial y} = \sum N_{I,y} \cdot w_I$$

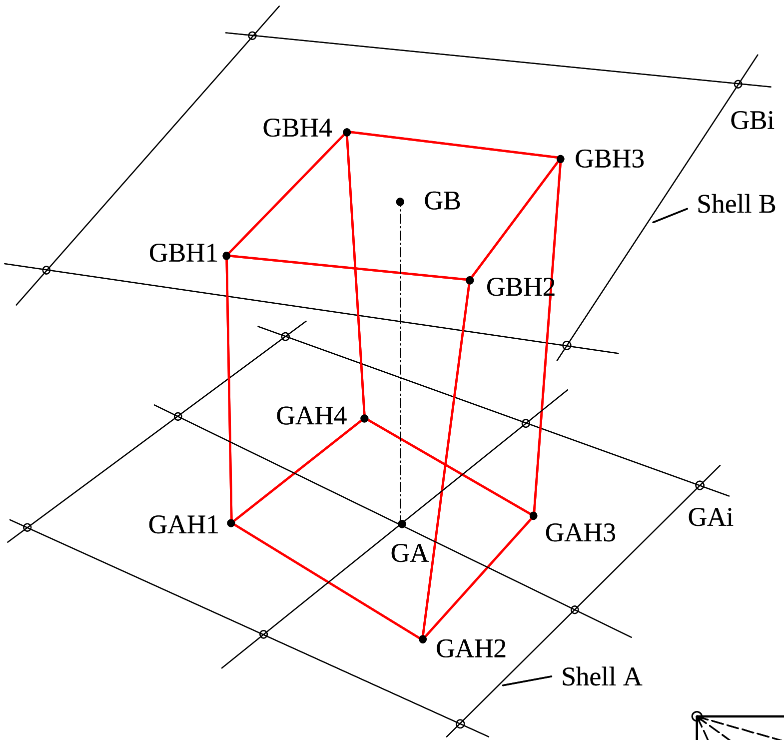
$$\theta_y^A = -\frac{\partial w}{\partial x} = -\sum N_{I,x} \cdot w_I$$

$$\theta_z^A = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(\sum N_{I,x} \cdot v_I - \sum N_{I,y} u_I\right)$$

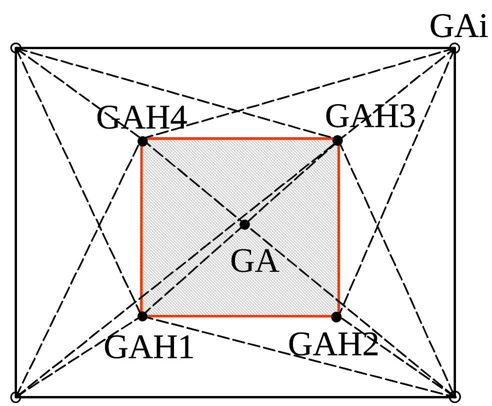


Beam connected to shell

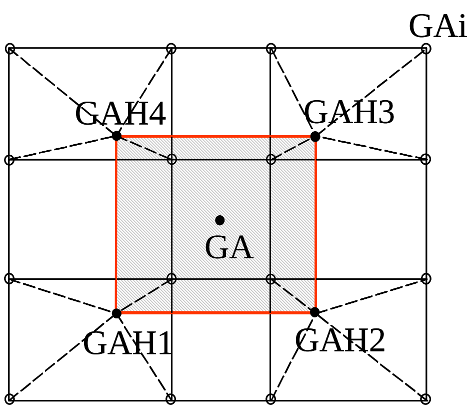
Connections Avoiding Rotations (cont.)



- **Spot weld is modeled with a HEXA element**
- **The HEXA is connected to the shell elements through interpolation of translational dof only**

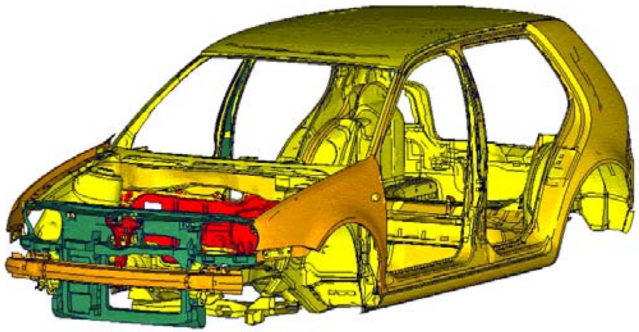


Coarse Mesh



Fine Mesh

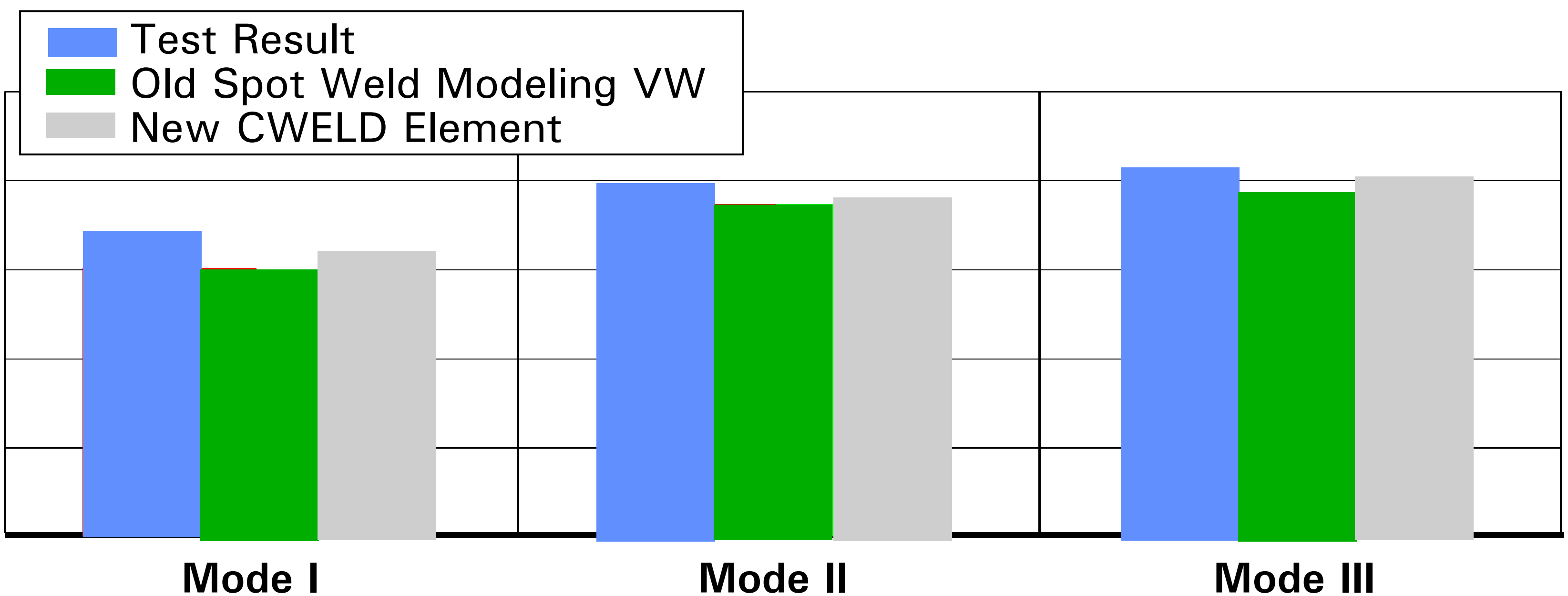
Modeling Connectors (Results)



Body in White Model

- 3,712 CWELD Elements
- 3,562 connecting two parts
- 150 connecting three parts

Eigenfrequency Analysis:



Conclusions

- **Shell elements with membrane rotations (6 dof per node) are a good choice in general**
  - Increase accuracy
  - Provide realistic stiffness in membrane rotations

- **For connectors (bolts, rivets, and spot welds), global results are more accurate when coupling to shell rotations is avoided**
  - Generate constraints which involve only translational dof in shells