# An analysis of the phase dispersion in the symmetric beam combiner

Hong Tang*, Feng Zhao
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena,
CA, USA 91109

## ABSTRACT

An astronomical beam combiner combines the two beams of starlight to form white-light fringes. It is desirable that the dispersion of the beam combiner be minimized across the observation wavelength range. We present here an analysis of the phase dispersion from coatings for a symmetric beam combiner. The sensitivity of the dispersion to a slight mismatch in beamsplitter coatings is also studied.

**Keyword:** Beam combiner, beamsplitter, coatings, dispersion, retardance.

## 1. INTRODUCTION

The simplest beam combiner consists of a single beamsplitter as shown in Figure 1. Compensators in the paths of both beams are omitted. At the beamsplitter a light beam is divided into two, the reflected and the transmitted beams. At port A the reflected portion of the beam 1 interferes with the transmitted portion of beam 2. Assume that beam 1 and beam 2 have the same electric field, $E_0$, before they enter the beamsplitter coating. The electric field of beam 1 at port A is $rE_0$, where r is the amplitude reflection coefficient of the beamsplitter coating. The electric field of beam 2 at port A is $tE_0$, where t is the amplitude transmission coefficient of the beamsplitter coating. The phase difference of the two interfering beams is then r-t. Similarly, the phase difference of the two interfering beams at port B is t-r. The dispersion of the phase difference of the two beams at port A is the variation of quantity r-t with wavelength (or wavenumber). Typically, a beamsplitter coating of 50/50 split ratio has the dispersion of hundreds nanometers peak-to-valley in the wavelength range of 400-1000 nm. This is not acceptable if phase knowledge of a few picometer is required.

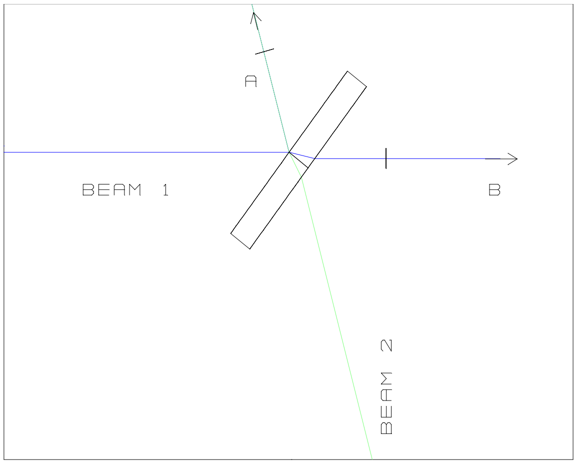

Figure 1. The simple beam combiner consists of one beamsplitter. Each beam of light is split into two at the BS. The split portion from beam 1 and beam 2 interfere at ports A and B.

To significantly reduce the phase dispersion, a symmetric beam combiner is proposed. In our combiner system, there are two additional beamsplitters in beams 1 and 2 that are used to split a portion of both beams to the angle tracker. We can use these two beamsplitters and the main beamsplitter to form a symmetric combiner system as shown in Figure 2. In the symmetric combiner, beams 1 and 2 that reach port A encounter the same number of reflection by a beamsplitter and

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*hong.tang@jpl.nasa.gov; phone 1 818 393-0786; fax 1 818 393-1370 same number of transmission through a beamsplitter and a compensator. The key assumptions are that all the beamsplitter and anti-reflection (AR) coatings are identical, all bulk substrates are of the same thickness and the angle of incidence (AOI) is the same for every optic. As it becomes evident in next section, the phase difference of beam 1 and beam 2 at port A is zero because of the symmetry.

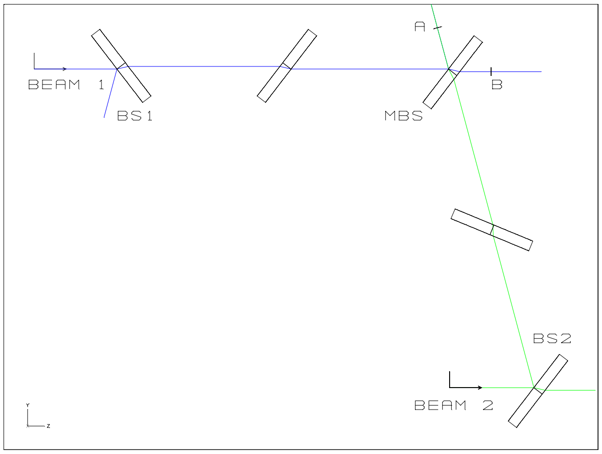

Figure 2. The layout of a symmetric beam combiner. It consists of three beamsplitters and two compensators. The two beams at port A have zero phase difference.

## 2. ANALYSIS

In this section, we present the analysis of the symmetric beam combiner with all identical beamsplitter and anti-reflection coatings. We then analyze the effects of thickness mismatch between the three beamsplitters.

### 2.1 The analysis of a perfect system

Let us first consider beam 1. It is split at beamsplitter 1 (BS1). The reflected portion is directed to the angle tracker. The transmitted portion of beam 1 acquires a phase from t, which is the amplitude transmission coefficient of the BS1 coating. It also transmits through three AR coatings (one of the back of BS1 and two of the compensator) and two piece of bulk material (BS1 and the compensator). To reach port A, beam 1 is reflected by the MBS coating. The total phase accumulation of beam 1 is $tt_{AR}^3G^2r$, where $t_{AR}$ is the amplitude transmission coefficient of the AR coating and G represents the phase of transmitting through glass substrate. Beam 2 is first split be BS2 where the transmitted portion is directed toward the angle tracker. The reflected portion is then transmitted through a compensator and the MBS. By the same token, the total phase accumulation of beam 2 when it reaches port A is $rt_{AR}^3G^2t$. It is obvious that the phase of beam 1 at port A is the same as that of beam 2 at port A. The phase difference of the two beams at port A is zero for all wavelengths. That is,

$$\Delta\phi_A(\lambda) = \phi_{1A} - \phi_{2A} = tr - rt = 0,$$

where λ represents wavelength. We refer port A as the “balanced” port.

The phase difference of the two beams emerging from port B is a little complicated. To reach port B, beam 1 propagates through BS1, a compensator and the MBS. The accumulative phase is $tt_{AR}^4G^3t$. It transmits through two BS coatings, four AR coatings and three pieces of bulk glass. Beam 2 first reflects off BS2 and transmits through a compensator. It then enters the substrate and reflects internally (within the substrate) off the MBS. The accumulative phase is $rt_{AR}^4G^3r'$, where r' is the internal amplitude reflection coefficient of the MBS coating and $G^2$ accounts for traversing the substrate of MBS twice. The phase difference of beam 1 and beam 2 at port B is,

$$\Delta\phi_B(\lambda) = tt - rr'.$$

It seems that we may not know the function of wavelength dependence of the above phase difference. However, by the argument of conservation of energy, if the beamsplitter coating is lossless, the phase difference of port B should equal to that of port A plus an odd number of $\pi$. That is when port A is of a bright fringe, port B is of a dark fringe. We have,

$$\Delta\phi_B(\lambda) = \Delta\phi_A(\lambda) + \pi.$$

It means that,

$$\Delta\phi_B = tt - rr' = \pi.$$

The phase difference of beam 1 and beam 2 at port B is also “balanced”. It is constant for all wavelengths.

## 2.2 Fringe visibility and photon efficiency

Once the fringes are formed at ports A and B, it is important to assess the fringe visibility (V). The accuracy of estimating the phase in the star light fringes (locating the zero OPD point) depends on the photon efficiency, which equals to $V^2$.

For the combiner system shown in Figure 1, there is only one beamsplitter. The electric field of beam 1 at port A is $rE_0$, where r is the amplitude reflection coefficient of the beamsplitter coating. The electric field of beam 2 at port A is $tE_0$, where t is the amplitude transmission coefficient of the beamsplitter coating. The irradiance distribution of the fringes at port A is

$$I = \left|rE_0 + tE_0\right|^2 = RE_0^2 + TE_0^2 + 2\sqrt{RT}\,E_0^2\cos(\omega_{ODL}t + \Delta\phi),$$

where $R = |r|^2$ is the reflectance of the BS coating, $T = |t|^2$ is the transmittance of the BS coating, $\omega_{ODL}$ is the modulation of the optical delay line and $\Delta\phi = \phi_r - \phi_t$ is the static phase difference between beam 1 and beam 2. The fringe visibility is

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = 2\sqrt{RT}.$$

We have used the relationship $R + T = 1$ for a lossless coating to derive the final form of the above equation. The photon efficiency is then

$$V^2 = 4RT.$$

It should be noted that the photon efficiency at port B is the same as that at port A since one beam is reflected by the BS and the other transmits through the BS, just like the two beams at port A. It can be seen that the field amplitude reflection coefficient r and amplitude transmission coefficient t are two key quantities. Their phases ($\phi_r$ and $\phi_t$) determine the static phase difference of the two interfering beams and hence the dispersion of this phase difference in the spectral range of our interest. The quantities squared, $R = |r|^2$ and $T = |t|^2$, are the reflectance and transmittance that determine the fringe visibility.

Let us now turn our attention to the symmetric combiner system shown in Figure 2. There are three beamsplitters in this system. Each beam encounters two beamsplitters before they are combined at ports A and B. The two beams at port A reflect off a BS and transmit through a BS. They have the same field amplitude and static phase, $rtE_0$. The visibility of fringes at port A is one. That is,

$$V_A = 1.$$

For the two beams at port B, if we omit the reflection losses at the AR coatings, beam 1 has a field of $ttE_0$ and beam 2 has a field of $rr'E_0$. The quantity r' represents the internal reflection at the MBS from the glass substrate side. The irradiance distribution at port B is

$$I = \left|ttE_0 + rr'E_0\right|^2 = T^2E_0^2 + R^2E_0^2 + 2RTE_0^2\cos(\omega_{ODL}t + \Delta\phi_B),$$

where $R = |r|\cdot|r'|$ and $T = |t|^2$. The fringe visibility at port B is

$$V_B = \frac{2RT}{R^2 + T^2}.$$

The photon efficiency at port B is then

$$V_B^2 = \frac{4R^2T^2}{(R^2 + T^2)^2}.$$

The average photon efficiency of the symmetric combiner is simply the average of ports A and B, $V_{ave}^2 = (1 + V_B^2)/2$. Figure 3 shows the photon efficiency for the combiner with one beamsplitter and the symmetric combiner with three beamsplitters. One can see that the photon efficiency of the symmetric combiner falls off faster than that of the single BS combiner as the split ratio deviates from 50/50. This is because each beam in the symmetric combiner encounters two beamplitters.

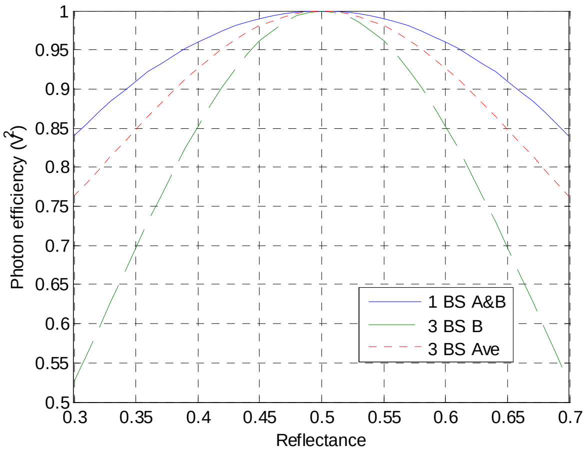

Figure 3. The photon efficiency at port A for the symmetric combiner (3 beamsplitters) equals to one ($V_A^2 = 1$) for all values of reflectance. The photon efficiency for the symmetric beam combiner at port B is plotted in the dashed line. The dotted line is the average of ports A and B for the symmetric combiner. The photon efficiency curves for the combiner with one beamsplitter are plotted in the solid line. In this case the photon efficiency of port A equals to that of port B.

## 2.3 Perturbation of the thickness in coatings

In last section we have seen that in a symmetric beam combiner, if all three beamsplitter coatings are identical, the phase difference of the two beams is independent of the wavelength. There is not dispersion. Since we live in a non-perfect world, there is always some difference in the layer thickness between the three beamsplitters. Sophisticated planetary systems in a coating chamber and well calibrated process can make the non-uniformity from optic-to-optic within 0.1%

of the layer thickness[1]. In order to analyze the effects of coating and substrate thickness mismatch and the angle of incidence mismatch, optical models must be built.

We have built two kinds of models using two different approaches. The first kind is a full physical model of the combiner system. Every optic is fully specified in this model, including distance from the previous element, material and thickness of the substrate and coatings. Polarization raytrace is carried out at a number of wavelengths. The phases of beam 1 and beam 2 in s and p polarizations are calculated at each wavelength. This approach accounts for all the errors in the system, the thickness and angle of incidence mismatch, the coating thickness mismatch and other errors such as thermal effects.

The second model is based on coating analysis. We know that a small amount of thickness mismatch in the bulk material in the two beams can be corrected by tilting one or both compensator plates. The optical paths of the two beams are equalized by introducing an optical delay in one beam. The dominant phase errors after adjusting the optical delay and compensator plate are due to coating thickness mismatch and AOI mismatch. The phase of each beam at a port after the MBS is the sum of phase acquired at each interface of coatings. Using this model, we can analyze quickly the effects of coatings at a number of wavelengths. It is obvious that for a perfect system the phase difference of beam 1 and beam 2 at port A is zero. We deduced that the phase difference at port B is $\pi$. However, it is not obvious that from the formulae of $\Delta\phi_B$. Let use carry out the analysis and examine the result.

Coating data can easily be generated by coating design software and optical design software if one enters the layer and material information of the coating. The data typically contain the reflectance, transmittance, absorption, phase in reflection and phase in transmission for both s and p polarizations at a number of wavelengths. At port B, beam 1 encountered two transmission and beam 2 encountered two reflections, one in air and one in glass. We add up the phases of each beam and then take the difference. The result is plotted in Figure 4. As one can see that the phase difference is half of the corresponding wavelength, that is, $\pi$.

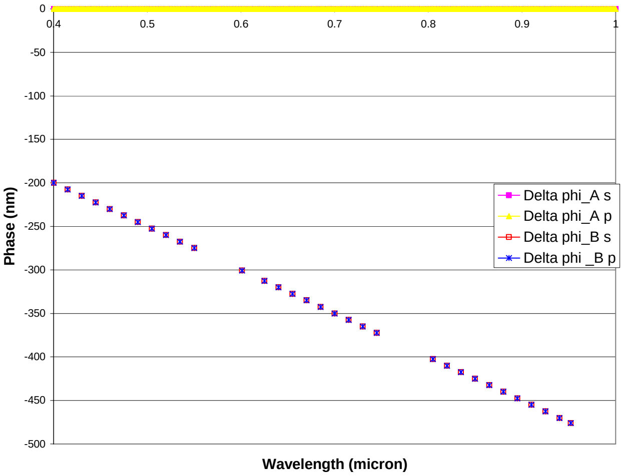

Figure 4. Phase difference at port A and port B calculated with the summation of phase at each encountered coating. For a perfect symmetric combiner, the phase difference of port A is zero and that of port B is $\pi$, or half wave.

We use the second approach to do the analysis that is described next. We use the phases of both beams at port A to calculate the variations when we perturb the coating thickness. It is much simpler to use port A because there is no internal reflection for either beam. To calculate the phase change upon the internal reflection, correct angle of incidence at each wavelength must be used to yield the right answer. This process is tedious. However, once we obtain the phase variations at port A, the phase at port B is simply is that of port A plus $\pi$. It is noted that to reach port A beam 2 transmit through MBS coating from the glass side. Since light propagation is reversible, the phase change of light transmitting through an interface must be the same regardless the direction of propagation. We can just use the phase of transmission from air to glass for the internal transmission from glass to air.

The beamsplitter coating MBS is perturbed such that the thicknesses of all layers are scaled by 1.001 to simulate the coating mismatch of 0.1%. The coatings at BS1 and BS2 stay unchanged. The phase of beam 1 at port A is $\phi_1 = \phi_t + \phi_{r1001}$ and the phase of beam 2 at port A is $\phi_2 = \phi_r + \phi_{t1001}$, where $\phi_{r1001}$ and $\phi_{t1001}$ are phases acquired by the light beams when they reflect off and transmit through the 1.001x scaled MBS coating. First we use an R = 50% BS coating with more than 10 layers. The phase difference is then calculated and plotted in Figure 5. It can be seen that the dispersion of the phase difference at port A is increase from zero for a perfect system to 3 nm peak-to-valley. This is a rather significant amount or increase for a beam combiner that is design to have performance of hundreds picometers when all error sources are considered. The differential retardance is defined as the difference of the phases between s light and p light at port A. It is plotted in Figure 6. One can see that the peak-to-valley variation is1.5 nm.

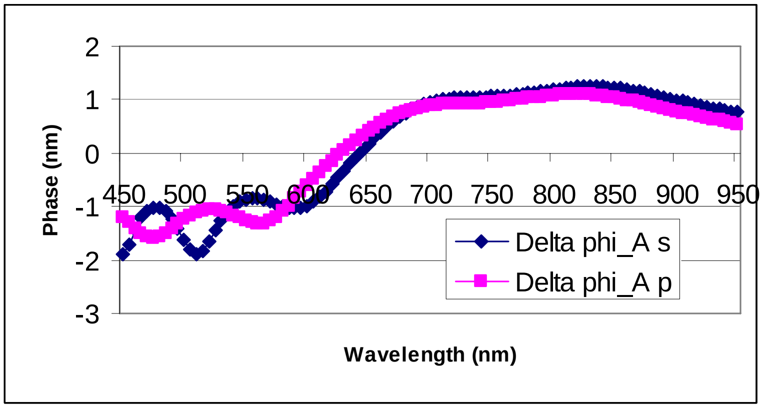

Figure 5. The dispersion of the phase difference at port A for the symmetric combiner when the MBS coating is scaled 1.001x.

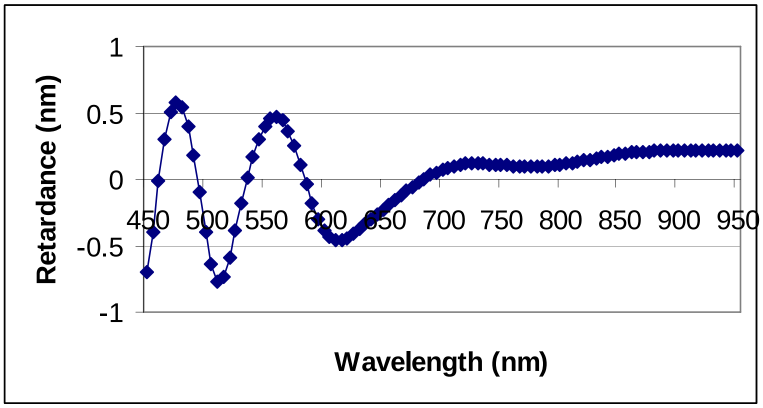

Figure 6. Differential retardance of the two beams at port A for the symmetric combiner when the MBS is scaled 1.001x.

One solution to the problem is to use simpler design for the beamsplitter coating. Simpler design means fewer layers. It is less sensitive to the perturbation of layer thicknesses. The drawback of a simpler BS coating is that it does not have the desired 50/50 split ratio. If we keep it close to R/T = 40/60 the impact to the photon efficiency is acceptable. A five-layer beamsplitter coating is designed and significant reduction in the dispersion and retardance is achieved.  The performance of the five-layer BS coating when perturbed with 1.001x scaling for the MBS is presented in Figures 7 and 8. It can be seen that the five-layer coating gives a reduction factor of ~10 in both dispersion and retardance of the phase difference at port A.

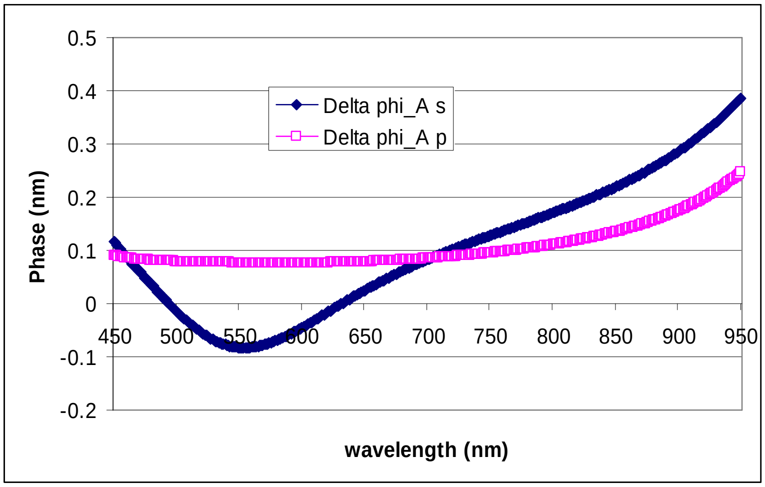

Figure 7. The dispersion of the phase difference at port A for the symmetric combiner with 5-layer BS coatings when the MBS coating is scaled 1.001x.

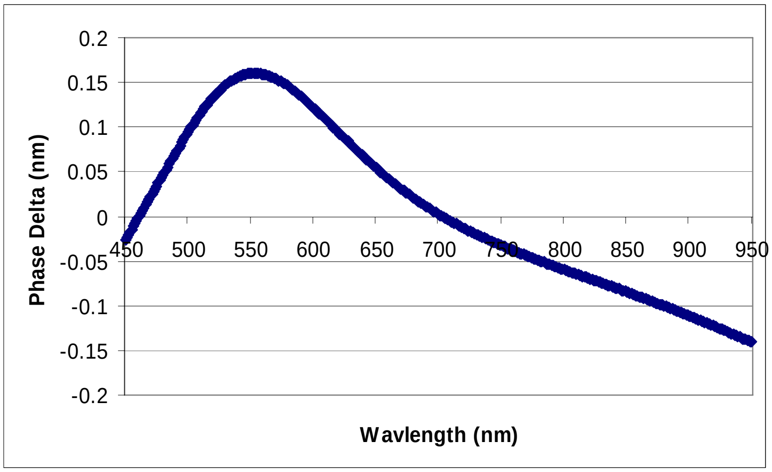

Figure 8. The retardance of the phase difference at port A for the symmetric combiner with 5-layer BS coatings when the MBS coating is scaled 1.001x.

## 3 Conclusions

A method of analyzing the performance of beam combiner based on coating performance data is presented. A symmetric beam combiner with three beamsplitters has zero dispersion and retardance in the phase difference of the interfering beams at exit ports when the beamsplitters coatings are identical. To simulate the reality, we perturb the MBS coating with 1.001x scaling factor for all layers. It reveals that the dispersion and retardance of the phase difference at port A are both increased drastically to ~ 1 nm level. Using a simpler five-layer beamsplitter coating design reduces the dispersion and retardance of the differential phase of the two beams at port A by a factor of ten to ~ 100 pm.

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