Accumulate Repeat Accumulate Codes

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Abstract — **An innovative channel coding scheme called "Accumulate Repeat Accumulate codes" (ARA) is proposed. ARA codes can be viewed as a subclass of Low Density Parity Check (LDPC) codes with fast encoder, and they have a projected graph or protograph representation. Using density evolution on their associated protographs, we find examples of rate 1/2 ARA codes with maximum variable node degree 5 for which a minimum bit SNR as low as 0.08 dB from channel capacity can be achieved as the block size goes to infinity. A family of high rate ARA codes with thresholds that stay uniformly close to their respective channel capacity thresholds are constructed. The ensemble weight distribution and ML threshold for rate 1/2 ARA codes were computed. For ARA with repeat 4, the ML threshold approaches within 0.005 dB of the ML threshold of random codes based on the existing tightest closed form bound.**

I. Precoding Gain

An ARA code can be viewed as precoded Repeat Accumulate (RA) code with puncturing or as precoded Irregular Repeat Accumulate (IRA) code, where simply a rate-1 accumulator (or punctured accumulator) is chosen as the precoder. Consider a rate 1/2, systematic serial concatenation of a repeat 3 outer code, and an accumulator with periodic puncturing pattern 00X, where "0" indicate puncturing positions, as an inner code. The iterative decoding threshold for this punctured RA code is 1.116 dB. Now precode this code with an accumulator as shown in Fig. 1. The precoded version has threshold of 0.516 dB. Thus there is a precoding gain of 0.6 dB. The smallest base-graph (projected graph [4] or protograph [1]) for this ARA code is shown in Fig. 1 for n=0 (also in Fig. 2). A larger ARA graph is obtained by "copy-and-permute" operation.

II. ARA code Family

We also propose a constructive method to design higher code rates from the rate 1/2 ARA code and its protograph. This family is obtained by just further puncturing the rate 1/2 ARA code. For the protograph, $2n$ additional variable nodes each with degree 3 are added. This looks like adding $2n$ repeat 3 RA codes to a single rate 1/2 ARA code. In this case the addition is done at the inner check nodes of rate 1/2 ARA, and one common inner accumulator is used. Such construction for high code rates can be done using any other rate 1/2 ARA protograph. One example of such construction to generate an ARA code family for various code rates is shown in Fig. 1. Other examples of rate 1/2 ARA codes and their simulation results are provided in Fig. 2. The ARA code with threshold 0.264 dB in Fig. 2 is better than the unstructured irregular LDPC [3] with maximum node degree of 5 by 0.45 dB.

[1]This research was carried out at UCLA, and JPL, Caltech, under a contract with NASA.

References

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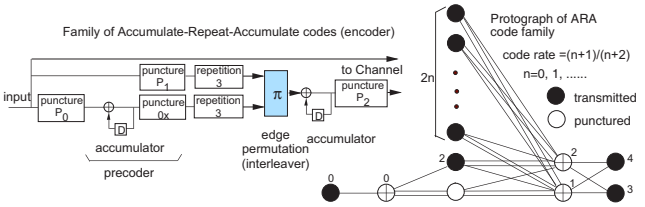
[3] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity-approaching irregular low- density parity-check codes," IEEE Trans. Inform. Theory, vol. 47, pp. 619-637, 2001.

[4] T. Richardson, and V. Novichkov,"Methods and apparatus for decoding LDPC codes," U.S. Patent 6,633,856, Oct. 14, 2003.



Code Rate	Protograph Threshold	Capacity	Difference	Puncturing Patterns		
				P_0	P_1	P_2
1/2	0.516	0.187	0.329	XX	X0	00X
2/3	1.288	1.059	0.229	00XX	XXX0	00000X
3/4	1.848	1.626	0.222	0000XX	XXXXX0	00000000X
4/5	2.277	2.040	0.237	000000XX	XXXXXXX0	000000000X
5/6	2.620	2.362	0.258	00000000XX	XXXXXXXXX0	0000000000000X
6/7	2.897	2.625	0.272	0000000000XX	XXXXXXXXXXX0	000000000000000X
7/8	3.129	2.845	0.284	000000000000XX	XXXXXXXXXXXXX0	00000000000000000X

Figure 1: ARA code family: encoder, protograph, and thresholds (dB) for rates 1/2 to 7/8

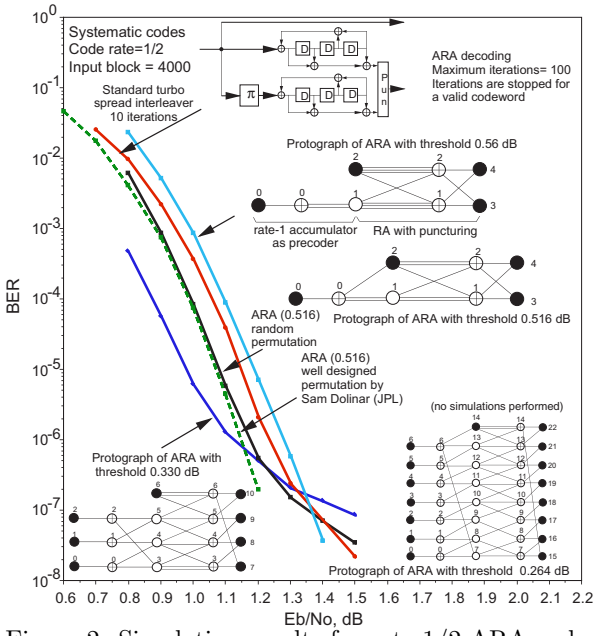


Figure 2: Simulation results for rate 1/2 ARA codes