

Phase control of electromagnetically induced transparency and its applications to tunable group velocity and atom localization

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ABSTRACT

We show that, by simple modifications of the usual three-level Λ -type scheme used in electromagnetically induced transparency (EIT) schemes, phase dependence in the response of the atomic medium to a weak probe field can be introduced. This gives rise to phase dependent susceptibility. By properly controlling phase and amplitudes of the drive fields we obtain variety of interesting effects. On one hand we obtain phase control of the group velocity of a probe field passing through medium to the extent that continuous tuning of the group velocity from subluminal to superluminal and back is possible. While on the other hand, by choosing one of the drive fields to be a standing wave field inside a cavity, we obtain sub-wavelength localization of moving atoms passing through the cavity field.

Keywords: Phase control, subluminal and superluminal group velocity, atom localization

1. INTRODUCTION

The study of quantum coherence and quantum interference in a atomic system has led to a variety of novel phenomena such as lasing without inversion,¹ absorption cancellation,² refractive index enhancement,³ electromagnetically induced transparency,⁴ ultra-slow,⁵ superluminal⁶ and even stored light,⁷ and unprecedented control of spontaneous emission.⁸⁻¹⁰ The quantum coherence effects observed in light-matter interactions have also been applied to an entirely different area of atomic manipulations, namely atom optics. There exist schemes for localization of moving atoms as they pass through the standing wave field of a cavity by monitoring spontaneous emission spectrum¹¹⁻¹⁶ and by monitoring absorption of a weak probe field.¹⁷

Noting that the optical fields, in general, have a carrying phase, one can expect that there will be phase dependence involved in atomic coherence effects where more than one fields are being used due to a relative phase difference between them. Utilization of the phase dependence of atomic properties such as coherence have been proposed in the manipulation of the spontaneous emission spectra^{15,18,19} and manipulation of the index of refraction and the group velocity of light.^{20,21} Phase dependent effects are fascinating due to the inherent tunability associated with them and they offer an easy to control parameter in an experiment. This tunability associated with manipulation of relative phase allows a whole range of possibilities in the spontaneous emission spectrum and group velocity manipulations. We have recently proposed a model to use phase control to obtain subluminal to superluminal light propagation in a single system,²¹ and sub-half-wavelength localization of atom passing through a standing wave field.²² In this article we revisit some of our recent results^{21,22} to study the phase dependence of susceptibility for a weak probe applied to a specially prepared four-level atomic medium and the applications of the scheme to tunable control of the group velocity and atom localization.

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We first discuss our model and brief discussion of the susceptibility calculations in Sec.. We then discuss how a continuous change of phase gives the possibility of switching the group velocity from subluminal to superluminal and vice versa in Sec.3. Later we discuss how the same scheme can be used for localization of atom as it moves through a cavity in Sec.4. Finally we conclude in Sec.5.

2. THE MODEL

The schematics of the proposed scheme are shown in Fig. 1. Part (a) of the figure shows the energy level structure of the atom as required by the proposal. Schematic of the group velocity manipulation scheme is shown in part (b) of Fig. 1. For the atom localization proposal we consider the setup as shown in Fig. 1(c). We consider an atom, moving in the z direction, as it passes through a classical standing-wave field of a cavity. The cavity is taken to be aligned along the x axis. The internal energy level structure of the atom is shown in Fig. 1(b). The

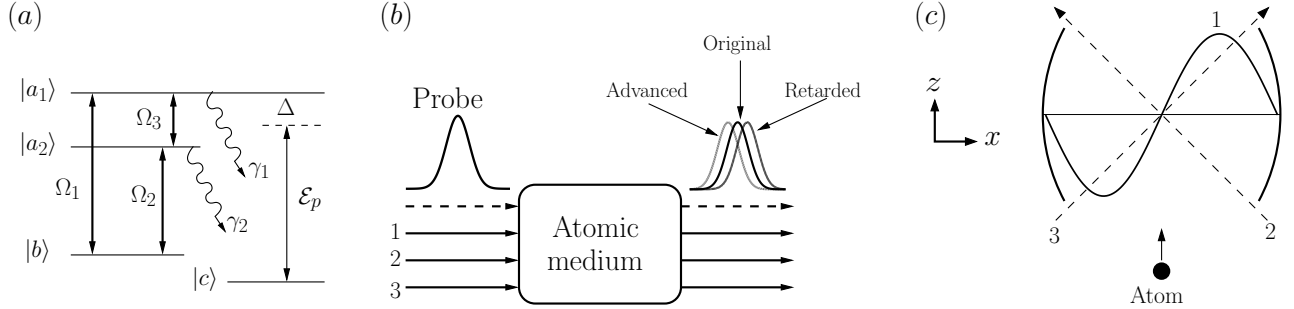


Figure 1. The Model: (a) The energy level structure of the atom. Probe field, denoted by \mathcal{E}_p , is detuned by an amount Δ from the $|a_1\rangle - |c\rangle$ transition. The fields (2, 3) shown in (b) and (c) part of the figure correspond to the fields with Rabi frequencies Ω_2 and Ω_3 respectively. The decay rates from the upper levels $|a_1\rangle$ and $|a_2\rangle$ are taken to be γ_1 and γ_2 respectively. (b) The schematic for the group velocity manipulation scheme. The three drive fields and the probe pulse all are taken to be co-propagating as they pass through the atomic medium. The relative phase of the three fields determines which one of the advanced, original or the retarded pulse will appear at the output. (c) The cavity supports the standing wave field (1) corresponding to Rabi frequency Ω_1 . Two other fields (2, 3) are applied at an angle as shown. The atom enters the cavity along the z axis and interacts with the three drive fields. The whole process takes place in the $x - z$ plane.

Hamiltonian for the problem can be written as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I, \quad (1)$$

where the self-energy \mathcal{H}_0 is given by

$$\mathcal{H}_0 = \hbar\omega_{a_1} |a_1\rangle \langle a_1| + \hbar\omega_{a_2} |a_2\rangle \langle a_2| + \hbar\omega_b |b\rangle \langle b| + \hbar\omega_c |c\rangle \langle c|, \quad (2)$$

and the interaction hamiltonian is

$$\mathcal{H}_I = -\frac{\hbar}{2} [\Omega_1 e^{-i\nu_1 t} |a_1\rangle \langle b| + \Omega_2 e^{-i\nu_2 t} |a_2\rangle \langle b| + \Omega_3 e^{-i\nu_3 t} |a_1\rangle \langle a_2| + \Omega_p e^{-i\nu_p t} |a_1\rangle \langle c| + \text{H.c.}]. \quad (3)$$

Here, ω_i correspond to the energy of state $|i\rangle$; the angular frequencies of the optical fields are denoted by ν_i ; and the subscript p stands for the quantity corresponding to the probe field. For the rest of the discussion we assume the Rabi frequencies Ω_1, Ω_2 to be real and allow Ω_3 to have a carrying phase, i.e., $\Omega_3 = |\Omega_3| e^{-i\phi}$. Since the three driving fields form a closed loop the phase can be imparted to any one of them and that will not change the result of the calculation. This will come clear at a later stage.

The coupling of the probe field of amplitude \mathcal{E}_p is governed by the corresponding Rabi frequency $\Omega_p = \mathcal{E}_p \wp_{a_1 c} / \hbar$. We note that $\wp_{a_1 c}$ is the dipole moment associated with the transition $|a_1\rangle - |c\rangle$. We construct the density matrix equations through

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] - \frac{1}{2} \{\Gamma, \rho\}, \quad (4)$$

where $\{\Gamma, \rho\} = \Gamma\rho + \rho\Gamma$. Here the decay rates are incorporated into the equations through the decay matrix Γ , which is defined by $\langle n|\Gamma|m\rangle = \gamma_n\delta_{nm}$. The so obtained density matrix equations can be easily solved at steady state with the initial condition that the atom starts in its ground state $|c\rangle$ and the probe field is weak compared to all the parameters of the system. Solution for the important density matrix element is obtained to be

$$\rho_{a_1c} = \frac{1}{Y\hbar}(\Omega_2^2 - 4\Delta^2 + i2\gamma_2\Delta)\mathcal{E}_p\varphi_{a_1c}\exp(-i\nu_p t), \quad (5)$$

where $Y = A + iB$, with

$$\begin{aligned} A &= -8\Delta^3 + 2\Delta(\Omega_1^2 + \Omega_2^2 + \Omega_3^2) + 2\gamma_1\gamma_2\Delta + \Omega_1\Omega_2\Omega_3(e^{i\phi} + e^{-i\phi}), \\ B &= 4\Delta^2(\gamma_1 + \gamma_2) - (\gamma_1\Omega_2^2 + \gamma_2\Omega_1^2). \end{aligned} \quad (6)$$

Now noting that the susceptibility can be written as

$$\chi = \frac{2N\varphi_{ca_1}\rho_{a_1c}}{\epsilon_0\mathcal{E}_p}e^{i\nu_p t} = \frac{2N|\varphi_{a_1c}|^2}{\epsilon_0} \frac{(\Omega_2^2 - 4\Delta^2 + i2\gamma_2\Delta)}{Y\hbar}, \quad (7)$$

where N is the atom number density in the medium. Separating the real and imaginary parts $\chi = \chi' + i\chi''$, we obtain

$$\chi' = \frac{2N|\varphi_{a_1c}|^2}{\epsilon_0\hbar Z} \{(\Omega_2^2 - 4\Delta^2)A + 2\gamma_2\Delta B\}, \quad \chi'' = \frac{2N|\varphi_{a_1c}|^2}{\epsilon_0\hbar Z} \{2\gamma_2\Delta A - (\Omega_2^2 - 4\Delta^2)B\} \quad (8)$$

where $Z = YY^*$. It is imperative to point out that the phase enters the susceptibility expression only through quantities A and Y . Moreover, the phase dependence of Y itself is only through that of A . We observe that the phase dependent term in A is $\Omega_1\Omega_2\Omega_3(e^{i\phi} + e^{-i\phi})$. Thus, this phase factor could very well have come from either of the three driving fields. Moreover, in the event that all the fields have a carrying phase, only the collective phase would occur in the susceptibility and there would be no dependence on individual phases. This collective phase can be easily determined to be $\phi = \phi_2 + \phi_3 - \phi_1$, by repeating the susceptibility calculation and noting that the Rabi frequencies are in general complex. Here ϕ_i is the phase of the complex Rabi frequency Ω_i of the i th driving field.

Noting the phase denendence of the atomic response to a weak probe field it can see that variety of effects are possible, we study two examples in the following sections.

3. PHASE CONTROL OF THE GROUP VELOCITY

For the discussion of this section the relevant quantity to consider is the group index $n_g = c/v_g$, where c is the speed of light in vacuum and the group velocity v_g is given by

$$v_g = \frac{c}{1 + 2\pi\chi'(\nu_p) + 2\pi\nu_p\partial\chi'(\nu_p)/\partial\nu_p}. \quad (9)$$

We observe that our model imparts unprecedented control over the group velocity of the probe pulse. The group velocity shows continuous tunability over a wide range of values ranging from subluminal to superluminal with just the change of the phase of one of the control fields while other parameters are kept constant. Important feature of our model being considerably less absorption accompanying the superluminal group velocities. Except for the first experimental realization⁶ where superluminality was gain assisted, most of the other proposals observe superluminality along with considerable absorption of the pulse as it passes through a specially prepared medium. We discuss our results with the help of the figures 2, 3.

In Fig. 2 (a)-(d) we plot the susceptibility (χ) and the group index ($c/v_g - 1$) as we vary the phase (ϕ) of the field corresponding to the Rabi frequency Ω_3 . We observe appearance and disappearance of a peak in the absorption profile in the center near $(\Delta/\gamma = 0)$. Thus there is resulting change in the sign of the slope of the dispersion with the phase giving rise to switch in the group velocity of the probe pulse from subluminal to superluminal. This change is continuous as depicted in Fig.2 (e). In this choice of parameters one has

superluminality accompanied by slight absorption. Note that we have chosen a modest value of the frequency of the probe field $\nu_p = 1000\gamma$. The derivative term being the dominating one in the group index, for real experimental parameters we expect the range of variation of the group velocity to be much larger than shown here. We note that in all the figures to follow, susceptibility χ and the group index $n_g - 1$ are plotted in the units of $2N|\rho_{a_1c}|^2/\epsilon_0\hbar$. Also, detuning Δ is represented by its dimensionless equivalent Δ/γ and the phase ϕ is in radians.

However, it would be desirable to have superluminal propagation with reduced absorption to make sure that the pulse is not attenuated considerably as it passes through the medium. We consider a different range of parameters in Fig. 3 that gives us essentially the same feature. To illustrate, we observe that the susceptibility has a small feature near the line center ($\Delta/\gamma = 0$) as shown in part (d) of the figure. We concentrate on this small feature and take advantage of the fact that the absorption is small to show phase tuning of the group velocity in parts (a)-(c). It turns out that having the probe field slightly detuned from the probe transition gives a wider range of group velocities (parts (a) and (c)) compared to zero detuning (part (b)). It is also instructive to notice that even though we are focusing on a small spectral feature of the medium the range of the group velocities available is no less than that observed in Fig. 2.

Now we contrast some of the features of the above considered model with some other approaches. A desirable feature in the group velocity control is intensity dependent tunability, demonstrated by Agarwal *et al.*²⁴ Our model has this feature as well and shows the intensity controlled tuning of the group velocity.²¹ Most of the proposals for the tunability of the group velocity are affected adversely due to the Doppler broadening of the medium. As pointed out by Goren *et al.*²⁵ the tunability from subluminal to superluminal is lost completely due to the Doppler broadening effects in some cases. It is however instructive to note that the model we propose here is naturally Doppler free if we consider all the drive fields and the probe field to be propagating colinearly. To put this in perspective we note that the major requirements of our model constitute maintaining the loop formed by the three driving fields and the two-photon resonance condition of EIT among the driving field Ω_1 and the probe field. We note that since $\nu_1 = \nu_2 + \nu_3$, the Doppler shifts for collinear propagation would satisfy $\Delta\nu_1 = \Delta\nu_2 + \Delta\nu_3$ maintaining the loop structure and phase sensitivity. With the assumption that the low lying levels $|b\rangle$ and $|c\rangle$ are very close to each other the frequency shift in the driving field corresponding to Ω_1 and the probe field are nearly the same thus maintaining the two-photon resonance with the transition $|b\rangle - |c\rangle$. Within the two-photon resonance regime, the only requirement for keeping the EIT medium Doppler free is to have sufficiently strong driving field.²⁶ Thus, noting that we work very close to the EIT condition our model is naturally Doppler free and there is very little absorption of any of the fields shining on the medium. Moreover, the model maintains the phase sensitivity as it preserves the loop structure even with the Doppler shifts.

4. SUB-HALF-WAVELENGTH LOCALIZATION

For the purpose of the discussion for this section we concern ourselves only with the imaginary part of the susceptibility which is given as

$$\chi'' = \frac{2N|\rho_{a_1c}|^2}{\epsilon_0\hbar Z} \{2\gamma_2\Delta A - (\Omega_2^2 - 4\Delta^2)B\}, \quad (10)$$

However, the quantities A and B are to be modified because the field 1 now corresponds to the standing-wave field of the cavity as shown in Fig. 1(c). Therefore the Rabi frequency Ω_1 is to be replaced by $\Omega_1(x) = \Omega_1 \sin \kappa x$. Thus the new definitions of the quantities A and B are given by

$$\begin{aligned} A &= -8\Delta^3 + 2\Delta (\Omega_1^2 \sin^2 \kappa x + \Omega_2^2 + \Omega_3^2) + 2\gamma_1\gamma_2\Delta + \Omega_1\Omega_2\Omega_3(e^{i\phi} + e^{-i\phi}) \sin \kappa x, \\ B &= 4\Delta^2(\gamma_1 + \gamma_2) - \gamma_1\Omega_2^2 - \gamma_2\Omega_1^2 \sin^2 \kappa x, \end{aligned} \quad (11)$$

and $Z = A^2 + B^2$. In the next section we consider the imaginary part of the susceptibility χ'' in detail and obtain various conditions for sub-wavelength localization of the atom.

We study the expression (10) for the imaginary part of the susceptibility on the probe transition in greater details in the following discussion. It is clear that χ'' , i.e., probe absorption depends on the controllable parameters of the system like probe field detuning, amplitudes and phases of the driving fields.

Noting the dependence of χ'' on $\sin \kappa x$, we can see that it is, in principle, possible to obtain information about the x -position of the atom as it passes through the cavity by measuring the probe absorption. Nevertheless, for precise localization of the atom the susceptibility should show maxima or peaks along the x -coordinate. We obtain the conditions for the presence of peaks in χ'' in the discussion to follow. In the case of $\gamma_2 = 0$, i.e., the level $|a_2\rangle$ is a metastable Eq. (10) can be simplified as follows:

$$\begin{aligned}\chi'' &= \frac{2N|\rho_{a_1c}|^2}{\hbar\epsilon_0} \frac{\gamma_1(\Omega_2^2 - 4\Delta^2)^2}{\gamma_1^2(\Omega_2^2 - 4\Delta^2)^2 + (8\Delta^3 - 2\Delta(\Omega_1^2 \sin^2 \kappa x + \Omega_2^2 - \Omega_3^2) - \Omega_1\Omega_2\Omega_3 \cos \phi \sin \kappa x)^2} \\ &= \frac{2N|\rho_{a_1c}|^2}{\hbar\epsilon_0} \frac{\gamma_1(\Omega_2^2 - 4\Delta^2)^2}{\gamma_1^2(\Omega_2^2 - 4\Delta^2)^2 + (\sin \kappa x - R_1)^2(\sin \kappa x - R_2)^2}\end{aligned}\quad (12)$$

where

$$R_{1,2} = \frac{1}{4\Delta\Omega_1} \left(-\Omega_2\Omega_3 \cos \phi \pm \sqrt{\Omega_2^2\Omega_3^2 \cos^2 \phi - 16\Delta^2[(\Omega_2^2 + \Omega_3^2) - 4\Delta^2]} \right). \quad (13)$$

Thus peaks will occur in χ'' at x -positions satisfying $\sin \kappa x = R_{1,2}$. In other words, χ'' peaks at the spatial positions defined by $\kappa x = \sin^{-1}(R_{1,2}) \pm n\pi$, where n is an integer, leading to localization of atoms conditioned on the detection of the probe absorption at that particular frequency corresponding to the value of Δ . In general, there will be four distinct peaks spread over the whole wavelength as both the roots $R_{1,2}$ appear twice in the denominator of Eq. (12).

As already discussed, the positions of maxima are strongly dependent on the probe field frequency through its detuning Δ . One of the main requirement for obtaining the novel result we mentioned in the introduction, namely sub-half-wavelength localization of the atom, would be to reduce the usual four peaks to two and to confine them to one of the half-wavelength regions along the standing-wave of the cavity. Once again observing expression (12) for the probe absorption we can see that two peaks are possible only if R_1 and R_2 coincide. This gives rise to the condition $\Omega_2^2\Omega_3^2 \cos^2 \phi - 16\Delta^2[(\Omega_2^2 + \Omega_3^2) - 4\Delta^2] = 0$. To simplify the discussion we consider a special case when $\Omega_2 = \Omega_3 = \Omega$. This assumption is general enough and easy to realize as the drive field strengths are controllable. This condition can be solved for Δ with $\phi = 0$, or π to obtain

$$\Delta = \pm \frac{\Omega}{4}(\sqrt{3} \mp 1) = \pm\delta_{1,2}. \quad (14)$$

For brevity we have introduced the notation, $\delta_{1,2} = (\Omega/4)(\sqrt{3} \mp 1)$, for the interesting values of the detunings. We plot the susceptibility in arbitrary units versus the κx in Figs. 4 and 5 for the detunings $\Delta = \pm\delta_1$ and $\Delta = \pm\delta_2$ respectively for the different values of the cavity field phase $\phi = 0, \pi/2$, and π for the choice of the drive field Rabi frequencies such that the localization peaks are sharp and well within a half-wavelength when sub-localization occurs.

Another interesting value of the detuning should be looked at as well corresponding to the phase value $\phi = \pi/2$, to check if there is a possibility of reducing the number of peaks from four to two in this case as well. In fact, it can be seen that one can obtain $R_1 = R_2$ for $\phi = \pi/2$ by solving, $-16\Delta^2[(\Omega_2^2 + \Omega_3^2) - 4\Delta^2] = 0$ for Δ . Thus, with $\Omega_2 = \Omega_3 = \Omega$ we obtain $\Delta = 0$, and $\Delta = \pm\Omega/\sqrt{2} = \pm\delta_3$. It can be easily seen that for these values of $\Delta = \pm\delta_3, 0$ there is no sub-half-wavelength localization possible. Now we discuss the results presented in Figs. 4-5 one by one.

We observe in Fig. 4, where we consider $\Delta = \omega_{a_1c} - \nu_p = \pm\delta_1$, that for the values of the phase of the cavity field $\phi = 0$ (boxes (a), (d)) and $\phi = \pi$ (boxes (c) and (f)) there is sub-half-wavelength localization possible. It can be seen that for a given value of the detuning changing the phase ϕ from 0 to π shifts the localization peaks from one-half of the wavelength of the cavity field to the other (See boxes (a) and (c) for $\Delta = +\delta_1$ and boxes (d) and (f) for detuning $\Delta = -\delta_1$). Similarly, for a given value of the phase ϕ changing the sign of the detuning moves the localization peaks from one half-wavelength to the other (See boxes (a) and (d) for $\phi = 0$ and boxes (c) and (f) for $\phi = \pi$). Exactly similar observation applies to Fig. 5 where we have chosen $\Delta = \pm\delta_2$. We notice that $\delta_1 < \delta_2$, thus different values for the Rabi frequencies are needed for the drive fields Ω_1 and Ω_2 to obtain well localization of the atom in a half-wavelength. This will be clear from the discussion to follow.

To clarify conditions to obtain good localization peaks well-within the half-wavelength region of the cavity field we remind ourselves that the sub-half-wavelength localization occurs when $R_1 = R_2$, i.e. when $\sin \kappa x = \pm \Omega_2 \Omega_3 / (4\Delta \Omega_1) = \pm \sin X_0$ with $\phi = 0$, or π . Note that the $+$ sign corresponds to $\phi = 0$ and $-$ corresponds to $\phi = \pi$. Thus the base peak occurs either the left or right side of $\kappa x = 0$ depending on the value of the phase ϕ . We notice that X_0 should be sufficiently far from zero so that the whole lineshape is contained within the given half-wavelength region. This observation governs the choice of the intensities or the Rabi frequencies of the drive fields at the chosen detuning Δ . We have chosen the appropriate values for both the figures 4 and 5 to obtain sub-half-wavelength localization. It is to be noticed that for the choice of parameters corresponding to sub-half-wavelength localization for $\phi = 0, \pi$, changing the phase to $\phi = \pi/2$, overall we still obtain two peaks. However these peaks are distributed over the full wavelength as can be seen from boxes (b) and (e) of both the figures 4 and 5. There is a central peak at $\kappa x = 0$ and two half-cut peaks at the boundaries $\kappa x = \pm\pi$.

Thus, we have shown how to obtain sub-half-wavelength localization through monitoring the probe absorption at a particular frequency. We note that the atom is to be prepared in its ground state to start with as opposed to the schemes based on the observation of the spontaneous emission spectrum (e.g., Ref.¹⁶), where the atom needs to be prepared in its excited state. Thus the preparation stage is considerably simplified in our model. Moreover, as we need monitoring the probe absorption as opposed to spontaneous emission as in Ref.¹⁶ we have distinct advantage to offer as the absorption measurements are straightforward to realize in an experiment compared to the measurement of spontaneous emission spectrum. In the following section we summarize our conclusions.

5. CONCLUSIONS

We have studied an extended Λ -type scheme and have demonstrated appearance of phase dependence in the response of the atomic medium to a weak probe field. We have further investigated application of the scheme to two situations one concerning the group velocity of the probe field and another allowing sub-wavelength localization of a moving atom.

The model shows a wide range of tunability in the group velocity of the probe field just by changing the phase of one of the control fields. The group velocity can also be switched from subluminal to superluminal through a continuous change of the phase. In contrast with most of the proposals where superluminality is accompanied with considerable absorption, we can reduce the absorption of the probe pulse substantially. The model imparts control of the propagation properties of the probe pulse by controlling the intensities of the laser fields as well. In addition, the model imparts natural Doppler-free situation when all the fields are propagating in a collinear fashion. Moreover, we work within the EIT domain thus with sufficiently strong drive fields one does not have to worry about the absorption of any of the fields incident on the medium.

By taking one of the drive fields as a standing-wave field of a cavity we can use our model for sub-wavelength localization of the atom as it passes through the standing-wave. Measurement of absorption of the probe field at a particular frequency localizes the atom. We have shown that the precision of the position measurement of the atom depends upon the amplitudes and the relative phase of the driving fields. The amplitude of standing-wave driving field when increased leads to line narrowing in the probe absorption, thus giving increased precision in the position measurement. Whereas the relative phase of the fields has important role in reducing the number of localization peaks leading to sub-half-wavelength localization, namely confinement of the localization peaks to one of the half-wavelength regions of the cavity field. As the method is based on the measurement of the probe absorption, it has two distinct advantages compared to the similar methods based on the observation of the spontaneous emission spectrum. Absorption measurements are much easier to perform in a laboratory compared to monitoring of spontaneous emission spectrum. Moreover, we do not require the atoms to be prepared in their excited states, in fact they are prepared in their natural ground state. Thus the preparation stage is fairly straightforward.

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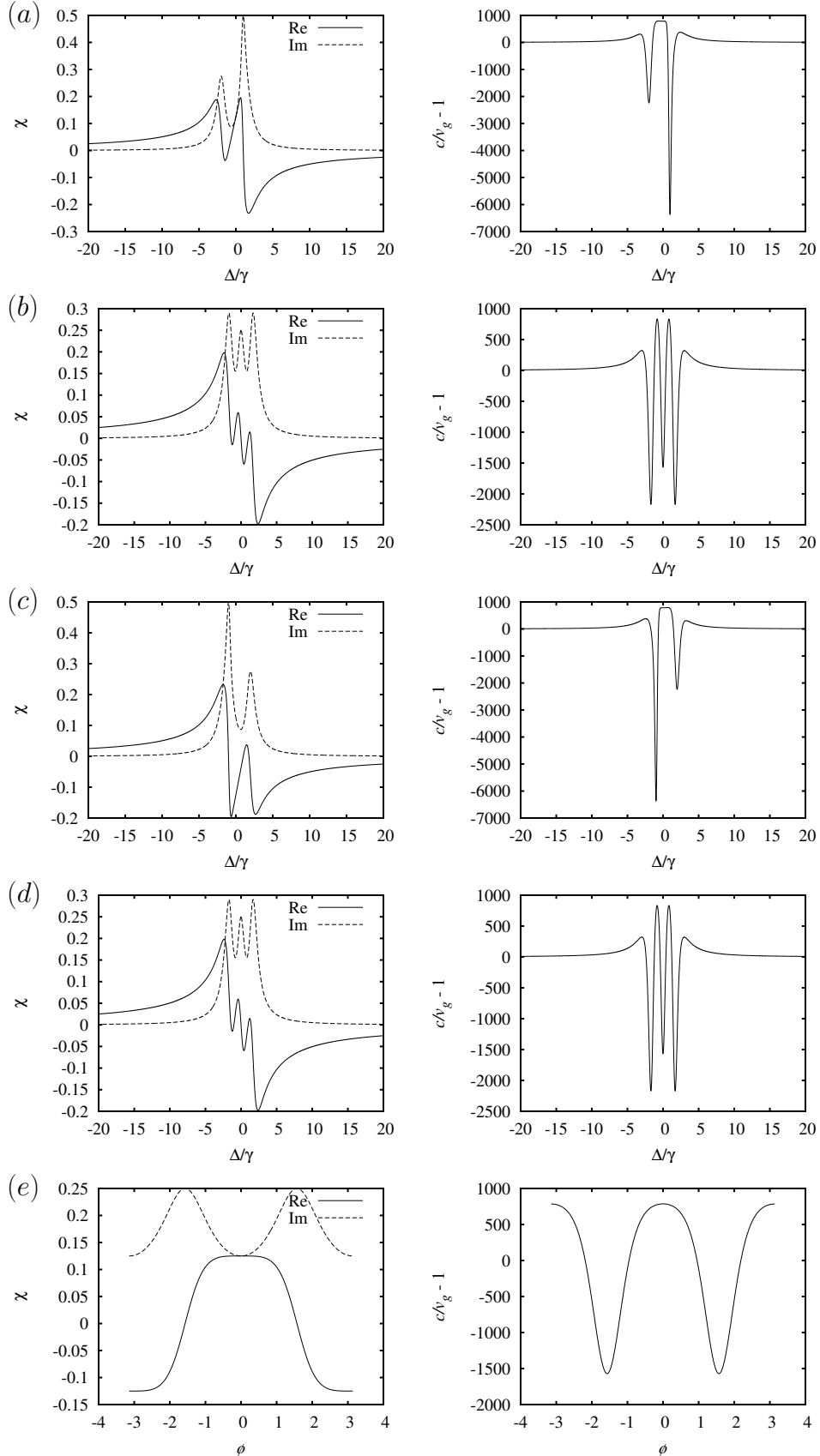


Figure 2. Phase variation of Group Velocity (Accompanied by absorption): The general parameters are $\Omega_1 = \Omega_2 = \Omega_3 = 2\gamma$, $\gamma_1 = \gamma_2 = 2\gamma$. (a) $\phi = 0$, (b) $\phi = \pi/2$, (c) $\phi = \pi$, (d) $\phi = 3\pi/2$, and (e) Variation of group index as a function of ϕ . Units: χ and $c/v_g - 1$ are in the units of $2N|\varphi_{a1c}|^2/\epsilon_0\hbar$, Δ/γ is dimensionless and ϕ is in radians. Please note that each label corresponds to both the plots appearing horizontally right to next to it.

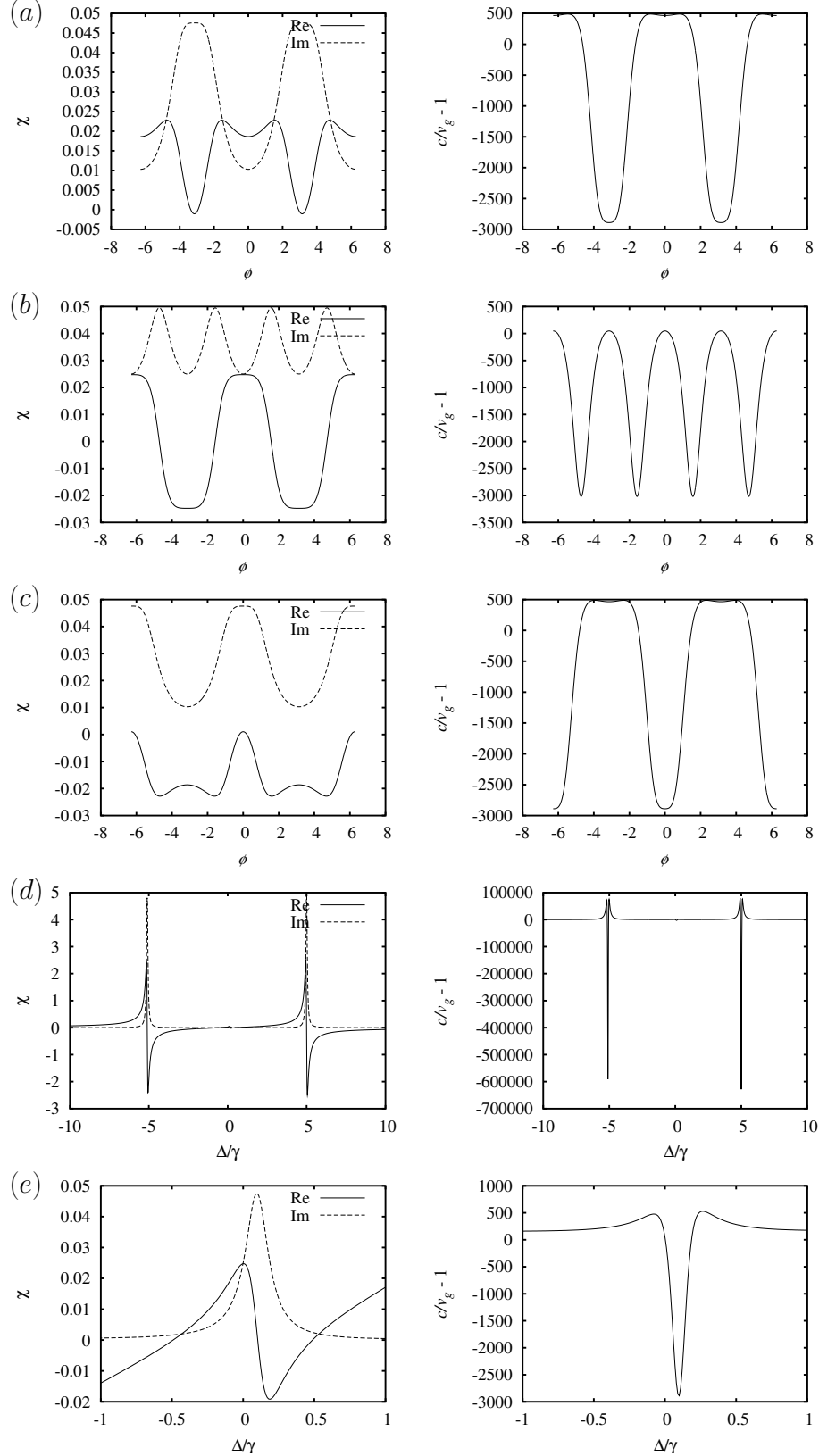


Figure 3. Phase variation of group velocity (Small absorption): The general parameters are $\Omega_1 = 10\gamma, \Omega_2 = \Omega_3 = \gamma, \gamma_1 = \gamma_2 = 0.2\gamma$. (a) $\Delta = 0.1\gamma$, (b) $\Delta = 0\gamma$, (c) $\Delta = -0.1\gamma$, (d) Variation as a function of Δ/γ for $\phi = 0$. We concentrate on the small feature near $\Delta/\gamma = 0$ in the susceptibility curves to obtain (a)-(c). This small feature seen in (d) is magnified and depicted in (e). Thus have superluminality at a considerably less absorption compared to most of the existing proposals. Another point to be noted here is that large variation of group velocity is available for $\Delta/\gamma \neq 0$ but close to 0. The units are the same as in Fig. 2.

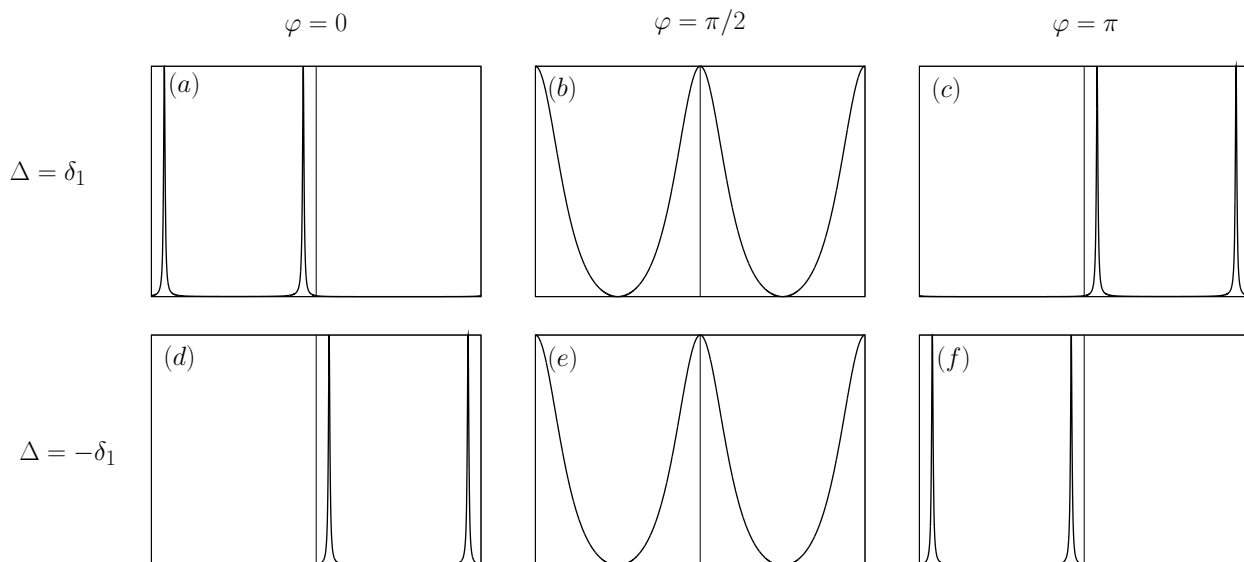


Figure 4. Phase dependence of the localization for the choice of the probe detuning $\Delta = \pm\delta_1$ defined in Eq.(14): Plot of the imaginary part of the susceptibility in arbitrary units vs the dimensionless x -coordinate κx along the standing wave in the cavity. κx runs from the values $-\pi$ on the extreme left to π to the extreme right in each box. A vertical line is drawn at $\kappa x = 0$ for the guiding of the eye. The common parameters are $\Omega_2 = \Omega_3 = \Omega = 20\gamma_1$, $\gamma_2 = 0$, and $\Omega_1 = 30\gamma_1$, and $\delta_1 = (\Omega/4)(\sqrt{3} - 1)$ unless specified otherwise. (a) $\phi = 0$ and $\Delta = \delta_1$ (b) $\phi = \pi/2$ and $\Delta = \delta_1$ (c) $\phi = \pi$ and $\Delta = \delta_1$ (d) $\phi = 0$ and $\Delta = -\delta_1$ (e) $\phi = \pi/2$ and $\Delta = -\delta_1$ (f) $\phi = \pi$ and $\Delta = -\delta_1$. Notice the presence of sub-half-wavelength localization for the boxes (a), (c), (d) and (f), where the two peaks are confined to either the range $\kappa x = \{-\pi, 0\}$ or $\kappa x = \{0, \pi\}$. The range in which the localization peaks appear depends on the value of ϕ and the sign of the detuning Δ . There is no sub-half-wavelength localization for boxes (b) and (e) as $\phi = \pi/2$. Notice that the lineshapes are much sharper for the case of $\phi = 0, \pi$ compared to $\phi = \pi/2$.

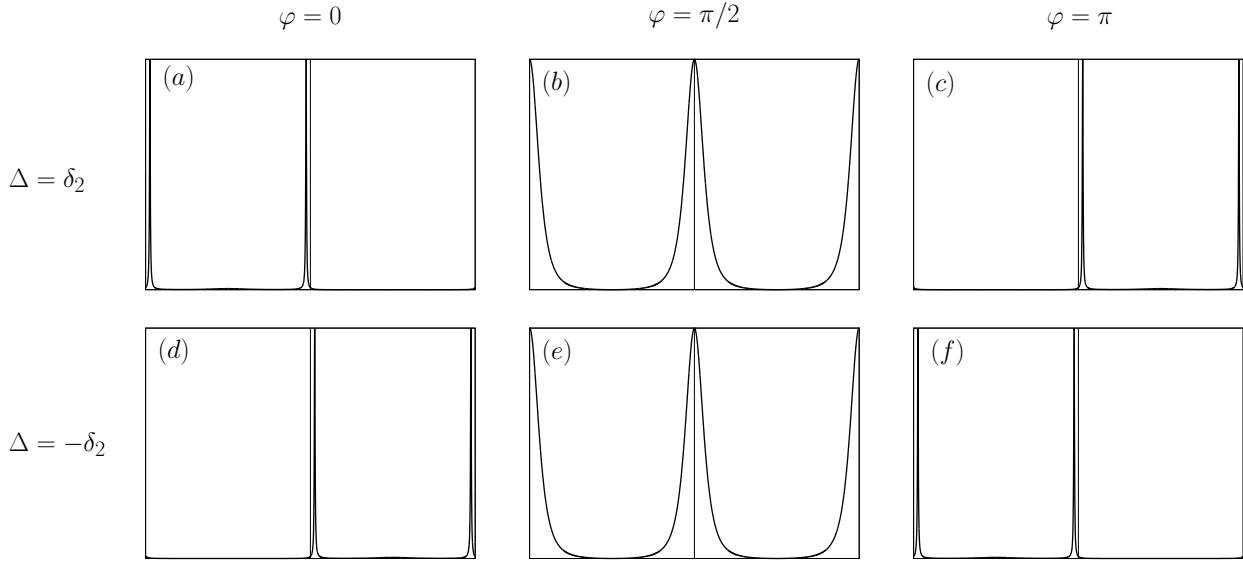


Figure 5. Phase dependence of the localization for the choice of the probe detuning $\Delta = \pm\delta_2$ defined in Eq.(14): Plot of the imaginary part of the susceptibility in arbitrary units vs the dimensionless x -coordinate κx along the standing wave in the cavity. κx runs from the values $-\pi$ on the extreme left to π to the extreme right in each box. A vertical line is drawn at $\kappa x = 0$ for the guiding of the eye. The common parameters are $\Omega_2 = \Omega_3 = \Omega = 50\gamma_1$, $\gamma_2 = 0$, and $\Omega_1 = 60\gamma_1$, and $\delta_2 = (\Omega/4)(\sqrt{3} + 1)$ unless specified otherwise. (a) $\phi = 0$ and $\Delta = \delta_2$ (b) $\phi = \pi/2$ and $\Delta = \delta_2$ (c) $\phi = \pi$ and $\Delta = \delta_2$ (d) $\phi = 0$ and $\Delta = -\delta_2$ (e) $\phi = \pi/2$ and $\Delta = -\delta_2$ (f) $\phi = \pi$ and $\Delta = -\delta_2$. Notice the presence of sub-wavelength localization for the boxes (a), (c), (d) and (f), where the two peaks are confined to either the range $\kappa x = \{-\pi, 0\}$ or $\kappa x = \{0, \pi\}$. The range in which the localization peaks appear depends on the value of ϕ and the sign of the detuning Δ . There is no localization for boxes (b) and (e) as $\phi = \pi/2$. Notice that the lineshapes are much sharper for the case of $\phi = 0, \pi$ compared to $\phi = \pi/2$. Also notice that the lineshapes in this figure are sharper as compared to the ones in Fig. 4 due to larger value of Ω_1 .