Full-Wave Effects in Coronagraph Masks

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

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Fourier Optics Analysis of Thin Masks

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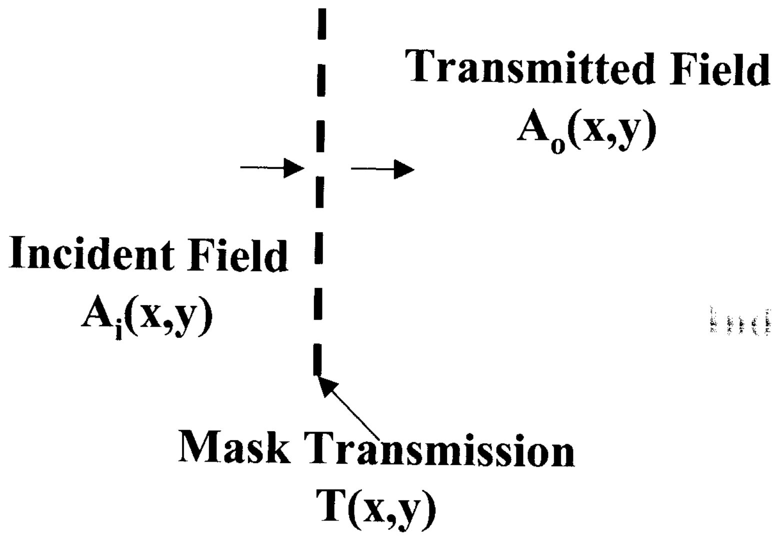
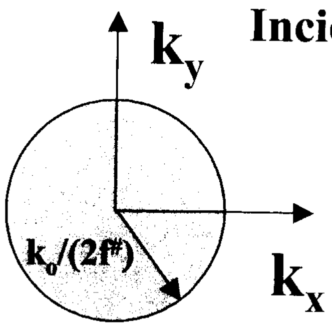


$$A_o(x,y) = A_i(x,y) \cdot T(x,y)$$

Assumes T(x,y) is
Independent of $A_i(x,y)$, Independent of λ,
Independent of Polarization ... !
[For Binary Masks T(x,y)=1/0]



Incident Field (Airy Disk) Spectrum

$$\widetilde{A}_i(k_x,k_y)$$

Fourier Transform of A_0 is equivalent to the far field radiated toward the Lyot stop.

$$\widetilde{A}_o(k_x,k_y) = \widetilde{A}_i(k_x,k_y) \otimes \widetilde{T}(k_x,k_y)$$

A convolution only for the simple T(x,y) behavior assumed above!

Effects Ignored in the Fourier Transform Analysis

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- Finite Conductivity of the Metal Layer
- Thickness of the Metal Layer [Due to Finite Conductivity]
- Dependence of the Transmission Coefficient on the Polarization of the Incident Field
- Dependence of the Transmission Coefficient on the Illumination Angle of the Incident Field
- Interaction Between Different Regions/Periods of the Mask
- Substrate Effects

Full-Wave Electromagnetic Effects in Thick Masks

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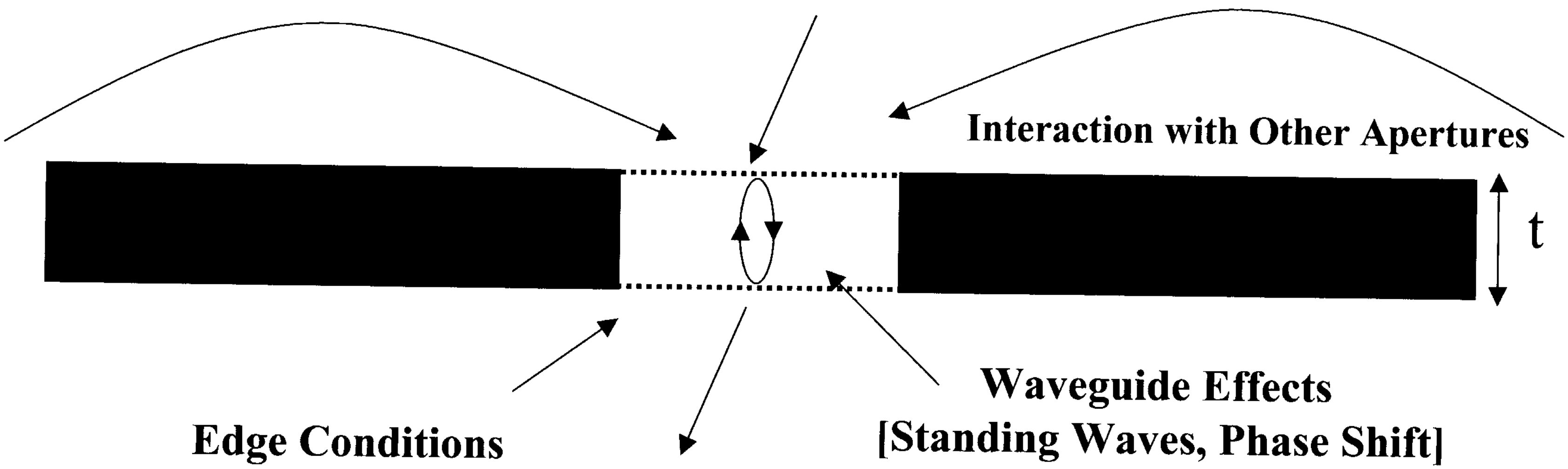

A Typical Periodic, Binary Mask

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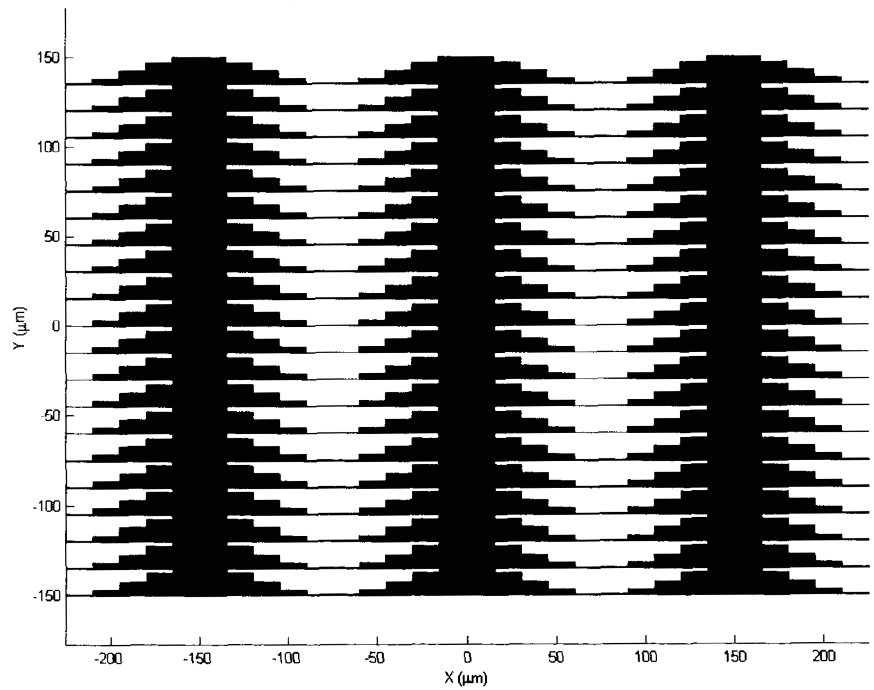
Binary Sin² Mask: F#=15, λ=1 μm, Total Transmission @ 5λ/D



Pupil-Plane Masks

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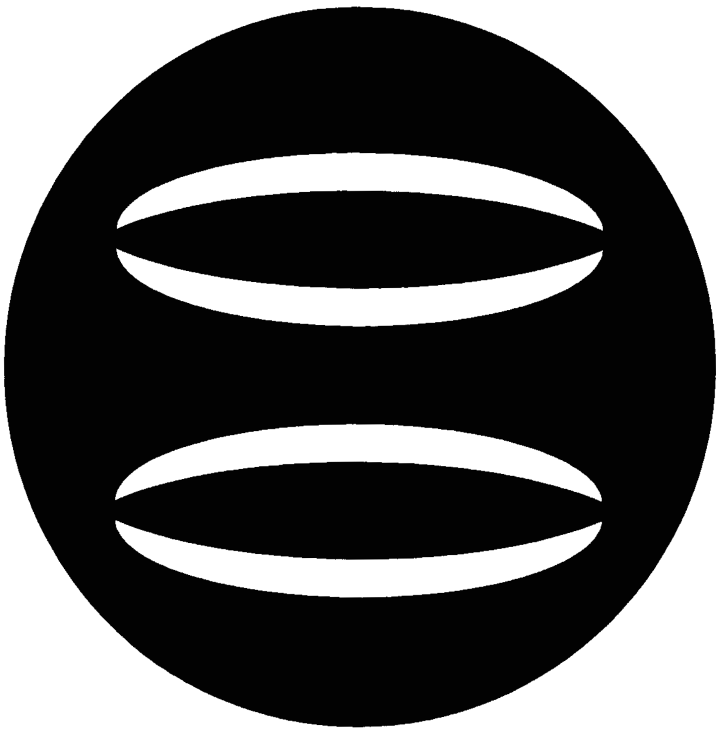
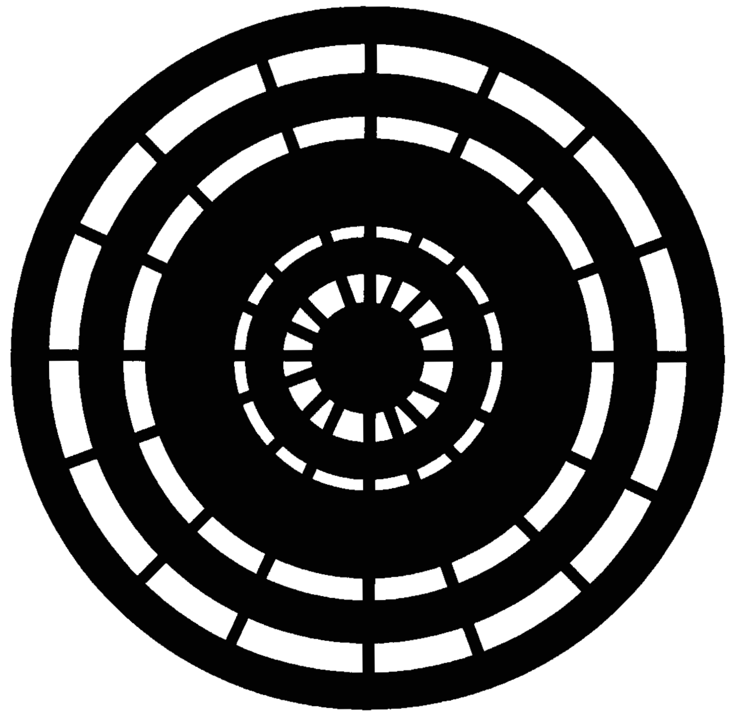


Shaped Pupil Mask



Spider-Web Mask

Comparison of Mask Types

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Mask Type	Electromagnetic Problem Type	$Scale_X$ (Search Direction)	$Scale_Y$ (Orthogonal to Search Direction)	Comments
Binary Sin^2, Sin^4, Periodic	Periodic in 2-D, with finite depth (Z)	2*IWA/(λ/D)*F#*λ	F#* λ	Airy Disk Illumination
Binary 1-$Sinc^2$, 1-$Sinc^4$ Non-Periodic	Periodic in 1-D, with finite depth (Z)	Infinite	F#* λ	Airy Disk Illumination
Shaped Pupil Masks	Finite but Large, with finite depth (Z)	D/M, M=Magnification	D/M, M=Magnification	Plane Wave Illumination
Spider Web Pupil Masks	Body of Revolution (BOR) or near-BOR with finite depth (Z)	D/M, M=Magnification	D/M, M=Magnification	Plane Wave Illumination

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EM Analysis of Masks

Method Outside	Method Inside	Mask Type	Unknowns	Dimensionality
2-D Periodic Green's Function	N/A Thin Sheet	Thin Binary Sin^2	Sheet Current	2-D
2-D Periodic Green's Function	Modal Expansion	Thick Binary Sin^2	Mode Amplitudes	2-D
2-D Periodic Green's Function	FEM	Thick Binary Sin^2	Cell Volume Fields	3-D
2-D Periodic Green's Function	Surface Current	Thick Binary Sin^2	Cell Surface Current	2-D
1-D Periodic Green's Function	N/A Thin Sheet	Thin Binary 1-$Sinc^2$	Sheet Current	2-D
1-D Periodic Green's Function	Surface Current	Thick Binary 1-$Sinc^2$	Cell Surface Current	2-D
Fourier Series Expansion	Modal Expansion	Spiderweb	Mode Amplitudes	2-D
Fourier Series Expansion	FEM	Spiderweb	Volume Fields	3-D
Fourier Series Expansion	Surface Current	Spiderweb	Surface Current	2-D
Free Space Green's Function	Modal Expansion	Shaped Pupil	Mode Amplitudes	2-D
Free Space Green's Function	FEM	Shaped Pupil	Volume Fields	3-D
Free Space Green's Function	Surface Current	Shaped Pupil	Surface Current	2-D
GTD/UTD	GTD/UTD	All	Ray Amplitudes	---
FDTD	FDTD	All	Volume Fields	3-D
Fast Multipole	Fast Multipole	All	Surface Current	2-D
Approximate	Approximate	All	Various Items	---

Analysis of Masks

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- Typical Dimensions for a single period of a Sin^2 Binary Mask: For F/15 (considered a minimum): $T_x=150\lambda$, $T_y=15\lambda$, and Thickness=0.6λ.
- Even this best-case (smallest mask) is a very large full-wave electromagnetic problem.
- Two-Pronged Approach
  - Pursue Exact Analysis using Available Solution Techniques: GTD/UTD, FDTD, MOM, FMM, …
  - Consider Several Types of Canonical Problems: Thick Periodic Gratings, Propagation Through Various Thick Apertures
    - Obtain Order-of-Magnitude Estimates for EM Effects
    - Use in "Local", Approximate Analysis of a Real Mask
    - Compare Various Mask Types Against Each Other

Thick Periodic Grating

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- Why a Thick Periodic Grating?
  - Thick One-Dimensional Grating will Demonstrate Propagation Effects of Gap Size and Incident Polarization
  - Periods of 30-100l (F# =30-100) Translate into a Reasonable EM Problem for which a Converged Result can be Obtained
  - Both the Periodic and Non-Periodic Binary Masks Appear as A Thick Periodic Grating when Examined Locally and will Share many of the Grating's EM Effects
  - Grating Transmission Coefficients can be used in an Approximate Analysis of the Binary Mask

Thick Periodic Grating

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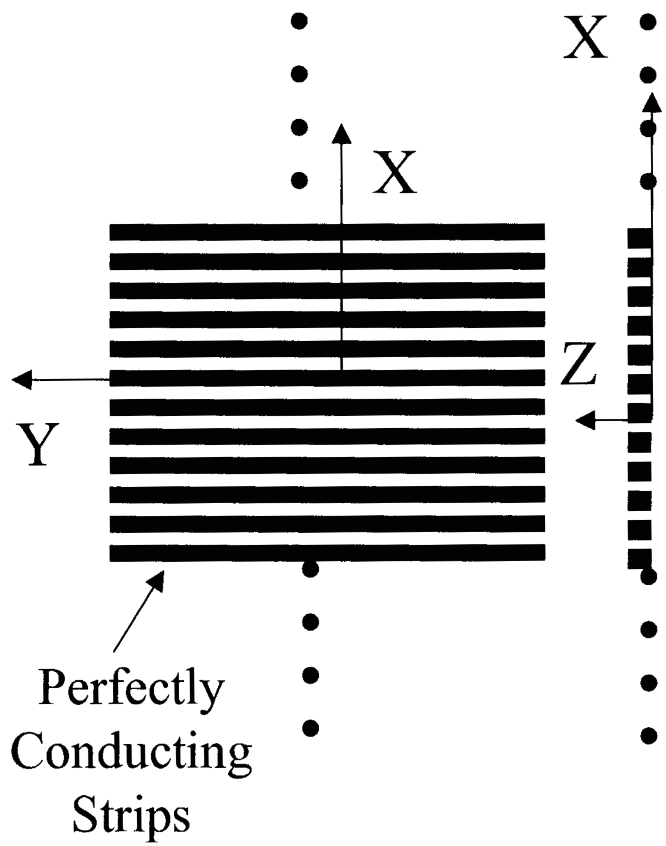

- Array of Thick Metal Strips
- Periodic in X (equals15 λ for F/15)
- Infinite in Y
- Assumed to be Perfect Conductors
- Finite Depth (Z Dimension)
- No Substrate (Can be added)
- Exact (Numerical) Solution
  - Periodic Free-Space Modes [Plane Waves]
  - Parallel Plate Waveguide Modes
  - Field Matching to Obtain Transmitted and Reflected Waves
- Arbitrary
  - Dimensions
  - Polarization
  - Angles of Incidence

Transmission Versus Gap Dimension, Thick Strip Grating

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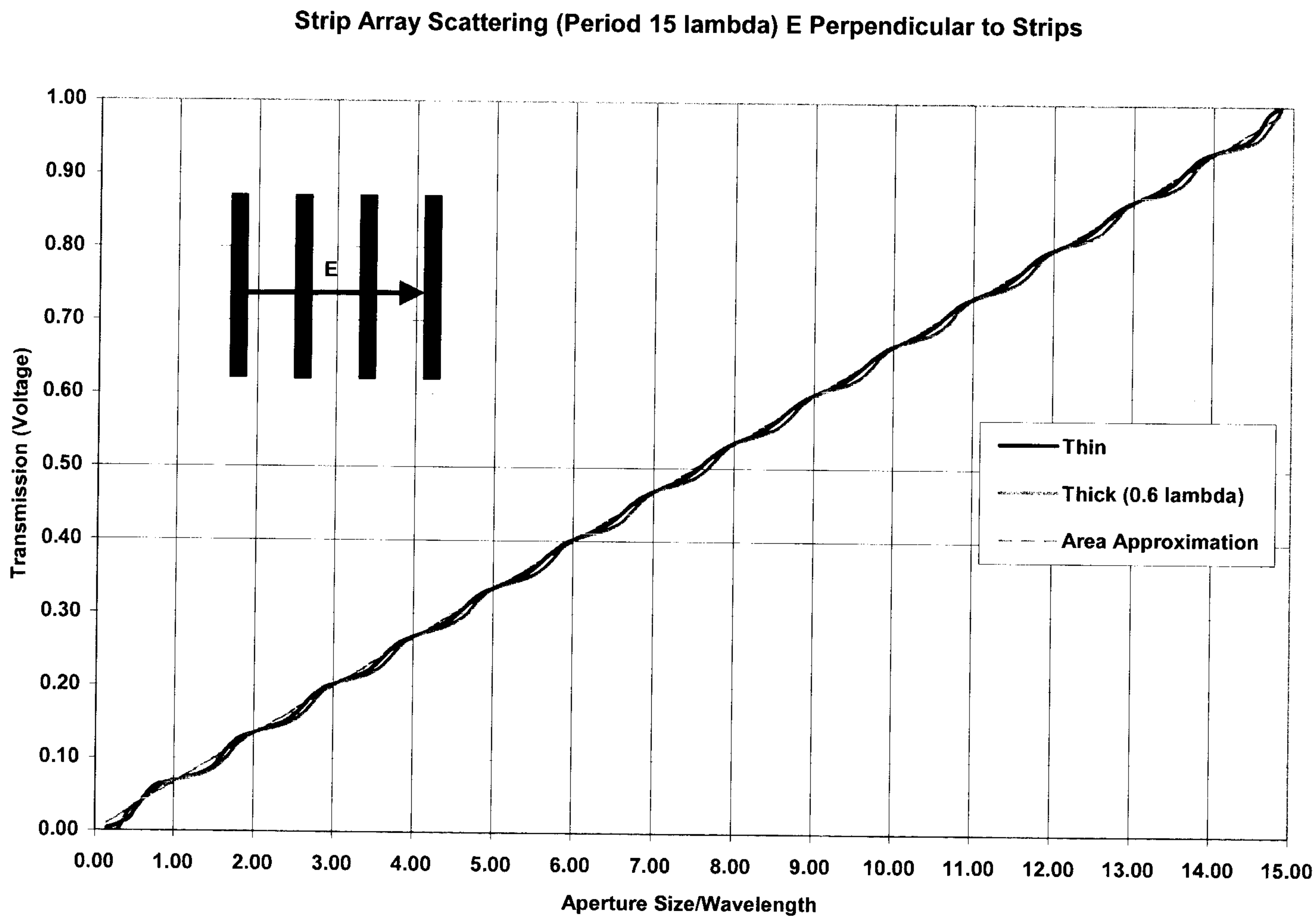

Transmission Phase Versus Gap Dimension, Thick Strip Grating

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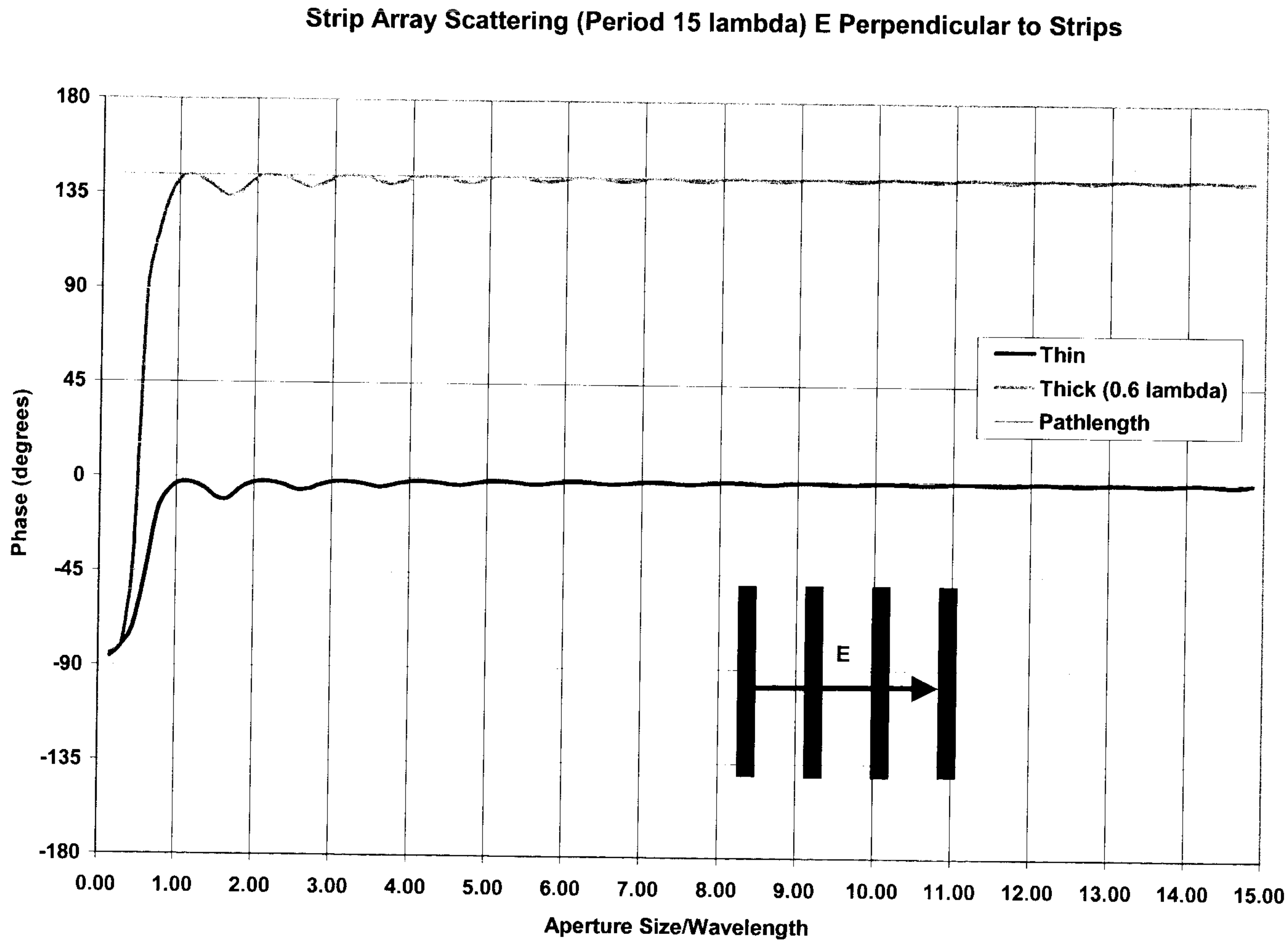

Relative Magnitude Versus Gap Dimension, Thick Strip Grating, 15λ Period

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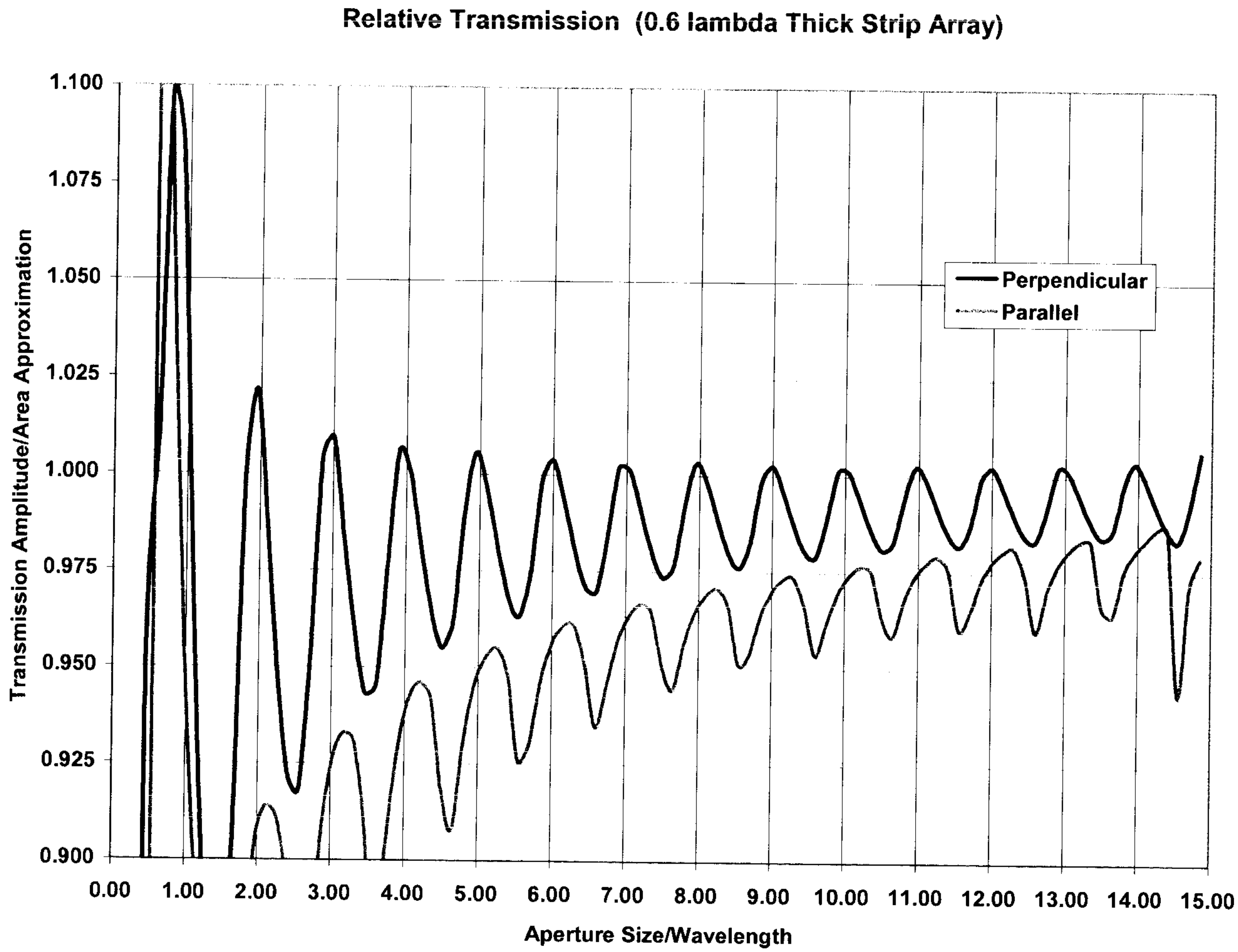

Relative Phase Versus Gap Dimension, Thick Strip Grating, 15λ Period

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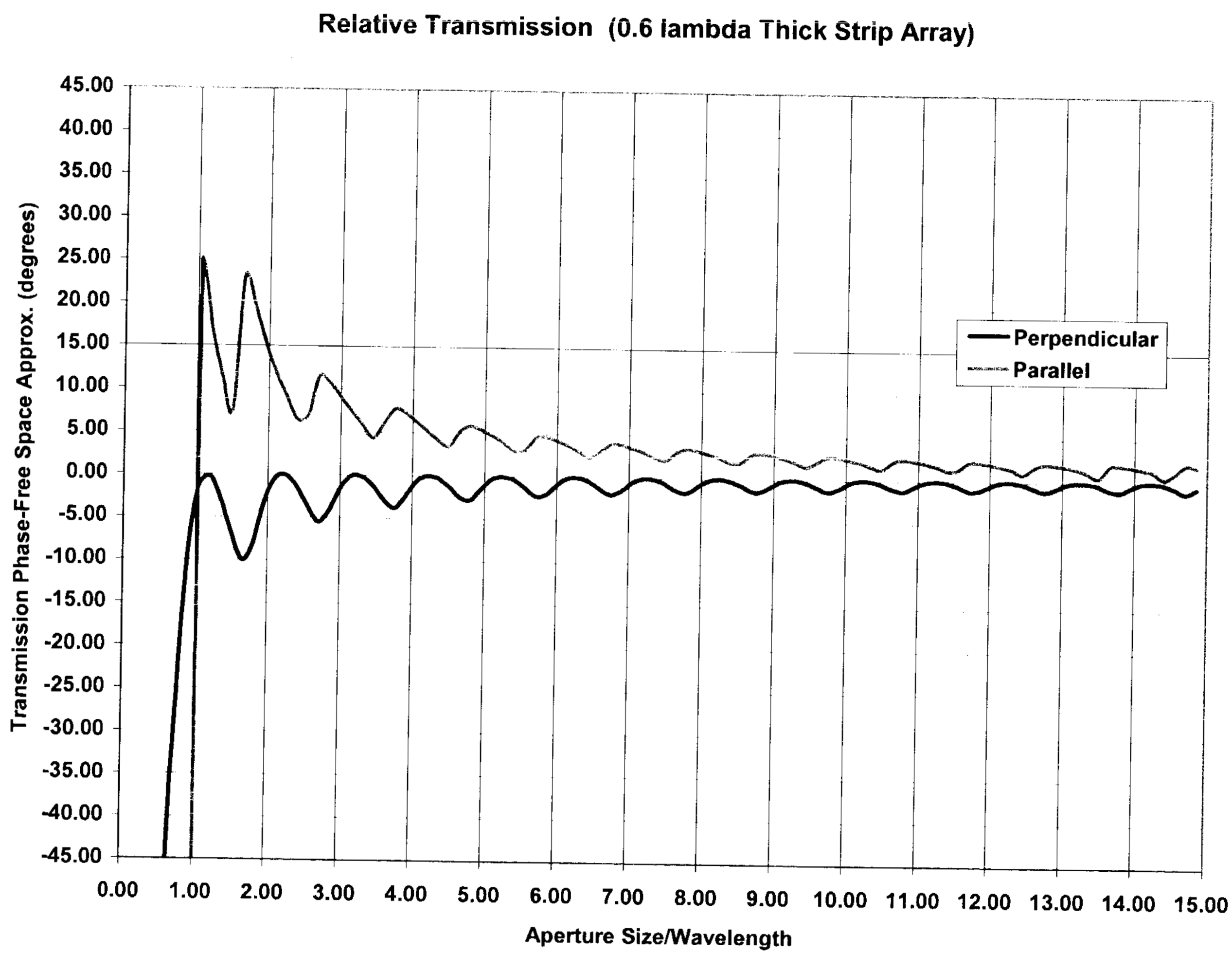

Relative Magnitude Versus Gap Dimension, Thick Strip Grating, 30λ Period

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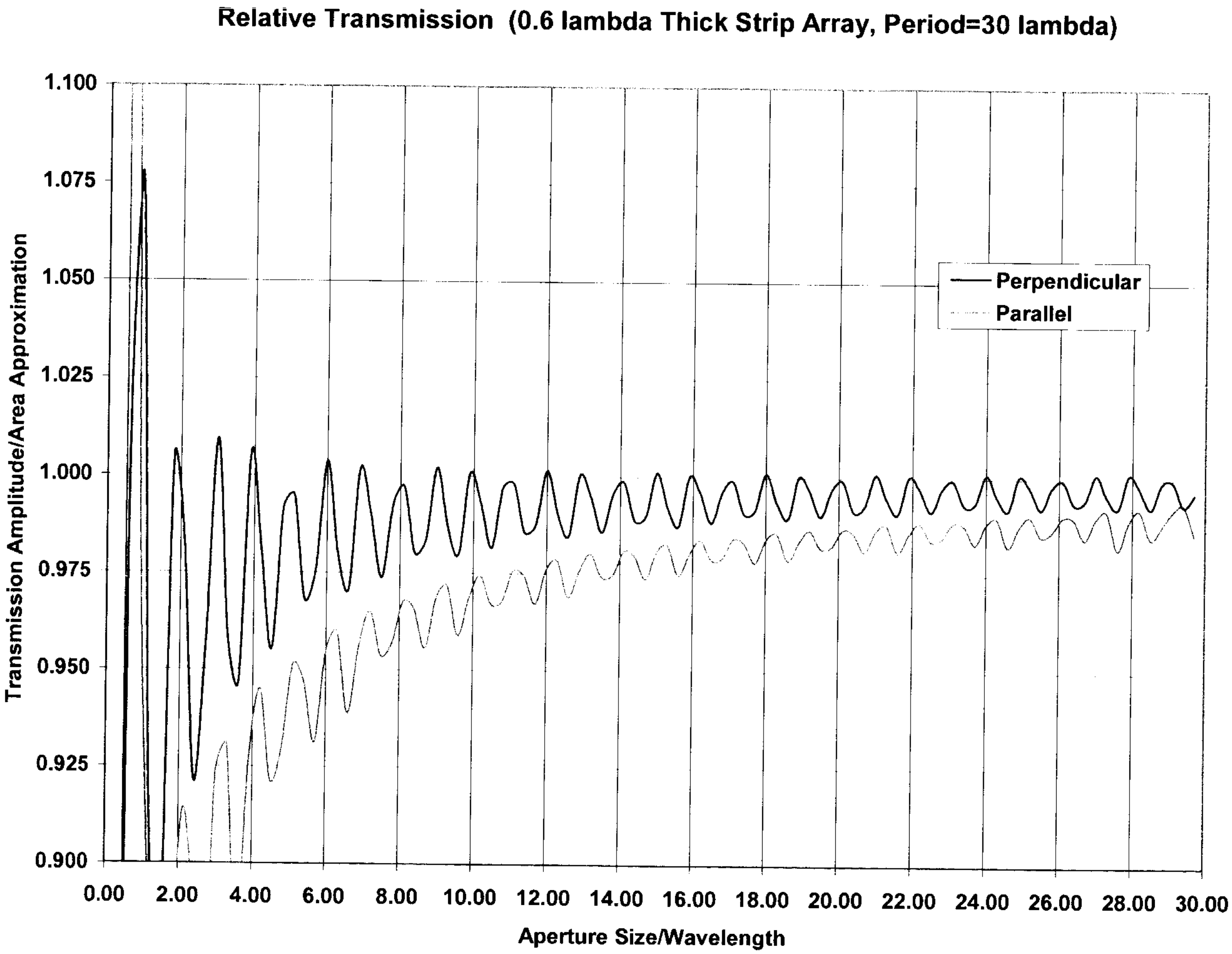

Relative Phase Versus Gap Dimension, Thick Strip Grating, 30λ Period

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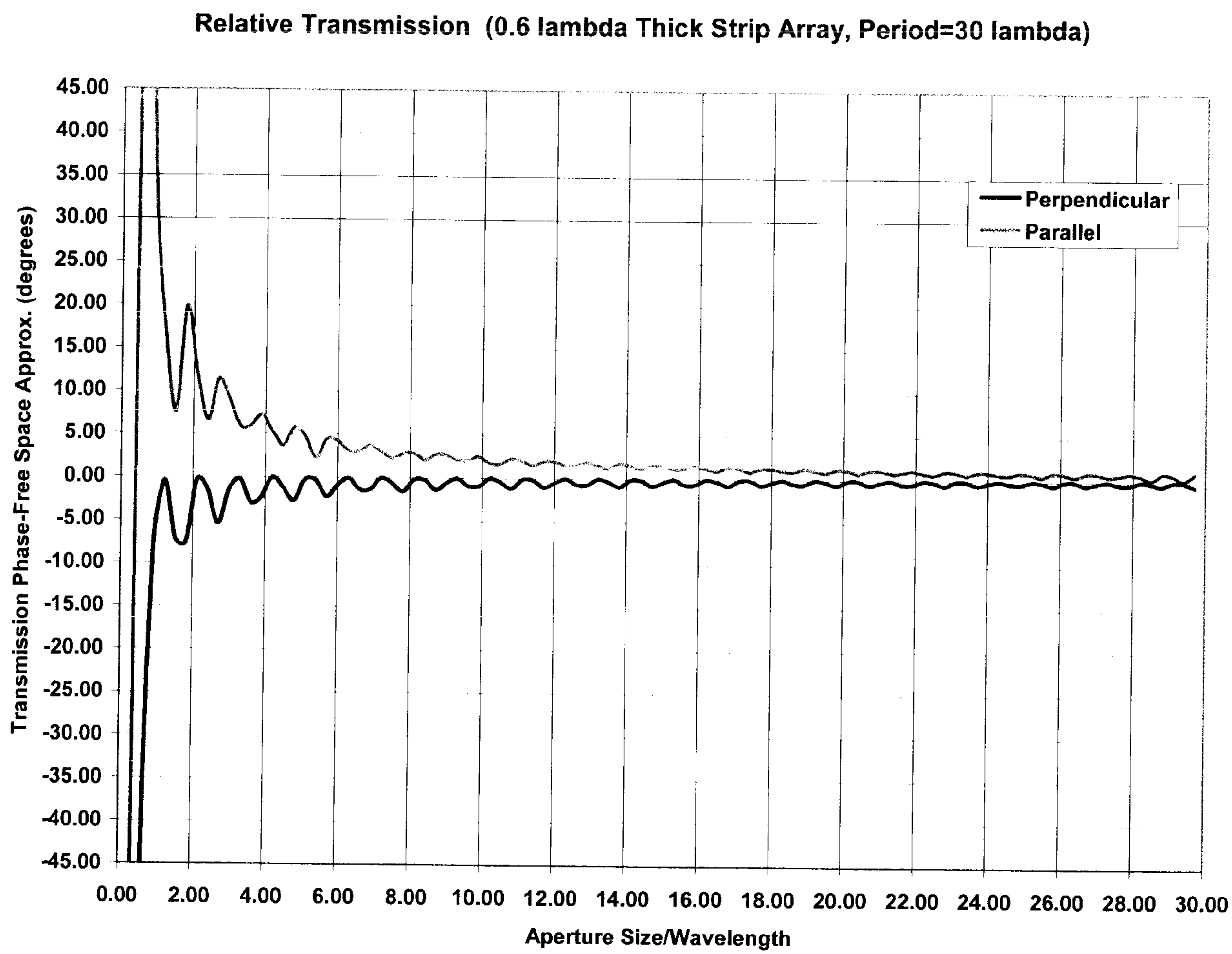

Major Effects

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- Significant effects for apertures less than a few λ wide.
- Larger effects for E parallel to the strips.
- Amplitude ripple of a few percent, even for large gaps.
- Phase ripple of a few degrees, even for large gaps.
- Different effects for the two polarizations.
- Although data only shown for normal incidence, non-normal results were also computed:
  - Significant amplitude and phase differences with respect to normal incidence values, particularly for gaps less than a few λ.

Approximate Analysis of an Example Binary Mask

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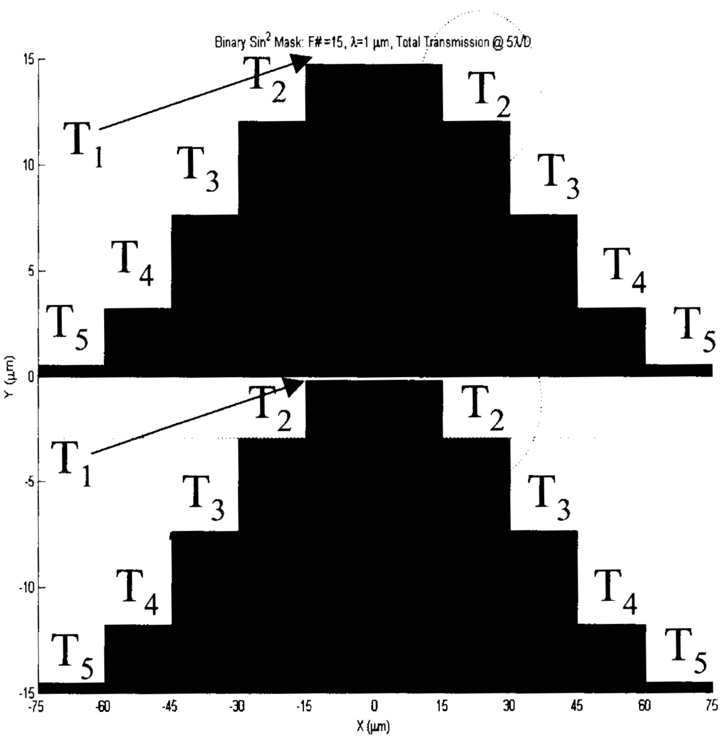

(Not to Scale)

$T_m \neq T_n \neq 1$ $\quad$ $T_m = |T_n| e^{j\phi m}$

- Assume each X-segment operates independent of others, and as if it were infinite in X and periodic in Y.
- Apply transmission coefficient, T_n, obtained from exact solution of appropriate strip problem to each segment
  - Includes thickness and wave-guiding effects
  - Effective Transmitted Amplitude in segment (not necessarily 1)
  - Effective Transmitted Phase in Segment (not necessarily equal)
  - Polarization dependence of above
  - Simplify by using normal incidence values of T for all spectral components

One Period of an Example Binary Mask, Width 150λ, Height 15λ, Thickness 0.6λ

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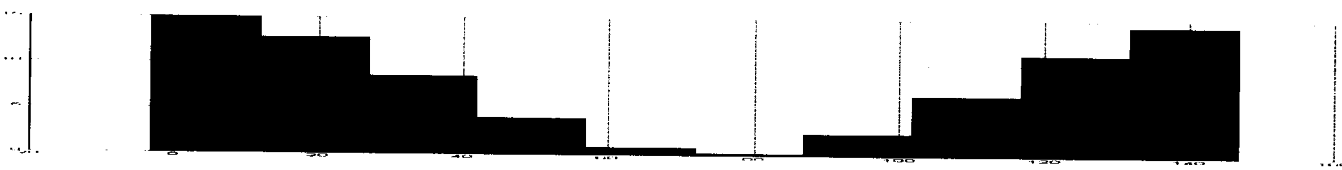

Lyot-Stop Fields, Transmission =1/0

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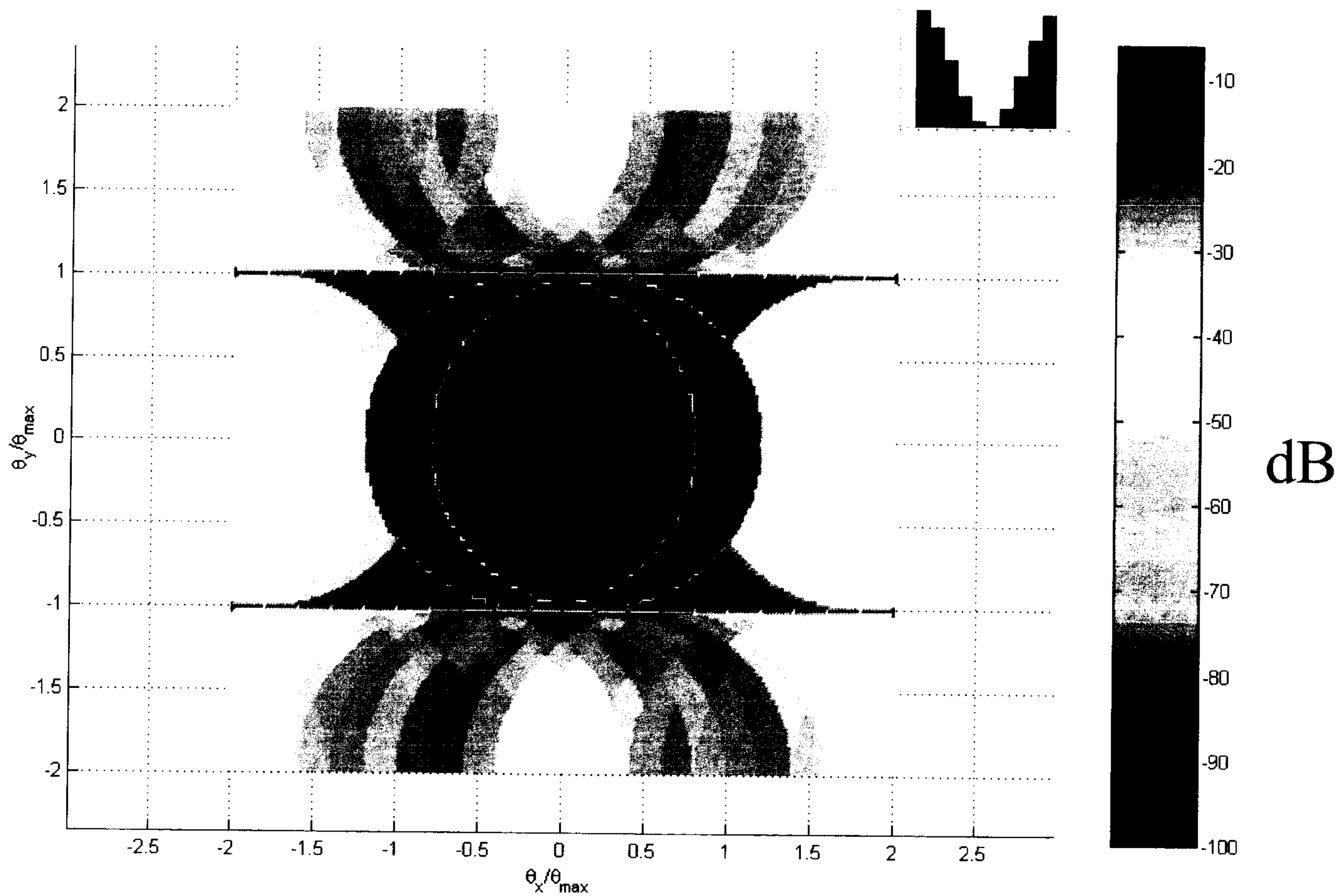

Approximate Gap Transmission Coefficients, Parallel Polarization

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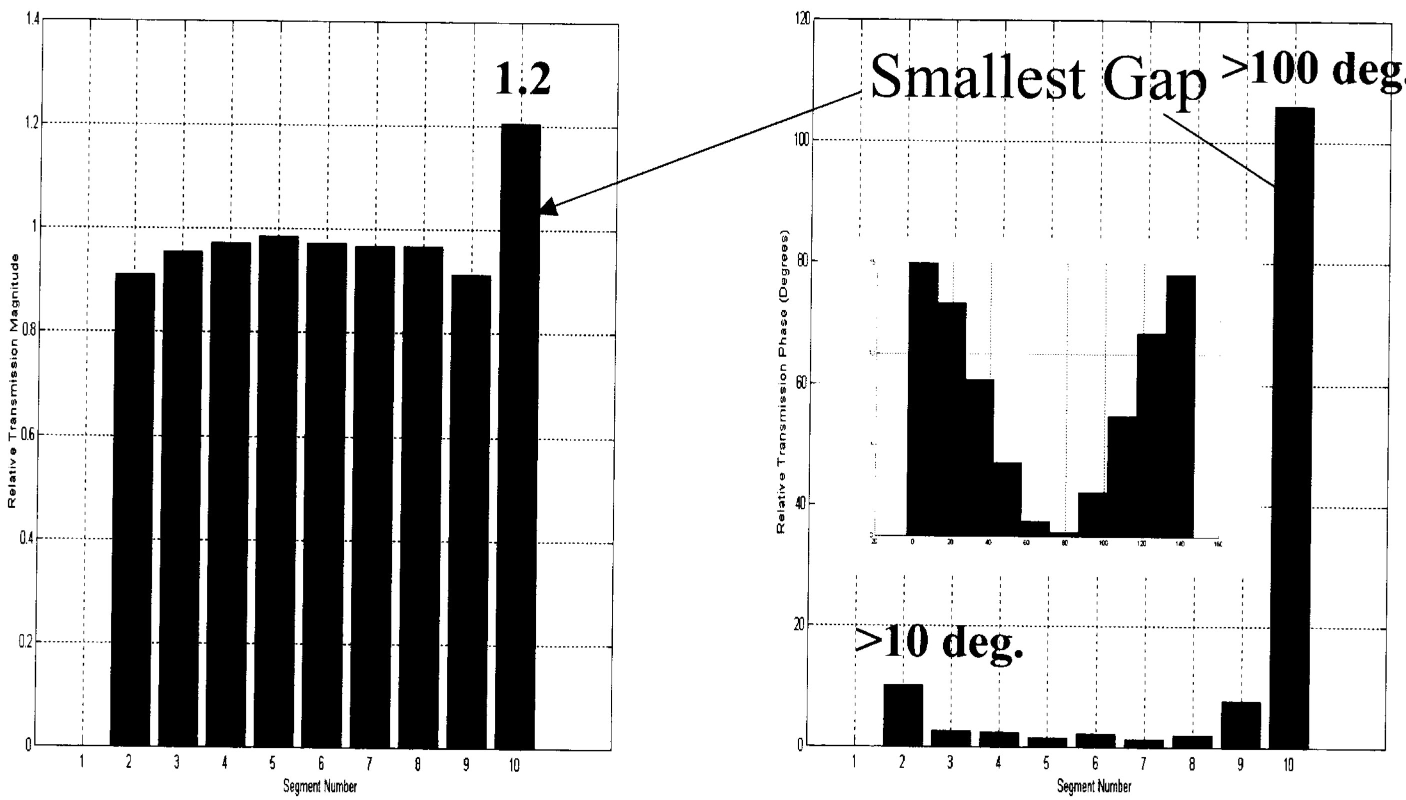

Relative Amplitude

Relative Phase

Lyot-Stop Fields, Using Approximate Transmission Coefficients

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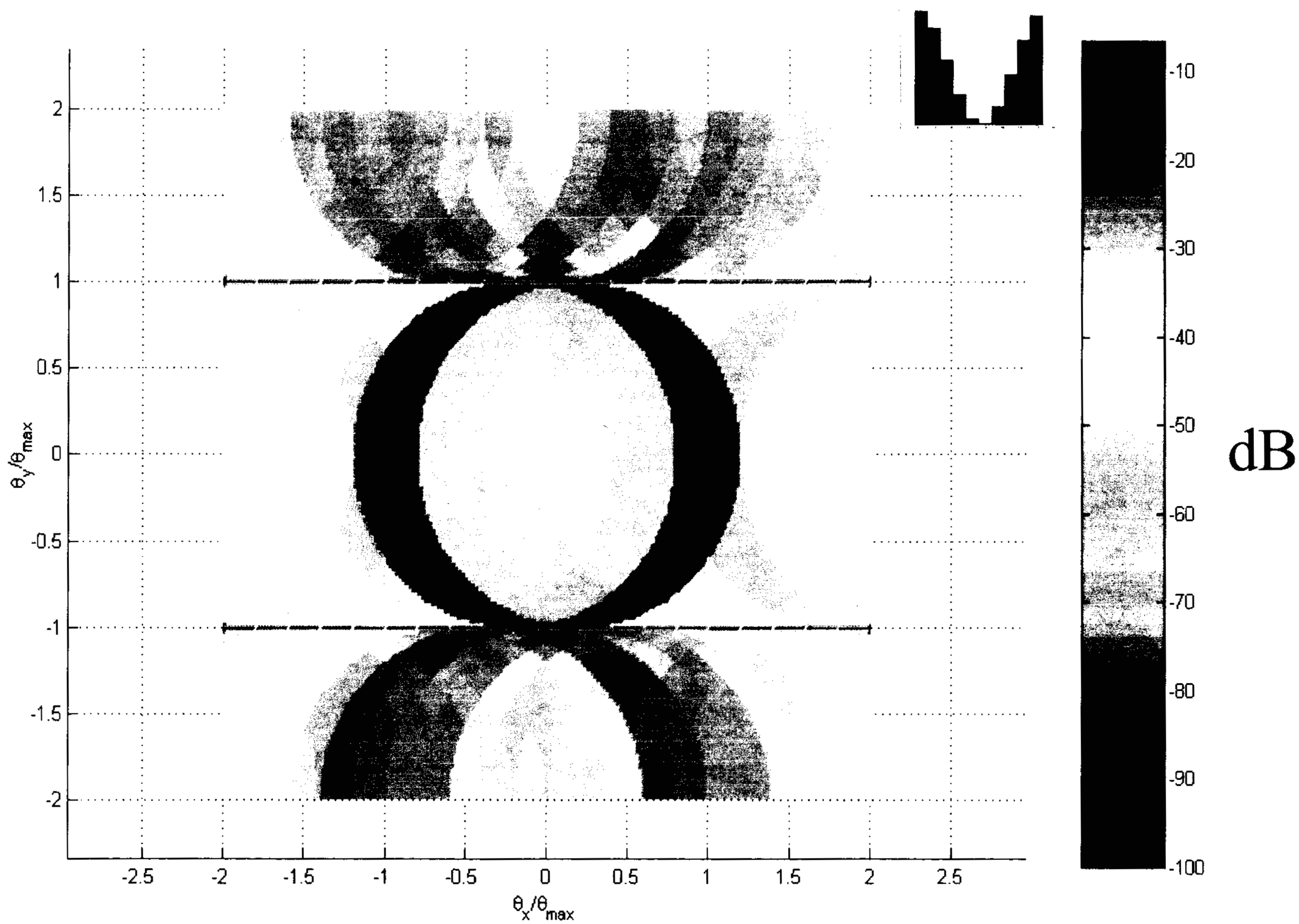

General Observations Based on a Group of These Very Approximate Mask Results

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- For a single wavelength and angle of incidence amplitude effects can be compensated for by adjusting the aperture sizes.
- Phase effects through the mask are likely to be the dominant effect, and are not easily compensated with mask dimensions alone.
- Although not shown here, the transmission coefficients change significantly over the angular range corresponding to F/15. (F/15$\rightarrow$1.9^{o})
- Effects such as the transmission phase vs. aperture size are also a function of wavelength. (phase ripple amplitude and location of maxima/minima)

Design Strategies (1)

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- Although the previous analysis was approximate similar, non-negligible effects should be expected upon a full-wave analysis of the complete mask.
- At a given wavelength it may be possible to compensate for the dependence of the transmission coefficients on angle of incidence by tailoring the incident field's amplitude and phase. Need to optimize the mask and incident field simultaneously.
- Polarization effects can be dealt with by providing separate trains for the two components and multiple polarization filters to maintain purity. Different mask, stop, and incident field for each.
- Wavelength effects are also significant and more difficult to deal with.

Design Strategies (2)

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- All of the undesirable effects mentioned previously can be reduced by using the largest F# possible.
  - Apertures in the mask scale as the F#. Larger features with respect to the operating wavelength are desirable.
  - The range of incidence angles contained in the incident field's spectrum scales as 1/F#. Narrower angular spectrums are also desirable.
- In addition to choosing a large F# the mask design (parent function and sampling) should be chosen to maximize the aperture dimensions. We have investigated this through numerical optimization of some masks.

Approximate Propagation Through a Thick Aperture

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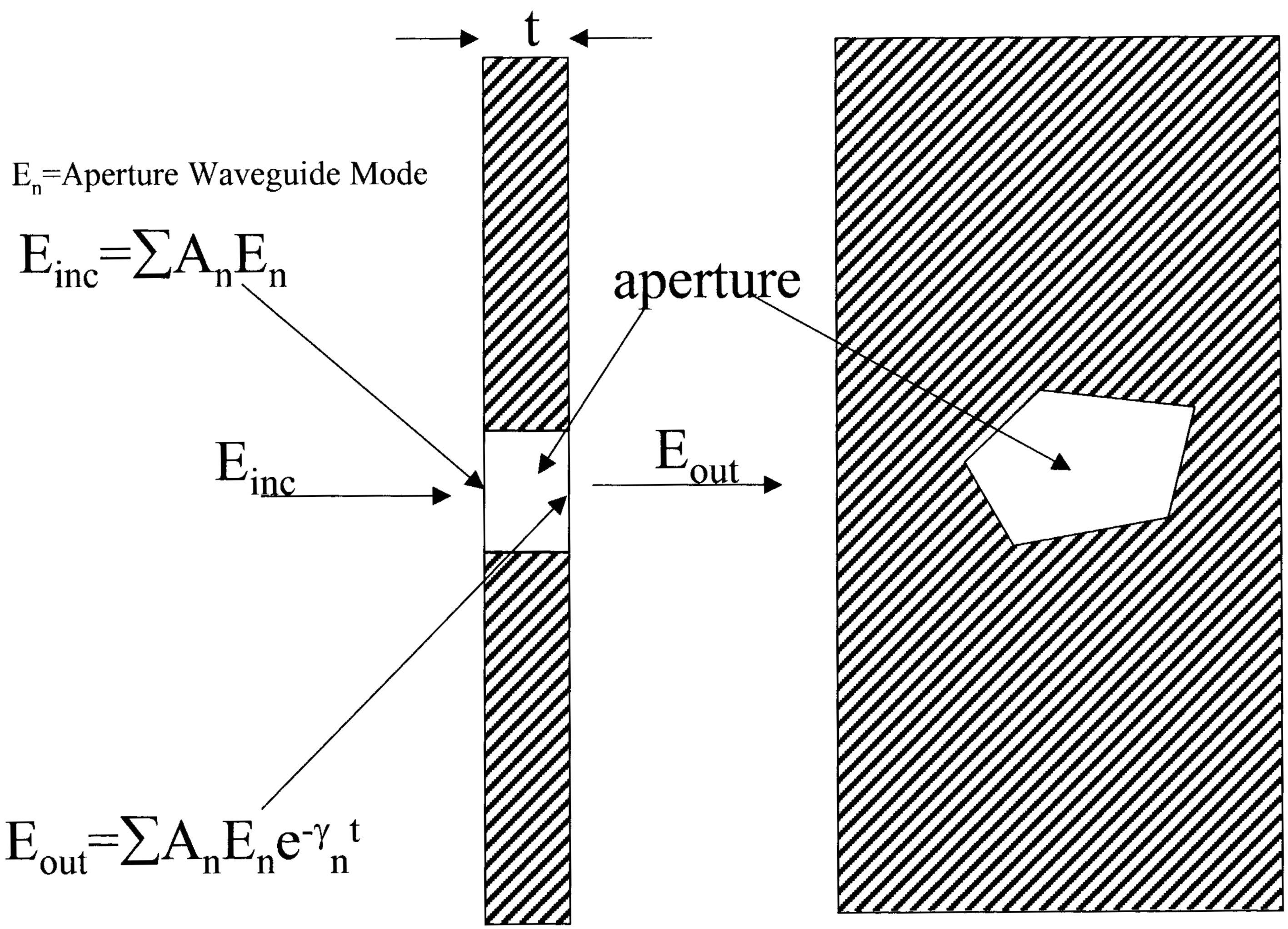

Approximate Propagation Through a Gap in a Thick Screen Using Modal Expansion

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- Apply preceding approach to the propagation of fields through a gap in a thick screen.
- Compare transmitted phase to that computed exactly in the thick results.
- Validates first-order modal expansion model for obtaining fields at the exit of the thick aperture.
- Appropriate only for parallel polarization, perpendicular polarization shows no first-order gap size effect. Need to consider edge effects in this case.
- Demonstrates fundamental result: Differential Phase Shift Over Mask Aperture Depends on Overall Scale and Ratio of Largest to Smallest *Significant* Feature Size

Average Phase Shift, Thick Screen, Parallel Polarization

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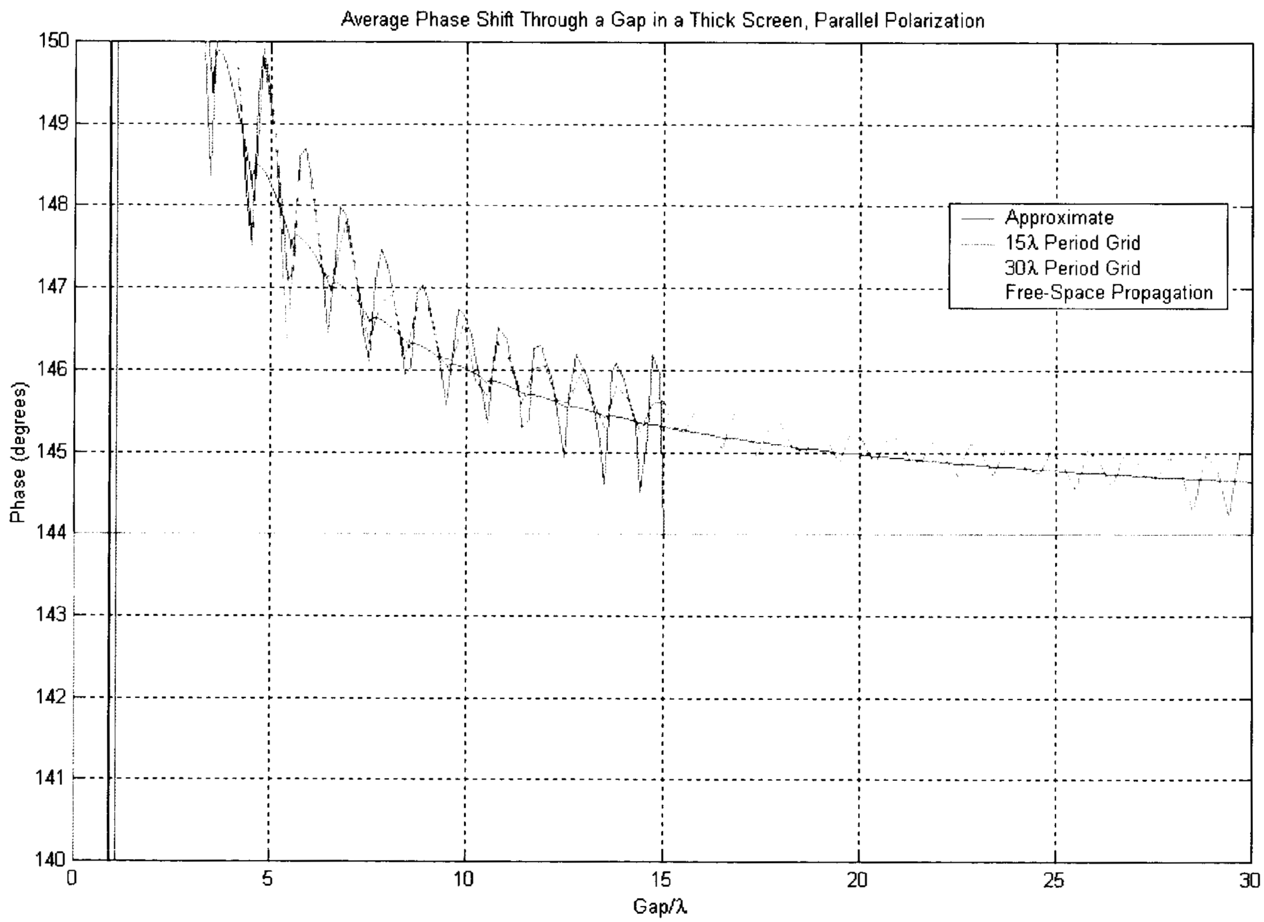

Average Phase Shift, Thick Screen, Parallel Polarization

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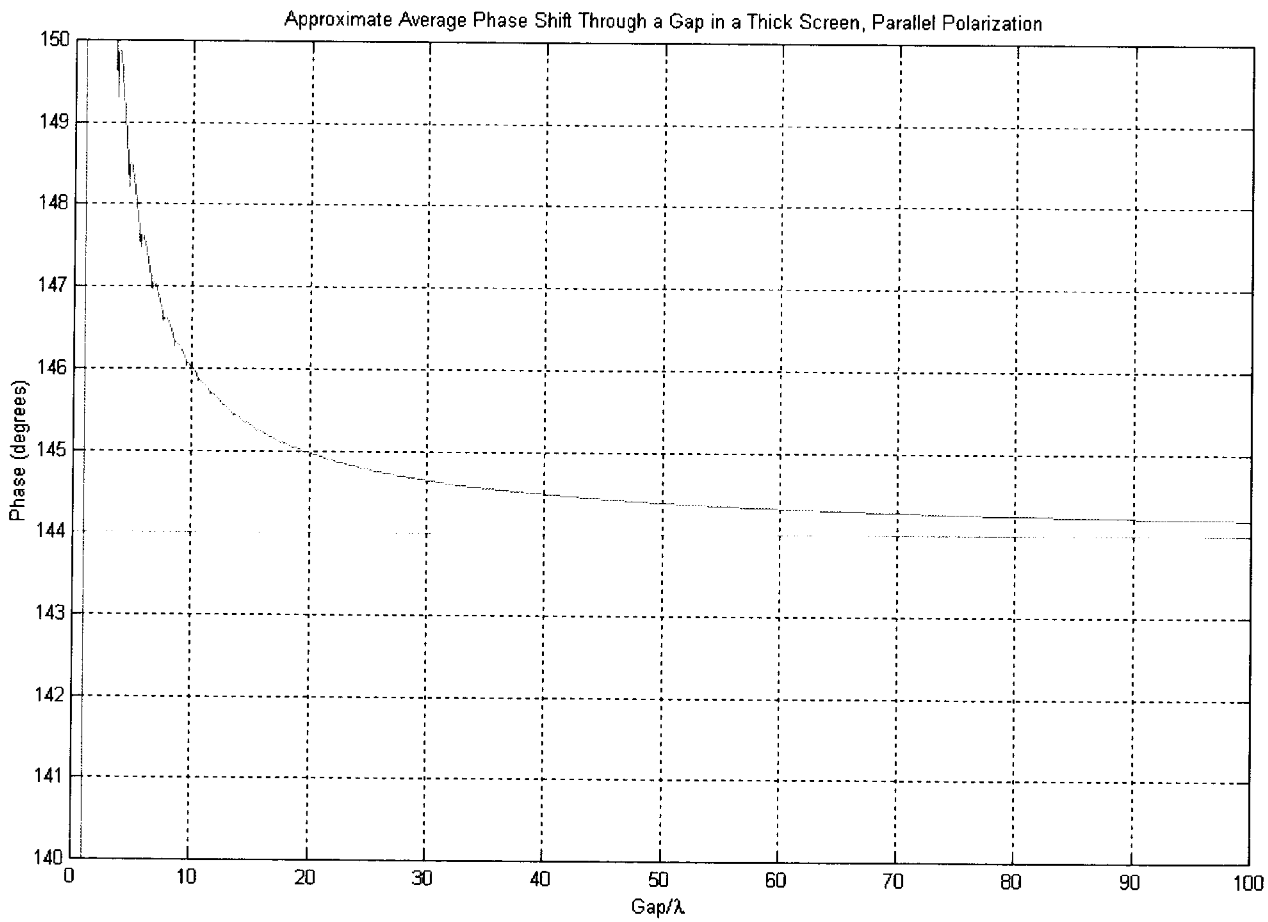

Sector-Shaped Aperture Calculation

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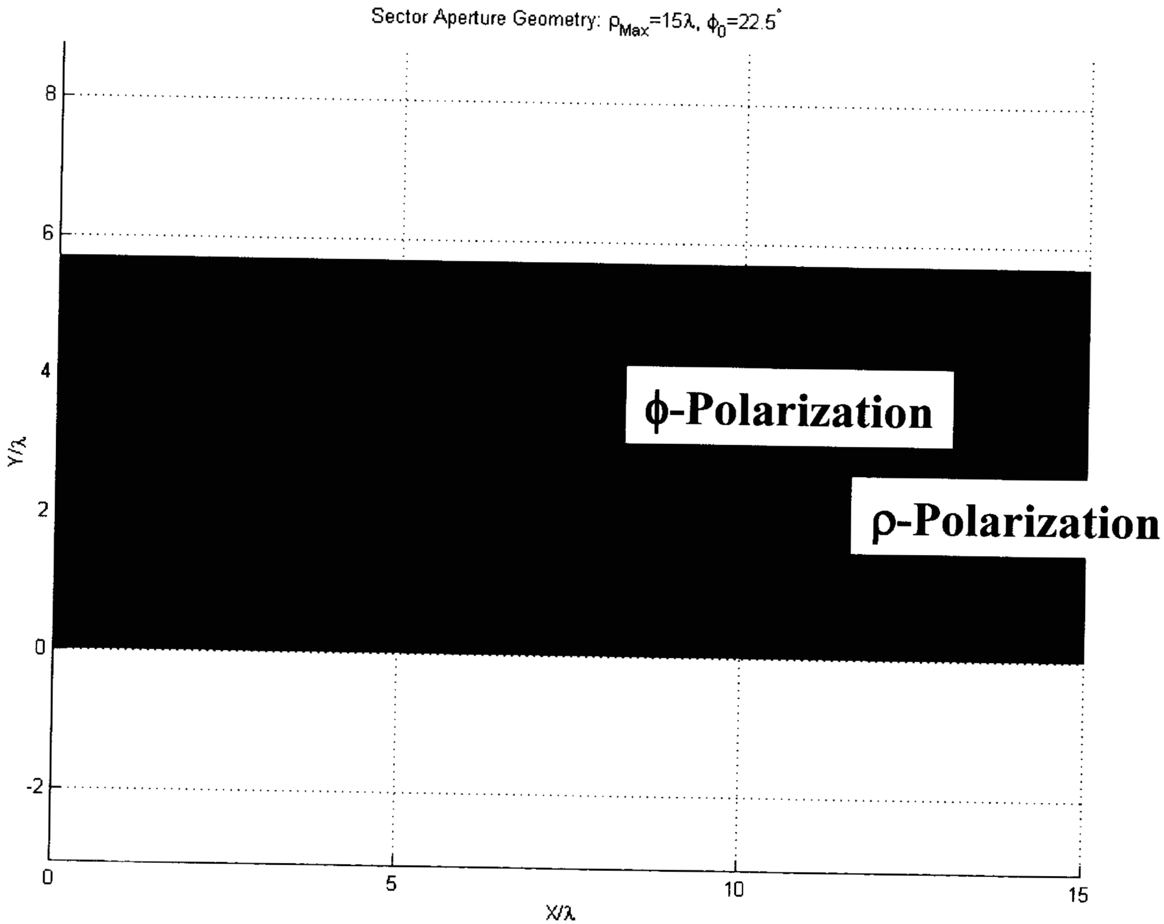
- Why a Thick Sector-Shaped Aperture?
  - Aperture has both large-scale and small scale features
  - Aperture has significant Polarization effects
  - Sector modes are analytic $\propto J_\alpha(k_\rho \rho)$, α non-integer
- Approximate Analysis Procedure
  - Expand incident plane wave (2 possible polarizations) into the waveguide mode set [match E-field only]
  - Include both propagating and evanescent modes
  - Propagate/decay modes through aperture thickness
  - Compute composite amplitude, phase, cross-polarization at output

Sector-Shaped Aperture



Sector Waveguide Modes

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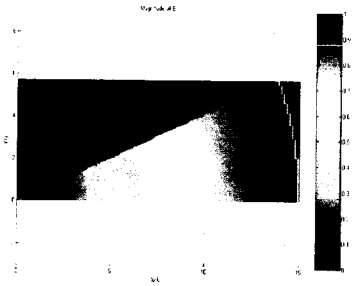
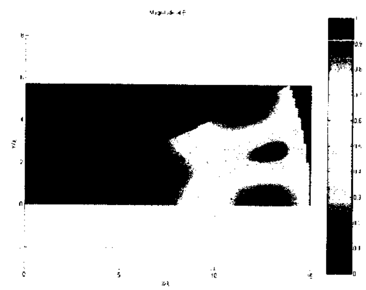
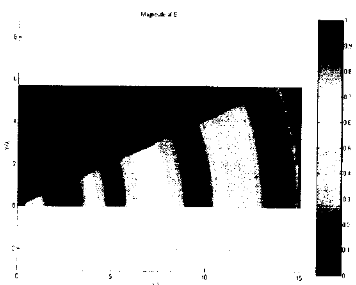


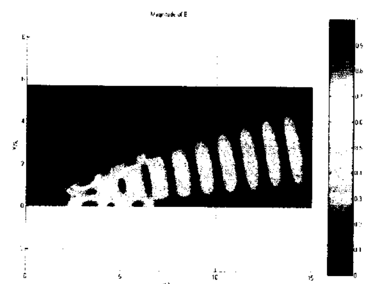


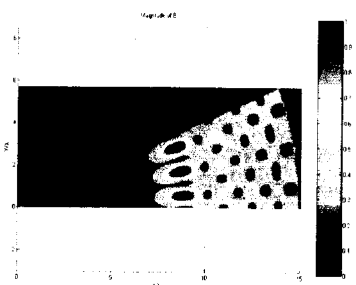
Low-Order

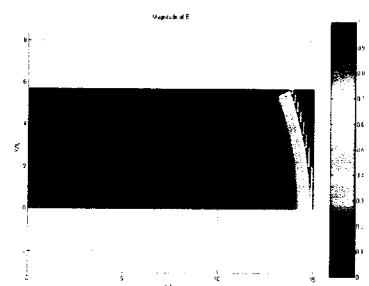


TE_z

TM_z







High-Order

ρ-Directed Field at Exit of Aperture, ρ-Directed Field Incident [Magnitude]

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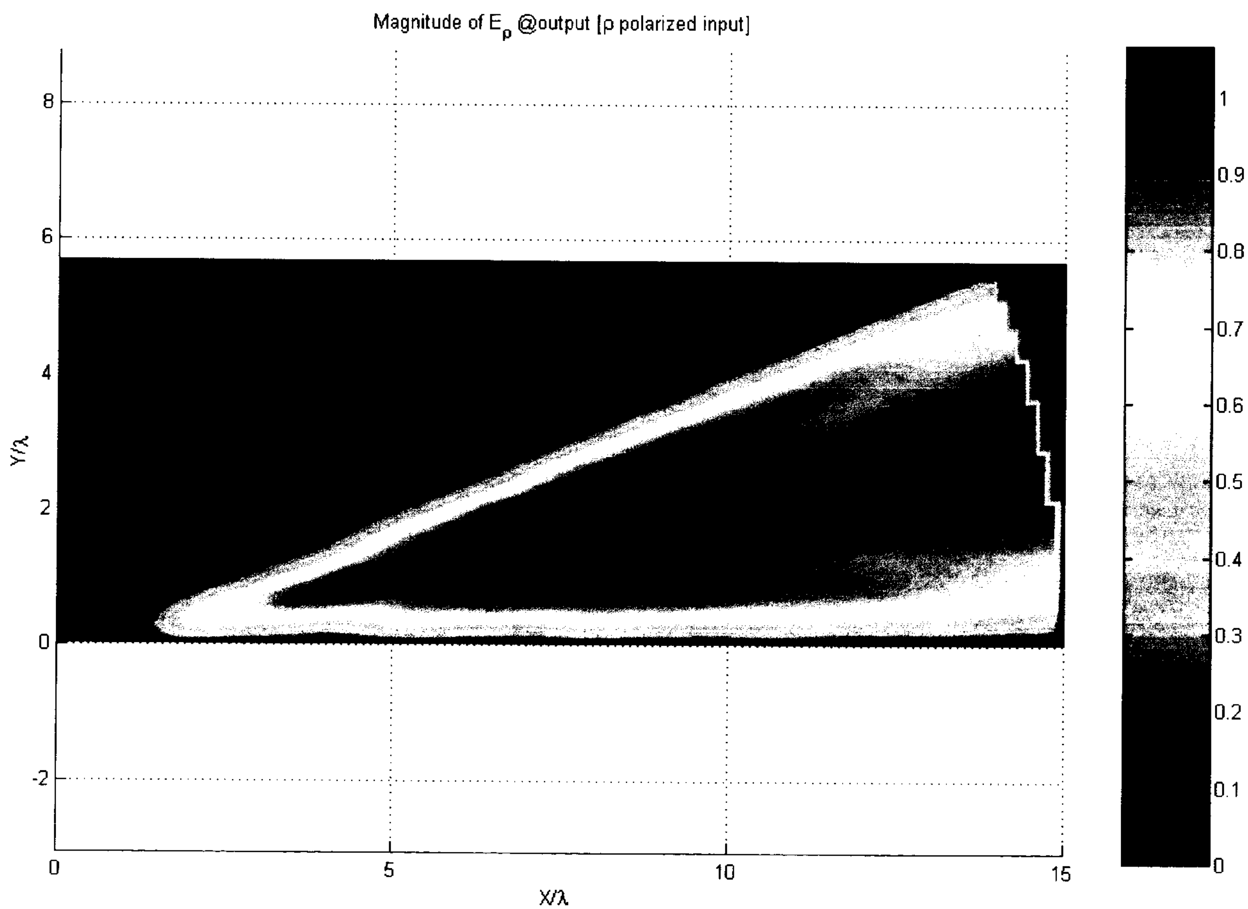

ϕ-Directed Field at Exit of Aperture, ρ-Directed Field Incident [Magnitude]

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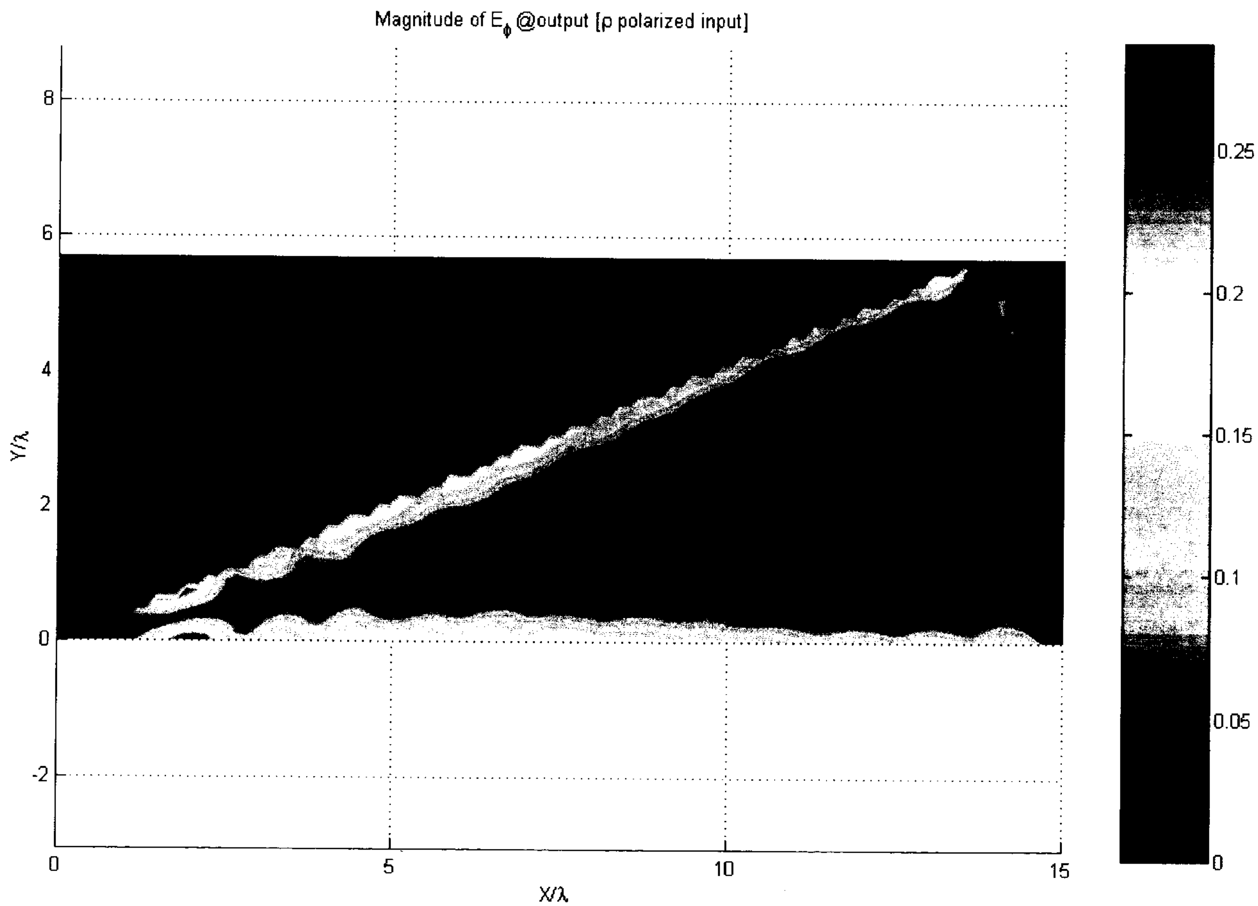

ρ-Directed Field at Exit of Aperture, ρ-Directed Field Incident [Phase]

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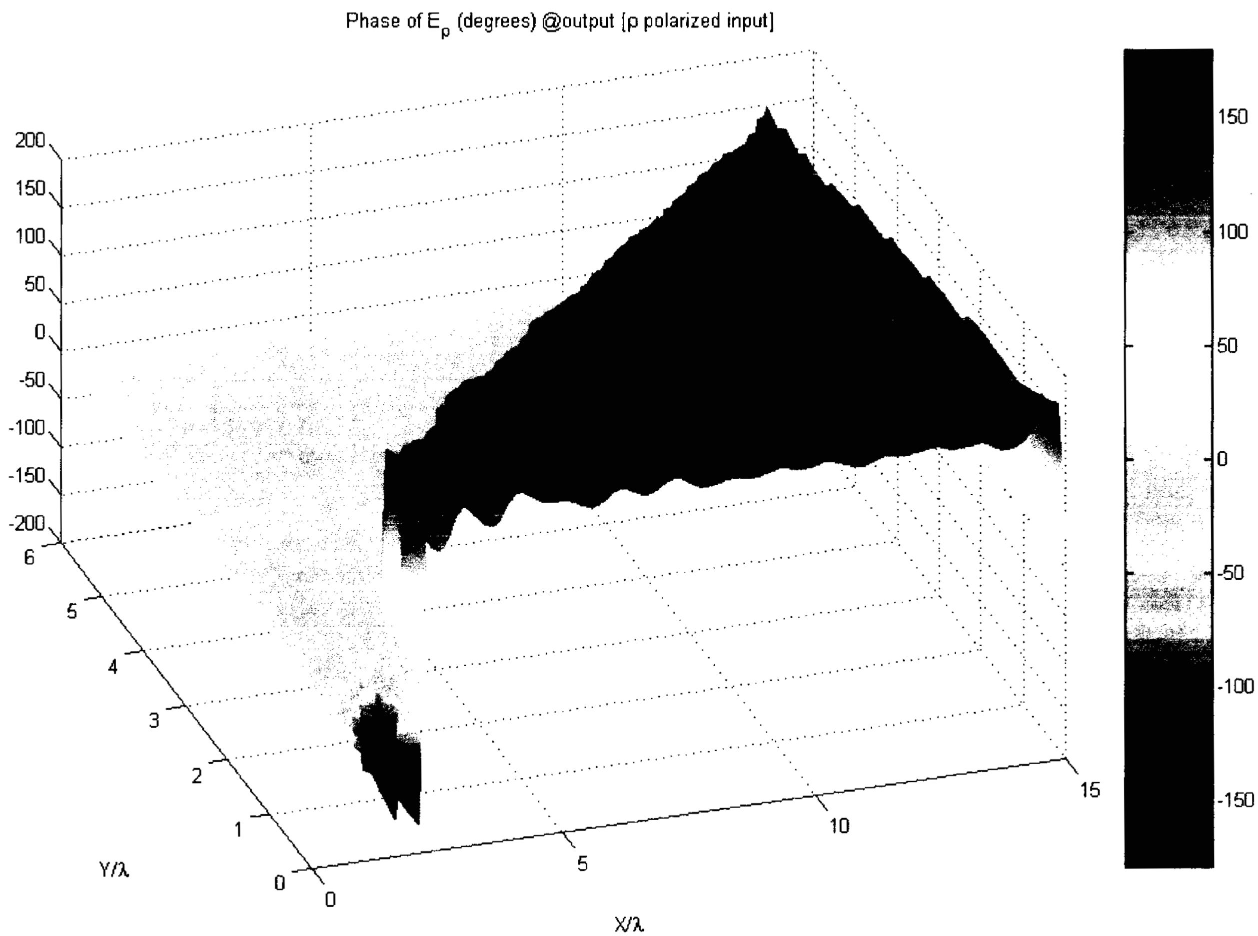

ϕ-Directed Field at Exit of Aperture, ϕ-Directed Field Incident [Magnitude]

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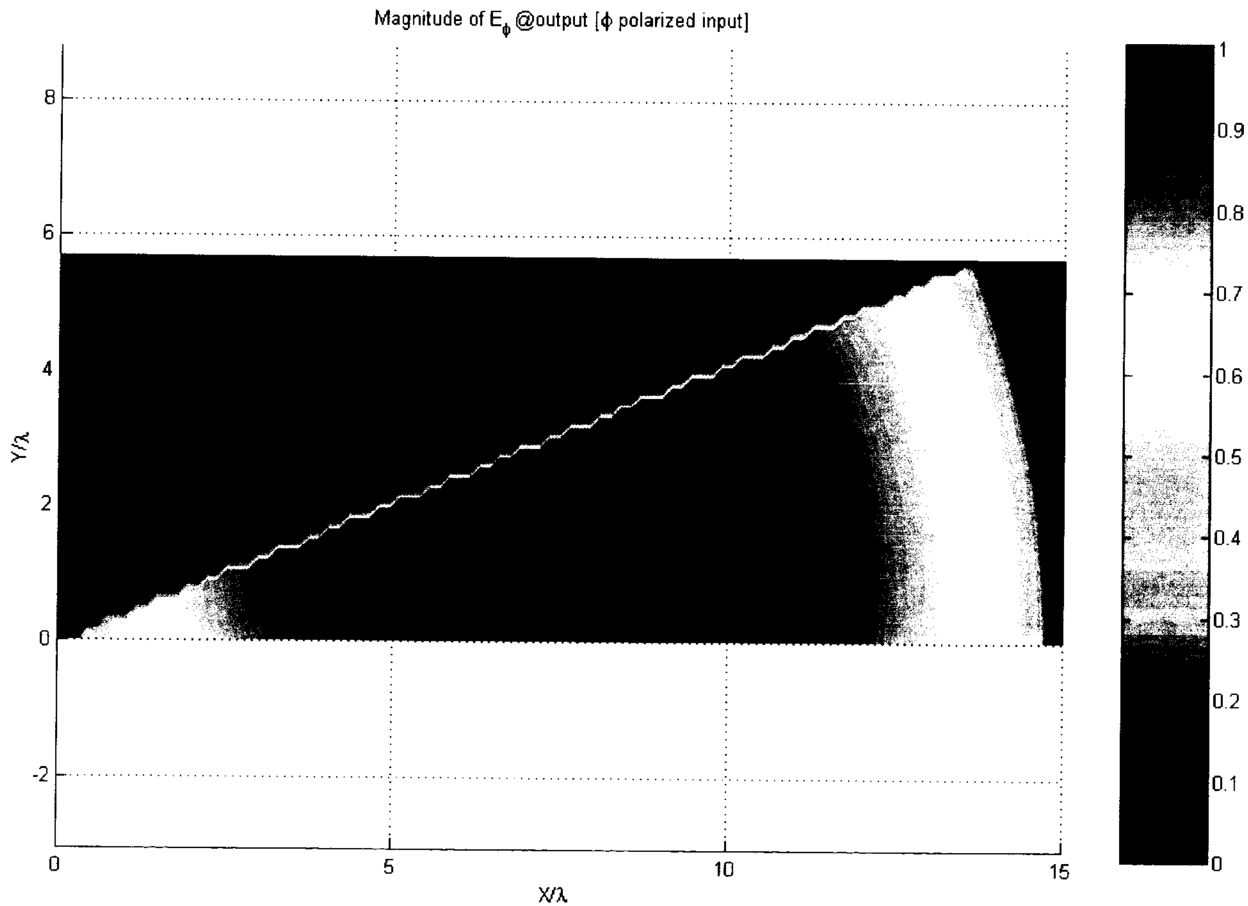

ρ-Directed Field at Exit of Aperture, ρ-Directed Field Incident [Magnitude]

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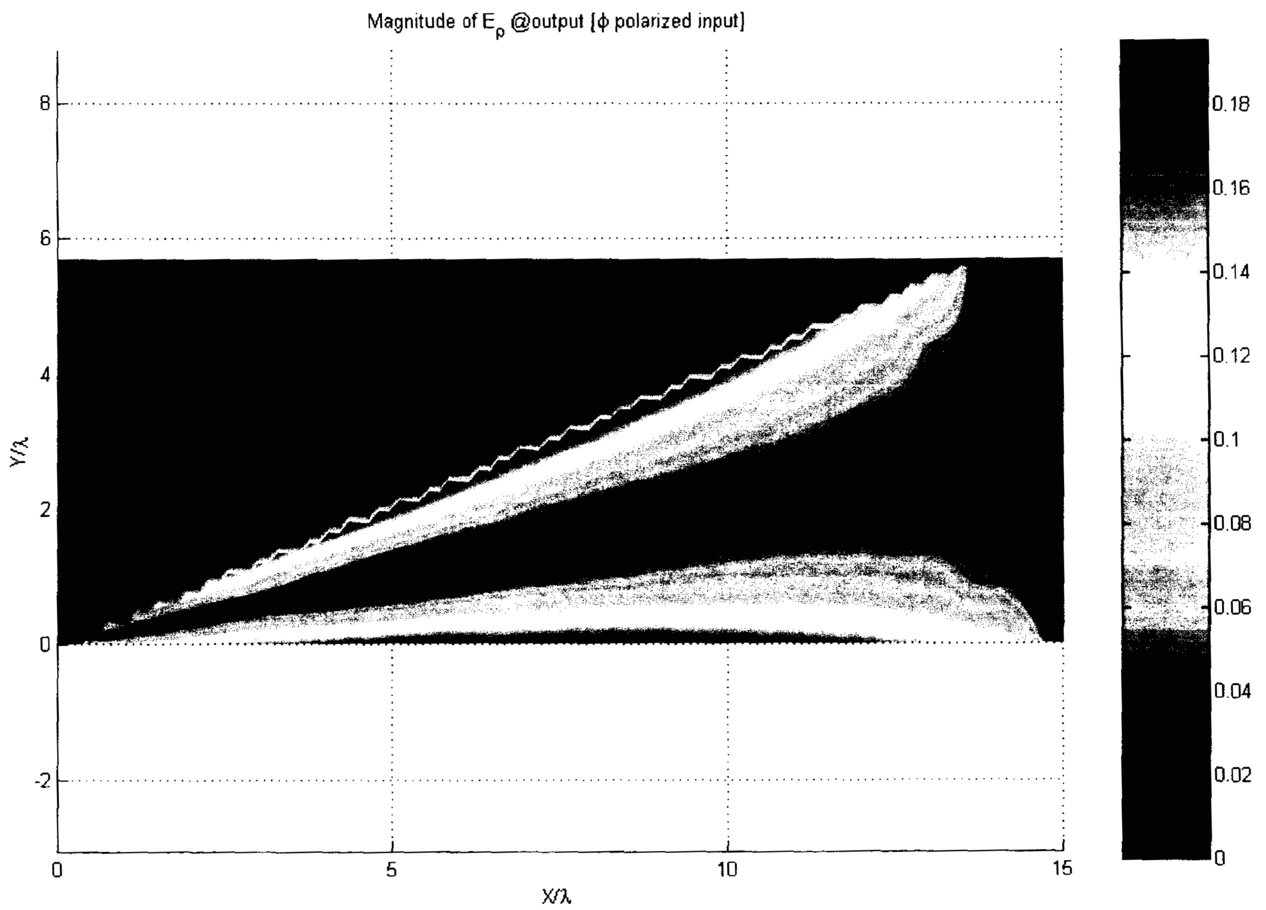

ϕ-Directed Field at Exit of Aperture, ϕ-Directed Field Incident [Phase]

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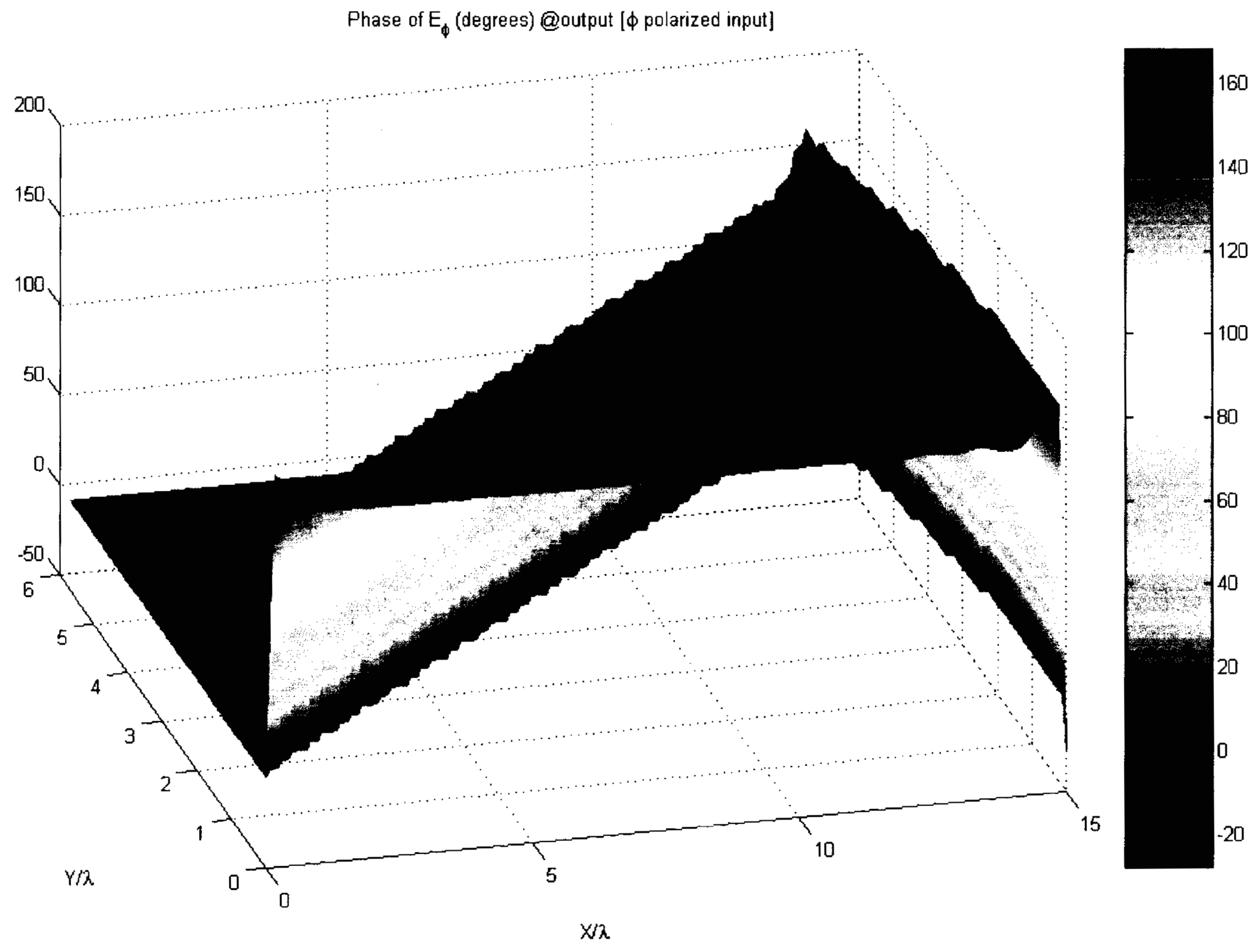

Extension of the Modal Expansion Approach to Complex Apertures

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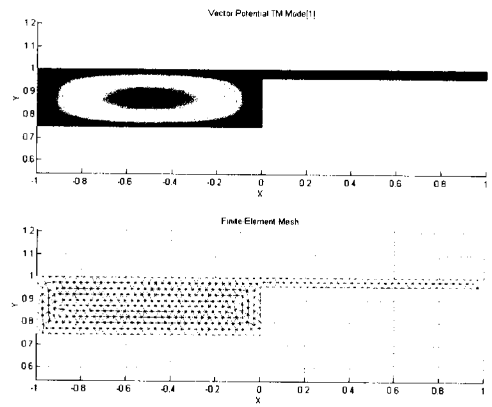
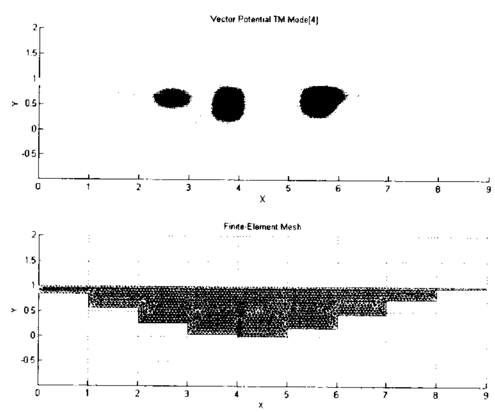


Subset of Unit Cell



Entire Unit Cell

- Determine Mode Set of Arbitrary Aperture using 2-D FEM
- Perform Simple Propagation Analysis on Arbitrary Aperture Shapes
- Solve Full Problem Exactly by Matching Both E and H Fields at Both Ends of Aperture
- Work is On-Going in this Area

Major Effects

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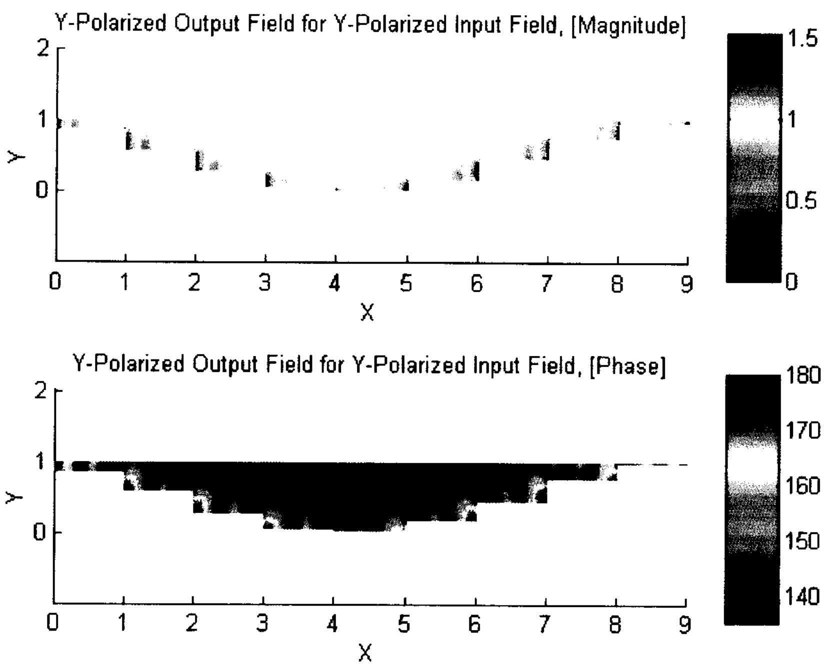
- Distortion of the Incident Field as it Propagates Through the Aperture is Due to:
  - Dispersion of the Propagating Aperture Modes
  - Decay of the Evanescent Aperture Modes
- Distortion is Most Significant Near The Aperture Edges for Which the Incident Field is Tangent
- Cross-Polarized Fields Will Appear in the Exit Aperture Near These Edges as Well
- These Effects are not Easily Controlled
- Masks that Are Tolerant Aperture Errors within a λ or so of their Edges are to be Preferred over those that are not

Binary Mask: Small-Scale Example

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California



- Small Scale (F/3) Example
- Aperture Size 30λx3λ
- Thickness 0.6λ
- Y (Perpendicular) Polarization
- FEM Solution/Modal Expansion into 300 Waveguide Modes
- Visible Effects:
  - High-Order Phase/Amplitude Ripple
  - Distortion/Shorting of Field Near PEC Edges

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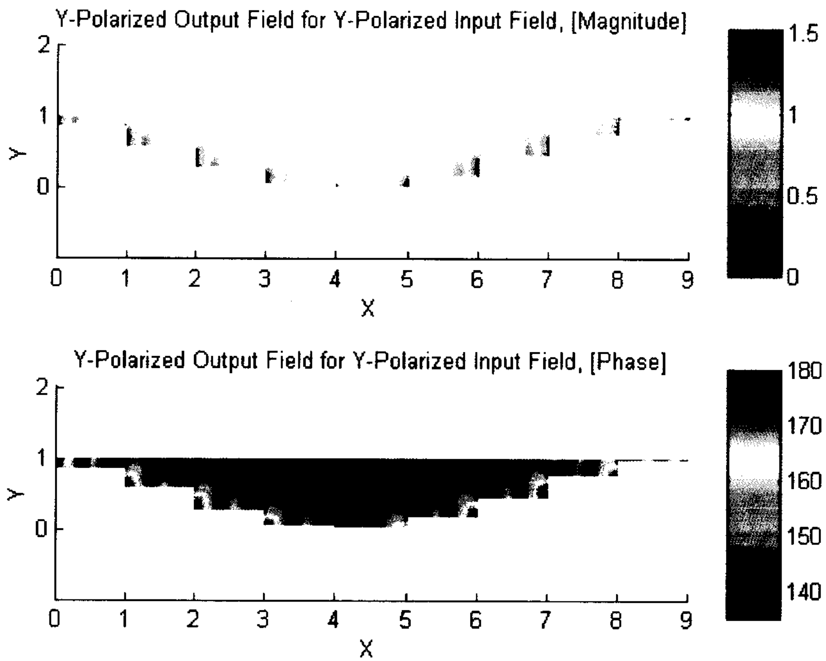

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