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Application of Dynamical Systems Theory to a Very Low Energy Transfer

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Abstract

We use lobe dynamics in the restricted three-body problem to design orbits with prescribed itineraries with respect to the resonance regions within a Hill's region. The application we envision is the design of a low energy trajectory to orbit three of Jupiter's moons using the patched three-body approximation (P3BA). We introduce the "switching region," the P3BA analogue to the "sphere of influence." Numerical results are given for the problem of finding the fastest trajectory from an initial region of phase space (escape orbits from moon A) to a target region (orbits captured by moon B) using small controls.

INTRODUCTION

Low energy trajectories have been increasingly investigated, due to the possibility of large savings in fuel cost (as compared to classical approaches) by using the natural dynamics arising from the presence of a third body. Recent work by our group gives a rigorous explanation of these phenomena by applying some techniques from dynamical systems theory to systems of n bodies considered three at a time.[1–3] We obtain a systematic way of designing ballistic lunar transfers and, more generally, trajectories with a predetermined future and past, in terms of transfer from one Hills region to another. One of the examples we have considered is an extension of the Europa Orbiter mission[4–6] to include an orbit around Ganymede.[7] More recently, we have considered a mission in which a single spacecraft would orbit three of Jupiter's planet-size moons—Callisto, Ganymede and Europa—one after the other, using very little fuel.[8] Our approach, which we have dubbed the "Multi-Moon Orbiter" (MMO), should work well with existing techniques, enhancing NASA's trajectory design capabilities for missions such as the Jupiter Icy Moons Orbiter.

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Our goal in this paper is not to construct flight-ready end-to-end trajectories, but to explore the orbital dynamics of spacecraft in the presence of the jovian moon system. In particular, we wish to determine the fuel consumption versus time of flight trade-off for the inter-moon transfer portion of a MMO mission.

By approximating a spacecraft's motion in the $n+1$ body gravitational field of Jupiter and n of its planet-sized moons into several segments of purely three body motion—involving Jupiter, the ith moon, M_i, and the spacecraft—we can design a trajectory for the spacecraft which follows a prescribed itinerary in visiting the n moons. An advantage of this patched three-body approach to mission design is that that the spacecraft will orbit a moon M_i in a capture orbit for a desired number of circuits, escape M_i in the direction of a neighboring moon M_k, become ballistically captured in a capture orbit, and so on. Instead of flybys lasting only seconds, a scientific spacecraft can orbit several moons for any desired duration. Furthermore, the total velocity change (ΔV) necessary is much less than that necessary using purely two-body motion segments.

We have found tours as low as $\sim$20 m/s vs $\sim$1500 m/s using previous methods.[4,5] In fact, this low ΔV is on the order of statistical navigation errors. The lowest energy MMO tour is shown in Figure 1. By using small impulsive maneuvers totaling only 22 m/s, a spacecraft initially injected in a jovian orbit can be directed into an elliptical capture orbit around Europa. Enroute, the spacecraft orbits both Callisto and Ganymede for long duration using a ballistic capture and escape methodology developed previously.[7] This way of designing missions is called the *patched three-body approximation* (P3BA) and will be elaborated upon further in this paper.

Trade-Off Between Fuel and Time Optimization. The dramatically low ΔV needed for the tour of Figure 1 is achieved at the expense of time—the present trajectory has a time of flight (TOF) of about four years, mostly spent in the inter-moon transfer phase. This is likely too long to be acceptable for an actual mission. With refinement, we believe the method could be applied to an actual mission, maintaining both a low ΔV for the tour and low accumulated radiation dose (a concern for an actual mission in the jovian system). Therefore, in this paper we will explore the ΔV vs TOF trade-off for the inter-moon transfer between Ganymede and Europa. What we observe is that for slightly larger ΔV, a reasonable TOF can be achieved.

THE BUILDING BLOCKS FOR DETERMINING THE ΔV VS TIME OF FLIGHT TRADE-OFF

In order to make this trade-off study computationally tractable, one needs to use simplified models. The forward-backward method in the restricted three-body problem phase space is used.[7,9] The influence of only one moon at a time is considered. Criteria are established for determining when the switch from one moon's influence to another occurs.

Much evidence[1–3] suggests that the use of invariant manifold structures related to L_1 and L_2 points (e.g., "tubes") yields fuel efficient impulsive trajectories. Using the planar circular restricted three-body problem as our baseline model, we will compute tubes over a range of three-body energies (aka, Jacobi constants). The tubes are the passageways leading toward or away from the vicinity of Lagrange points. The tubes have the numerically observed property that the larger the energy, the further the tube travels from its associated

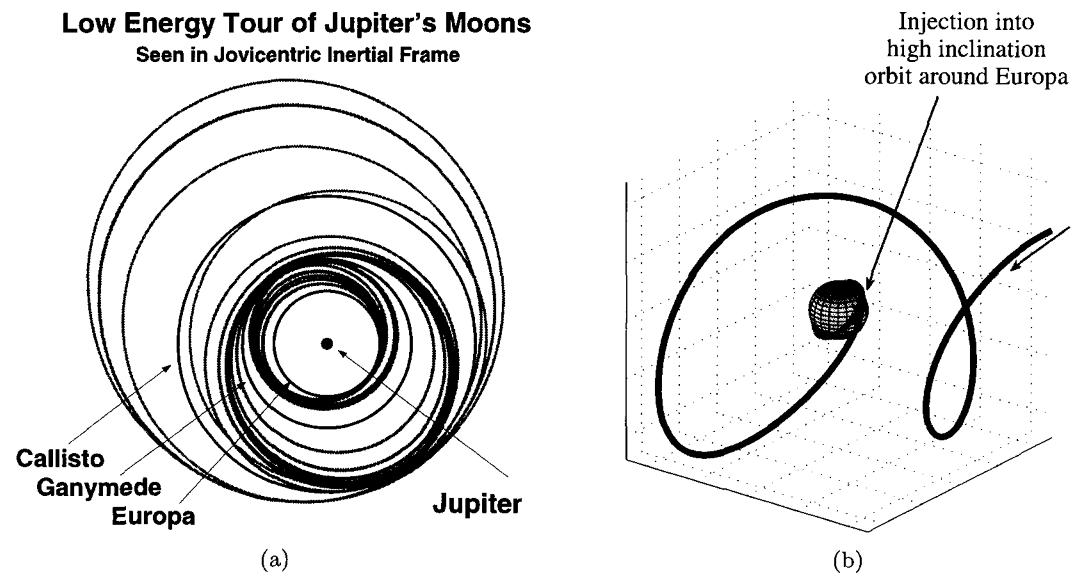


Figure 1: **The Multi-Moon Orbiter space mission concept for the jovian moons** involves long duration orbits of Callisto, Ganymede, and Europa, allowing for extensive observation. Starting in an elliptical jovian orbit with perijove near Callisto's orbit, the spacecraft trajectory gets successively reduced in jovicentric energy by resonant gravity assists with the various moons, effectively jumping to lower resonances at each close approach, as shown in (a). The trajectory has its jovicentric energy reduced by Callisto, Ganymede, and Europa, in sequence. As the orbit converges upon the orbit of Europa, it will get ballistically captured by Europa. Small corrections during the tour add up to a total ΔV of about 20 m/s, on the order of statistical navigation errors. At the end of the tour phase, the spacecraft is at a 100 km altitude periapse with respect to Europa. A ΔV of approximately 450 m/s is then needed to get into a 100 km altitude orbit about Europa, with an inclination of about 45°, as shown (b).

Lagrange point in a fixed amount of time. We will use this property to find the TOF between Ganymede and Europa as a function of the energy in their respective three-body systems. These energies can be used to calculate the ΔV of escape from each moon under certain assumptions.

First, we review the P3BA and the dynamics in the circular restricted three-body problem.

The Patched Three-Body Approximation (P3BA)

The P3BA discussed by Ross, Koon, Lo, and Marsden[8] considers the motion of a particle (or spacecraft) in the field of n bodies, considered two at a time, e.g., Jupiter and its ith moon, M_i. When the trajectory of a spacecraft comes close to the orbit of M_i, the perturbation of the spacecraft's motion away from purely Keplerian motion about Jupiter is dominated by M_i. In this situation, we say that the spacecraft's motion is well modeled by the Jupiter-M_i-spacecraft restricted three-body problem.

Tube Dynamics: Ballistic Capture and Escape. Stable and unstable invariant manifold tubes associated to bounded orbits around the libration points L_1 and L_2 are phase space structures that mediate motion to and from the smaller primary body (e.g., mediating spacecraft motion to and from Europa in the Jupiter-Europa-spacecraft system), and between primary bodies for separate three-body systems (e.g., spacecraft motion between Europa and Ganymede in the Jupiter-Europa-spacecraft and the Jupiter-Ganymede-spacecraft systems).[1]

These invariant manifold tubes can be used to produce new techniques for constructing spacecraft trajectories with interesting characteristics.

The design of a MMO of the jovian system is guided by two main ideas.[7,8]

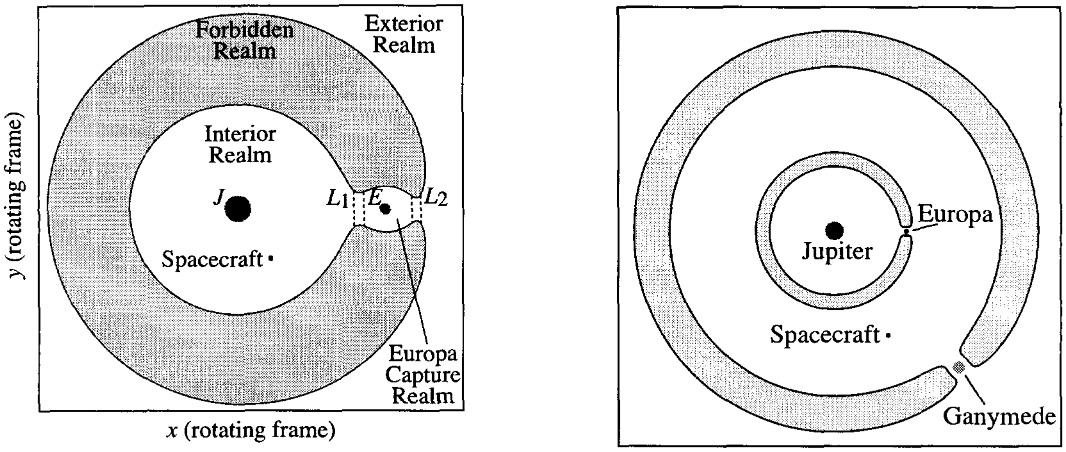
1. The motion of the spacecraft in the gravitational field of the three bodies Jupiter, Ganymede, and Europa is approximated by two segments of purely three body motion in the circular, restricted three-body model. The trajectory segment in the first three body system, Jupiter-Ganymede-spacecraft, is appropriately patched to the segment in the Jupiter-Europa-spacecraft three-body system.

2. For each segment of purely three body motion, the invariant manifolds tubes of L_1 and L_2 bound orbits (including periodic orbits) leading toward or away from temporary capture around a moon, as in Figure 2, are used to construct an orbit with the desired behaviors. This initial solution is then refined to obtain a trajectory in a more accurate four-body model.

Inter-Moon Transfer. During the inter-moon transfer–where one wants to leave a moon and transfer to another–the control problem becomes one of performing appropriate small ΔV's to decrease the jovicentric orbit energy by jumping between orbital resonances with a moon, i.e., performing resonant gravity assists. This is illustrated in the schematic spacecraft trajectory shown in Figure 3.

After the spacecraft escapes from the vicinity of the outer moon, the outer moon's perturbation is only significant over a small portion of the spacecraft trajectory near apojove (A). The effect of the moon is to impart an impulse to the spacecraft, equivalent to a ΔV in the absence of the moon. The strategy to achieve consecutive gravity assists is to maneuver the spacecraft to pass through apojove a little *behind* the moon. As illustrated in Figure 3(a), the result is a *decrease in the perijove* of the spacecraft's orbit, while the apojove remains (mostly) constant in inertial space due to the conservation of the three-body energy. As long as the spacecraft's trajectory repeatedly targets apojove a little behind the moon, it will decrease its perijove once more, and so on.

Switching Orbit. During the inter-moon transfer trajectory, there comes an arc of the spacecraft's trajectory at which the spacecraft's perturbation switches from being dominated by moon M_1 to being dominated by a nearby moon, M_2. A rocket burn maneuver need not be necessary to effect this switch. The set of possible "switching orbits," which we will refer to as the "switching region," is the analogue to the "sphere of influence" concept used in the patched-conic approximation, which guides a mission designer regarding when to switch the central body for the model of the spacecraft's Keplerian motion.



(a) (b)

Figure 2: **The patched three-body approximation.**(a) The co-orbiting frame with Europa is shown, otherwise known as the *rotating frame.* The spacecraft's motion in each Jupiter-moon-spacecraft rotating frame is limited to the region in white due to constant energy in that system (constant Jacobi integral). We work with three-body energy regimes where the region surrounding the moon's orbit (shaded) is energetically forbidden to spacecraft motion. Note small opening near moon, permitting capture and escape. (b) The four-body system approximated as two nested three-body systems. This picture is only a schematic, as the spacecraft's motion conserves the three-body energy (aka, Jacobi integral) in only one three-body system at a time.

Consider again Figure 3(a). Once the spacecraft orbit comes close to grazing the orbit of the inner moon, the inner moon takes "control" (has the dominant effect) and the outer moon no longer has much effect. The spacecraft orbit where this occurs is denoted E in Figures 3(a) and 3(b). The spacecraft now gets gravity assists from the inner moon at perijove (P). Once again, we use small maneuvers to maintain the near-resonance condition, i.e., pass through perijove a little *ahead* of the moon. This causes the *apojove to decrease* at every close encounter with the inner moon, causing the spacecraft's orbit to get more and more circular, as in Figure 3(b). When a particular resonance is reached, the spacecraft can then be ballistically captured by the inner moon at M_2.[1] We note that a similar phenomenon has been observed in previous studies of Earth to lunar transfer trajectories.[9,10]

Resonant Structure of Phase Space. In order to obtain very low energy trajectories like the Multi-Moon Orbiter shown in Figure 1, it is critical to consider the resonant structure of the phase space in the interior and exterior Hill's regions. This is because the switching region between neighboring pairs of moons can only be accessed by traversing several subregions of a Hill's region, otherwise known as "resonance regions," where the resonance is between the periods of the spacecraft and the moon around Jupiter, respectively.

Early investigation into the phase space of the restricted three-body problem using Poincaré maps revealed a phase space consisting of mixed regular and chaotic motion, described as a series of overlapping resonance regions.[9,11] Lobe dynamics provides a general theoretical framework, based on invariant manifold ideas from dynamical systems theory,

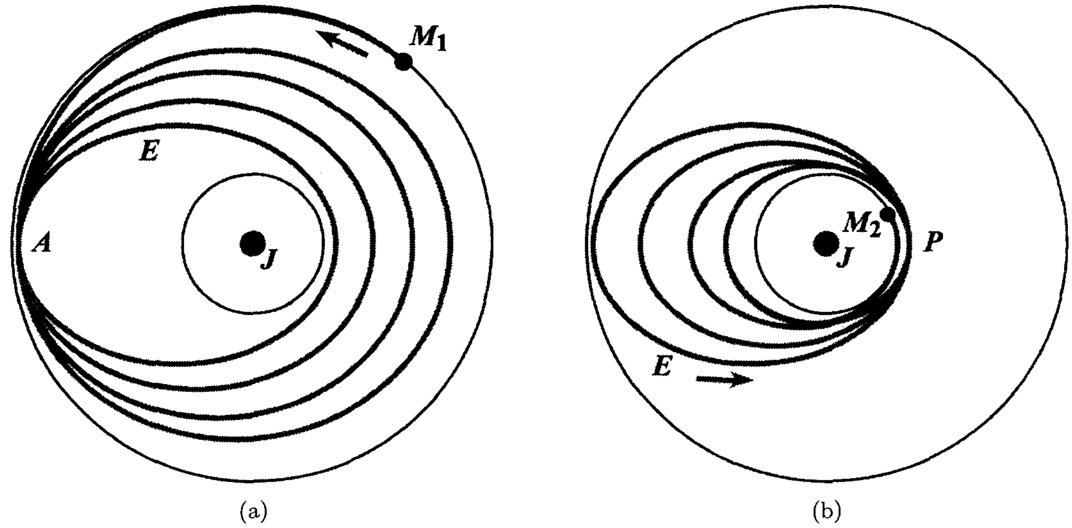


Figure 3: **Inter-moon transfer via resonant gravity assists.** (a) The orbits of two Jovian moons are shown as circles. Upon exiting the outer moon's sphere-of-influence at M_1, the spacecraft proceeds under third body effects onto an elliptical orbit. The spacecraft gets a gravity assist from the outer moon when it passes through apojove (denoted A). The several flybys exhibit roughly the same spacecraft/moon geometry because the spacecraft orbit is in near-resonance with the moon's orbital period and therefore must encounter the moon at about the same point in its orbit each time. Once the spacecraft orbit comes close to grazing the orbit of the inner moon, the inner moon takes "control." The spacecraft orbit where this occurs is denoted E. (b) The spacecraft now receives gravity assists from the inner moon at perijove (P), where the near-resonance condition also applies. The spacecraft is then ballistically captured by the inner moon at M_2.

for discussing, describing and quantifying organized structures in a mixed regular/chaotic phase space and determining their influence on transport. In particular, using lobe dynamics we can discover, describe, and quantify barriers, transport "alleyways," and statistical quantities of transport.[12]

In other words, lobe dynamics tells us the most important spacecraft trajectories, i.e., the uncontrolled trajectories which traverse a set of subregions in the shortest time. Consequently, it should prove useful for designing low energy spacecraft trajectories. In this study, we will use tube dynamics along with lobe dynamics to design orbits which quickly traverse the space between moons during the inter-moon transfer phase. Essentially, the lobes guide pieces of the tube across resonances. We can numerically determine the fastest trajectory from an initial region of phase space (e.g., orbits which have just escaped from moon M_i) to a target region (e.g., orbits which will soon be captured by a neighboring moon M_k). This yields the ΔV vs. TOF trade-off for the inter-moon transfer between Ganymede and Europa.

NUMERICAL RESULTS FOR THE ΔV VS TIME OF FLIGHT TRADE-OFF

Method Description. In order to do a trade study of transfers between orbits around Ganymede and Europa, we can initially consider an impulsive transfer from a Ganymede L_1 orbit (denoted Ga_{L1}) to a Europa L_2 orbit (Eu_{L2}). If we find such a transfer, we know that a transfer between orbits around Ganymede and Europa is nearby in phase space.[1] We can break the transfer into two pieces.

1. In the first piece, we consider the transfer along the unstable manifold tube of a Ga_{L1}, which we denote $U(\mathrm{Ga}_{L1})$.[a] The set of all Ga_{L1}'s is parameterized by the energy E_{Ga}, one of our tunable parameters. For each E_{Ga}, one can compute the Ga_{L1} and $U(\mathrm{Ga}_{L1})$. One can determine the trajectory within $U(\mathrm{Ga}_{L1})$ which takes the least time to transfer to a perijove distance r_p, equal to the approximate radial distance from Jupiter of Europa's L_2 point. In Figure 4, we illustrate the method for numerically constructing natural trajectory arcs which will switch control from Ganymede to Europa. The time of flight of this portion of the inter-moon transfer trajectory, T_{Ga}, is seen to be a function of E_{Ga}.

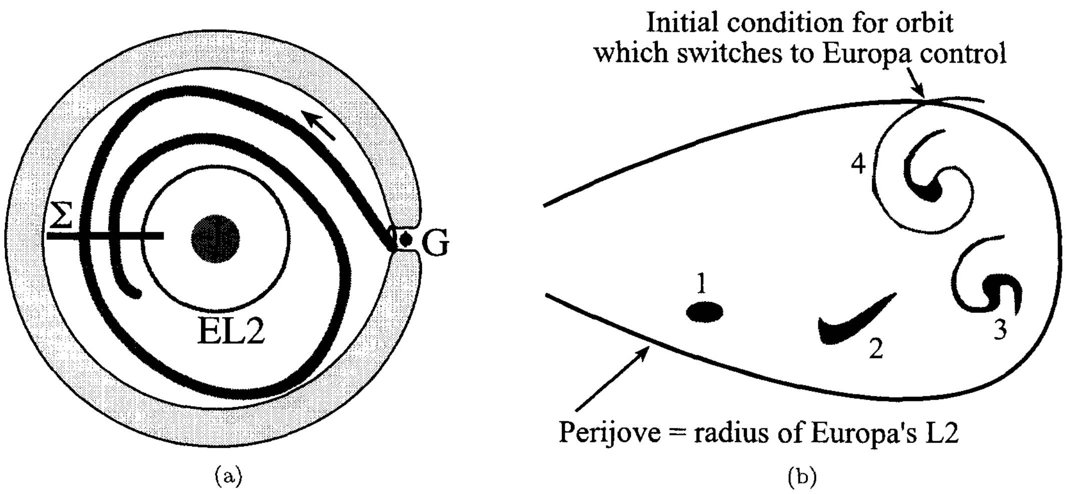


Figure 4: **Numerical construction of natural trajectory arcs which will switch control from Ganymede to Europa.** Suppose we want to find trajectories which begin near Ganymede (G in the figure) and escape toward Europa, finally getting naturally captured by Europa. The first step is to numerically construct the Ganymede L2 tube heading toward Europa, or $U(\mathrm{Ga}_{L1})$ in the terminology of the text. We take a Poincaré section, Σ, at the position shown in (a). We show only two crossings of Σ, but there are an infinite number. We also show the radial distance of Europa's L_2, labeled EL2. In (b), we show the initial cross-section of the tube on Σ, labeled 1. The successive crossings are labeled 2, 3, ... In this schematic, we also show the line corresponding to a perijove equal to the radial distance of Europa's L_2. Any spacecraft trajectory in the Jupiter-Ganymede-spacecraft system which crosses this line can be assumed to "switch" control to Europa, meaning the Jupiter-Europa-spacecraft system becomes a good approximation.

[a] $U(\mathrm{Ga}_{L1})$ has two branches, but we consider the one heading initially in the direction of Europa's orbit.

2. For the second piece, we consider the transfer along the stable manifold tube of a Eu_{L2}, which we denote $S(\mathrm{Eu}_{L2})$.[b] The set of all Eu_{L2}'s is parameterized by the energy E_{Eu}, one of our tunable parameters. For each E_{Eu}, one can compute the Eu_{L2} and $S(\mathrm{Eu}_{L2})$. One can determine the trajectory within $S(\mathrm{Eu}_{L2})$ which takes the least time to transfer to an apojove distance r_a, equal to the approximate radial distance from Jupiter of Ganymede's L_1 point. The time of flight of this trajectory, T_{Eu}, is seen to be a function of E_{Eu}.

The sum, $\mathrm{TOF} = T_{\mathrm{Ga}} + T_{\mathrm{Eu}}$, is an approximate inter-moon transfer time. The total fuel expenditure, ΔV_{tot}, needed to perform the transfer can be estimated as follows. We assume only two impulsive maneuvers, ΔV_{Ga} and ΔV_{Eu}.

ΔV_{Ga} = the ΔV to escape from the scientific orbit around Ganymede can be estimated from the difference between the energy of the transfer away from Ganymede, E_{Ga} and the energy of the scientific orbit at Ganymede, E_{GaO}.

ΔV_{Eu} = the ΔV to enter the scientific orbit around Europa can be estimated from the difference between the energy of the transfer toward Europa, E_{Eu}, and the energy of the scientific orbit at Europa, $E_{\mathrm{Eu}}O$.

The total fuel expenditure is the sum, $\Delta V_{\mathrm{tot}} = \Delta V_{\mathrm{Ga}} + \Delta V_{\mathrm{Eu}}$. We suppose that E_{GaO} and E_{EuO} are given. We can then perform this method for a range of tunable parameters (E_{Ga} and E_{Eu}), to determine the fuel consumption (ΔV_{tot}) versus time of flight (TOF) trade-off.

Computing the Delta-V's. We assume that portions of each tube quickly reach a periapse of 100 km altitude above each moon, and that the solutions which do this are close in phase space to the transfer solutions found, assumptions justified by earlier work.[1,13] Given these assumptions, we can estimate ΔV_{Ga} and ΔV_{Eu} as follows. In the rotating frame of a Jupiter-moon-spacecraft three-body system, a spacecraft with a velocity magnitude v has a three-body energy

$$E = \frac{1}{2}v^2 + \bar{U}, \tag{1}$$

where the effective potential, a function of position, is

$$\bar{U} = -\frac{1}{2}r^2 - \frac{1-\mu}{r_J} - \frac{\mu}{r_M}, \tag{2}$$

where μ is the mass ratio $\frac{m_M}{m_J + m_M}$, r_J is the spacecraft's distance from Jupiter's center, r_M the distance from the moon's center, and r the distance from the Jupiter-moon center of mass, which is very close to Jupiter. At a distance of 100 km altitude above the moon, we are very close to the moon. Therefore, $r \approx r_J \approx 1$, and we can approximate Eq. (2) as

$$\begin{aligned} \bar{U} &\approx -\frac{1}{2}(1)^2 - \frac{1-\mu}{1} - \frac{\mu}{r_M}, \\ \bar{U} &\approx -\frac{1}{2} - 1 - \frac{\mu}{r_M}, \\ \bar{U} &\approx -\frac{3}{2} - \frac{\mu}{r_M}. \end{aligned}$$

[b] $S(\mathrm{Eu}_{L2})$ has two branches, but we consider the one heading initially in the direction of Ganymede's orbit.

Using Eq. (1), the velocity can then be approximated as

$$v \approx \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E)}. \tag{3}$$

Therefore, the approximate ΔV to go between energies E_1 and E_2 while at the same distance r_M close to the moon is

$$\Delta V \approx \left| \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E_1)} - \sqrt{2(\frac{\mu}{r_M} + \frac{3}{2} + E_2)} \right|. \tag{4}$$

We can use the above equation to compute ΔV_{Ga} given $E_1 = E_{\mathrm{Ga}}$ and $E_2 = E_{\mathrm{GaO}}$. For this study, we take E_{GaO} to be the energy of L_1 in the Jupiter-Ganymede-spacecraft system. This corresponds to a bound elliptical orbit around Ganymede which is just below the energy threshold of escape. We can perform similar calculations for ΔV_{Eu}.

The result of tabulating $\Delta V_{\mathrm{tot}} = \Delta V_{\mathrm{Ga}} + \Delta V_{\mathrm{Eu}}$ for each TOF $= T_{\mathrm{Ga}} + T_{\mathrm{Eu}}$ for several cases is given in Figure 5(a). We find a near linear relationship between ΔV and time of flight. The data for the Europa Orbiter endgame from Ludwinski et al.[5] is shown for comparison and labeled EO. For this study we looked at a range of energies in both three-body systems. The highest energy (and lowest TOF) transfer we computed is shown in

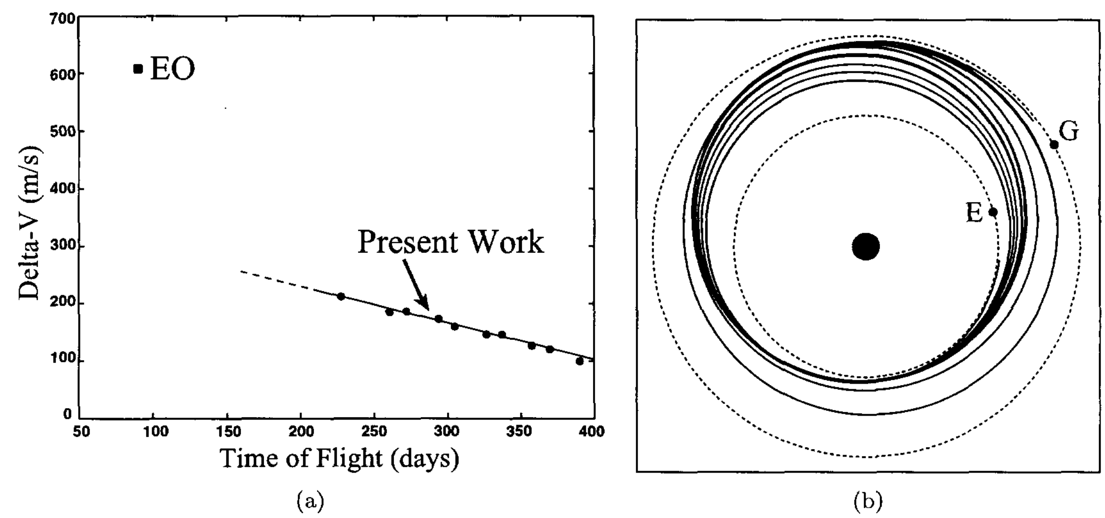


Figure 5: **Fuel consumption versus flight time trade-off for the inter-moon transfer phase of a multi-moon orbiter mission.** (a) The ΔV vs time of flight plot for several transfer trajectories from Ganymede to Europa are shown. For the several cases run, we find a near linear relationship between ΔV and time of flight. The data for the Europa Orbiter endgame from Ludwinski et al.[5] is shown for comparison and labeled EO. For this study we looked at a range of energies in both three-body systems. The highest energy (and lowest TOF) transfer we computed is shown in (b) in inertial coordinates, where G labels Ganymede's orbit and E labels Europa's. This transfer had a TOF of 227 days and a ΔV of 211 m/s. Beyond this lower TOF limit to our computations, we speculate that the linearity will continue for a while then the curve will bend upward, joining the branch of patched-conic solutions, as in Figure 9 of Ross et al.[8]

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FUTURE WORK

In addition to exploring the important compromise between time and fuel optimization, future studies will investigate the following.

- *The use of low-thrust continuous propulsion and optimal control:* The maturity of current ion engine technology has brought low thrust controls into the practical world of mission design in industry and in NASA (cf. the *Deep Space 1 mission*). Similar work is being done at the European Space Agency as well. Therefore, low thrust trajectory control is of great interest to current mission design. Our current work on the MMO considers several small impulsive burns. But an actual mission may want to save on spacecraft weight by using low thrust propulsion. How could our method be modified to incorporate low-thrust? Theoretically, one of the most favored approaches is to use optimal control in generating low thrust trajectories. We have found that a good first guess is often vital for numerical optimization algorithms, especially for an n-body problem, which is numerically very sensitive. Dynamical systems theory can provide geometrical insight into the structure of the problem and even good approximate solutions, as we found in an earlier paper.[14] There is evidence that optimal trajectories using multiple low thrust burns are "geometrically similar" to impulsive solutions.[10,15] Thus, multiple burn impulsive trajectories that we construct for the MMO can be good first guesses for an optimization scheme which uses low thrust propulsion to produce a fuel efficient mission.

- *Radiation effects:* The current model does not include radiation effects. Evidence suggests it is desirable to keep the spacecraft outside of a 12 R_J from Jupiter, in which the radiation may destroy sensitive electronics on board the spacecraft. The orbit of Europa is located at 10 R_J, so the transfer between Ganymede and Europa must minimize the time spent near its perijove for the final resonant gravity assists that lead to a capture by Europa.[c] One needs to determine what is the best way to minimize radiation effects and still achieve a very low thrust transfer. On the other hand, the strong magnetic field of Jupiter may make the use of tethers a viable propulsion or power generation option.

- *More control over operational orbits for scientific observation:* For a mission to Europa and the other moons, some control strategy is necessary to maximize desirable scientific observation and avoid collisions with the moon surface or escape from the moon's vicinity. Exotic strategies might be considered. For instance, what is the optimal thrusting strategy during the ballistic capture approach in order to achieve an operational orbit which maximizes observation time over an interesting portion of a moon's surface? Also, what is the optimal station-keeping strategy for elliptical

[c]The perijove for this portion of the tour is just outside the radial distance of Europa's L_2 point, which is located slightly within 12 R_J.

operational orbits? In the short term, it may be desirable to target particular stable operational orbits, but still save fuel using third body effects.[13,16]

- *Autonomous on-board navigation and control:* A trajectory of this type, which is sensitive to ΔV errors and modeling errors, will need to have the capability of autonomous on-board navigation and control. The first step toward this which one can look at is the trajectory correction maneuver problem, in which errors are modeled and a control algorithm corrects for those errors.[14]

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