NONLINEAR BAROTROPIC INSTABILITY IN THE STRATOSPHERE

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1. INTRODUCTION

Several linear studies suggest that planetary scale waves in the stratosphere may arise from barotropic instability. Features such as the eastward moving 4-day wave 1 (Hartmann 1983; Manney 1991; Randel and Lait 1991; and references therein), and the eastward traveling wave 2 and wave 3 in the southern hemisphere winter mid-stratosphere (Manney et al. 1991, and references therein) have characteristics suggesting they may arise from barotropic instability. Manney et al. (1988) showed that barotropically unstable modes may also arise from an easterly jet during summer with periods and spatial structures similar to those of eastward moving waves observed near the mesospheric easterly jet (Burks and Leovy 1986, and references therein).

It has long been recognized (see, for example, Pedlosky 1981) that the most unstable mode in a linear model is not necessarily the one that attains largest amplitude when nonlinearity becomes important. A nonlinear model also provides additional information, such as the relative amplitudes of different waves and the zonal mean flow. Kwon and Mak (1988, hereafter KM) examine equilibration of a barotropic system for various renal forcing and dissipation conditions, for a midlatitude easterly jet in a ~-plane model. Here, idealized stratospheric jets like those studied by Hartmann (1983) and Manney et al. (1988) are subjected to a similar analysis in a spherical system. This **highly** idealized system provides some insight into the type of behavior expected when barotropically unstable waves in the stratosphere grow to finite amplitudes.

2. MODEL AND INITIALIZATION

Nondivergent barotropic flow cm a sphere with friction and forcing is governed by the vorticity equation,

$$\frac{\partial \zeta}{\partial t} + \vec{V} \cdot \vec{\nabla}(\zeta + f) = -\eta \nabla^2(\psi - \psi_0)\,, \qquad (1)$$

where $\zeta = \nabla^2\psi$ is the relative vorticity, ψ is the streamfunction, η is a Rayleigh type friction coefficient, and $\nabla^2\psi_0$ is a vorticity source. ψ_0 is a streamfunction derived from a zonally symmetric jet. The above equation is written in nondimensional form with the length scale being the radius of the earth, and the time scale 1/(2['1). The Rayleigh friction coefficient in d^{-1} is given by $\eta/2/(2\pi\, radians/day)$. Thus, η = ,002 corresponds to a Rayleigh friction coefficient of 1/(40d), η = .004, 1/(20d), and so on. This system is solved using a spectral transform technique as described by Washington and Parkinson (1986). A rhomboidal truncation with M = 8, N = 35 is used. Manney et al. (1988) discuss the effect.. of truncation and find that a considerably higher meridional than zonal resolution is necessary in linear studies to resolve unstable modes like those discussed here. A leapfrog time scheme is used with a time step of 1/2 hour.

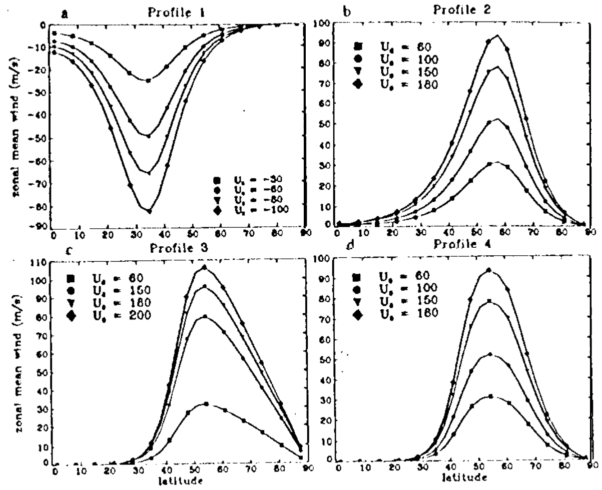

Figure 1. Zonal mean wind profiles used for initialization and forcing for the nonlinear model. (a) Profile 1 -- an easterly midlatitude jet; (h) Profile 2-- a westerly high latitude jet; (c) Profile 3 -- a westerly midlatitude jet; (d) Profile 4-- a westerly jet combining profiles 2 and 3.

The four jet structures used for initial states and ψ_0 are shown in Fig. 1 for several wind speeds: an easterly jet hyberbolic secant jet (Hartmann 1983) with a than.gc in sign of the vorticity gradient in mid-latitudes, similar to the summer jet in the upper stratosphere and mesosphere (profile 1); a westerly hyperbolic secant jet with a sign change in the vorticity gradient at high latitudes (profile 2); a westerly hyperbolic tangent jet (Hartmann 1983) with a sign change in the vorticity gradient at mid-latitudes (profile 3); and a combination of profiles 2 and 3, with a sign change in the vorticity gradient both at mid-latitudes and in the polar regions (profile 4). Profiles 2, 3 and 4 resemble idealized versions of the wintertime stratospheric polar night jet. Linear stability characteristics of these jets have been discussed in some detail by Hartmann (1983) and Manney et al. (1988). Initial states used for nonlinear integrations use the streamfunction of the linearly most unstable mode with a small amplitude of .001 times the maximum amplitude of ψ_0, similar to the "white noise" initial state described by KM. Further details of the model and initial states are given by Manney (1993).

3. NONLINEAR INTEGRATIONS

Nonlinear integrations were done for many values of η and wind speed for each profile. For profiles 1 and 3, for which the vorticity gradient changes sign in mid-latitudes, an equilibrated steady state resulted for a large range of parameters, dominated by one wavenumber; this behavior is similar to that shown by KM in /?-plane studies. A fcw cases also showed limit-cycle behavior similar (o that seen by KM. For profile 2, incoherently fluctuating states resulted for all wind speeds and friction parameters. For profile 4, for which the vorticity gradient changes sign both at mid and high latitudes, equilibrated states were dom - nated by midlatitude modes, although polar modes were still present. The details of these regime studies, and calcula- tions of domain-averaged kinetic energy and phase speeds, arc given by Manney (1993). Here wc show selected repre- sentative results for each profile.

3.1 Profile 1 -- midlatitude easterly jet

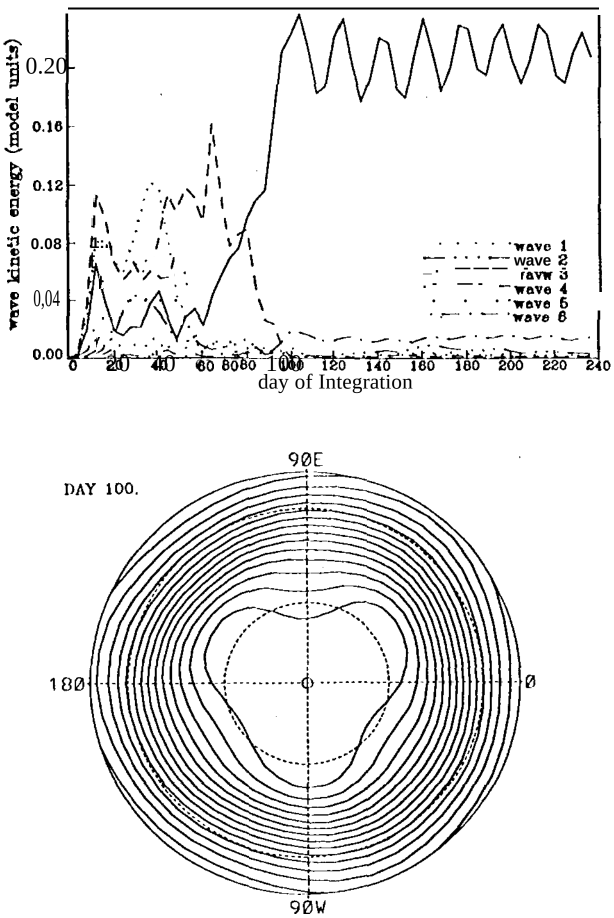

Figure 2. (a) Domain averaged kinetic energy for run with profile 1 with U_0 = -80 ms^{-1}, η = .016, for waves 1 through 6. Kinetic energy is in model units. (h) Streamfunction on day 100 of this run, in model units.

Figure 2 shows domain averaged kinetic energy as a function of time for waves 1 through 6 for profile 1 with U_0 = -80 m/s and η = .016, and the streamfunction cm day 100 of the integration. A stable limit cycle dominated by wave 3 results. The streamfunction shown in Fig. 2b strongly resembles observations of the 2-day westward moving wave 3 shown by Rodgers and Rata (1981). The zonal mean flow at this time has been altered such that it is somewhat more stable (Manney 1993). The periods for the waves resulting from nonlinear integrations using profile 1 are slightly shorter than those of the linearly unstable modes.

3.2 Profile 2-- high latitude westerly jet

All nonlinear integrations using profile 2 resulted in incoherently fluctuating states, dominated by wave 1 and sometimes wave 2. Figure 3 shows shows a series of plots of the streamfunction including waves 1 through 6 with U_0 = 1501111s^{-1}, η = .002. Identifiable "blobs" can be sccu mov- ing around with a period ucar 4 days. These patterns arc qualitatively very similar to those shown by Lait and Stan- ford (1988) from observations. Linearly unstable modes for profile 2 are quasi-nondispersive, and nonlinear waves retain this characteristic, with nearly the same periods.

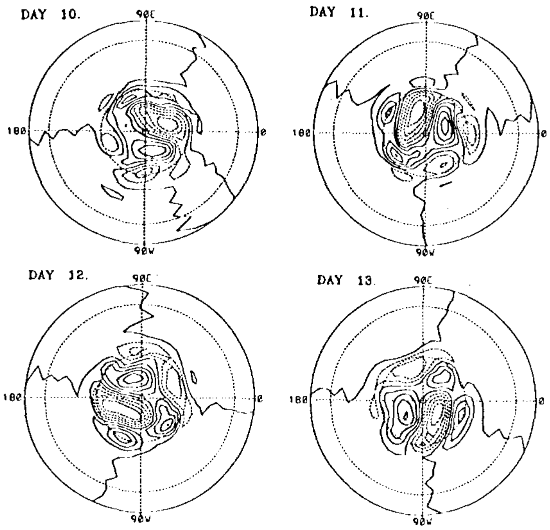

Figure 3. An 4 day series of synoptic plots including the wave 1 through wave 6 components of the streamfunction field for a run with profile 2 with U_0 = 150 ms^{-1}, η = .002.

3.3 Profile 3 -- midlatitude westerly jet

The kinetic energy for waves 2, 3 aud 4, for an inte- gration case where the equilibrated state is dominated hy wave 2 in a situation where wave 3 is the linearly most unstable mode (U_0 = 200 m/s, η = .016) is shown in Fig. 4a. Figures 4h shows a time/longitude plot of wave 2 for the first 4(I days of this integration. The wave 2 period increases to approximately twice the value for the linear mode within approximately the first ten clays, when wave 3 amplitude becomes very large. The background state upon which wave 2 grows has been altered, both by the broadening and weakening of the jet as energy from the zonal mean flow is transferred to wave 3, and by the presence of a wave 3 mode with large amplitude. Experiments with the linear stability of the altered jet suggest (bat most of the effect on wave 2 is duc to the presence of the large wave 3 while wave 2 is growing.

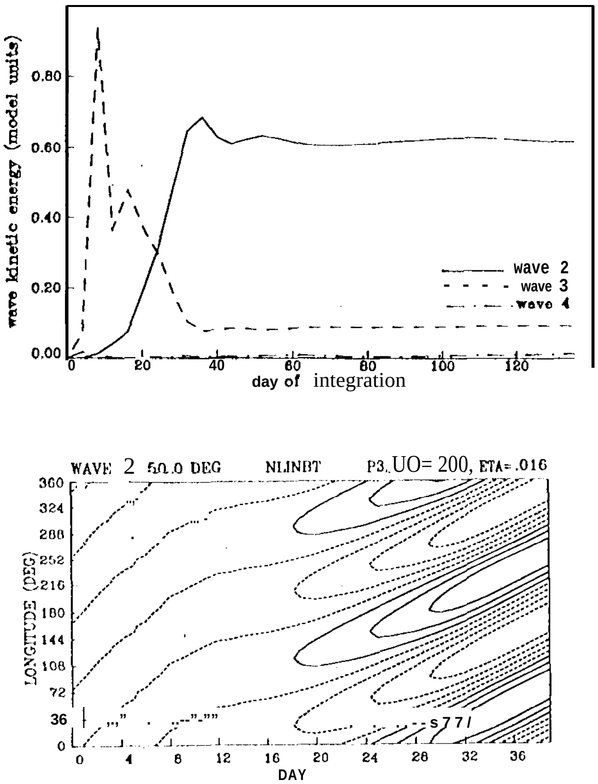

Figure 4. (a) Domain averaged kinetic energy for run with profile 3 with $U. = 200\ ms^{-1}$, $\eta = .016$, for waves 2, 3 and 4. Kinetic energy is in model units. (b) Time/longitude plot of wave 2 for the first 40 clays of this run, at 50° latitude.

3.4 Profile 4-- combined westerly jet

Profile 4 is a combination of profiles 2 and 3, designed so as to give both poleward and equatorward linear modes that arc similar to those obtained separately from profiles 2 and 3. Figure S shows lougituclc/time plots for a run using this profile that results in a state dominated by wave 3, for wave 1 at 70° latitude, wave 3 at 50° latitude, and wave 3 at 70° latitude. The heavy lines on the plot of wave 3 at 70° show the propagation of wave 3 at 50° (solid line), and wave 1 at 70" (dashed line). As the wave 3 amplitude varies at 70° it appears to move with similar phase speed to wave 1 (polar modes arc quasi-nondispersive) at times, and at other times, with the similar phase speed to wave 3 at SO° (near the peak of wave 3 amplitude). This suggests that both the polar and midlatitude modes of wave 3 arc present; a time-series analysis (not shown) confirms the presence of both modes of wave 3. In this case, tbc nonlinear result was an incoherently fluctuating state. The same analysis has been done in cases where the final result was a steady wave 3 or 4; polar modes continue to be present, but the amplitudes arc considerably smaller than for similar cases with profile 2.

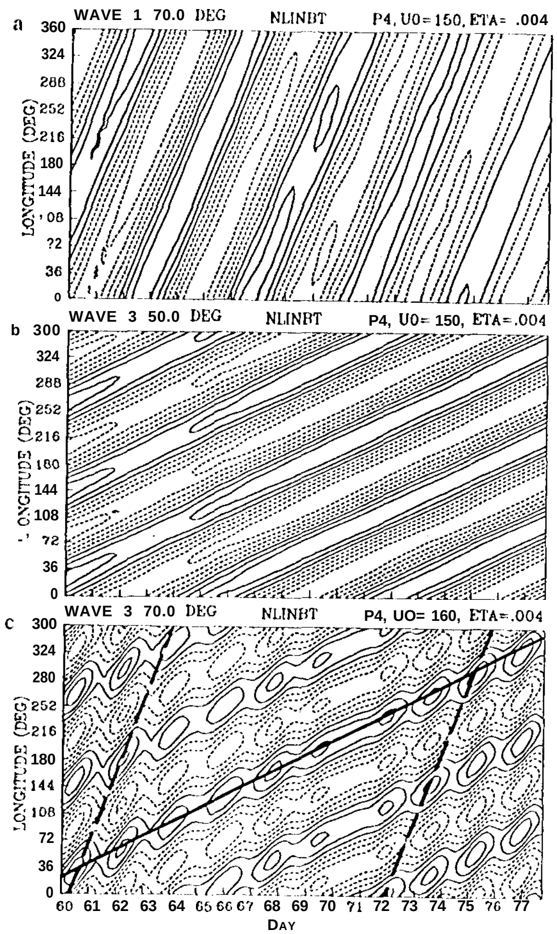

Figure S. Time/longitude plots for the case with profile 4 with $U. = 150\ ms^{-1}$, $\eta = .004$, (a) wave 1 at 70° latitude, (b) wave 3 at 50° latitude, and (c) wave 3 at 70° latitude. Heavy solid line in (c) shows the period of wave 3 at 50°, heavy dashed line shows the period of wave 1 at 70°.

4. SUMMARY AND CONCLUSIONS

The behavior of barotropically unstable waves at finite amplitude was examined in a spherical, nonlinear barotropic model using idealized wind profiles representative of stratospheric jets. Wind profiles for which the vorticity gradient changes sign in midlatitudes result in steady wave states dominated hy one wavenumber for a wide range of wind speeds and Rayleigh friction parameters. Longer wavelengths arc dominant for stronger jets and smaller friction parameters. Limit cycle behavior also occurs for specific cases (Manney 1993). A westerly jet where the vorticity gradient changes sign in high latitudes results in incoherently fluctuating waves for all cases. Periods and spatial str uctures of nonlinear waves Usually resemble those of linearly unstable modes; however, a case is shown where the nonlinear integration results in a dominant wave 2 when wave 3 is linearly most unstable, and the wave 2 period is approximately doubled. Steady wave states resulting from a wind profile that is unstable to both polar and midlatitude modes are dominated by midlatitude modes, with polar modes partially suppressed; for incoherentl y fluctuating states, midlatitude and polar modes co-exist, with little apparen t effect on each other.

The behavior of these barotropically unstable waves at finite amplitude reinforces the correspondence between barotropically unstable waves and some observed planetary scale waves in the stratosphere, Amplitudes and latitudinal phase tilts (Manney 1993), as well as periods and spatial str ucture, in the model are in reasonable agreement with characteristics of several observed features,

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