

Emulating the Lateral Dynamics of A Range of Vehicles
Using A Four-Wheel-Steering Vehicle

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Abstract

The concept of Variable Dynamic Testbed Vehicle (VDTV) has been proposed as a tool to evaluate collision detection/avoidance systems, to perform vehicle-related human factors research, and to support other Automated Highway System programs. The goal of this study is to analytically investigate to what extent a VDTV with four-wheel-steering can emulate the lateral dynamics of a broad range of vehicle models. Using the Ford Taurus as a baseline vehicle, our study indicated that a Taurus with a closed-loop four-wheel-steering control system can emulate the lateral response characteristics (including control sensitivity, 90% rise time, yaw rate bandwidth, and others) of a range of vehicles (from a "small" Ford Escort to a "full-size" Mercury Marquis) reasonably well over a speed range of interest. A novel steering control configuration that has the potential to improve further the degree that a variable dynamic vehicle can emulate the lateral characteristics of other vehicles has also been proposed.

Key Words: Four-wheel-steering, Variable Dynamic Testbed Vehicle.

1. Introduction

The office of Crash Avoidance Research (OCAR) under the National highway Traffic Safety Administration (NHTSA) has the responsibility of correlating vehicle characteristics with driver commands relative to crash avoidance. To this end, OCAR has at its disposal a comprehensive set of tools and facilities. These include the Vehicle Test and Research Center, and the (currently being developed) National Advanced Driving Simulator. To augment these tools and facilities, OCAR has defined their concept of a Variable Dynamics Testbed Vehicle (VDTV). This vehicle would be capable of emulating a broad range of automobile dynamic characteristics, and could be used in collision detection/avoidance concepts development, in vehicle human factors research, and to validate the vehicle models of driving simulators, among others.

To emulate the dynamics of a broad range of vehicles, from small to large, the steering, suspension, braking, and powertrain subsystems of the VDTV will be made "programmable." This will be achieved by replacing components, or by changing appropriate control algorithms via software. In this study, we investigate the extent the lateral dynamics of a broad range of vehicles can be emulated by a single "variable dynamic" vehicle. Both the steady state and transient dynamics of the vehicle's lateral-directional characteristics are to be varied via the steering of the rear wheels of the vehicle.

2. Vehicle Dynamics Model

A vehicle handling model that the author had developed, VEHDYN, is used in this study. In VEHDYN, the lateral dynamics of a vehicle are modeled using the approach proposed in Ref. 1. The model includes both the vehicle's yaw and roll degrees of freedom, and since the pitch degree of freedom does not significantly affect handling, it was not included in this model. The "states" of the model are: yaw rate, side-slip angle, roll rate, and roll angle.

For simplicity, VEHDYN uses a linear tire model. Lateral forces and aligning torques generated by the tires are computed as functions of the tires' slip and camber angles. The model also includes the effects that the vehicle's roll angle as well as tires' lateral forces and aligning torques have on both the camber and tire angles. Results obtained with this model are accurate up to approximately 0.3 g's of lateral acceleration. Beyond that, models which include both the tire saturation effects and suspension nonlinearities

should be used.

In our study, VEHDYN is augmented with the following; actuator dynamics models:

$$\begin{aligned}\tau_f \dot{\delta}_f + \delta_f &= \delta_{fc}, \\ \tau_r \dot{\delta}_r + \delta_r &= \delta_{rc}.\end{aligned}\tag{1}$$

Here, δ_f and δ_r are the front and rear tire angles, while δ_{fc} and δ_{rc} are commands to the front and rear actuators, respectively. The front tire command δ_{fc} is related to the steering wheel angle δ_{SW} by the steering ratio, $\delta_{fc} = \delta_{SW}/N_S$. For two-wheel-steering vehicles, there is no rear tire command (i.e., $\delta_{rc} = 0$). For four-wheel-steering (4WS) vehicles, the rear tire command is determined by a four-wheel-steering control algorithm (cf. Section 4), which is typically implemented using an on-board micro-processor. The time constants of the front and rear actuators are τ_f and τ_r , respectively. In our study, the bandwidths of these actuators are both estimated at 4 Hz.

Estimated values of vehicle parameters needed by VEHDYN, for four U.S. designed and manufactured passenger sedans: Ford Escort, Buick Skylark, Ford Taurus, and Mercury Marquis, are summarized in Table 1. These vehicle models span a spectrum of vehicle sizes, from small to full-size vehicles. Estimates of vehicle parameters tabulated in Table 1 are made based on data given in Refs. 2-6. Linearized tire parameters are estimated from data found in Reference 7, and are summarized in Table 2.

Table 1 Estimated Values of Vehicle Parameters

Vehicle Model	Ford Escort	Buick Skylark	Ford Taurus	Mercury Marquis
Class	small	compact	mid-size	large
Year	1985	1986	1988	1984
Drive	FWD	FWD	FWD	RWD
Wheel base [l ₁] (m)	2.39	2.62	2.69	2.90
Track width (m)	1.40	1.40	1.55	1.58
Weight [M] (kg.wt.)	1007	1262	1419	1750
Weight front (%)	65.3	64.0	64.6	57.0
e.g. distance from front axle [a] (m)	0.83	0.94	0.95	1.22
e.g. height (m)	0.51	0.54	0.56	0.57
Roll inertia (kg-m ²)	328	431	573	717
Pitch inertia (kg-m ²)	1535	2032	2553	3848
Yaw inertia [I_{zz}] (kg-m ²)	1545	2082	2687	3907
Front tire excursion (deg)	±32.8	±31.7	±26.5	±30.0
Front/Rear roll stiffness (Nm/deg)	684	828	1206	865
	490	381	935	294
Front and Rear roll damping (Nm-sec/deg)	42.7	53.5	60.1	74.2
Steering ratio (-)	17.0	17.6	17.0	18.0

Table 2 Estimated Tires Characteristics

Vehicle Model	Ford Escort	Buick Skylark	Ford Taurus	Mercury Marquis
Class	small	compact	mid-size	large
Year	1985	1986	1988	1984
Tire	P185 /60R14	P185 /75R14	P205 /65R15	P215 /70R15
Front /Rear Loading (kg.wt.)	658 / 349	808 / 454	917 / 502	998 / 752
Front/Rear Cornering Stiffness (N/deg, per wheel)	633.2 / 433.4	704.7 / 509.2	1051.0 / 794.0	1092.4 / 954.3
Front/Rear Aligning Torque Stiffness (Nm/deg, per wheel)	11.77 / 6.26	14.46 / 8.13	16.41 / 8.99	17.86 / 13.47
Front/Rear Camber Stiffness (N/deg, per wheel)	20.95 / 9.60	27.43 / 13.16	54.50 / 41.03	87.98 / 71.69
Front/Rear Aligning Torque per Camber (Nm/deg, per wheel)	1.18 / 0.63	-1.45 / 0.81	1.64 / 0.90	1.79 / 1.35

3. Steady-state and Transient Characteristics of Vehicle Models

Using VEHDYN, and the estimated values of vehicle parameters given in Tables 1 and 2, both the steady-state and transient characteristics of the four selected vehicle models can be computed. Results obtained are depicted in Figure 1,

The control sensitivity of a vehicle at a given forward speed is defined as its steady-state lateral acceleration (at the vehicle's c.g.) per 100 degrees of steering wheel excursion. It is sometimes called the vehicle's steering sensitivity or lateral acceleration gain. As depicted in Fig. 1, this gain generally increases with the vehicle's forward speed.

Transient responses of the selected vehicle models are compared in terms of the 90% rise times and "percent overshoots" of their acceleration responses. The 90% rise time is a measure of the vehicle's "speed of response" when it is subjected to a "step" front steering wheel command. Since a true "step" is physically impossible, the steering command is ramped to its steady-state value over a time period of, for example, 0.15 seconds. The 90% rise time is then defined as the time it takes the vehicle's lateral acceleration to reach 90% of its steady-state value, measured from the time the steering command reaches 50% of its

steady-state value. The percent overshoot is a measure of the amount of ‘clamping’ in the vehicle’s yaw responses. It is defined as the peak acceleration response measured from the steady-state acceleration level, as a percent of that steady-state level. As depicted in Fig. 1, the 90% rise time and percent overshoot generally decrease and increase, respectively, with the vehicle’s forward speed.

The lateral dynamics of a vehicle can also be measured using frequency-domain performance metrics. The vehicle’s yaw rate-based bandwidth (BW) is the frequency at which the magnitude of the transfer function, from the steering wheel to the vehicle’s yaw rate, has dropped below 70.7% (-3 dB) of its steady-state value. As depicted in Fig. 1, this bandwidth increases monotonically with the vehicle’s forward speed.

With reference to Fig. 1, both the steady state and transient dynamics of the selected vehicle models differ from one another. From Fig. 1, we also note that the Taurus model has relatively “good” lateral performance: low acceleration rise time and percent overshoot, and high yaw rate-based bandwidth. For these reasons, it is selected as the “baseline” vehicle in our study. The idea behind the variable dynamic testbed vehicle concept is then one of making “modifications” to the baseline Taurus model so that it can closely emulate the lateral dynamics of other vehicle models. The only modification considered in our study is the addition of a four-wheel-steering subsystem near the rear axle of the vehicle to allow the steering of the rear wheels of the vehicle. The details are given below.

4. Four-Wheel-Steering Control Algorithms

The lateral dynamics of a vehicle can be substantially altered by steering its rear wheels in conjunction with those at the front. For example, the control sensitivity of a four-wheel-steering vehicle at a given forward speed can be increased/decreased by steering the rear wheels out-of-phase/in-phase with the front wheels. Additionally, the transient characteristics of the vehicle can also be “manipulated” via carefully designed control algorithms. Hence, we can use a 4WS vehicle to conveniently emulate the directional characteristics of a range of vehicles via changes in the control algorithms.

In this study, simple 4WS control algorithms are used to change the lateral dynamics of a Taurus to make it emulate the lateral dynamics of a range of vehicles. The control

algorithm has the following structure:

$$\delta_{rc} = K_{\delta}(U) \left(\frac{1 + \tau_1 s}{1 + \tau_2 s} \right) \delta_{fc} - K_r(U) r . \quad (2)$$

Here, the speed-dependent feed-forward gain, $K_{\delta}(U)$, is approximated by the relation: $K_{\delta}(U) = a_0 + a_1 U + a_2 U^2$, U being the vehicle forward speed. Similarly, the speed-dependent feedback gain $K_r(U)$, is approximated by the relation: $K_r(U) = b_0 + b_1 U + b_2 U^2$. The variable r is the filtered yaw rate of the vehicle. The parameters τ_1 and τ_2 of the lead-lag compensator are in general speed-dependent. In our study, constant values are used for these time constants throughout the speed range of interest (80 to 170 km/h). Typically, the quantities a_i ($i = 0, \dots, 2$), are used to alter the steady-state condition of the vehicle while the time constants are used to change the transient characteristics of the vehicle. The quantities b_j ($j = 0, \dots, 2$) will affect both the steady-state and transient characteristics of the vehicle. Appropriately selected, these control parameters will allow us to alter the lateral dynamics of Taurus to approximate those of Escort, Skylark, and Marquis.

It should be noted here that more complex open-loop or closed-loop controller structures could have been employed for our purpose. See, for example, those described in Ref. 8. Potentially, their uses can improve the degree to which the 4WS Taurus can emulate the lateral dynamics of other vehicle models. But the designs of these controllers are outside the scope of this study,

The addition of a steering actuator near the rear axle of a vehicle can increase the mass and yaw moment of inertia of the vehicle's unsprung mass, move the vehicle's e.g., and alter the tires' cornering and camber stiffnesses. These perturbations, while small, are all taken into account in our study,

Consider the addition of a concentrated mass m_R at a distance l_R behind the rear axle. Here, the subscript "R" denotes the rear axle of the vehicle. The increased total mass of the vehicle is $M + m_R$, where M is the total mass of the "nominal" vehicle (cf. Table 1). Also, the nominal location of the vehicle's e.g. is shifted rearward by A :

$$\frac{\Delta}{L} = \left(\frac{m_R/M}{1 + m_R/M} \right) \left(\frac{b + l_R}{L} \right), \quad (3)$$

where the parameters L and b are defined in Table 1. The new vehicle's e.g. is now located at (\bar{a}, \bar{b}) where $\bar{a} = a + \Delta$, and $\bar{b} = b - A$. The increased yaw moment of inertia of

the sprung mass about the new vehicle's e.g. is: $I_{zz} = m_R(l_R + \bar{b})^2 + M\Delta^2$. The altered loadings on the front and rear tires will change both the cornering and camber stiffnesses of the vehicle's tires. Let F_Z (kg.wt.) denotes the altered loading on either the front or rear tire of the vehicle. The estimated cornering stiffness (C_α) and camber stiffness (C_γ) of the Taurus's tires (P205/65R15) with a loading of F_Z are given by the following approximate relations:⁷

$$\begin{aligned} C_\alpha &= -51.97 + 4.536F_Z - 0.00465F_Z^2, \\ C_\gamma &= 0.2171F_Z - 0.000223F_Z^2. \end{aligned} \quad (4)$$

In our study, we estimate the weight of a four-wheel-steering actuator to be on the order of 15 kg.wt. ($m_R = 15$ kg.wt.), and is located right on the rear axle ($l_R = 0$).

5. Simulation Results

In our study, the Ford Taurus is selected as the baseline vehicle. Its dynamics are then altered using a 4WS system. Control parameters, including $a_i, i = 0, \dots, 2, \tau_1$, and τ_2 are then iteratively determined so that the characteristics of the "modified" Taurus closely approximate those of the Escort, Skylark, and Marquis, over the speed range of interest. Via a trial-and-error process, sets of parameter values were found (cf. Table 3) for the Escort, Skylark, and Marquis models. Alternatively, the simplex methodology employed in Ref. 11 could be used to optimally select the values of the "control" parameters that produced the best match between the lateral dynamics of the modified Taurus and the targeted sedan model. This is a topic for future research.

Table 3 Selected Values of Control Parameters

Control Parameters	Unit	Ford Escort	Buick Skylark	Mercury Marquis
$a_0 \times 10$	-	0.522	-1.185	-2.278
$a_1 \times 10^3$	(km/h) ⁻¹	-1.613	2.933	5.918
$a_2 \times 10^5$	(km/h) ⁻²	0.543	-0.855	-1.725
$b_0 \times 10$	sec	0.760	1.693	1.920
$b_1 \times 10^3$	sec (km/h) ⁻¹	0.000	0.292	0.480
$b_2 \times 10^5$	sec (km/h) ⁻²	0.000	-0.300	-0.360
τ_1	sec	-0.267	+0.359	+0.161
$\tau_2 \times 10$	sec	0.478	0.344	3.980

Graphical comparisons between the lateral characteristics of the Ford Escort and the Variable Dynamics Testbed Vehicle (VDTV) are depicted in Fig. 2. Those for the Buick Skylark and Mercury Marquis are given in Figs. 3 through 4, respectively. Emulation errors in the vehicle's control sensitivity, 90% rise time, percent overshoot, and yaw rate-based bandwidth, over the speed range of interest are summarized in Table 4.

Table 4 Mean Emulation Errors

Mean Emulation Errors	Ford Escort	Buick Skylark	Mercury Marquis
error in control sensitivity (%)	2.96	10.22	7.13
error in 90% rise time (%)	0.49	12.91	12.09
error in percent overshoot (%)	1.92	1.87	2.99
error in yaw rate-based BW (%)	2.12	2.22	2.32

6. Concluding Remarks

The concept of Variable Dynamic Testbed Vehicle (VDTV) has been proposed as a tool to evaluate collision detection/avoidance systems, to perform vehicle-related human factors research, and to support other Automated Highway System programs. The goal of this study is to analytically investigate to what extent a VDTV with four-wheel-steering can emulate the lateral dynamics of a broad range of vehicle models.

Using the Ford Taurus as a baseline vehicle, our study indicated that a Taurus with a closed-loop four-wheel-steering control system can emulate the lateral response characteristics (including control sensitivity, 90% rise time, yaw rate bandwidth, and others) of a '(small)' Ford Escort very well over a speed range from 80 to 170 km/h. The degrees to which the "compact" Buick Skylark and "(full-size)" Mercury Grand Marquis can be emulated by a four-wheel-steering Taurus are poorer. The levels of approximation can potentially be improved with better 4WS controller designs.

With reference to Fig. 5, a VDTV with both the four-wheel-steering and steer-by-wire features will provide us with additional "degree-of-freedom" to tailor approximate the lateral characteristics of the baseline vehicle to that of the "target" vehicle. The potential of a vehicle with such a novel steering control configuration in emulating the lateral

characteristics of a broad range of vehicles is an important topic for future study.

7. References

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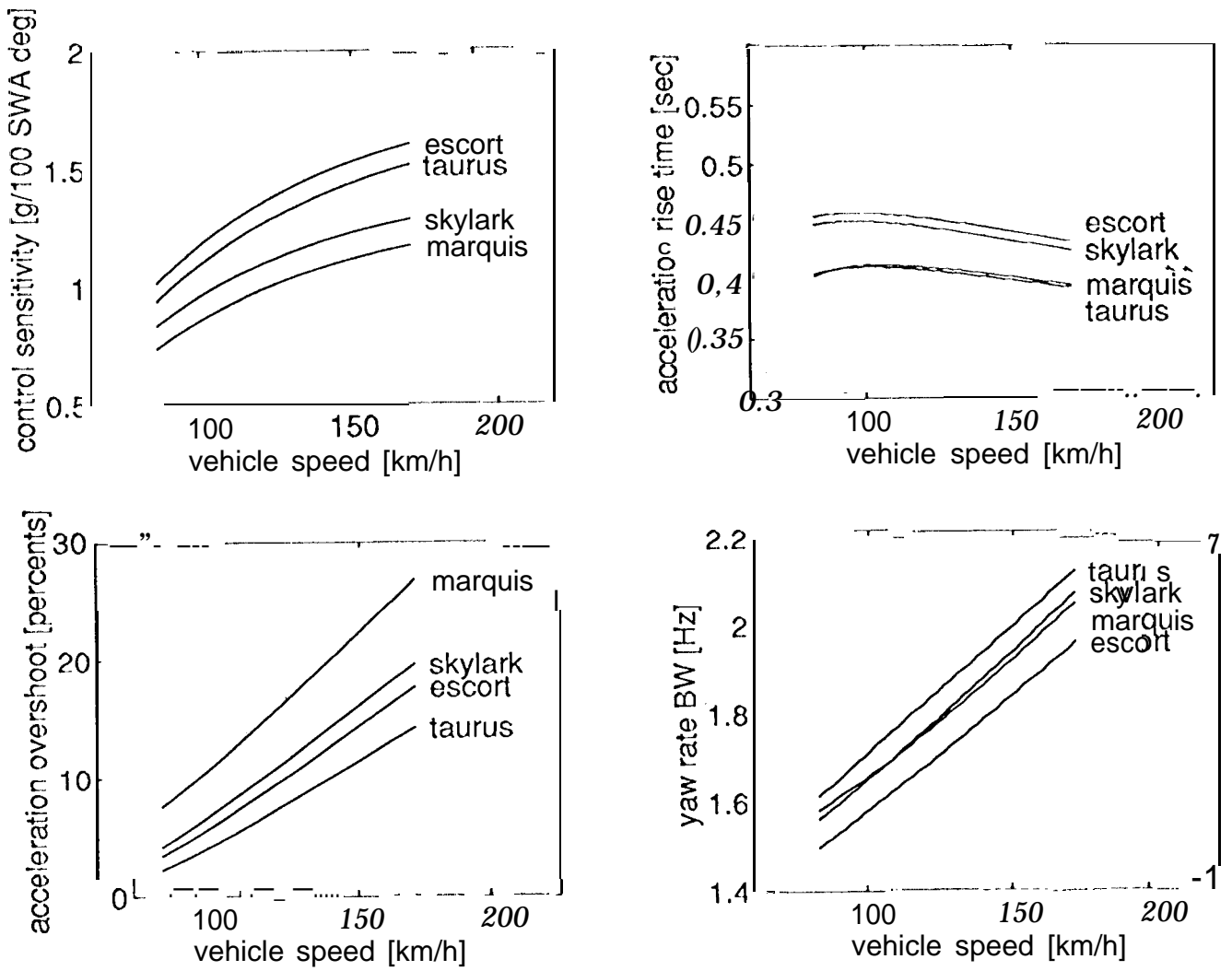


Fig. 1 Lateral characteristics of four vehicle models

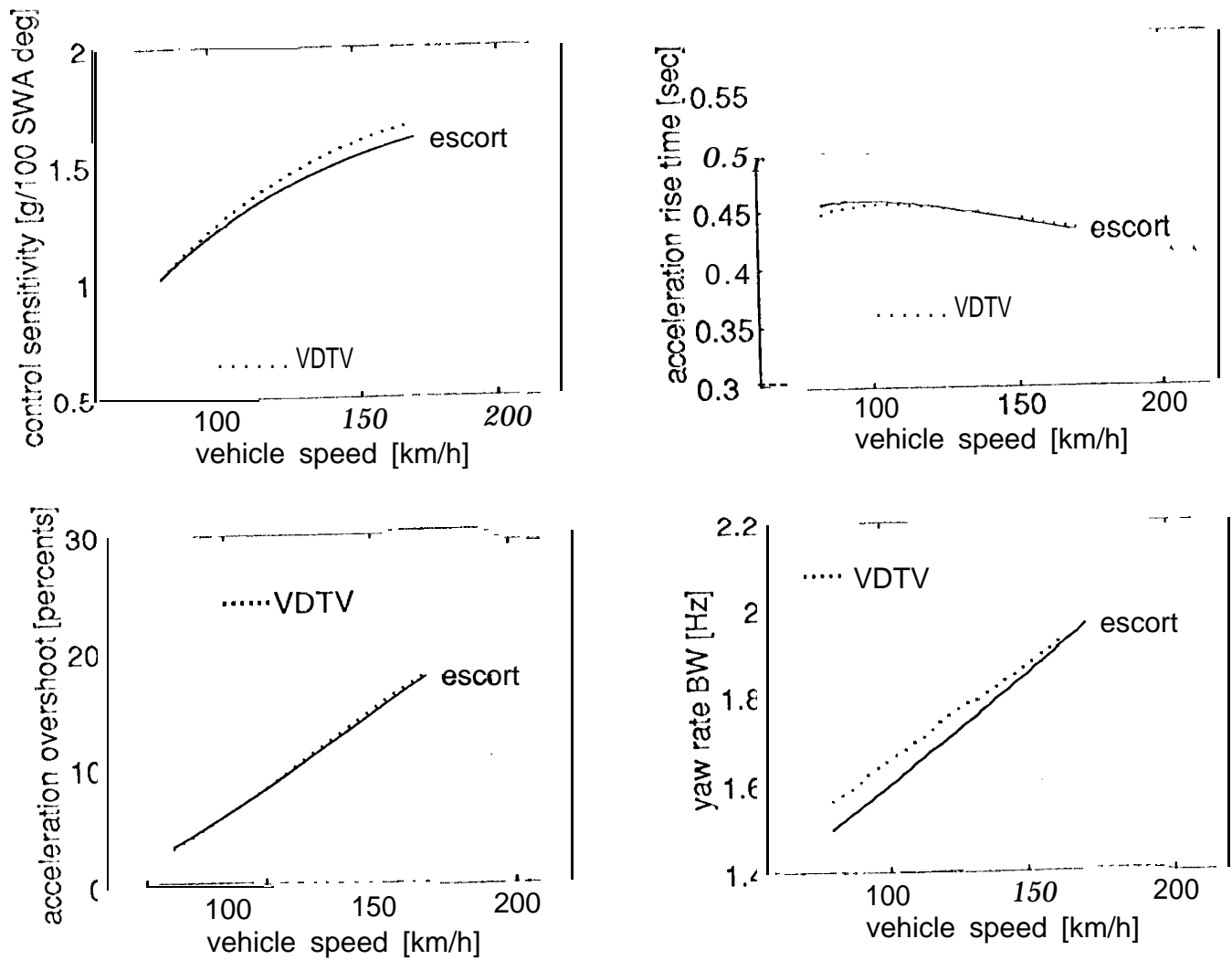


Fig. 2 Emulating the lateral characteristics of Escort using a VDTV

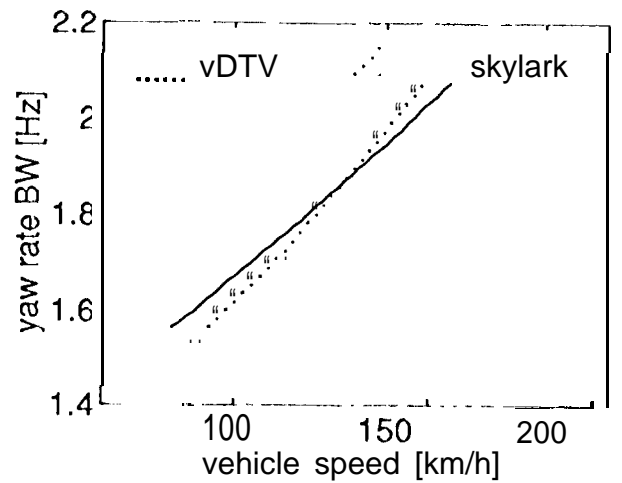
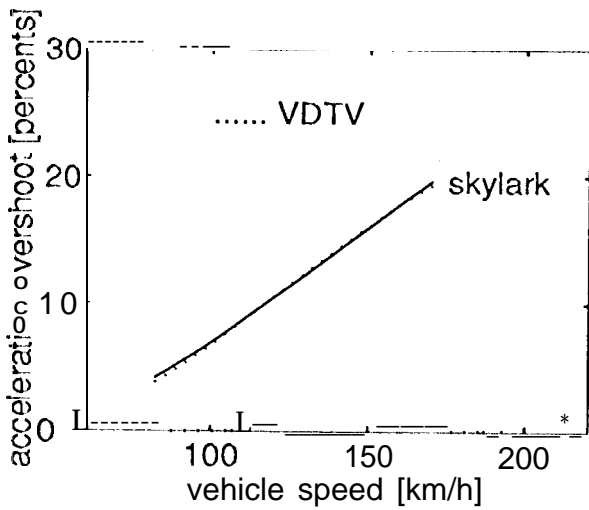
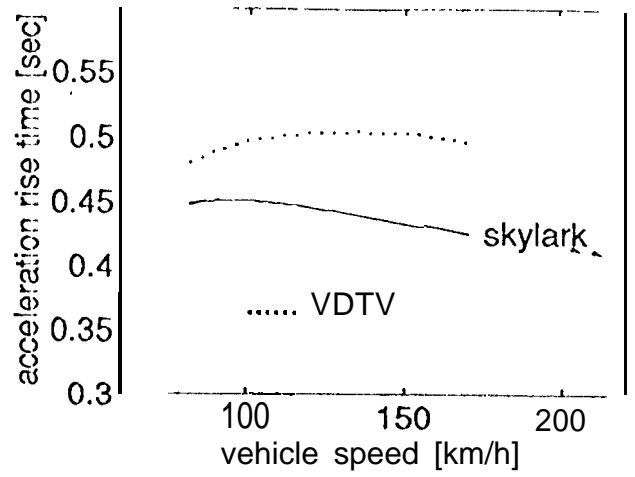
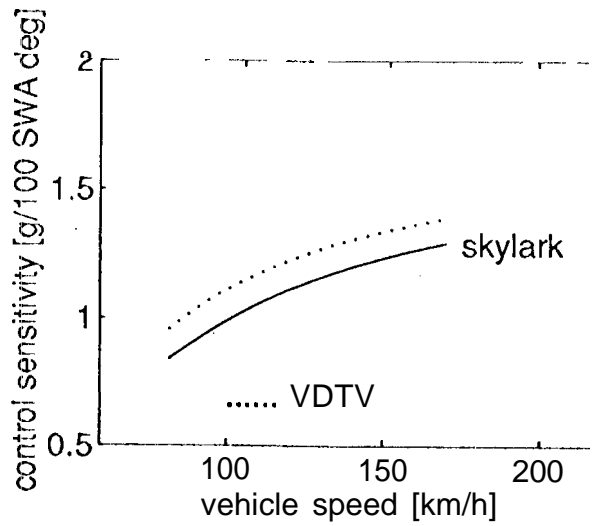


Fig. 3 Emulating the lateral characteristics of Skylark using a VDTV

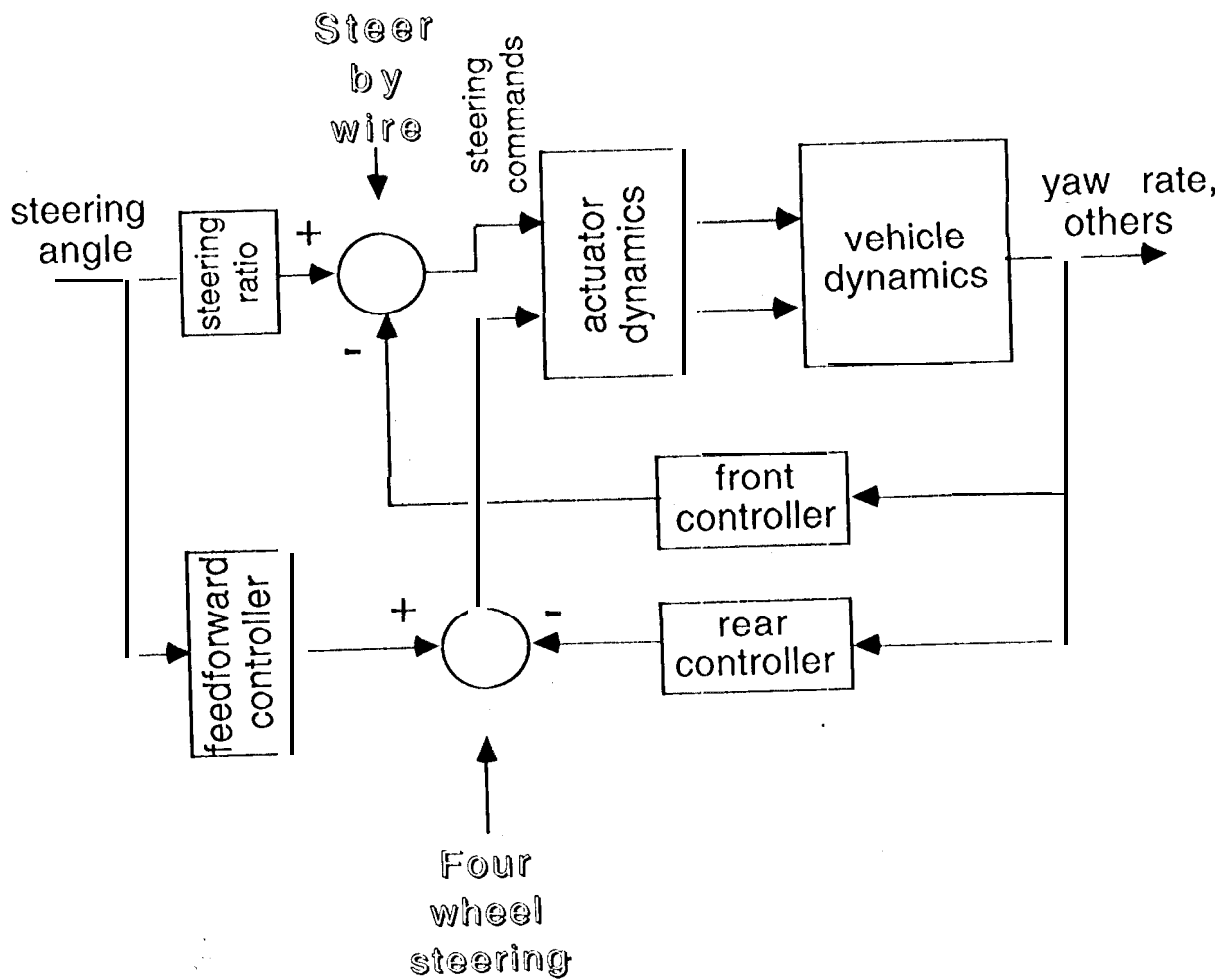


Fig. 5 A VDTV with steer-by-wire and four-wheel-steering

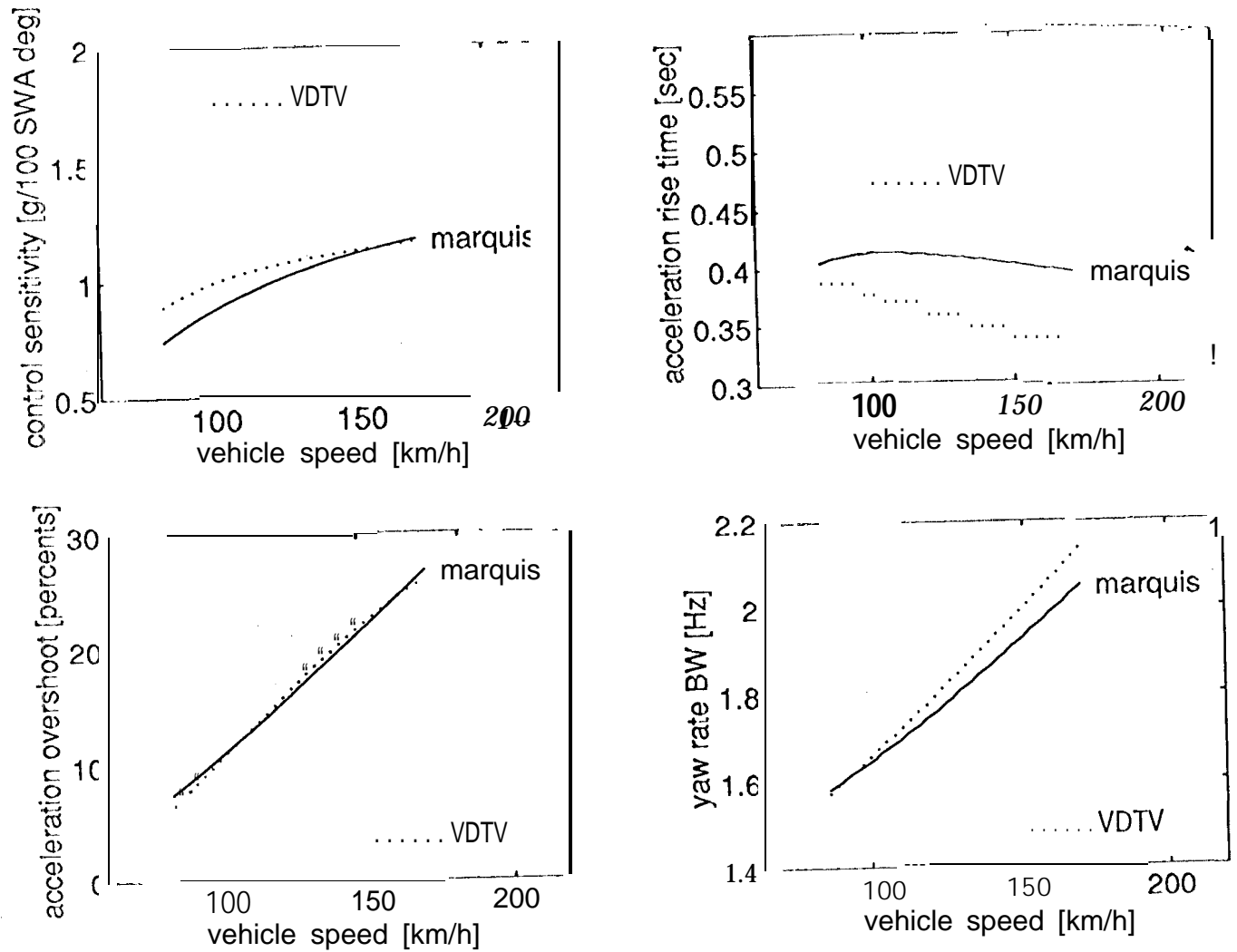


Fig. 4 Emulating the lateral characteristics of Marquis using a VDTV