IMPROVED OPTIMIZATION METHOD FOR ARRAY FEED CALCULATIONS

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ABSTRACT

Reflector antenna optimization schemes using array feeds have been used to recover antenna losses resulting from antenna distortions and aberrations and to generate contour coverage patterns. Historically these optimization have been carried out using the antenna far-field scattered patterns. The far-field patterns must be calculated separately for each of the array feed elements. For large or complex antennas the far-field calculation times can be prohibitive. This paper presents a method that allows the optimization to be carried out in the antenna focal region where the scattering calculation needs only to be done once independent of the number of the elements in the array.

Keywords: Focal Plane, Array Feed, Antennas, Gain Optimization, Reflector Antenna.

1, INTRODUCTION

Array feeds for reflectors have a number of important uses which include (1) generating contour coverage patterns, (2) correction for reflector distortions, and (3) improved wide angle scan. Typical methods for optimizing the array feed for each of these applications are very efficient when a fixed array geometry is utilized and only the feed excitation coefficients are optimized. For this case only one calculation of the radiating fields is required from each array clement. For example, to maximize gain in a given direction, the optimization can be as simple as taking the complex-conjugate of the secondary fields resulting from the illumination of the reflector in the given direction by each of the array feed elements. For most existing methods, an optimization which allowed the element spacing and size to vary would be extremely time consuming since a radiation integral evaluation would be required for each feed element at each step of the optimization process.

A new method of computing array feed performance is presented here that obviates the need to re-compute the reflector radiation fields when the feed element size or spacing is varied. This allows the optimization technique to efficiently include size and spacing as parameters. The mathematical formulation is based upon the usc of the Lorentz reciprocity theorem which convolves the focal-plane field distribution of the reflector system with the feed element aperture field distribution to obtain the element response. Thus the time-consuming reflector system radiation integral evaluation is only done once for a given scan direction or reflector surface distortion for all array feed geometries and types considered.

Examples are given using the technique to design an array feed for the correction of gravity-induced distortions of a large dual-shaped ground antenna and to design an array feed for improved wide-angle scan.

2. FOCAL PLANE ANALYSIS

The calculation of the gain of an antenna system by evaluating the fields in the antenna focal plane consists first of computing the focal fields produced by a plane wave impinging upon the aperture of the antenna. Second, the aperture fields of the feedhorns located at the focal plane are determined, and these fields are then convolved with the focal plane fields to provide the antenna gain. The process can bc explained as follows. Consider a reflector antenna fed by a horn. We wish to determine the gain of this system in a given direction, (θ_0, ϕ_0), in the receive mode. First, consider the Lorentz reciprocity theorem

$$-\iint_S \{\bar{E}_a \times \bar{H}_b - \bar{E}_b \times \bar{H}_a\} \bullet \overline{ds} = \iiint_V \{\bar{E}_a \bullet \bar{J}_b - \bar{H}_a \bullet \bar{M}_b - \bar{E}_b \bullet \bar{J}_a + \bar{H}_b \bullet \bar{M}_a\} dv \quad (1)$$

In this expression, $\bar{E}_a$ and $\bar{H}_a$ are fields radiated by a set of sources $\bar{J}_a$ and $\bar{M}_a$ and also $\bar{E}_b$ and $\bar{H}_b$ are fields radiated by sources $\bar{J}_b$ and $\bar{M}_b$. The left integral is over a closed surface, which encloses the vohrrne defined by the integral on the right side. Over an infinite region, the surface integral becomes zero. The Lorentz reciprocity theorem can therefore be rewritten as

$$\iiint_V \{\bar{E}_a \bullet \bar{J}_b - \bar{H}_a \bullet \bar{M}_b\} dv \cdot \iiint_V \{\bar{E}_b \bullet \text{ ja-. fib } \bullet \bar{M}_a\} d \quad (2)$$

which holds that as long as the relationship between the fields and their source currents holds true, the results will bc the same wherever the integrals are evaluated in the region. Let us redefine the "a" sources as sources $\bar{J}_{ha}$ and $\bar{M}_{ha}$ to be associated with the feedhom apertures and generating the fields $\bar{E}_{ha}$ and $\bar{H}_{ha}$ in the aperture plane of the feed. In turn, let us redefine the "b" sources as sources $\bar{J}_{fp}$ and $\bar{M}_{fp}$ to be associated with the antenna reflector system when illuminated by an incident plane-wave source from a direction (θ_0, ϕ_0), and evaluated in the reflector system focal plane. The feedhorn apertures are defined to be co-planar with the antenna focal plane.

Since the integration is limited to the aperture plane/focal plane, the integrals reduce to surface integrals. Each integral is proportional to the feedhom output voltage (Ref. 1). 'l'he left-hand equation is used since the program that generates the focal-plane equivalent currents outputs currents and the program that computes the feedhorn aperture distributions outputs fields. The expression relating the feedhorn outputs v_{r_A} to the currents from the antenna reflector system and the feedhorn aperture fields is then

$$v_{r_A} \propto \iint_s \{\bar{E}_{ha} \bullet \bar{J}_{fp} - \bar{H}_{ha} \bullet \bar{M}_{fp}\} ds \qquad (3)$$

$\bar{E}_{ha}$ and $\bar{H}_{ha}$ should be determined in the presence of the antenna reflector system and the focal-plane currents $\bar{J}_{fp}$ and $\bar{M}_{fp}$ should be obtained when the feedhorn is present. Such computations would require that the interactions between the feedhorn and the reflectors be taken into consideration. Taking into account these interactions seriously complicates the analysis and increases the computational time. Often $\bar{E}_{ha}$ and $\bar{H}_{ha}$ are approximated to the aperture fields of a horn radiating into an infinite homogeneous free space (no reflector) and the focal-plane currents $\bar{J}_{fp}$ and $\bar{M}_{fp}$ of the antenna reflector system are also obtained in the absence of a feed. This is a reasonable assumption when the feed and antenna reflector system are widely separated in terms of wavelengths.

As will be seen later, there is also a need to obtain the performance of a feedhorn in the presence of a plane-wave incident field arriving from a direction (θ_p, ϕ_p) and in the absence of the antenna reflector system. In the same manner as presented above, it can be shown that the output voltage for such a feedhom is

$$v_{r_P} \propto \iint_s \{\bar{E}_{ha} \bullet \bar{J}_{pw} - \bar{H}_{ha} \bullet \bar{M}_{pw}\} ds \qquad (4)$$

where $\bar{J}_{pw}$ and $\bar{M}_{pw}$ arc the currents in the feed-aperture plane due to the incident plane-wave field.

In Equations (3) and (4), the proportionality constants should be the same, a function of the horn aperture characteristics. The proportionality constants can be eliminated by performing the ratio of Equations (3) and (4) as follows

$$\frac{v_{r_A}}{v_{r_P}} = \frac{\iint_s \{\bar{E}_{ha} \bullet \bar{J}_{fp} - \bar{H}_{ha} \bullet \bar{M}_{fp}\} ds}{\iint_s \{\bar{E}_{ha} \bullet \bar{J}_{pw} - \bar{H}_{ha} \bullet \bar{M}_{pw}\} ds} \qquad (5)$$

Let us now consider two transmit situations. First let the feedhorn radiate in the absence of the antenna reflectors and let the radiated field at (r, θ_p, ϕ_p) be E_h and the power be P_o. Then the gain of the feedhorn in the direction (θ_p, ϕ_p) is

$$G_h = \frac{4\pi r^2}{\eta P_o} |E_h|^2 \qquad (6)$$

Next, let the horn illuminate the reflector system. The scattered field at (r, θ_o, ϕ_o) is E_a. Assume that the power that is radiated by the feedhorn is still P_o. Then the gain of the complete antenna system in the direction @o, $\phi_0)$ is

$$G_a = \frac{4\pi r^2}{\eta P_o} |E_a|^2 \qquad (7)$$

and consequently

$$G_a = G_h \frac{|E_a|^2}{|E_h|^2} \qquad (8)$$

From reciprocity wc know that the radiated fields and horn output voltages are related by

$$\frac{|E_a|^2}{|E_h|^2} = \frac{|v_{r_A}|^2}{|v_{r_P}|^2} \qquad (9)$$

Therefore by combining Equations (5) and (9) the overall gain of the reflector antenna system can be found in the receive mode from

$$G_a = G_h \cdot \left[\frac{\left| \iint_s \{\bar{E}_{ha} \bullet \bar{J}_{fp} - \bar{H}_{ha} \bullet \bar{M}_{fp}\} ds \right|}{\left| \iint_s \{\bar{E}_{ha} \bullet \bar{J}_{pw} - \bar{H}_{ha} \bullet \bar{M}_{pw}\} ds \right|} \right]^2 \qquad (10)$$

It should be noted that (θ_0, ϕ_0) and (θ_p, ϕ_p) need not be the same and therefore (θ_p, ϕ_p) has been set to (0.0,0.0) for simplicity of analysis when evaluating the interactions of the array feed with an incident plane wave.

3. OPTIMIZATION TECHNIQUE

The optimization technique used in obtaining the maximum gain for an antenna reflector system illuminated by a group or an array of feed elements is referred to as the conjugate weight or match method. Consider a reflector antenna with an array feed operating in the transmit mode and with each feed element excited with equal signal levels. In some direction (00, $\phi_0)$, in which the maximum output fields are desired, the output field for each feed element is determined. Let f_i represent the complex output field voltage of the antenna for feed element i in the direction (θ_0, ϕ_0). Then the maximum or optimum output field in that direction would be

$$v_t = \sum_{i=1}^{N} f_i^* f_i \qquad (11)$$

If in the receive mode c_i is the output voltage from the i-th feed element of the reflector antenna system when illuminated with a plane wave arriving from the direction (θ_0, ϕ_0), then the total received signal from the antenna where each feed element is weighted by its complex conjugate is

$$v_r = \sum_{i=1}^{N} c_i^* c_i \qquad (12)$$

From reciprocity, $\frac{c_i}{f_i} = const$ for all i. Therefore, except for a constant, the expressions that are based on the complex weighting in the receive mode are identical to those in the transmit mode. Thus v_r also represents an optimum gain solution. In this analysis, the effects of mutual coupling between array elements has been ignored. For the size and type of feed elements considered in this study, this is not a limitation.

If the integral portion of Equations (3) and (4) are rewritten as follows

$$c_i = \iint_S \left\{ \overline{E}_{ha_i} \bullet \overline{J}_{fp} - \overline{H}_{ha_i} \bullet \overline{M}_{fp} \right\} ds \qquad (13)$$

$$d_i = \iint_S \left\{ \overline{E}_{ha_i} \bullet \overline{J}_{pw} - \overline{H}_{ha_i} \bullet \overline{M}_{pw} \right\} ds \qquad (14)$$

and by the use of Equation (12), Equation (10) can be rewritten as

$$G_a = G_h \cdot \left[\frac{\sum_{i=1}^{N} \hat{c}_i^* c_i}{\left| \sum_{i=1}^{N} c_i^* d_i \right|} \right]^2 \qquad (15)$$

This expression is used to determine the optimum antenna gain simply by knowing the focal-plane currents of the antenna, the array feed geometry, and the feed element aperture fields. G_h, the gain of the array feed in the absence of the antenna reflectors, is obtained by first performing a physical optics integration over the E_{ha} fields in each feed element aperture to obtain thefar-field pattern for each element, Then the power and peak fields are computed from the total fields from all elements in the conventional manner and used to compute G_h.

The analysis method consisted of computing the focal-plane currents of a reflector system using reverse (receive mode) scattering programs. Physical optics is used for a single-reflector antenna design and a combination of geometrical optics off the main reflector and physical optics off of a subreflector is used in a dual-reflector antenna system. Both electric and magnetic focal-plane currents arc computed on a fixed grid over which the array feed element aperture distributions are superimposed. This grid becomes the integration grid over which the convolution of the focal-plane currents and feed aperture field distributions arc integrated. The feed aperture fields are in effect interpolated to fall on the points established by the reverse scattering program's focal-plane grid. To generate the feed element aperture fields at the required grid locations, the far-field element patterns are expanded into a set of circular waveguide modes. These modes can then be evaluated at the required grid location to obtain the feed-aperture fields.

4. USE OF OPTIMIZATION TO MINIMIZE BEAM SCAN LOSS

One application of the optimization approach described in this paper is to minimize the scan losses of an antenna where the antenna beam has been scanned off the axis of symmetry. The test case used consisted of a 423.55-cm-diameter parabolic reflector with an F/D of 0.5. The compensating array feed consisted of 37 elements 1.588 cm in diameter. The main beam is scanned 6.15 degrees off the axis of symmetry. The primary array element that provides the required beam scan is located 24,356 cm off the antenna axis. The frequency is 11.8 GHz. The gain of the antenna with the single feed element is 44.8 dB. With complex conjugated weights applied to 37 elements, the gain is 51.9 dB, for a net improvement of 7.1 dB. For a 19-element version of the array, a gain of 50.4 dB is obtained, for an improvement of 5.6 dB. For a seven-element array, a gain of 49.1 dB is obtained, for an improvement of 4.3 dB.

5. COMPENSATING REFLECTOR DISTORTIONS BY OPTIMIZATION

As was mentioned in the introduction, the main purpose of the focal-plane optimization technique was to reduce the amount of computer time required to arrive at a solution This is particularly true for studying large complex antennas such as the 34-meter beam-waveguide dual-shaped reflector antennas operating at 33.67 GHz that are used at the Jet Propulsion Laboratory/NASA deep space tracking network. In the conventional far-field optimization approach (Ref. 2), a full scattering calculation is required for each array feed element, for each antenna configuration, and for each array geometry. In the focal-plane analysis, a scattering calculation is required only for each antenna configuration. Any number of array geometries and any number of feed elements can be studied with out any further scattering calculations. With the large number of reflectors involved with beam-waveguide antennas and with the analysis being carried out at Ka-band, computation times of 6 hours or more are common, thus limiting the number of scattering calculations for this type of applications is important. Although the ultimate application of this technique is for beam-w waveguide antennas, to expedite the validation of the technique, the initial calculations were limited to the focal-plane of a dual-reflector antenna and these results are reported here. The goal was to compensate the 34-meter antenna for gain losses that result from gravity-induced distortions as a function of the antenna's elevation angle *where* the surface is adjusted for minimum distortion at a 45-degree elevation angle.

The case used for the evaluation consisted of an array feed of seven equal sized elements in a circular cluster on a triangular grid. The first step was to select an element size that minimized the antenna loss at one of the distortion extremes such as at an elevation angle of 7.5 degrees. A set of 13 element diameters were selected ranging from 2 cm to 13 cm. For each element diameter, the focal-plane optimization technique was used to determine the feed-element weights that gave the best performance improvement. Figure 1 contains two curves. The first curve shows the performance of the antenna that results from a single on-axis clement as a function of element size. The second curve shows the best performance for an array of elements. From the curve it was found that an array of elements with diameters of 9.26 cm gave the best performance. Calculations using the optimization technique were repeated at a series of antenna elevation angles using the 9.26-cm-diameter elements to determine the best performance that can be obtained based on the optimum element size determined at 7.5 degrees. Figure 2 summarizes the result. The first curve illustrates the performance that would be expected due to reflector distortions as a function of elevation angles if a single feedhom is used. The second curve shows the improvement in performance that can be obtained with the seven-element array using the optimum element weights computed by the optimization technique.

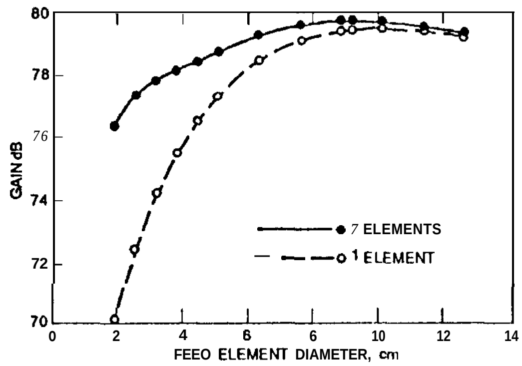

Figure 1. *Optimum antenna gain vs. feed element diameter at 7.5-deg elevation angle*

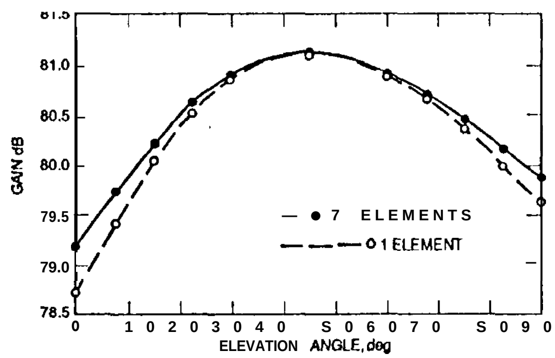

Figure 2. *Optimum antenna gain vs. antenna elevation angle for 9. 26-cm-diameter feedhorn*

6. CONCLUSIONS

The focal-plane optimization technique was shown to provide a method of recovering performance losses for two antenna problems: the aberration losses due to scanning the beam of an antenna and the losses associated with antenna reflector distortions that result from movements of the antenna. The second case demonstrates the power of the technique that performs the optimization in the antenna focal plane. For the antenna configuration used in the second case, it takes 5.6 hours on a Cray Y-MP2 computer to perform one forward scattering calculation to determine the far-field pattern for a single feed element. The scattering calculation must be repeated seven times to include the effect of each array feedhorn, requiring 39.2 hours pcr antenna configuration, The time to compute the optimum gain from the far-field patterns for each feed element is small compared to the scattering calculation time and will be neglected. If the process is repeated for thirteen feed sizes, as was done in the example in this paper, a total time of 509.6 hours is needed! The computation time required for the focal-plane optimization technique can be determined as follows. Assume that the time to do the reverse scattering calculation through the antenna and beam-waveguide optical system is the same as for the forward scattering case, or 5.6 hours. The time required to calculate the currents in the focal plane to the required resolution is 3.3 hours. This gives a one-time total time of 8.9 hours, which does not have to be repeated unless the antenna geometry changes. The time required to perform the focal-plane optimization that must be repeated for each feed size or array geometry ranged from 2 minutes for the smallest feed size to 33 minutes for the largest feed size and 22.5 minutes for the optimum feed size. The total time to perform the optimization calculation for all 13 feed element sizes studied was 2.7 hours. The grand total time, including the scattering calculations, comes to 11.6 hours. This reduction in computation time from 509.6 hours to 11.6 hours illustrates the usefulness of this technique. The question can be raised whether so many element sizes really need to be considered in order to determine the optimum element size, but that is not the issue. There might be a need to also trade off the array geometry and horn type such as single mode, dual mode, hybrid mode, or any other type, resulting in the need to analyze more cases than the 13 element sizes considered in this paper. The point is that significant time savings can be obtained by doing the optimization in the focal plane for situations where the scattering calculation times arc significant, such as is the case for beam-waveguide antennas where five or more large scattering surfaces in terms of wavelengths must be analyzed.

7. REFERENCES

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