Automated Modeling and Control Synthesis Using 'The MACSYN Toolbox [1]

D. S. Bayard, R. E. Scheid, R. Y. Chiang, A. Ahmed, E. Mettler

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109-8099

Abstract

This paper describes the *Modeling and* Control Synthesis (MACSYN) Toolbox, which is an integrated software package for system identification, modeling, and robust control. 'Using MACSYN, the plant is identified in state-space form, and modeling errors are characterized in terms of norm-bounded weighting functions in a form that can be used with modern robust control design software packages. An automated framework simplifies the modeling and control design procedure, and enables robust high-performance feedback designs for systems which would be otherwise difficult to control reliably. MACSYN is implemented as programmable M-files in MATLAB.

1 Introduction

The *Modeling and Control* Synthesis (MACSYN) Toolbox provides a methodology and software tools to identify a linear time-invariant plant and its uncertainty bounds in a form directly useful for robust control design. The plant is estimated in state-space form, and uncertainty models are generated in terms of norm-bounded weighting functions, to precisely capture (to a specified statistical confidence) the *empirical plant set*. Here, the empirical plant set is defined as the set of plants which can't *be* discounted based on the measured data. With the plant and uncertainty in the desired form, standard software packages such as the MATLAB Robust Control Toolbox can be used to design robust controllers with the desired properties.

The main idea is that robust controllers designed with respect to the empirical plant set will tend to be more reliable than those designed with respect to less systematic or ad-hoc specifications of the uncertainty set. This approach enables robust high-performance feedback design for systems which would be otherwise difficult to characterize and control reliably. The MACSYN toolbox is used in conjunction with MATLAB's Robust Control Toolbox and Control System Toolbox.

The MACSYN toolbox can be divided into 4 functional modules (Fig. 1):

Module 1: Input excitation design, Spectral estimation routines

Module 2: State-space system identification routines (Curve fitting, State-space realization, etc.)

Module 3: Model uncertainty characterization and overbounding

Module 4: Control law synthesis routines utilizing plant and uncertainty models from Module 2 & 3 to call the H^∞-based methods from Robust Control Toolbox

The following sections will elaborate each module.

2 Overview

A linear multivariable plant $\mathcal{P}(z)$ is identified in the representation shown in Fig. 2. Many modern robust control methods are applicable to uncertainty expressed in this form. Here, $\hat{P}(z)$ is *a nominal* estimate of the true plant $\mathcal{P}(z)$; Δ_A is the additive uncertainty defined as $\Delta_A = \mathcal{P} - \hat{\mathcal{P}}$; C(z) is the digital controller under consideration; d is a disturbance, and $W_d(z)$ is a frequency weighting filter which characterizes the effect of $d(k)$ on the open-loop plant output $y(k)$. For control design purposes, it is desirable to represent the additive uncertainty in the form $\Delta_A = \Delta W_A$ such that Δ is norm bounded, i.e., such that $||\Delta||_\infty \leq 1$. The filter W_A *is* then typically incorporated into the control design, to ensure robustness properties over the additive uncertainty set. Alternatively, for square plants, a multiplicative uncertainty representation $\Delta_M = (P - \hat{P})\hat{P}^{-1}$ can be used, and the associated weighting filter W_M computed where $\Delta_M = \Delta W_M$ with Δ norm bounded. This toolbox identifies a nominal plant estimate $\hat{P}$, and a weighting filter W_A such that the relation $P = \hat{P} + \Delta W_A$ holds (or a filter W_M such that $P = (I + \Delta W_A)\hat{P}$ holds), for some $||\Delta||_\infty \leq 1$ to a specified statistical confidence" $1 - \kappa$ specified by the designer. Robust control synthesis methods can then be used to find a compensator C that has desirable stability /performance properties for all plants in the uncertainty set defined by P and W_A (or $\hat{P}$ and W_M).

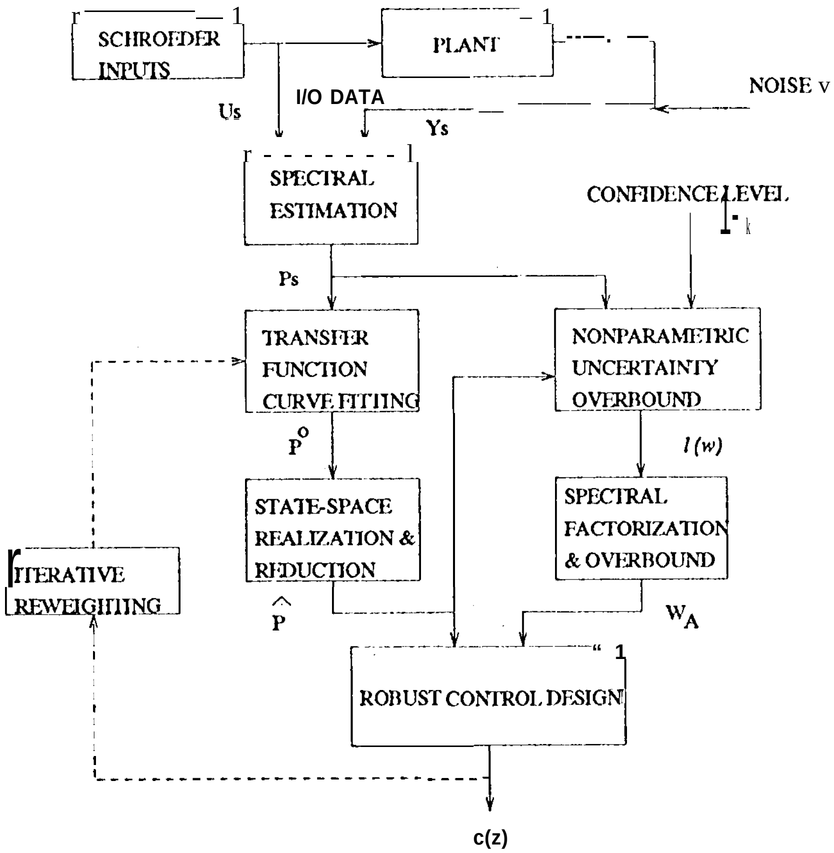

Figure 1: MACSYN Concept: Frequency Domain ID and Robust Control Design.

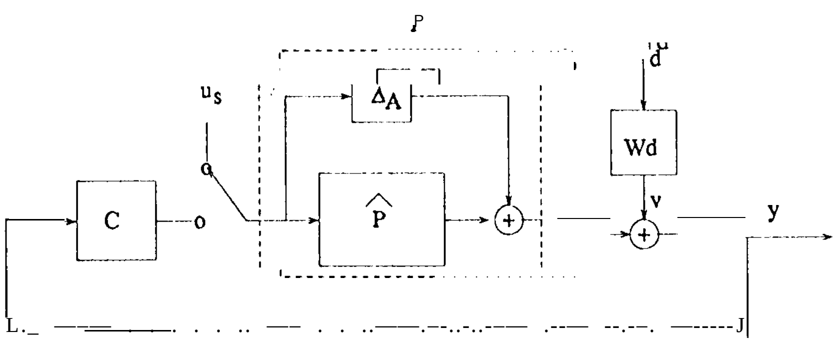

Figure 2: Canonical representation for identification and robust control

3 *Design* Example

A single example is carried throughout the paper and used to demonstrate each module. The example problem concerns identification and control design for the CRAF/Cassini spacecraft High Precision Scan Platform (HPSP) Articulation/Pointing control system. The CRAF spacecraft design with boom-mounted HPSP is **shown** in Fig. 3.

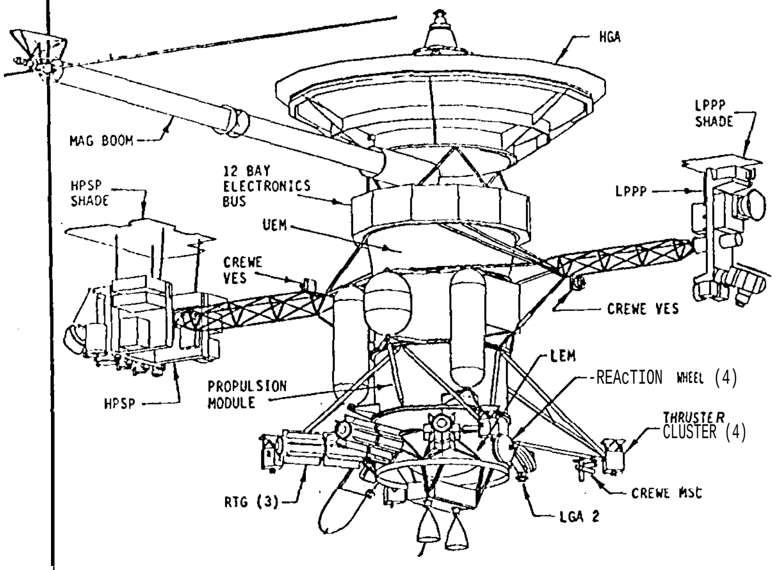

Figure 3: CRAF Spacecraft with Righ Precision Scan Platform

In this example, a 2-input/2-output transfer function is to be identified and controlled, where the azimuth (AZ) and elevation (EL) HPSP motor torques are used as inputs, and the AZ and EL angular positions (i.e., *integrated* rate from a 3-axis Inertial Reference Unit) are chosen as outputs. A Bode magnitude plot of the 2 x 2 nonminimum-phase HPSP transfer function is shown in Fig. 4, as determined using physical modeling techniques, This transfer function will serve as a ‘truth” model for demo purposes.

4 MODULE 1

4.1 input Excitation

The approach begins with the generation of plant input-output data. For this purpose, a multisinusoidal signal u_s is used for plant excitation, of the general form,

$$u_s(k) = \beta \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\ cos(\omega_i kT + \phi_i) \quad (1)$$

where T is the sampling period, $\omega_i = 2\pi i/T_p$, $T_p = N_s T, n_s \leq N_s/2$. For efficient computation using an Fast Fourier Transform (FFT) the total number of frequency grid points N_s should be chosen as a power of 2. This signal has a non-zero power spectrum over a finite grid of points in frequency domain. The amplitudes α_i can be arbitrarily assigned to make any desired profile in the frequency domain. The phases ϕ_i arc chosen (as a function of α_i), to minimize peaking in time, using a formula first given by Schroeder [6].

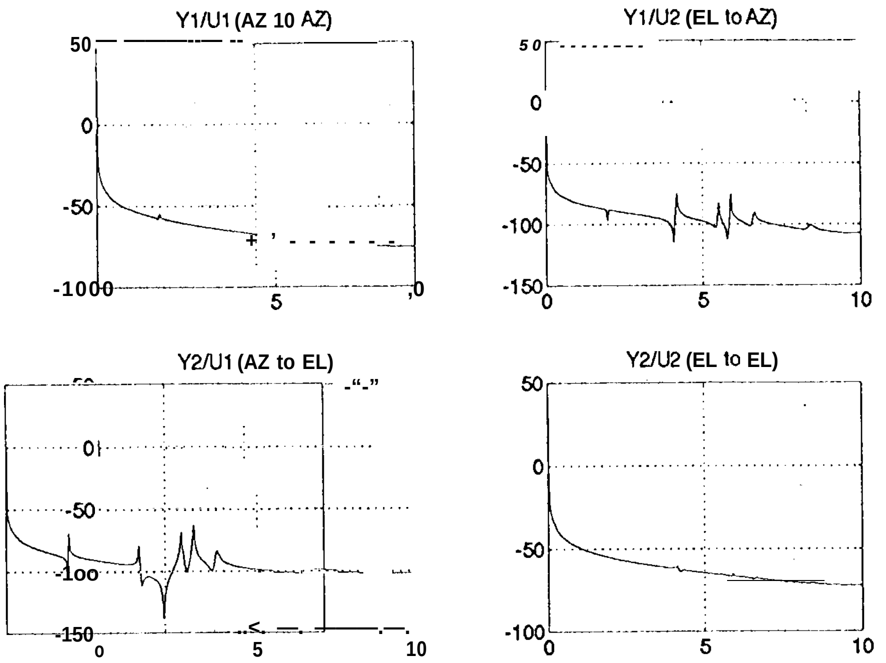

Figure 4: Bode magnitudes of 2 x 2 HPSP transfer function

4,2 Spectral Estimation

The signal u, is applied to the plant until the output reaches steady-state. Input/output data are gathered in steady-state, and averaged to produce a spectral estimate P_s of the plant, a statistical overbound on the error $\overline{\sigma}(P_s - \mathcal{P})$, and a nonparametric estimate of the noise coloring profile W_d (cf., [6][7]). MIMO data are gathered in separate SIMO experiments, one for each input. Responses are measured and processed to give raw multivariable frequency domain estimates.

The input design and spectral estimation is implemented using the following MATLAB functions:

schroeder	multisinusoidal input designs
tfest	frequency domain estimation

4 . 3 Design Example (Revisited)

MIMO data is acquired in two separate SI MO experiments. Based on the response data, the" plant and associated uncertainties are characterized and a robust controller is designed.

An input excitation for each SIMO experiment is designed using routine **schroeder**, with sampling time T=. 05 and number of points Ns=512.

For the first SIMO experiment the input excitation signal is designed using a saturation limit of usat=. 5, and flat spectrum design. A plot of the input design and its spectrum is shown in Figure 5. The flatness of the excitation spectrum is clearly seen in this figure. 'l'he excitation is applied to the first input, and nskip=5 windows of data are skipped, in order to wait for the system to reach periodic steady-state. Once in periodic steady-state, nwin=20 windows of data are collected.

Likewise, a SIMO experiment is performed from the second input. For

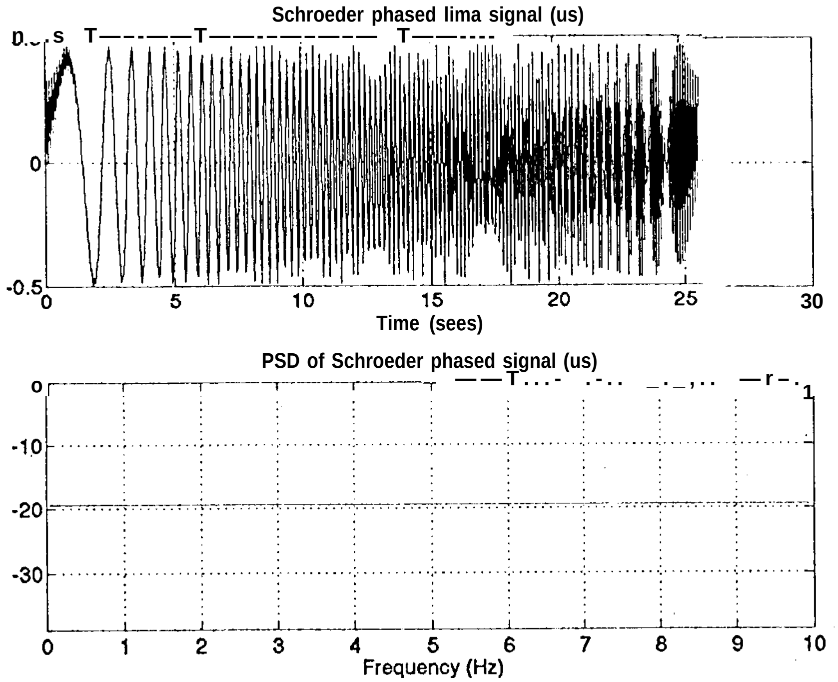

Figure 5: Schroeder excitation for input one (AZ)

this experiment, the input excitation is designed with an actuator saturation level of **usat=1** and a flat spectrum design. As in the first experiment, **nksip=5** windows are skipped, and **nwin=20** windows of data are collected.

The two SIMO data sets are stacked into a single MIMO data set. The MIMO data is passed to the spectral estimation routine tfest, which computes a plant spectral estimate, the spectral estimation error (to 95% confidence) and disturbance spectrum. The plant spectral estimates are shown in Fig. 6. Plots are restricted to the first column of the 2 x 2 transfer function due to space limitations.

5 MODULE 2

A state-space model is determined using the State-Space Frequency Domain (SSFD) algorithm [5] involving a two-step procedure of complex curve fitting and state-space realization.

5.1 Complex Curve Fitting

First, the spectral estimate P_s is curve fitted to obtain a rational transfer function model $P^{o}(z) = Ii(z)/a(z)$ where B(z) is a matrix polynomial, and a(z) is a scalar polynomial,

$$P^{o}(z) = \frac{B(z)}{a(z)} \tag{2}$$

$$B(z) = B_0 + B_1 z^{-1} + \ldots + B_n z^{-n} \tag{3}$$

$$a(z) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \tag{4}$$

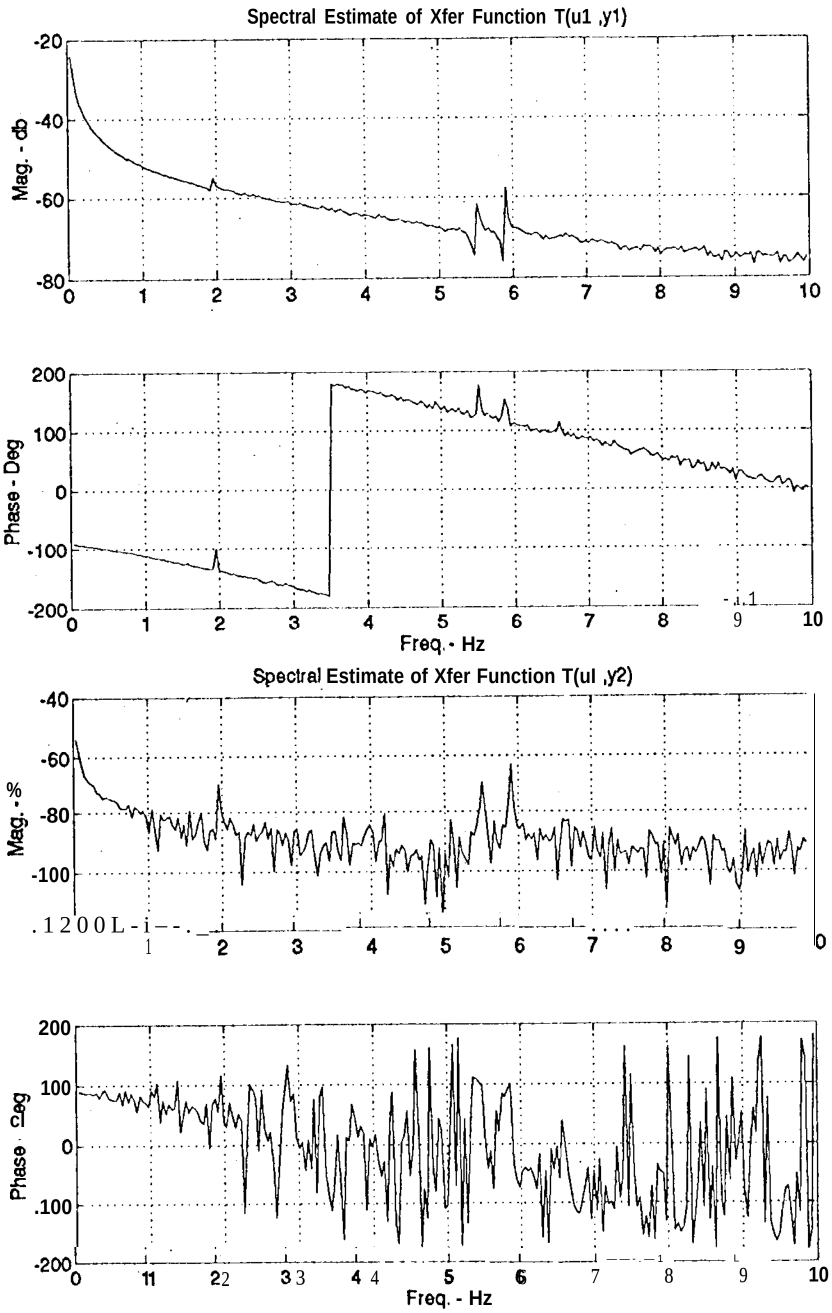

Figure 6: Spectral estimates of transfer functions Y1/U1, Y2/U1

‘I’he curve fitting approach based on minimizing a weighted 2-norm error criteria of the form,

$$\min_{P^o} \sum_{i=1}^{n_s} w^2(\omega_i) \left\| P_s(\omega_i) - P^o(e^{j\omega_i T}) \right\|_f^2 \tag{5}$$

where the Frobenious norm is defined as,

$$||X||_f^2 = Tr\{X^* X\} \tag{6}$$

and “ * “ denotes the complex conjugate transpose.

The minimization procedure consists of a Gauss Newton (GN) algorithm, initialized by a suboptimal fixed point iteration, denoted as the SK iteration, due originally to to Sanathanan and Koerner. The algorithm uses sparse matrix methods to take advantage of the special structure associated with large MIMO problems (cf., Bayard [4]). The frequency weighting $w(\omega_i)$ can be chosen to reduce error in frequency regions which are most relevant to the control design.

5.2 State-Space Realization

Using a singular value decomposition of the Hankel matrix of Markov parameters associated with $P^o(z)$, the transfer function matrix is realized/balanced/reduced to give the state-space model,

$$\hat{P}: \quad x_{k+1} = Ax_k + Bu_k \tag{7}$$

$$y_k = Cx_k + Du_k \tag{8}$$

Curve fitting and state-space realization are performed using the following MACSYN functions:

zinfit	complex MIMO curve fitting
22ss	state-space realization
z2ssi	state-space realization (interactive)

5.3 Design Example (Revisited)

In the curve fitting step of the SSFD algorithm, a rational transfer function is fit to the plant frequency data by the routine zinfit. The curve fit is chosen with denominator order na=22, uniform frequency weighting, and niter=4 iterations of the SK algorithm are specified.

In the second step, a state-space realization is performed by the routine p2ssi. This routine in interactive. It provides the user with a plot of the Hankel singular values and asks the user to specify the number of singular values to retain. The number specified will correspond to the *number of states* in the final state-space realization. The singular values for the demo problem are shown in Fig. **7.**

There is a large break in the singular values after 44, since there exists an exact state-space realization of this order. The value of 44 is used to continue the demo, which corresponds to a negligible model reduction error.

The Bode plot of the state-space realization of order 44 is plotted in Fig. 8 against the spectral estimate for comparison. The plot is restricted to the first column of the transfer function due to space limitations.

6 MODULE 3

6.1 Modeling Errors

An additive uncertainty bound $\ell_A^{1-\kappa}(\omega) \geq \bar{\sigma}(\mathcal{P} - \hat{P})$ is characterized *in* non-parametric form to within statistical confidence $(1-\kappa)$ x 100% (specified by

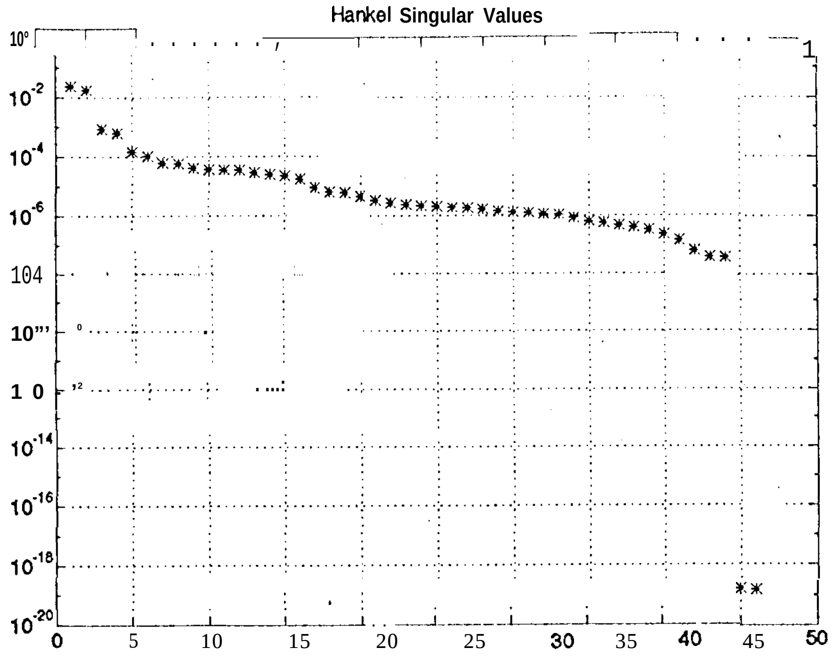

Figure 7: Hankel singular values for state-space realization

the' designer). The basic idea is to use the triangle inequality,

$$\overline{\sigma}(\mathcal{P}(\omega_i) - \hat{P}(\omega_i)) \leq E_{ss} + E_{se} \leq \ell_A^{1-\kappa}(\omega_i) \tag{9}$$

$$E_{ss} = \overline{\sigma}(P_s(\omega_i) - \hat{P}(\omega_i)) \tag{10}$$

$$E_{se} = \overline{\sigma}(\mathcal{P}(\omega_i) - P_s(\omega_i)) \tag{11}$$

Here, E_{ss} is the error in the state-space model (with respect to the spectral estimate), and can be calculated exactly since P_s and $\hat{P}$ are known. In contrast, the spectral estimation error E_{se} is a probabilistic quantity. It is not known exactly (because the true plant is unknown), but can be bounded statistically to any desired confidence level $(1 - \kappa) \cdot 100\%$ from the spectral estimation errors [6][7]. The resulting bound on the additive error is denoted as $\ell_A^{1-\kappa}$. A similar bound on the multiplicative error can be determined, and is denoted as $\ell_M^{1-\kappa}$.

For visualization purposes, it is useful to use the triangle inequality a second time, and split the error E_{ss} as,

$$E_{ss} = \overline{\sigma}(P_s(\omega_i) - \hat{P}(\omega_i)) \leq E_{fit} + E_{rom} \tag{12}$$

$$E_{fit} = \overline{\sigma}(P_s - P^o) \tag{13}$$

$$E_{rom} = \overline{\sigma}(P^o - \hat{P}) \tag{14}$$

The term E_{fit} denotes the error incurred in the curve fitting step, and the term E_{rom} denotes the error from reducing the order between the curve fit model P^o and the state space model $\hat{P}$. Both terms can be computed exactly and displayed to indicate the relative contributions of the various errors.

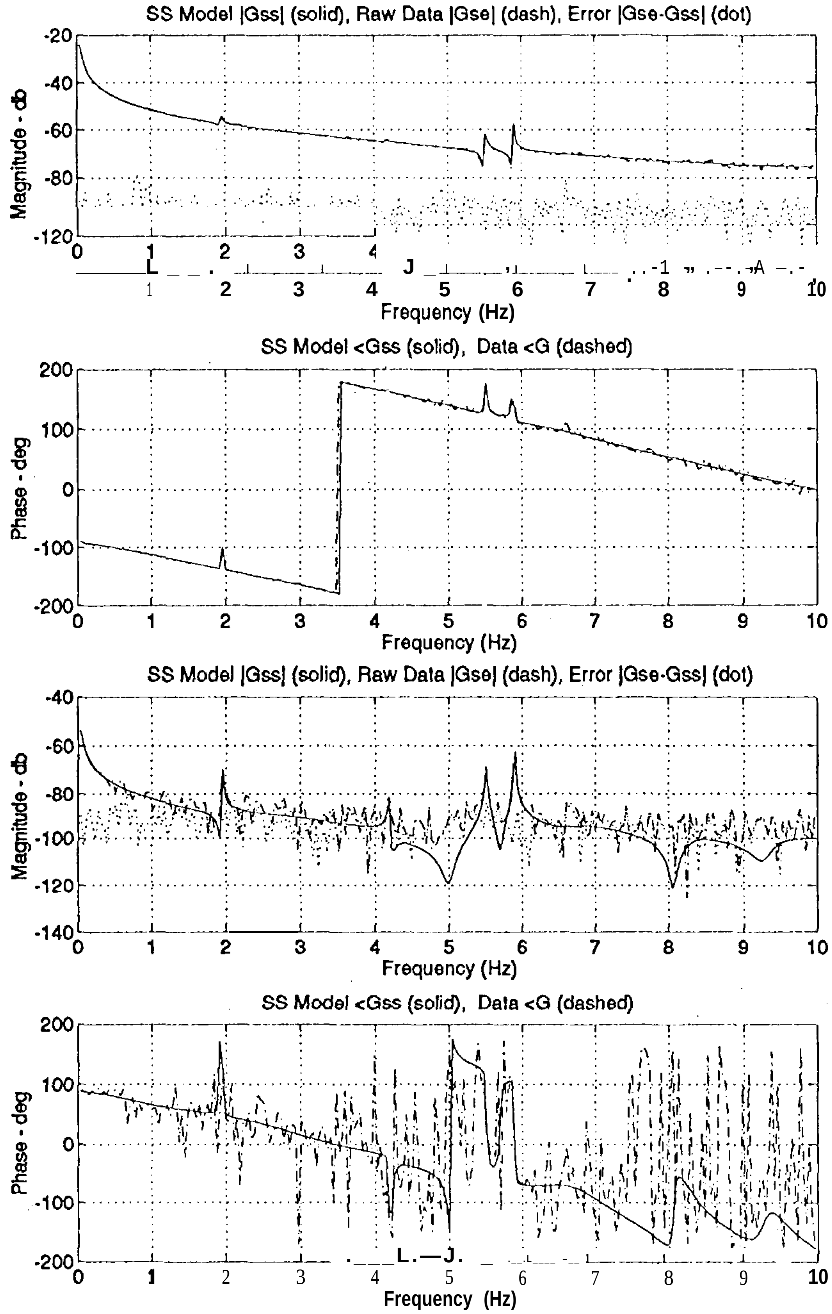

Figure 8: Identified State Space model and spectral estimate, Y1/U1 and Y2/U1 transfer functions

6.2 Uncertainty Overbounding

The nonparametric additive uncertainty profile $\ell_A^{1-\kappa}(\omega)$ in (9) is overbounded (tightly) by a parametric minimum-phase transfer function W_A of specified order, i.e.,

$$|W_A| = b(z)/a(z) \geq \ell_A^{1-\kappa}(\omega) \qquad (15)$$

The method used for overbounding is the linear programming spectral overbounding and factorization (LPSOF) algorithm [10]. This algorithm finds a tight (i.e., globally minimax) overbound W_A on the additive error (or overbound W_M on the multiplicative error), of specified order. With this construction, the additive uncertainty has the form $\Delta_A = \Delta W_A$ (or the multiplicative uncertainty ha-s the form $\Delta_M = \Delta W_M$), where $\|\Delta\|_\infty \leq 1$ is norm bounded. Model error characterizations are determined by the following MATLAB functions,

moderr	componentwise modeling error
bounder	**uncertainty characterization**
overbound	**uncertainty overbound**
factor. mimo1	**uncertainty spectral factorization**

6.3 Design Example (Revisited)

Components of the modeling error are calculated by the routine moderr, and are summarized in the plot shown in Fig. 9. This plot shows the relative contributions of the spectral estimation error, ese (dotted); the curve-fit error, efit(dashed); and the *model reduction error,* erom (o) to the total additive uncertainty, eadd (+).

As a rule of thumb, in order to achieve a control design of a specified bandwidth, it is desired for the additive uncertainty (+) not to exceed $\underline{\sigma}(\hat{P})$ (solid) (thus ensuring a multiplicative error less than unity), except possibly at frequencies where $\hat{P}$ is small in magnitude (i.e., providing small loop gain at those frequencies). With this guideline, it is seen from Fig. 9, that a control bandwidth of 5 or 6 Hz is reasonable.

If greater control bandwidth is desired, the uncertainty must be reduced accordingly. From Fig. 9, one can discern the dominant error components in each frequency regime. For example, the reduced order modeling error erom (o) is seen to be negligibly small due to the full 44 states retained in the earlier state-space realization step. The largest component is the spectral estimation error (dotted), which is the same order of magnitude aa (and obscured in the plot by) the total additive uncertainty (+). Hence, for this demo problem, improved control performance requires reducing and/or reshaping the spectral estimation error, which in turn requires taking additional experimental data and possibly redesigning the frequency shaping profile of the input excitation.

The multiplicative uncertainty is also calculated by moderr and is shown in Fig. 10. The control design can be computed in terms of either the additive or multiplicative uncertainties.

For this demo, the multiplicative uncertainty description will be used. The total multiplicative uncertainty is overbounded by a stable, minimum phase, rational transfer function W_M. The overbound is constructed by calling the routine bounder with order=2. The overbound should be as tight as possible in order to avoid conservatism in the final robust control design. The tightness of the overbound is manipulated by specifying data points where the tightness of the overbound is optimized and/or by constructing a ceiling that puts a cap on the overbound.

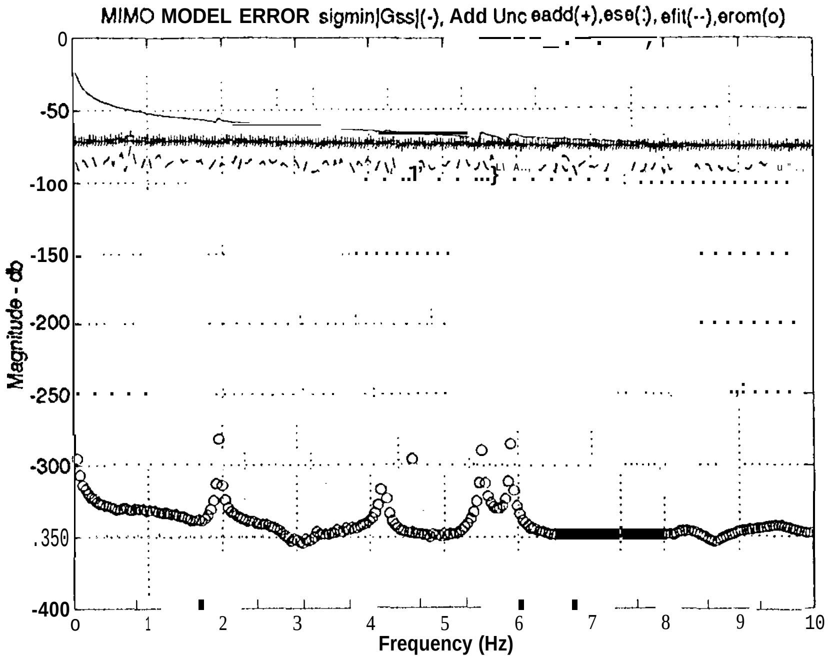

Figure 9: Model uncertainty (Additive)

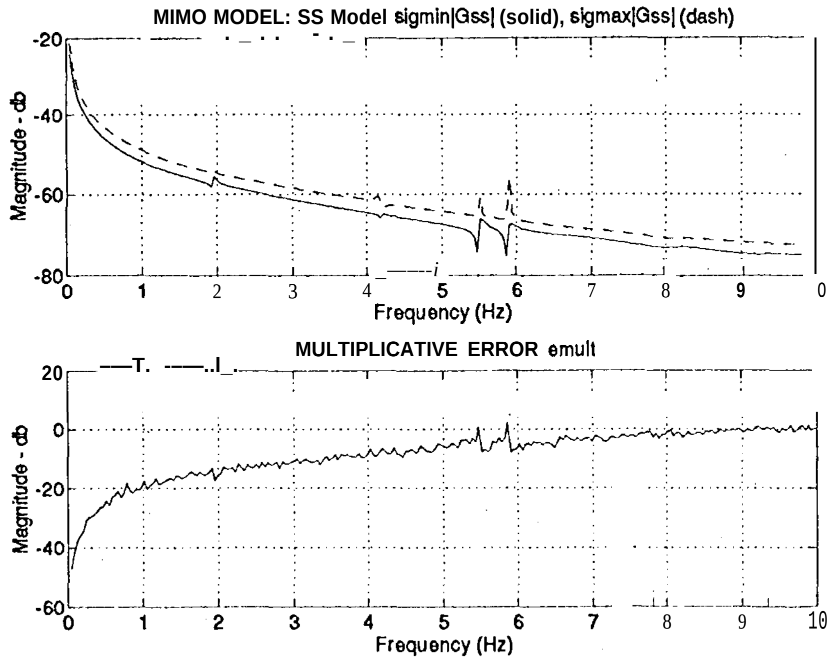

Figure 10: Model Uncertainty (Multiplicative)

The multiplicative uncertainty and overbound for the demo problem, obtained using the bounder routine are shown in Fig. 11.

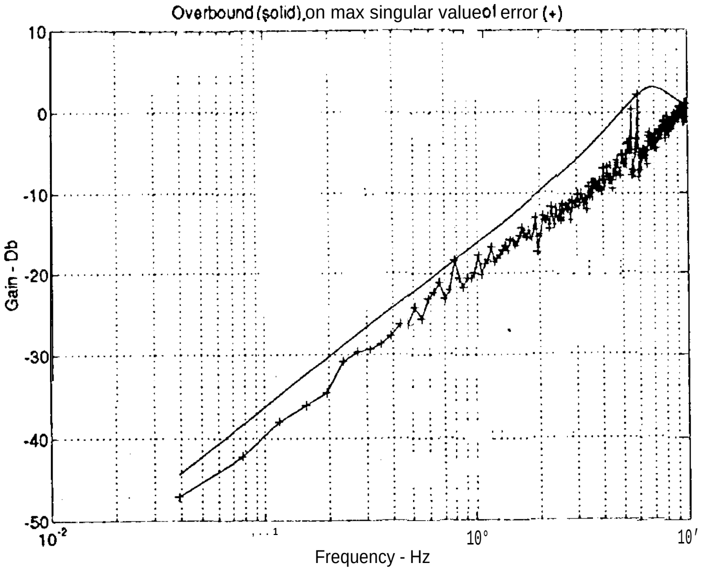

Figure 11: Optimal minimax overbound on multiplicative uncertainty

7 MODULE 4

7.1 Robust Control Synthesis

Robust control synthesis tools are used to find a robust controller using the identified values of $\hat{P}$ and W_A (or W_M). Three key fundamental issues in any control system design, *stability*, robustness and *performance*, can all be captured in one robust control problem formulation and solved as one single H^2 and/or H^∞ optimization problem [2].

The following is a popular two-block mixed-sensitivity example illustrating a standard problem formulation of the robust control design (Ref. Fig. 12) [3]:

$$\min_C \left\| \begin{bmatrix} W_1 S \\ W_3 T \end{bmatrix} \right\|_\infty < 1 \qquad (16)$$

where $S = (I + \hat{P}C)^{-1}$ is the sensitivity junction, $T = \hat{P}C(I + \hat{P}C)^{-1}$ is the *complementary* sensitivity function, C is the compensator, and W_1 and W_3 are weighting functions that penalize *performance* and robustness, respectively. To ensure robustness with respect to the estimated plant set, weighting W_3 is chosen as the multiplicative uncertainty weighting W_M derived above.

Robust control loop shaping tools available in Robust Control Toolbox [2] can be called directly from MATLAB environment:

(d)hinf	(discrete) continuous H^∞ synthesis
(d)h2lqg	(discrete) continuous H^2 synthesis
musyn	μ-synthesis method
ssv	mixed real/complex robustness analysis
bstschmr,...	singular value based model reduction

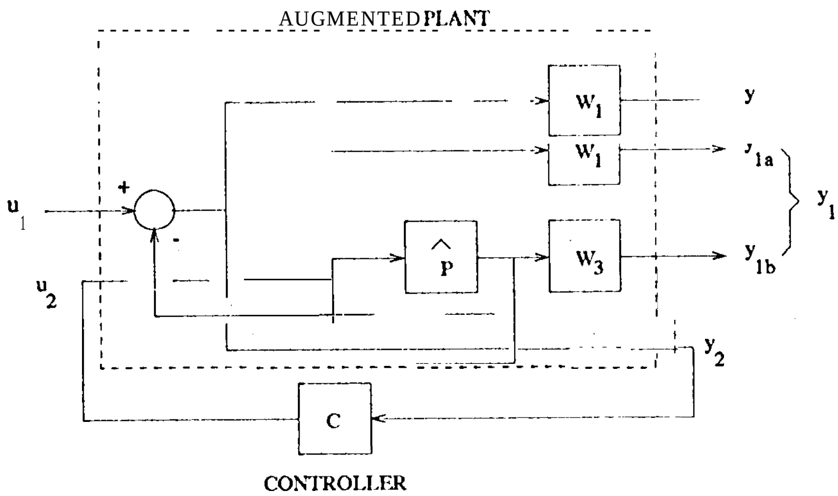

Figure 12: Mixed-Sensitivity Problem Formulation.

7,2 **Design Example (Revisited)**

The weighting functions W_1 and W_3 are displayed in Fig. 13 for the two-block mixed-sensitivity formulation. Weighting W_3 is chosen as the estimated multiplicative uncertainty bound W_M.

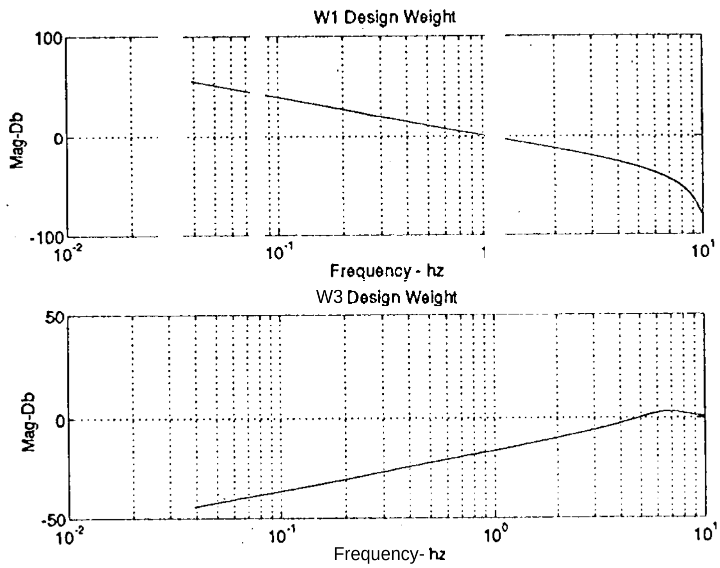

Figure 13: Weighting functions W_1 and W_3.

Routine **dhinf** from the Robust Control Toolbox [2] performs a series of tests for the existence of solutions of the required Riccati equations and the H-infty controller. Once these tests are complete, an H_∞ controller is designed. The resulting 52-state, 2 x 2 robust H_∞ control design is shown in Fig. 14. Included are four plots for controller evaluation.

The upper left plot (1 ,1) is the Hinf Cost function. The closer the value is

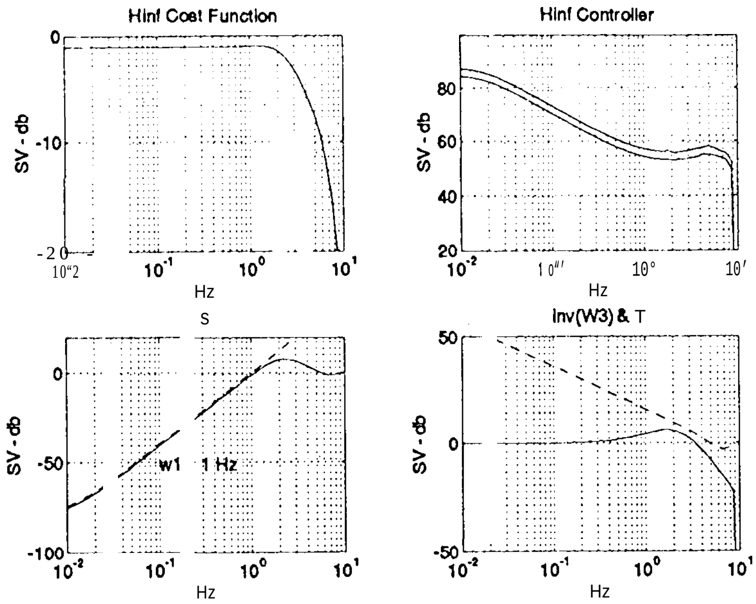

Figure 14: Evaluation of controller.

to O db, the closer the desired loop shaping is achieved for the given weightings. The upper right plot (1,2) is the Hinf controller singular value plot. The lower left plot (2,1) is the sensitivity weighting W_1^{-1} and sensitivity function S. Ideally, these curves should "hug" each other closely at low frequencies. The small value of sensitivity at low frequency guarantees disturbance rejection up to particular frequency range and good tracking of the platform command.

The lower right plot(2,2) is the inverse weighting W_M^{-1} *and* the *achieved* complementary sensitivity function *T* (closed loop transfer function from input to output). The peak of the complementary sensitivity function *T* determines system robustness against multiplicative uncertainty. The roll-off rate indicates the system bandwidth, robustness against high frequency uncertainty, etc. A small gap is intentionally left for additional robustness, beyond that required to capture the effects of identification errors.

The final controller achieves the design specifications, and ensures robustness with respect to the plant set consistent with the measured data. This serves to illustrate the end-to-end automated ID and control synthesis capability using the MACSYN toolbox.

8 Summary

The Modeling *and Control Synthesis* (MACSYN) Toolbox is an integrated software package for system identification, modeling, and robust control. The main idea is to identify the plant and the modeling uncertainty in a form which is directly usable by modern robust control design formulations. Specifically, the plant is identified in state-space form, and modeling errors are characterized in terms of norm-bounded weighting functions. An automated framework simplifies the modeling and robust control design procedure.

Since MACSYN presently assumes that the plant is linear and time-irrvariant, not all plant uncertainties and modeling errors are captured in the empiracle plant set. Hence, the presence of time varying plant parameters, nonlinearities, and other complexities may require certain modifications of the approach. For example, the designer may wish to add extra uncertainty blocks to increase the robustness in the final control design. Hence, " like any tool, the MACSYN toolbox should be used with the best engineering judgement, and is flexible enough to be combined with other approaches to obtain the best final control design.

The MACSYN concept was originally developed at NASA's Jet Propulsion Laboratory, for applications involving the identification and control of high-order multivariable systems over wide bandwidths (e.g., large space structures, adaptive optics applications, etc.). However, the underlying functions are generically useful and can serve the needs of various industrial ap placations such as process control, aircraft flight control, and automotive, robotic and manufacturing control systems.

Acknowledgement The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

References

[1] E. Mettler, D. S. Bayard, Y. Yarn and R. Scheid, "In-Flight System Identification for the CRAF/CASSINI Spacecraft," *Proc. of First IEEE Conf. on Control Application*, Dayton, Ohio, Sep. 13-16, 1992.

[2] R. Y. Chiang and M. G. Safonov, *Robust Control Toolbox, Version* 2. The MathWorks, Inc. 1988-1992.

[3] R. Y. Chiang, "Theory and Weighting Strategy of Mixed Sensitivity H^∞ Synthesis on a Class of Aerospace Applications," Submitted to *IFAC*, Symposium on *Automatic Control in Aerospace*, Palo Alto, CA, Sep. 12-16, 1994.

[4] D.S. Bayard, "Multivariable frequency domain identification via 2-norm minimization," *Proc. of American Control Conf.*, pp. 1253-1257, Chicago, Illinois, June 1992.

[5] D.S. Bayard, "An algorithm for state-space frequency domain identification without windowing distortions," *Proc. 31st IEEE Conf. on Decision and Control*, pp. 1707-1712, Tucson, Arizona, December 1992.

[6] D.S. Bayard, " Statistical plant set estimation using Schroeder-phased multisinusoidal input designs," J. Applied Mathematics and Computation, Vol. 58, pp. 169-198, 1993; also, American Control *Conf.*, 2988-2995, Chicago, Illinois, June 1992.

[7] D.S. Bayard and F.Y. Hadaegh, "Multivariable plant set estimation using multisinusoidal input design," to be presented at the System Identification (SYSID) Conference, Copenhagen, Denmark, July 3-6, 1994.

[6] D.S. Bayard and Y. Yam, "Frequency domain identification for robust control design," in *The Modeling of* Uncertainty in *Control Systems*, Roy S. Smith & Mohammed Dahleh (Eds.), Springer- Verlag: New York, 1994.

[9] D.S. Bayard, Y. Yarn, E. Mettler, " A criterion for joint optimization of identification and robust control," IEEE Transactions on Automatic Control, Special hfini-issue on *System Identification for* Control Design, Vol. 37, No. 7, July 1992.

[10] R.E. Scheid, D.S. Bayard and Y. Yam, "A linear programming approach to characterizing norm-bounded uncertainty," *Proc. American Control Conf.*, pp. 1956-1958, Boston, Mass, June 1991.

[11] D.S. Bayard and R.Y. Chiang, "A frequency domain approach to system identification, uncertainty characterization, and robust control design," *32nd IEEE Conference* on Decision *and Control*, San Antonio, Texas, December 1993.