Trellis Coding of Non-coherent Multiple Symbol

Full Response M-ary CPFSK with Modulation Index 1/M

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Abstract

This paper introduces a trellis-coded modulation (TCM) scheme for non-coherent multiple full response M-ary CPFSK with modulation index 1/M. A proper branch metric for the trellis 'decoder' is obtained by employing a simple approximation of the modified Bessel function for large signal-to-noise ratio (SNR). The pair-wise error probability of coded sequences is evaluated by applying an equivalent squared distance to the Rigvan random variable. Examples are presented for trellis codings of non-coherent binary CPFSK by using the Ungerboeck's set partitioning method. Asymptotic upper bounds on bit error probability are evaluated for the given coded systems, and simulation results are also presented.

1 Introduction

Trellis Coded Modulation (TCM) [1] is developed to obtain coding gain without increasing bandwidth. Also multidimensional TCM [2], [3] and Multiple TCM [4] are developed for power and bandwidth efficiency. These schemes are originally developed for a coherent modulation system. TCM applied to multiple symbol differential phase shift keying (MDPSK) can be found in [5]. The paper [6] considers the multiplex response continuous bias frequency shift keying with non-coherent detection. This paper introduces the equivalent squared distance (ENSD). The ENSD of a non-coherent system plays the same role as that of the coherent system for evaluating the bit error probability. We show that a combination of a trellis encoder and a non-coherent N-consecutive M-ary CPFSK can potentially yield a significant improvement in performance, even for small N, over the uncoded system. For the analysis we use a linear approximation for the Kacanik random variable to evaluate the pair-wise error probability. We show that the pair-wise error probability of coded sequences can be expressed as a function of the sum of ENSDs. We introduce the equivalent squared free distance, (which represent the smallest distance between coded sequences leaving from the same trellis state at a given time and remerging again later on). (ENSD) is a design tool for the trellis encoder with a non-coherent system as the Euclidean squared free distance is for a coherent system.

2 System Model for TCM

Figure 1 is a simplified block diagram of system under investigation. Input bits, b_k, occurring at a rate R_b, are passed through a trellis encoder with code rate r, producing an encoded bit stream c_m at a rate, R_m = r R_b. These encoded bits, c_m, are converted to a sequence, u_m = (u_m,0, u_m,1, ..., u_m,N-1), where u_m,i ∈ {0, 1, 2, ..., M-1}. The N-dimensional vector, u_m, is mapped into an N-consecutive continuous phase encoder (NCPE) [7], where the state of NCPE is denoted by v_m. The output is mapped into an N-consecutive M-ary CPFSK waveform. This generation of CPFSK is explained in [2].

We assume that an arbitrary phase offset, θ_m, introduced by the channel during the reception of u_m is constant and uniformly distributed in [0, 2πr]. Furthermore the sequence of random variables are assumed to be independent. This assumption is valid if interleaving and deinterleaving are used after trellis encoder and before the Viterbi algorithm. At the receiver, the noise corrupted signal is non-coherently detected, and the resulting computed metrics, denoted by (T_i), are then used for the branch values of the trellis. For coherent detection, a metric based on the squared Euclidean distance between received and transmitted waveforms, is optimum in the sense of a minimum probability of error sequence. For non-coherent detection, by the suitable modification, the appropriate metric can be interpreted as an equivalent normalized squared distance (ENSD).

We denote a coded symbol sequence of length L corresponding to the output sequence of trellis encoder, u = (u_0, u_1, ..., u_L), where the m+1st element of u is u_m = (u_m,0, u_m,1, ..., u_m,N-1). The state of NCPE, v_m, and the input vector, u_m, which specify a transmitting N-consecutive M-ary CPFSK waveform during the mth time interval [m T, (m + l)T) where TN = NT and T is the duration of each coded symbol. The complex received baseband signal r(t) can be represented as

\[ r(t) = s(t) + n(t) \]

and

\[ s(t) = A e^{j \phi(t)} \]

where

\[ \phi(t) = \frac{2 \pi f_c t}{T} \]

and

\[ f_c = \frac{M}{T} \]

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\[ \mu(t) = \frac{2 \pi f_c t}{T} \]

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algorithm can be implemented with the same number of states as required for the trellis encoder.

The conditional probability of $\mathbf{r}(t)$ over the interval $[0, (L - 1)TN]$, given an input sequence of length $L$, $\mathbf{u}$, and carrier phase offsets, $\theta_1, ..., \theta_{L - 1}$, can be expressed as

$$P_r(\mathbf{r}(t)|u, \theta_0, \theta_1, ..., \theta_{L - 1}) = C \prod_{m=0}^{L-1} \exp\left\{-\frac{1}{N_0} \int_{0}^{TN} |\mathbf{r}(t - mTN)|^2 dt \right\}$$

which can be expressed in a simplified form

$$P_r(\mathbf{r}(t)|u, \theta_0, \theta_1, ..., \theta_{L - 1}) = C e^{-2LTN/N_0} \exp\left\{-\frac{1}{N_0} \int_{0}^{TN} |\mathbf{r}(t)|^2 dt \right\}$$

where

$$\beta_m = \int_{0}^{TN} \mathbf{r}(t + mTN)e^{-j\theta_N(t + mTN, \omega_m)} dt.$$ (5)

Averaging (5) over $\theta_0, \theta_1, ..., \theta_{L - 1}$ (assuming interleaving and deinterleaving is used) we get

$$P_r(\mathbf{r}(t)|u) = F \prod_{m=0}^{L-1} I_0\left(\frac{2A}{N_0} |\beta_m| \right),$$

where $F$ is a constant which is independent of input sequence $u$. An exact evaluation of (6) in a closed form is difficult if not impossible. To get decision statistic which can be used to implement the Viterbi algorithm, we have employed a simple approximation of the modified Bessel function $I_0(\cdot)$ for large SNR as follows:

$$\ln I_0(x) \approx |x|.$$ (7)

Therefore the decision variable behaves as

$$L(\mathbf{r}(t)|u) = \sum_{m=0}^{L-1} |\beta_m|.$$ (8)

Thus, the appropriate decision rule for the coded sequence $\mathbf{u}$ is the following:

choose $\hat{\mathbf{u}} = \mathbf{u}''$ as the coded sequence if

$$\max_{\mathbf{u}''} \left( \sum_{m=0}^{L-1} |\beta_m| \right) = \sum_{m=0}^{L-1} \left| \int_{0}^{TN} \mathbf{r}(t + mTN)e^{-j\theta_N(t + mTN, \omega_m)} dt \right|,$$

where $\mathbf{u}'' = (u_0, u_1, ..., u_{L - 1})$. The decision metric in (10) can also be used for no interleave case which results in a suboptimum metric.

3 Evaluation of an Upper Bound on the Bit Error Probability

An error event of length $L$ can be described by considering two $L$-tuples of coded symbols. Let $\mathbf{u}_L = (u_0, u_1, ..., u_{L - 1})$ denote transmitting $L$-tuple and $\hat{\mathbf{u}}_L = (\hat{u}_0, \hat{u}_1, ..., \hat{u}_{L - 1})$ denote another $L$-tuple. Each component of $\mathbf{u}_L$ and $\hat{\mathbf{u}}_L$ consists of $N$ coded symbols. An error event with length $L$ occurs when demodulator chooses, instead of transmitted sequence $\mathbf{u}_L$, another sequence $\hat{\mathbf{u}}_L$ of channel symbols corresponding trellis path that splits from the correct path at a given time, and remerges exactly $L$ discrete times later.

The union bound provides the following inequality for the bit error probability:

$$P(e) \leq \frac{1}{b} \sum_{L=0}^{\infty} \sum_{m=0}^{L-1} w(\mathbf{u}_L, \hat{\mathbf{u}}_L) P(\mathbf{u}_L)P(\mathbf{u}_L \to \hat{\mathbf{u}}_L).$$ (10)

where $P(u_L \to \hat{u}_L)$ is the pairwise error probability and $b$ denotes the number of information bits transmitting every TN sec. and $w(\mathbf{u}_L, \hat{\mathbf{u}}_L)$ denotes the hamming distance between two binary sequences corresponding to $\mathbf{u}_L$ and $\hat{\mathbf{u}}_L$.

To find the upper bound on bit error probability in (10), we must first find the pairwise error probability which represent the probability of choosing the coded sequence $\hat{\mathbf{u}}_L = (\hat{u}_0, \hat{u}_1, ..., \hat{u}_{L - 1})$ instead of $\mathbf{u}_L = (u_0, u_1, ..., u_{L - 1})$. Let $|\beta_m|$ denote the maximum likelihood metric for the $m + 1$st trellis branch of the correct data sequence, computed from (5). Then the pairwise error probability is given by

$$P_r(\mathbf{u}_L \to \hat{\mathbf{u}}_L) = P_r\left( \sum_{m=0}^{L-1} |\beta_m| > \sum_{m=0}^{L-1} |\beta_m| |u_L| \right).$$ (11)

Here, $|\beta_m|$ denotes the metric computed for the data sequence associated with the $m + 1$st trellis branch of the incorrect path. To evaluate (11), we use a linear approximation. Random variable $\beta_m$ can be expressed as

$$\beta_m = ATu(m) \cdot 1 - \hat{n}_2,$$ (12)

where

$$u(m) = \frac{1}{T} \int_{t}^{T} e^{j\theta(t + mTN, \omega_m)} dt,$$ (13)

and $\Delta u_m = u_m - \hat{u}_m$. We denote the zero mean complex Gaussian random variable, $\hat{n}_2$, as follows:

$$\hat{n}_2 = \int_{0}^{TN} \hat{n}(t + mTN)e^{-j\theta_N(t + mTN, \omega_m)} dt.$$ (14)

For large SNR we can make a linear approximation for $|\beta_m|$ as follows:

$$|\beta| \approx |ATu| \sqrt{1 + \frac{u^* \hat{n}_2 + \hat{n}_2^*}{|ATu|^2}} = |ATu| \left( 1 + \frac{u^* \hat{n}_2 + \hat{n}_2^*}{|ATu|^2} \right) \approx |ATu| + \frac{1}{2|u|^2} u^* \hat{n}_2 + \hat{n}_2^*.$$ (15)

Similarly approximation for $|\beta_m|$ can be obtained by setting $\Delta u_m = 0$ which results in $u(m) = N$ and using this in the above expression.

The statistic of $\sum_{m=0}^{L-1} |\beta_m| - \sum_{m=0}^{L-1} |\beta_m|$ can be evaluated approximately as a Gaussian random variable. Define a new random variable $Y$ as

$$Y \triangleq \sum_{m=0}^{L-1} |\beta_m| - \sum_{m=0}^{L-1} |\beta_m|.$$ (16)
can be approximated as a sum of independent Gaussian variables. We now evaluate the mean, \( Y \), and variance of \( \gamma \) as follows:

\[
Y = \sum_{m=0}^{L-1} A T(N - |u(m)|),
\]
and

\[
\sigma_Y^2 = \sum_{m=0}^{L-1} N_0 T(N - |u(m)|).
\]

Therefore we can rewrite (11) as

\[
P_r(u_L \rightarrow \bar{u}_L) = P_r(Y < 0|u_L) \approx Q\left(\frac{E_s}{N_0} N^{L-1} \sum_{m=0}^{L-1} (N - |u(m)|)\right)
\]
\[
= Q\left(\frac{E_s}{N_0} \sum_{m=0}^{L-1} (N - |u(m)|)\right)
\]
\[
= Q\left(2N \sum_{m=0}^{L-1} d_{e,m}^2\right)
\]

where

\[
d_{e,m}^2 \triangleq 2(N - |u(m)|).
\]

The equivalent normalized squared distance, \( d_{e,m}^2 \), defined in (19), plays the same role as the normalized squared Euclidean distance of coherent detection for evaluating the error probability of the coded case as well as the uncoded case.

### 4 Design of Trellis Encoder

It is our goal in this section to design a trellis encoder shown in Figure 1 so that we can get the smallest error probability. As we discussed in the previous section, the pairwise symbol error probability of trellis coded sequences can be expressed as a function of the accumulated ENSD. We define the equivalent squared free distance \( d_{f,\text{free}}^2 \), which plays the same role as the squared Euclidean distance in coherent detection, \( d_{e,\text{free}}^2 \), represents the smallest value of the accumulated ENSD between sequences. Therefore we should find the encoder having the largest \( d_{f,\text{free}}^2 \). To pursue this goal we use Ungerboeck's set partitioning approach and computer search.

#### 4.1 2-consecutive Binary CPFSK with Modulation Index 1/2

We use the set partitioning method for the set of waveforms of 2-consecutive binary CPFSK. Each waveform is denoted by a two dimension vector \( u_m = (u_m,0,u_m) \). As shown in Table 1 we use two level of partitioning denoted by the Cartesian product. The set of vectors \( m \) level 1 is partitioned into 2 subsets of size 2.

<table>
<thead>
<tr>
<th>Level</th>
<th>Partitioning A</th>
<th>Partitioning B</th>
<th>ENSD0</th>
<th>ENSD1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,1) x (0,1)</td>
<td>(0,1) x (0,1)</td>
<td>1.454</td>
<td>1.63</td>
</tr>
<tr>
<td>2</td>
<td>0 (0,1) x 0</td>
<td>(0,1) x 1</td>
<td>1.63</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Then \( Y \) can be approximated as a sum of independent Gaussian variables. We now evaluate the mean, \( Y \), and variance of \( \gamma \) as follows:

\[
Y = \sum_{m=0}^{L-1} A T(N - |u(m)|),
\]
and

\[
\sigma_Y^2 = \sum_{m=0}^{L-1} N_0 T(N - |u(m)|).
\]

Therefore we can rewrite (11) as

\[
P_r(u_L \rightarrow \bar{u}_L) = P_r(Y < 0|u_L) \approx Q\left(\frac{E_s}{N_0} N^{L-1} \sum_{m=0}^{L-1} (N - |u(m)|)\right)
\]
\[
= Q\left(\frac{E_s}{N_0} \sum_{m=0}^{L-1} (N - |u(m)|)\right)
\]
\[
= Q\left(2N \sum_{m=0}^{L-1} d_{e,m}^2\right)
\]

where

\[
d_{e,m}^2 \triangleq 2(N - |u(m)|).
\]

The equivalent normalized squared distance, \( d_{e,m}^2 \), defined in (19), plays the same role as the normalized squared Euclidean distance of coherent detection for evaluating the error probability of the coded case as well as the uncoded case.

The generation of binary CPFSK is explained in two stages, a 2-consecutive phase encoder 2CPE and a memoryless modulator (MM) as shown in Figure 2. There are two binary input lines in 2CPE. Therefore it is possible to design a binary convolutional encoder with a code rate 1/2, cascaded with 2CPE. We implement this convolutional encoder in systematic feedback form. We write an input sequence, \( b_j(D) \), and an output sequence, \( c_j(D) \) for \( j = 0, 1 \), in polynomial notation. Here it is a delay operator. The information sequence is \( c_0(D) = \hat{b}_0(D) \) and the parity check sequence, \( c_1(D) = \hat{b}_1(D) \). The parity check equation of an encoder describes the relation in time of the encoded bit streams. For an encoder with a code rate 1/2, the parity check equation is

\[
H(D)c(D) = O(D)
\]

is a parity matrix, and

\[
C(D) = [c_1(D), c_0(D)]
\]

is an output sequence vector. We define the constraint length, \( u_c \), to be the maximum degree of all the parity check polynomials \( H_j(D) \) for \( j = 0, 1 \).

To search for good codes we implement the parity check polynomial in the following form:

\[
H_0(D) = h_0^1 \cdot D^r + h_0^0 \cdot D^{r-1} + \cdots + h_0^1 + 1
\]
\[
H_1(D) = h_1^1 \cdot D^r + h_1^0 \cdot D^{r-1} + \cdots + h_1^1 + 1
\]

The equivalent squared free distance, \( d_{f,\text{free}}^2 \), can be obtained from the remaining coefficients of \( H_j(D) \) for \( j = 0, 1 \), to maximize the equivalent squared free distance. The exhaustive search for all the remaining coefficients of \( H_j(D) \) for \( j = 0, 1 \), has been made by means of a computer program.

The results are presented in Table 2 for the number of convolutional encoders with number of states ranging from 2 to 16 states. Only one set of results has been reported for each convolutional encoder results in Table 2, more than one convolutional encoder results in Table 2.

We assign level 2 subset in Set Partitioning A of Table 1 to the paths leaving the same state of the trellis corresponding to the trellis encoder. This assignment ensures that the ENSD between paths leaving a given state is at least 1.63. This condition is implemented by the connection between the outputs of the convolutional encoder and the inputs of 2CPE as shown in Figure 2. Therefore the state of the remaining most dominant encoder are also set partitioned into two level. We use Ungerboeck's set partitioning approach and computer search.
Table 2: Optimal Code Rate $1/2$ Convolutional Encoder Cascaded with 2CP

<table>
<thead>
<tr>
<th>Code Rate</th>
<th>Encoder Type</th>
<th>Encoder with</th>
<th>Encoder with 4 States</th>
<th>Encoder with 8 States</th>
<th>Encoder with 16 States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>Systematic</td>
<td>Encoder with 2 States</td>
<td>2.578</td>
<td>4.887</td>
<td>6.341</td>
</tr>
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<td></td>
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<td>6.341</td>
</tr>
</tbody>
</table>

4.2 3-consecutive Binary CPFSK with modulation index 1/2

Table 3 shows a set partitioning for the set of waveforms with 3-consecutive binary CPFSK, i.e., $M = 2$ and $N = 3$. Each signal is denoted by a three-dimensional vector $u_m = (u_{m0}, u_{m1}, u_{m2})$, where $u_{m0} \in \{0, 1\}$. We write the set of vectors in the Cartesian product. The set of eight vectors in level 1 is successively partitioned into 2 and 4 subsets of size 4 and 2 respectively.

There are three binary input lines in 3CPE. Therefore it is possible to design a binary convolutional encoder with a code rate 2/3, cascaded with 3CPE. We implement this convolutional encoder in systematic feedback form. We write the input sequences, $b^t(D)$ for $j = 1, 2$, and the output sequences, $c^t(D)$ for $j = 0, 1, 2$ in polynomial notation. Here $D$ is a delay operator. The information sequences are $c^t(D) = 1(D)$ for $j = 1, 2$ and the parity sequence, $c^0(D)$, is a function of itself and $b^j(D)$ for $j = 1, 2$. The parity check equation of an encoder describes the relation in time of the encoded bit streams. For an encoder with a code rate 2/3, the parity check equation is

$$H(D)c^t(D) = 0(D)$$

where

$$H(D) = [H^t(D), H^1(D), H^0(D)]$$

is a parity matrix, and

$$C(D) = [c^2(D), c^1(D), c^0(D)]$$

is an output sequence vector. We define the constraint length $\nu$ to be the maximum degree of all the parity check polynomials $H^t(D)$ for $j = 0, 1, 2$.

To search for good codes we implement the parity check polynomial in the following form:

$$H^t(D) = h_t^tD^t + h_{t-1}^t D^{t-1} + \ldots + h_1^t + 1$$

$$H^1(D) = h_t^1 D^t + h_{t-1}^1 D^{t-1} + \ldots + h_1^1 + 1$$

$$H^0(D) = h_t^0 D^t + h_{t-1}^0 D^{t-1} + \ldots + h_1^0 + 1$$

We assign each subset in level 2 in Table 3 to the paths leaving the same state of the trellis corresponding to the encoder. This assignment ensures that the ENSD between paths leaving a given state is at least 2.8. This condition is implemented by setting $h_j^t = 1$ for $j = 0, 2$ and $h_j^t = 0$, Observe that subsets in level 2 are selected by the value of $u_{m0} \oplus u_{m1} \oplus u_{m2}$ as shown in Figure 4 where $h_j^t = 0$ and $h_j^t = 1$ for $j = 0, 2$. The exhaustive search to find the remaining coefficients of $H^t(D)$ for $j = 0, 1, 2$, which maximize the $d_{s, \text{free}}$ has been made by means of a computer program. The results are presented in Table 4 for convolutional encoder with the number of statea ranges from 2 to 16 states. Only one solution has been reported for cases where more than one trellis encoder with maximum $d_{s, \text{free}}$ was obtained.
To evaluate the performance of coded systems, we use two dominant terms in (10) to get an upper bound on the bit error probability. With 2-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \leq 0.75Q(1.328 \frac{E_b}{N_0}) + Q(1.521 \frac{E_b}{N_0}). \quad (32)$$

Observe that $E_b = 2/3E_b$ because the code rate is 2/3. With 4-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \leq 0.75Q(1.647 \frac{E_b}{N_0}) + 0.5Q(1.841 \frac{E_b}{N_0}). \quad (33)$$

With 8-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \leq 0.5Q(2.248 \frac{E_b}{N_0}) + 0.19Q(2.374 \frac{E_b}{N_0}). \quad (34)$$

With 16-state trellis encoder in Table 4, we obtain an asymptotic upper bound as follows:

$$P_b \leq 0.313Q(2.655 \frac{E_b}{N_0}) + 0.875Q(2.762 \frac{E_b}{N_0}). \quad (35)$$

Figure 5 shows asymptotic upper bounds and simulation results for 2, 4, 8, and 16 states trellis coded 2-consecutive binary CPFSK systems. The dashed line represents the asymptotic probability of the 3-consecutive CPFSK which obtained by using $r = 1$, 1-state encoder. By employing trellis coding with a code rate 2/3, we obtain power gains 1.68, 2.63, and 4.69 dB for 2, 4, and 8 states, respectively. The dotted line represents the asymptotic upper bound on bit error probability, but we lose the information rate by 2/3. The dotted line represents a better performance in bit error probability than MSK at an expense of reducing information rate by 2/3.

5 Discussion and Conclusion

We have obtained the coded system with non-coherent detection which has better bit error probability than coherent CPFSK at the expense of reducing the information rate by a factor equal to the code rate. Furthermore, we may achieve a better system than coherent MSK in bandwidth and power efficiency by considering the trellis coding with 4-ary or 8-ary CPFSK with a larger number of states.

Acknowledgments

This work was partially performed at the Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration.

References


