

## IMPROVED BOUNDS ON NON-LUMINOUS MATTER, IN SOLAR ORBIT

John D. Anderson, Eunice L. Lau, and Timothy P. Krisher

*Jet Propulsion Laboratory  
California Institute of Technology, Pasadena, California 91109*

Duane A. Dicus

*Center for Particle Physics and Department of Physics  
University of Texas, Austin, Texas 78712*

Doris C. Rosenbaum and Vigdor L. Teplitz

*Physics Department  
Southern Methodist University, Dallas, Texas 75275*

### ABSTRACT

We improve, using a larger set of observations including Voyager 2 Neptune flyby data, previous bounds on the amount, of dark matter (DM) trapped in a spherically symmetric distribution about the sun. We bound DM by noting that such a distribution would increase the effective mass of the sun as seen by the outer planets and by finding the uncertainty in that effective mass for Uranus and Neptune in fits to the JPL Developmental Ephemeris residuals, including optical data and those two planets' Voyager 2 flybys. We extend our previous procedure by fitting more parameters of the developmental ephemerides. Additionally, we present here the values for Pioneer 10 and 11 and Voyager 1 and 2 Jupiter ranging normal points (and incorporate these data as well).

# I Introduction

## A. Background

The purpose of this work is to use Deep Space Network (DSN) radio tracking data from the Voyager Neptune and Uranus encounters to investigate limits on the amount of non-luminous solar halo matter in a spherically symmetric distribution. In an earlier work (Anderson et al. 1989) we analyzed tracking data during the Voyager 1986 Uranus encounter. That encounter permitted reduction of the  $1\sigma$  uncertainty in Uranus' range (at the encounter time) from about 1500 km to 1 km. This new constraint on the orbit of Uranus led to a bound of  $3 \times 10^{-6} M_{\odot}$  on the amount of spherically symmetric, non-luminous matter in solar orbit interior to the radius of Uranus's orbit, an improvement of at least an order of magnitude from the bound without the Voyager ranging data. After that work was completed, we continued the analysis by adding more data (optical and radio from various sources) and by adding the ranging normal point from the Voyager 2 flyby of Neptune. This paper presents the improvements in bounds on spherically symmetric, non-luminous matter that follow from these new data.

## B. Method of Analysis

The basic idea is to compare the effective solar mass felt by the inner planets to the effective solar masses felt by Uranus and Neptune. If there is a spherically symmetric distribution of unseen matter not included in ephemerides fitting programs, then, when the effective solar mass,  $M_{eff}$ , is considered a free parameter for a planet, the value determined for  $M_{eff}$  should be sensitive to the matter interior to its orbit not otherwise included in the fitting program. The bound on the difference between  $M_{eff}$  as determined by this method from the motion of an outer planet and  $M_{\odot}$  then constitutes a bound on the total mass in a spherically symmetric distribution between the inner planets and that outer planet.  $M_{\odot}$  is determined from the fit to the entire Solar System in which the value of  $M$  is driven by the much more accurately and precisely known motions of the inner planets.

While one may determine the collection of solar system ephemerides, with  $M_{\odot}$  different for each planet, we first, adopted a more modest approach. We determined a value of  $M_{eff}$  from fitting all solar system ephemerides without provision for a varying  $M_{eff}$ , i.e. we used

the JPL ephemerides, and the solar mass determined by it. We then found new values for  $M_{eff}$  for Uranus and Neptune by refitting for just their ephemerides and the two  $M_{eff}$  values with a data set consisting of the residuals for the Uranus and Neptune observations (observed minus computed). A statistically significant difference between the  $M_{eff}$  value and the  $M_{\odot}$  value would constitute detection of spherically symmetric non-luminous matter; bounds on the difference constitute bounds on the mass of such a distribution. In the present work we first use the same method and then extend the fitting of residuals to include the full set of orbital parameters used in the ephemerides program.

### C. Motivation

A major reason for investigating solar halo dark matter is the general desirability of observing directly as much as possible about the Solar System. Beyond this, there are a wide range of specific reasons for attempting to detect non-luminous solar halo matter or, failing detection, to place observational bounds on the amount of such matter. First, there are many reasons for believing that dark matter exists. These include the cosmological dark matter problem, the galactic cluster dark matter problem, the galactic halo dark matter problem, and the short-period comet question. A recent concise summary of cosmological dark matter problems is given by Turner (1991) and further background may be found in Kolb and Turner (1990); the galactic disk dark matter problem is reviewed by Bahcall (1984, 1992); and the short period comet question has been recently discussed by Weissman (1990).

The cosmological dark matter problem stems from the relative proximity of the observed cosmic density to the critical density, sharpened by Guth's observation (1981) that the apparent isotropy, homogeneity, and flatness of the universe could be explained by a period of exponential inflation. One consequence of inflation is that  $\Omega = \rho/\rho_c$ , the ratio of cosmic mass-density to the critical density, should be one. Since  $\Omega_L$ , the ratio for luminous matter, is observed to be of the order of 0.01 and the ratio for baryonic matter is bounded by cosmic nucleosynthesis constraints at about 0.1, ten times as much, it is useful to search for signs for non-luminous (dark) matter, both baryonic and non-baryonic, in as many places as possible. Observational evidence for non-luminous matter comes from rotation curves in galaxies of Rubin et al. (1985), Hoffman et al. (1993) and properties of clusters of galaxies including

galactic motion and hot gas distributions. See Mulchaey et al. (1993) for recent results.

These issues may need more than one kind of dark matter for resolution. Indeed the recent COBE results (Smoot et al., 1992) observing large scale anisotropies encourage speculation that there may be both hot and cold dark matter (DM relativistic and non-relativistic at recombination). Many kinds of DM have been conjectured, including ordinary baryonic matter in non-luminous form, axions, supersymmetric particles, massive neutrinos, black holes, and more exotic particles. Most candidates are weakly interacting in order to explain the lack of luminosity, but cross sections vary according to other desiderata; Press and Spergel (1985) and Faulkner and Gilliland (1985), for example, use "cosmions" to address simultaneously the solar neutrino and dark matter problems. They would have cross sections about  $10^4$  times weak cross sections and hence could dissipate and be trapped in solar orbit. Particle detector searches, however, have left the cosmion dead, or very nearly so: The results of Caldwell et al. (1990) "exclude nearly all of the mass range possible for cosmions" – at least for models in which cold non-nuclear cross sections scale roughly as the square of the number of nucleons. Many believe that the most likely candidate is the Lightest Supersymmetric Particle (LSP). Supersymmetry assigns to each "ordinary" particle of integral (half integral) spin a supersymmetric partner of half integral (integral) spin; there is conservation of the total number of supersymmetric particles in most models. The existence of an LSP should be decided early next century from experiments at the Large Hadron Collider if it is constructed. Other particle physics candidates are the axion or a massive neutrino. See, for example, Kane (1992).

It may be possible, in some of these cases, that a significant density of non-luminous matter could condense into a halo about a newly forming star. The conditions on particle masses and interaction cross sections under which this would be the case, taking into account gravitational interaction mechanisms in star formation, have not been worked out in detail but it is difficult to envision mechanisms that would lead to capture of significant amounts of weakly-interacting DM particles. This is because, without some dissipation mechanism, dark matter cannot concentrate in the galactic disk, be enhanced in giant molecular clouds or condense sufficiently in star formation. Nevertheless dissipation is not impossible. A characteristic feature of at least some superstring models as noted by Gross et al. (1985) and recently discussed by, for example, Khloper et al. (1991) and Hodges (1993) would have

dark matter composed of mirror or shadow baryons that only interact with normal baryons gravitationally but could dissipate by emission of undetected shadow photons. While this model appears far from compelling, it has an interesting history of thought behind it (much of it cited by Khloper et al., 1991 and Hodges, 1993). (It is however, in serious disagreement with cosmological Helium "observations"). Were it true, there would appear to be reasonable likelihood of some concentration of DM particles in the disk and in giant molecular clouds and perhaps about the sun. Indeed, Khloper et al. cite estimates of  $10^{-7}$  to  $10^{-6}$  solar masses of shadow matter being captured by a normal matter star. There are at least two other models that permit some dissipation in principle, but in practice are severely constrained over most of their parameter space: SIMPs (Strongly interacting Massive Particles) which are reviewed by Starkman et al. (1990) and CHAMPs (Charged Massive Particles) limits on which are given by Gould et al. (1991).

A different DM candidate may have been detected. Recent reports (Alcock et al., 1993) have cited observations by two different groups of what appears to be "microlensing" by a MACHO (Massive Compact Halo Object) in the halo of our galaxy of a star in the Large Magellanic Cloud. MACHOs, such as brown dwarfs, are an important baryonic DM candidate, but were they to yield an appreciable fraction of the closure density they would be in serious contradiction with the lower limit on cosmological deuterium production (because high baryon density leads to "complete" burning of deuterium into  ${}^4\text{He}$ ). In short the DM situation is complex and fluid.

A different motivation for bounding non-luminous material trapped in solar orbit is the need for observational limits on solar system components. Tremaine (1990) has reviewed the subject of dark matter in the Solar System. He discusses techniques for measuring DM, including the one used by Anderson et al. (1989), and lists limits set by each. Tremaine reviews models which would account for DM being trapped in the formation of the planetary system. The planets are believed to have been formed from a disk of gas and dust surrounding the Sun. As the disk cooled, non-volatile material condensed into "planetesimals" many of which are incorporated in the cores of the giant planets. DM in the solar system could be in the form of a spherically symmetric population of residual planetesimals. Various forms of DM, including such bodies should be absent from the inner solar system because of gravitational perturbations by Jupiter and the inner planets. However, residual baryonic DM may be

present in at least two locations. One is the generally accepted Oort cloud of perhaps 70 to 100  $M_{\oplus}$  at  $r > 2 \times 10^4$  AU (Oort 1950). The second location is the controversial Kuiper belt (Kuiper 1951), perhaps the inner boundary of a flattened core of comets inside  $2 \times 10^4$  AU. It is speculated that the protoplanetary disk may have extended well beyond Neptune's semi-major axis of 30 AU. It is possible that there is a residual mass, the Kuiper belt, located in the area 30 – 45 AU, or so, and made of matter that was not depleted in the formation of Neptune. This hypothesized comet belt would be in the plane of the ecliptic with total mass of the order of  $M_{\oplus}$ . The Jupiter-family of comets with periods of less than 20 years (the so called short-period comets) gives indirect evidence for such a belt. It should be noted, that, since Tremaine's review, several objects, about 1.6 billion kilometers beyond Neptune have been detected (Jewitt and Luu, 1993). These may be the first observations of members of the postulated Kuiper belt.

Even though composed of ordinary matter and therefore having large cross sections with ordinary planets, such planetesimals would probably not have been accreted onto the planets in the age of the solar system. Writing

$$dM/dt = \pi R^2 v p$$

where  $p \sim M_{DM}/(4/3 \pi r^3)$  with  $R$  the radius of Uranus,  $r \sim 20$  AU, and  $v$  Uranus' orbital velocity gives

$$1/M_{DM} \times dM/dt \times 4.5 \times 10^9 \text{ years} \sim 10^{-2}.$$

Thus, even ordinary matter (planetesimals) might have survived (although likely not in spherical distribution inside 104 AU) since solar system formation. This ordinary matter is probably not sufficiently luminous to be detected with current instruments at the low densities under consideration, even in the infrared (Backman and Gillett, 1987).

For all these specific theoretical reasons and more generally, as noted above, because it is of interest to search for any additional existing matter that might possibly be in the Solar System, it is desirable to use all available data to detect non-luminous matter in solar orbit or, failing detection, to put bounds on the magnitude of such matter.

Section 11 below describes our procedure and presents the results. Section 11 I contains discussion. Quantitative work in the paper is restricted to the cases of spherically symmetric

distributions of JDM in solar orbit. Only a very rough statement is made on the Kuiper belt question.

## 11 Limits on Trapped Non-luminous Matter

### A. Analysis

We refer the reader to the detailed discussion in Anderson et al. (1989) of our fitting procedure. Here we review briefly the essence of the method and the extensions and improvements incorporated into the current work.

Reduced to its simplest terms, the planet's position vector is approximated by the following two-body expression.

$$\mathbf{r}' = a(\cos E - e)\hat{P} + a\sqrt{1-e^2}\sin E\hat{Q} \quad (1)$$

where  $a$  and  $e$  are the semi-major axis and eccentricity of the Kepler ellipse, while  $\hat{P}$  and  $\hat{Q}$  are the orthogonal unit vectors in the orbit plane with  $\hat{P}$  directed to the perihelion. The eccentric anomaly  $E$  is related to the time  $t$  by,

$$E - e \sin E = u_0 + nt \quad (2)$$

where  $u_0$  is the mean anomaly at the epoch and the fundamental orbital angular frequency  $n$  is related to  $a$  and the central mass  $M$  by  $GM = n^2 a^3$ .

For purposes of gaining insight into what is being measured, we linearize equation (1) with respect to  $a$  and  $n$ .

$$\Delta \mathbf{r}' = \bar{\mathbf{r}}' \frac{\Delta a}{a} + t \frac{d\bar{\mathbf{r}}'}{dt} \frac{\Delta n}{n}. \quad (3)$$

It is apparent from equation (3) that  $a$  is determined by observations in the radial direction, while  $n$  is determined by observations along the velocity vector. The angular frequency  $n$ , or equivalently the sidereal period  $2\pi/n$ , is determined by ground-based astrometric observations of the planetary motion on the sky. However astrometric observations provide only a weak determination of  $a$  through the heliocentric parallax. It is the ranging data that provide a good determination of  $a$ .

We recall that the mean orbital radius averaged over time is not the semi-major axis  $a$ . Instead, the time average of  $1/r$  is  $1/a$ . Therefore for a central mass distribution, the circular

velocity  $v_c(a)$  at orbital radius  $a$  is just  $v_c = na$ , a product determined by astrometric and ranging data. Our data analysis yields either  $v_c(a)$ , or equivalently the effective mass of the Sun  $GM_{eff}$  interior to orbital radius  $a$ . In the absence of ranging data over a complete orbital revolution, the two parameters  $a$  and  $n$  will be correlated. The full accuracy of the ranging data will not map directly into the determination of  $GM_{eff}$ . Therefore in setting one-sigma error estimates from the data analysis, we compute the formal covariance matrix for the  $N_P$  parameter least squares fit, and then multiply the formal errors by a factor of three. We are reasonably confident that by using three-sigma formal errors for one-sigma realistic errors, we have introduced sufficient safeguards against any unmodeled systematic effects.

Standish (1993) has pointed out the difficulty of characterizing hypothetical gravitational sources, in particular Planet X, using optical observations. Regarding systematic error, our concern is that we not claim a smaller error than the optical observations can deliver. The limiting accuracy for a meridian circle observation is about one arcsecond. From Eq. 3, we conclude that a small positive change in solar mass will cause the angular planetary position on the sky to advance linearly with the time. In the worst case, the fractional accuracy in solar mass will be limited by,

$$\sigma(M)/M = \sqrt{3} \frac{T}{\pi t} \sigma(\theta)$$

where  $T$  is the planet's sidereal period,  $t$  is the observational time interval, and  $\sigma(\theta)$  equals one arcsecond. But this is the absolute worst case, in the sense that the systematic error exactly mimics the signal we are measuring. Over several decades of observations, it is unlikely we will be that unfortunate. We expect that the error will be smaller by some factor  $1/\sqrt{N}$ , where for white noise  $N$  is equal to the number of observations. In the final results reported in Table 2, we are assuming  $N = 36$  for Uranus,  $N = 11$  for Neptune, and  $N = 250$  for Jupiter. Note that our three-sigma criterion for setting realistic error is most optimistic for Jupiter, but it should be because we have optical data over six full orbital revolutions. For Uranus, and particularly for Neptune, where we have optical data over less than one orbital period and only one ranging measurement, we are being quite conservative in our assumptions on the number  $N$  of statistically independent optical observations.

The Uranus and Neptune radial errors in the JPL2000 ephemeris were relatively large because of errors in outer-planet masses. For Uranus the one-sigma error was 1500 km (Anderson et al., 1989), while for Neptune it was 8700 km. Using the Voyager flyby mass results, one

could reduce the radial errors to 500 km for Uranus and 2600 km for Neptune. However, the Voyager 2 flyby determinations of orbital radii are much more accurate (one- $\sigma$  error equal to one km). We recommend the use of these Voyager radii in future ephemerides. Note from Table 1 that the actual DE200 radial errors as determined by Voyager were 147 km ( $0.1\sigma$ ) for Uranus and 8224 km ( $0.9\sigma$ ) for Neptune. With regard to the ranging measurements in our earlier works (Anderson et al. 1989) we assumed a 500 m accuracy for the distance determination to Uranus. After doing a similar analysis of Voyager data for Neptune, we are comfortable with a 1000 m error estimate (one  $\sigma$ ) for both Uranus and Neptune. In all analysis in this paper, we assumed the error estimates given in Table 1.

### B. Astrometric and Ranging Observations

Ideally, we would like to have both astrometric and ranging observations over a complete orbital period. Given such data, our determination of each planet's orbital radius  $a$  and angular frequency  $n$  would be uncoupled. However our data are incomplete in two ways. First, we have a limited amount of recent outer-planet VLA (Very Large Array) radio-interferometric data. Over a longer time interval dating from 1830, we have less-accurate meridian circle (transit) observations. When carefully reduced, these data are accurate to about 1.2 arcsec before the introduction of the impersonal micrometer in 1911, and to about 0.4 arcsec after that. We have used only the post 1911 data in this work. Consequently we have astrometric data on Uranus over slightly less than one orbital period, and on Neptune over about one-half its orbital period. We have downweighted the radio-interferometric data by a factor of 1000, effectively removing it from our fit.

Secondly, our data are incomplete because outer-planet ranging data are presently available only during spacecraft flybys. Thus we have only one range fix on Uranus and Neptune from the respective Voyager 2 flybys. Doppler and ranging data generated by the DSN with Voyager 1 and 2 during their outer-planet flybys are archived in the National Space Science Data Center (NSSDC). The Pioneer 10 and 11 spacecraft were not equipped with a ranging transponder, but during their flybys of Jupiter we introduced a ramp into the DSN's radio transmission and obtained a rough measure of range by autocorrelating the received and transmitted ramps. These Pioneer 10 and 11 Doppler data are also archived in the NSSDC.

Our reductions of all the currently available flyby data yield the ranging residuals displayed in Table 1. In the future we expect to supplement these reduced data with existing DSN Doppler and ranging data generated during two Voyager flybys of Saturn and one Ulysses flyby of Jupiter, as well as with anticipated Jupiter data from the two-year Galileo orbital tour (December 1995 to December 1997), and four years of Saturn data during the Cassini tour scheduled for the years 2004 to 2008. However within the next decade, at least, we expect no qualitative improvements comparable with those of this work in limits on a spherical DM distribution.

The numerous data sets included in recent JPL ephemerides have been reviewed by Standish (1990). These sets include data that were unavailable in 1980 when JPL constructed the fundamental planetary and lunar ephemerides (DE200/LE200) for the *Astronomical Almanac* (Standish et al., 1992). For the analysis summarized here, we used a 1993 reference planetary and lunar ephemeris DE242, along with its associated astronomical constants, and determined corrections to the parameters by the method of weighted least squares. In our previous analysis using DE192 (Anderson et al., 1989) we determined corrections to the orbits of Uranus and Neptune only, along with the effective solar mass for each planet. In the current analysis, recognizing that a solution for only two planets produces an ephemeris that is dynamically inconsistent, we expanded the parameter set to include all the planets, except Pluto, and all 194 parameters that went into the construction of DE242. Although we doubted that our previous dynamically inconsistent method would significantly alter our conclusions, we nevertheless obtained the dynamically consistent solution with little additional effort.

We express residuals with respect to the *Astronomical Almanac's* planetary ephemerides (DE200/LE200) available on magnetic tape for the period 1600-2200. We feel it is more useful to refer residuals to the universal JPL available DE200, rather than the temporary JPL ephemeris DE242 used in this paper. The Voyager Jupiter residuals are larger than Pioneer because DE200, created in 1981, included ranging data from the Pioneer flybys in 1973 and 1974, but not the Voyager flybys in 1979. The two Pioneer points were in the fit, the Voyager points were not.

In summary, we used reduced right ascension and declination observations of all the planets except Pluto. These included optics meridian transit observations of the Sun and planets from Washington (USNO) between 1911 and 1982, and from Herstmonceux between 1957 and

1982, from Bordeaux between 1985 and 1992, from Tokyo between 1986 and 1988, photoelectric meridian transits from La Palma between 1984 and 1992, astrolabe observations from seven observatories between 1969 and 1985, and stellar occultation timings of Uranian rings between 1977 and 1983 and Neptune's disk between 1981 and 1985.

in addition to the optical data, we used reduced radar ranging data for the inner planets Mercury and Venus, and spacecraft ranging for Mars from the 1971-1972 Mariner 9 orbiter, 1976-1982 Viking Landers, and 1989 Phobos 2 orbiter. Lunar laser ranging between 1969 and 1991 were included implicitly by means of information arrays (least-squares normal equations). We used reduced ranging data provided by spacecraft flybys of Mercury by Mariner 10 (1974 and 1975) and of Venus by the 1990 Galileo flyby.

But the crucial flyby data for our dark-matter search were the DSN Doppler and ranging data generated with Pioneer 10/11 and Voyager 1/2 at the outer planets. Because of their importance, and to collect them in one place, we list in 'table 1 the reduced data in two formats. in the first we express the ranging data near the flyby time as a geometric coordinate distance between the center of Earth and center of the planet. The coordinates for the geometry are isotropic metric coordinates as described by Standish et al. (1992). in the second format, we list the ranging residuals (observed minus computed) referred to J2000. An advantage of the residuals is that they remain essentially constant over the duration of the flybys, while the geometric distances apply only to the precise TT times listed in 'table 1.

### c. **Dark Matter Bounds**

'table 2 gives the results of our fits. Line 1 gives the results of Anderson et al., (1989)  $M_{DM}(r < r_U) < 2.8 \times 10^{-6} A_0$  and  $M_{DM}(r < r_N) < 114 \times 10^{-6} A_0$ . Line 2 shows, for comparison, the improvement that results from the new fitting procedure used without the Neptune ranging. Lines 3-5 show the dramatic effect of including the Neptune ranging data: the bound on  $DM$  in spherically symmetric distribution in orbits interior to the orbit of Neptune falls from over  $30M_{\oplus}$  to about  $M_{\oplus}$ .

Lines 3-5 show that the results are not affected by adding data (or parameters) for Jupiter. Note that the minus sign under  $DM$  interior to Neptune's orbit is quite provocative. If it were statistically significant, which we cannot claim, one interpretation would be that it is

the effect on Neptune's motion of a non-spherically symmetric mass distribution exterior to, but relatively close to, Neptune's orbit (i.e., a Kuiper belt).

### 1). The Isothermal Sphere

It is tempting to ask for the limit that can be placed on non-luminous  $DM$  under the assumption of a given radial distribution, and the isothermal sphere is an obvious distribution choice.

For such an analysis we would assume that the distribution of mass in the solar system consists of a central condensed source (the Sun) surrounded by a spherically symmetric dark halo approximated by an isothermal ideal gas sphere. Mutual gravitational attraction by the planets will perturb this configuration, but in a completely deterministic fashion which could be accounted for in the data analysis. At sufficiently large distances from the isothermal core, the density distribution approaches the power law  $r^{-2}$ . Therefore the effective mass of the Sun at orbital radius  $a$  is

$$GM(a) = GM_{\odot} + 2\sigma^2 a \quad (4)$$

where  $\sigma$  is the velocity dispersion (rms deviation from the mean) in one direction.

$GM_{\odot}$  for the Sun is determined from ranging data for the inner planets, so the only unknown in the model of equation (4) is the velocity dispersion. Without the isothermal-sphere constraint, we determine all outer-planet  $GM$ 's as independent parameters. As an alternative, we could impose the constraint given by equation (4) and refer all  $GM$  determinations to the orbital radius  $a_7$  of the seventh planet Uranus. The linear relation between an arbitrary  $GM$  at the orbital radius  $a$  and  $GM_7$  for Uranus is

$$\Delta [GM(a)] = \left( \frac{a}{a_7} \right) \Delta [GM(a_7)] . \quad (5)$$

Therefore, we could impose the isothermal-sphere constraint by multiplying the linear coefficient for each  $GM$  by  $a/a_7$  for Jupiter and Neptune, and by replacing the three independent  $GM$ 's by a single  $GM_7$  for Uranus in the least-squares fit. If we had obtained a statistically significant determination of  $\Delta(GM_7)$ , we would have obtained a determination of the density  $\rho$  of dark matter at the orbital radius of Uranus.

$$\rho(a_7) = \frac{\Delta[GM(a_7)]}{4\pi G a_7^3} \quad (6)$$

and the velocity dispersion (constant temperature) throughout the sphere would be

$$\sigma^2 = \frac{\Delta[GM(a_7)]}{2a_7} \quad (7)$$

We have investigated this procedure, but do not consider its results meaningful. Any assumed *DM* interior to the orbit of Jupiter is almost certainly fictional since gravitational perturbations from Jupiter would eject it in a short time. On the other hand the progression from Uranus to Neptune implies a best-fit decreasing  $M(r)$  (after subtracting out the masses of the planets themselves) which is inconsistent with the assumption of an isothermal distribution, or any other spherical mass distribution.

## 111 Discussion

There is debate as to the extent to which bodies of normal baryonic matter formed at the time of formation of the sun, interior to the orbit of Neptune, would be expected to survive. Modern theories of comets (see Bailey, Clube and Napier, 1990, for a review) are based on formation of the Oort cloud by means of ejection of such bodies from interior to the orbit of Neptune by the outer planets. The efficacy of such a mechanism was shown by Fernandez (1978). It has been shown, however, by Duncan, Quinn and Tremaine (1989) that stable circular orbits exist interior to Neptune. Thus our bound on the amount of normal matter interior to Neptune's orbit may be applicable to models of Oort cloud formation.

Our result in this paper -- any spherically symmetric distribution of nonluminous matter must be less than a few times  $10^{-6}$  solar masses out to Neptune -- shows rather clearly that the sun could not have captured all the dark matter the Bahcall analysis requires in the solar neighborhood ( $0.1 M_{\odot}/\text{pc}^3$  with "neighborhood" defined as within 0.1 pc) into any distribution as centralized as those considered here. That is, the Bahcall analysis says that the *DM* density should be about equal to the density of luminous matter, but this much *DM* about the sun captured during its formation and retained past Saturn is inconsistent with our result. In this connection, note that, as pointed out by Tremaine (1991), tidal forces from passing stars would not be effective in displacing dark matter interior to the Oort belt at 104 AU. Our result may focus the Bahcall dark matter problem by decreasing the possibility of its being resolved by small bodies of normal matter. It argues for either: (1) "new particle physics", e.g. elementary particles that cannot radiate but can dissipate

sufficiently to condense in the galactic disk but not sufficiently to be captured by the sun during its formation; or else (2) “new astrophysics,” e.g. large numbers of brown dwarfs.

We consider now the question of how much DM the sun could be expected to capture gravitationally during its formation. Conditions for capture during formation of the **sun** of a weakly interacting particle must be

$$v^2/2 < G/r (dM/dt)\Delta t; \quad v\Delta t < r. \quad (8)$$

That is, to be captured a particle must be moving slowly enough that it (a) does not leave the scene during formation of the sun and (b) has a velocity less than the escape velocity. Taking from Shu et al. (1987) that half the mass of the sun accumulates in  $2.5 \times 10^5$  yr, one sees that the sun would be expected to capture all dark matter within 0.1pc moving slower than about 0.3 km/s.

Thus our result puts no constraint on dark matter that is weakly interacting only, spread relatively uniformly over a spherical galactic halo, and moving with a gaussian distribution about the galactic virial velocity of 300 km/s. This is because the halo density is expected to be about  $10^{-4} M_\odot / \text{pc}^3$  so the amount captured should be  $10^{-9-3-4} M_\odot \sim 10^{-16} M_\odot$ . While it is not completely clear that relaxation mechanisms cannot enhance gravitationally the density of weakly interacting DM particles in the galactic disk, in the Appendix we present a calculation that makes such a scenario highly doubtful.

We can provide one possible direction, beyond those discussed in Section I above, in which particle models with dissipation may be found (although whether nature chooses one of them is a very different question). If particle  $X$  dissipates energy by scattering, it should have a cross section  $\sigma$  such that it will scatter and release some energy at least once in a time  $t$ , on the order of 109 years. For scattering off protons, electrons, or Hydrogen we can calculate the cross section needed for dissipation since we know the proton density ( $n$ ), Assuming a virial velocity,  $v$ , for  $X$ -particles, we have from

$$n\sigma vt_1 = 1$$

with

$$V_{gal} \sim 10^7 \text{ cm}^3$$

$$l_{\perp} \sim 0.01 \text{ cm}^{-3}$$

$$v \sim 300 \text{ km/s}$$

a cross section of

$$\sigma \sim 10^{-22} \text{ cm}^2.$$

Such a large cross section is ruled out of course.

Now consider the universe to be dominated by a very light abundant particle, for example the axion ( $a$ ) with a dissipation mechanism in an interaction  $a + a \rightarrow a + a + Y$ , with  $m_Y \ll m_a$ . If  $m_a \sim 10^{-5} \text{ eV}$  (see, for example, Kolb and Turner (1990)) and  $\Omega_a \sim 1$  then

$$\rho_a = \rho_c \sim 10^3 \text{ eV cm}^{-3}$$

which implies

$$\bar{n}_a \sim 10^8 \text{ cm}^{-3}$$

The axion number density in the galaxy could be as large as

$$n_a(\text{galactic}) \sim \frac{\bar{n}_p(\text{galaxy})}{\bar{n}_p(\text{universe})} \bar{n}_a.$$

Since

$$\bar{n}_p(\text{universe}) = 0.01 \frac{\rho_c}{m_p} = 10^{-8} \text{ cm}^{-3}$$

the number of axions in the galaxy is  $10^{16}$  larger than the number of protons. If a cross section  $10^{16}$  smaller than the  $10^{-22} \text{ cm}^2$  above would give significant dissipation. A zero mass Majorana would be a candidate for  $Y$ . In short, one direction for  $DM$  models with dissipation is that of  $\Omega$  dominated by a very light, and hence very abundant, but non-relativistic, particle with significant inelastic scattering.

Finally, we note that our analysis can be extended to address the question of the existence of a belt of cometary matter in the region just past the orbit of Neptune. As noted, such a belt has been postulated by a number of authors, Kuiper (1951), Duncan et al. (1988), in order to explain the high relative frequency of short period comets. Interest in the possibility of such a belt has increased recently with the observation of candidate objects by Jewett and Luu (1993). The techniques of the present work, generalized to mass distributions that are not spherically symmetric, should be able to place limits on the mass and location of such a

belt or to detect its presence. Such an effort is under way. In the meantime, a gross estimate of a bound can be made by equating the approximate attraction of such a belt on Neptune to the attraction of a spherically symmetric density of DM sufficient to saturate our present bound on DM interior to Neptune's orbit. In the approximation that the distance  $r_{BN}$  of Neptune to the belt is much smaller than  $r_N$  the semi-major axis of orbit of Neptune, we have, for a belt 10 AU past Neptune

$$F \sim 2G/r_{BN} \times M_B/2\pi (r_{BN} + r_N) \leq G\Delta M/r_N^2$$

or

$$M_B \leq \pi\Delta M r_{BN} (r_{BN} + r_N)/r_N^2 \sim \Delta M.$$

In this crude approximation, a belt 10 AU past Neptune must be less than a few Earth masses. However this approximate calculation does not take into account more sensitive effects of such a belt such as precession of the line of nodes of Neptune. (See, for example, Whipple, 1964.)

In summary, we recapitulate the principal results of this paper and the earlier one, Anderson et al. (1989), in Table 2. We note that we now have a limit on the amount of dark matter in orbit about the sun in a spherically symmetric distribution interior to Neptune of less than an earth mass and a Uranus limit of about 1/6 of an earth mass.

## Appendix: Gravitational Scattering and Disk Dark Matter Density

We investigate here whether the density in the galactic disk of weakly-interacting DM might be enhanced over the density in the galactic halo. The mechanism in question would be that of repeated soft gravitational scattering of galactic halo dark matter particles off giant molecular cloud complexes. This would be essentially the inverse of the mechanism of Spitzer and Schwarzschild (1951) by which scattering off giant molecular cloud complexes explains the greater velocity dispersion of older stars. We show this does not work, a result, that may be intuitive from thermodynamics.

We approximate the galaxy as a slab of clouds of mass  $m_c$ , with density  $n_c$ , traveling with constant velocity  $v$ . We find the effect of this distribution of clouds

$$f_c(v_c) = n_c \delta(\vec{v}_c - \vec{v}_1) \tag{A.1}$$

on an initial gaussian distribution (in velocity space) of halo DM particles

$$j(\mathbf{v}) = A e^{-v^2/v_0^2}. \quad (\text{A.2})$$

We calculate within the local approximation to the “master equation” as formulated in Binney and Tremaine (1987). We have then

$$df(v)/dt = \Gamma(f) = - \sum_1^3 \frac{\partial}{\partial v_i} [f(v)D(\Delta v_i)] + \frac{1}{2} \frac{\partial^2}{\partial v_i \partial v_j} [f(v)D(\Delta v_i \Delta v_j)] \quad (\text{A.3})$$

where Appendix 8.A of Binney and Tremaine (1987) gives

$$D(\Delta v_i) = -4\pi G^2 m_a^2 \ln A \int \frac{f_c(v_c)}{v_0^3} v_{ci} d^3 v_c \quad (\text{A.4})$$

$$D(\Delta v_i, \Delta v_j) = +4\pi G^2 m_a^2 \ln A \int \frac{f_c(v_c)}{v_0^3} \left( \delta_{ij} - \frac{v_{ci} v_{cj}}{v_0^2} \right) d^3 v_c. \quad (\text{A.5})$$

Here  $\mathbf{V} = \mathbf{v} - \mathbf{v}_c$  and the “Coulomb logarithm,”  $\ln A$ , is of the order in  $(R_{\text{Gal}}/R_{\text{cloud}})$ . Substituting (A.4, 5) into (A.3) gives

$$d f(v)/dt = K \left\{ \frac{\partial}{\partial v_i} \left[ e^{-v^2/v_0^2} \frac{2(v - v_1)_i}{|v - v_1|^3} \right] + \frac{1}{2} \frac{\partial^2}{\partial v_i \partial v_j} \left[ e^{-v^2/v_0^2} \frac{(\mathbf{V} \cdot \delta_{ij})_i (v - v_1)_j}{|v - v_1|^3} \right] \right\} \quad (\text{A.6})$$

where  $K = 4\pi G^2 m_c^2 (\ln A) n_c$ . Performing the differentiations in (A.6) gives

$$df(v)/dt = K e^{-v^2/v_0^2} \left\{ \frac{-2(v^2 - v_1 \cdot v_2)}{v_0^2 |v - v_1|^3} + 4\pi \delta(\vec{v} - \vec{v}_1) \left[ \frac{2}{V^2} \frac{(v^2 - \vec{v} \cdot \vec{v}_1)^2}{|v - v_1|^2} - \frac{1}{v_0^2} \frac{2(v^2 - \vec{v} \cdot \vec{v}_1)}{|v - v_1|^3} - \frac{1}{|v - v_1|^3} \frac{(2\vec{v} - \vec{v}_1) \cdot (\vec{v} - \vec{v}_1)}{|v - v_1|^3} \right] - 4\pi \delta(\vec{v} - \vec{v}_1) \right\}. \quad (\text{A.7})$$

The first square bracket comes from the first term in (A.6). (A.7) becomes

$$df(v)/dt = \frac{K e^{-v^2/v_0^2}}{v_0^2 |v - v_1|^3} \left\{ 2 [v^2 v_1^2 - (\vec{v} \cdot \vec{v}_1)^2] - v_0^2 [3\vec{v} \cdot \vec{v}_1 - 2v_1^2 - v^2] \right\}. \quad (\text{A.8})$$

Letting  $v = v_1 + \eta$  in (A.8) gives

$$df(v)/dt = \frac{ke^{-v^2/v_2^2}}{\eta^3 v_2^4} \left[ v_2^2 (\hat{v}_1 \cdot \hat{\eta}) - (2v_1^2 - v_2^2)\eta^2 - (\hat{v}_1 \cdot \hat{\eta})^2 \right]. \quad (\text{A.9})$$

Equation (A.9) is our principal result. We see that the  $\eta^{-3}$  factor provides an enhancement to the rate for scattering of DA4 particles off molecular clouds when these particles have small velocities relative to the clouds. However the first term in brackets in (A.9) merely removes *DM* particles with velocities somewhat less than  $v_1$ , and adds *DM* particles with velocities somewhat greater, with no net difference in total density. The other terms in (A.9) are negative ( $2v_1^2$  being greater than  $v_2^2$ ). Thus the net effect of (A.9) is to remove *DM* particles from the galactic disk by scattering off clouds, not to add to the density of disk *DM*. Such a result might be expected on the basis of general principles of statistical mechanics: increasing the density in the two-dimensional disk corresponds to decreasing the entropy of the three-dimensional *DM* system.

## Table Captions

### **Table 1. Range Points to Jupiter, Uranus and Neptune.**

We collect the range points to Jupiter, Uranus and Neptune. The analysis is described in Standish (1990). The Voyager points for Jupiter and Neptune have not been previously published.

### **Table 2. Limits on Dark Matter.**

Line 1 reproduces the results of Anderson et al. (1989); line 2 includes the Neptune Range Point with the same fitting procedure; lines 3-5 use a new, improved fitting procedure as described in the text.

**Table 1**

Spacecraft	Date (JED)	Geometric Distance (1-way Km) $\pm$	DE200 Residual (1-way Km) $\pm$
Jupiter Pioneer 10	2442020.50	825852471.1 $\pm$ 12	-5.6 $\pm$ 12
Jupiter Pioneer 11	2442384.50	731437233.5 $\pm$ 3	6.0 $\pm$ 3
Jupiter Voyager 1	2443938.00	678931390.1 $\pm$ 4	114.14''4
Jupiter Voyager 2	2444064.50	932054679.9* 4	96.1 $\pm$ 4
Uranus Voyager 2	2446455.25	2965361517.0 $\pm$ 1	147.3 $\pm$ 1
Neptune Voyager 2	2447763.67	4425522117.1 $\pm$ 1	8224.0 $\pm$ 1

**Table 2**

Limits on Dark Matter (in units of  $10^6 A_{4c}$ )

Spherically symmetric Distribution	Uranus	Neptune	Jupiter
Anderson et al. (1989) Uranus ranging	$0.4 \pm 2.8$	$-0.4 \pm 114$	
Uranus ranging with all planets refit	$0.32 \pm 0.49$	$38 \pm 108$	
Uranus, Neptune ranging with all planets refit	$0.32 \pm 0.49$	$-1.9 \pm 1.8$	
Uranus, Neptune, Jupiter ranging	<b><math>0.334:0.49</math></b>	<b><math>-1.93: 1.8</math></b>	<b><math>0.12 \pm 0.027</math></b>
Including Jupiter ranging with $M_{\odot}$ for Jupiter fixed by inner planets	$0.26 \pm 0.49$	$-2.0 \pm 1.8$	

## Acknowledgements

It is a pleasure to acknowledge stimulating conversations with Professor E.W. Kolb, as well as helpful interchanges some years ago with Professor David Spergel and valuable correspondence with Professor S. Tremaine on the appendix. We have benefited from several communications with E. Myles Standish, Jr. We would also like again to thank Dr. Jeffrey Briggs for bringing together the JPL and non-JPL authors in 1987. J.D.A. and E.L.L. acknowledge support from the Pioneer Project Office, NASA Ames Research Center, under Letter of Agreement ARC/PI '01 7; and support from the Ultraviolet/Visible/Gravitation Astrophysics office of the NASA Astrophysics Division. Their work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA. The work of DAD was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-93ER40757. The SMU work was supported in part by the Texas National Research Laboratory Commission.

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