A SIMPLE PHYSICAL OPTICS ALGORITHM PERFECT FOR PARALLEL COMPUTING ARCHITECTURE

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Abstract

A reflector antenna computer program based upon a simple discreet approximation of the radiation integral has proven to be extremely easy to adapt to the parallel computing architecture of the modest number of large-grain computing elements such as are used in the Intel iPSC and Touchstone Delta parallel machines. It has also proven to be very efficient since, with reasonable size reflectors, parallel efficiencies approaching 98% have been demonstrated.

introduction

One of the simplest reflector antenna computer programs is based upon a discrete approximation of the radiation integral (Reference 1). This calculation replaces the actual reflector surface with a triangular facet representation so that the reflector resembles a geodesic dome. The Physical Optics (PO) current is assumed to be constant in magnitude and phase over each facet so the radiation integral is reduced m a simple summation. This program has proven to be surprisingly robust and useful for the analysis of arbitrary reflectors, particularly when the near-fickl is desired and surface derivatives are not known.

Because of its simplicity, the algorithm has proven to be extremely easy to adapt to the parallel computing architecture of a modest number of large-grain computing elements such as are used in the Intel iPSC and Touchstone Delta parallel machines.

For generality, wc consider a dual-reflector calculation, which can be thought of as three sequential operations: (1) compute the currents on the first reflector using the standard PO approximation; (2) utilizing the currents on the first reflector as the field generator, compute the currents on the second reflector; and (3) compute the required field values by summing the fields from the currents on the second reflector. The most time-consuming part of the calculation is the computation of the currents on the second reflector due to the currents on the first, since for N $\times$ N triangles on the first reflector, each of the M x M triangles on the second reflector required an N^2 sum over the first, However, since each calculation requires the identical number of operations, the N^2 triangles can be evenly distributed over the nodes, and the sum done in parallel for each of the M^2 triangles on the second reflector (also evenly distributed over the nodes). In addition, the output field values can be calculated in parallel with each node, summing its respective triangles, and the final output field obtained by summing the field in each of the nodes.

For reasonable size reflectors, parallel efficiencies approaching 98% have been demonstrated.

Physical optics Algorithm

The analysis method utilized is a straightforward numerical integration of the physical optics radiation integral. Since the incident magnetic field is required to evaluate the PO surface current on the second reflector (see Figure 1), we choose the following form for the radiation integral (although the method is identical if the E field is required):

$$H(r) = -\frac{1}{4\pi}\int_{\Sigma}\left(jk+\frac{1}{R}\right)\hat{R}\times J_s(r')\frac{e^{-jkR}}{R}\,ds' \tag{1}$$

in which r designates the field point, r' the source point, R $= |r - r'|$ is the distance between them, and $\hat{R} = (r - r')/R$ is a unit vector. The PO current on the surface J_s is expressed as

$$J_s(r') = 2\hat{n}\times H_s(r') \tag{2}$$

with $H_s(r')$ the incident magnetic field.

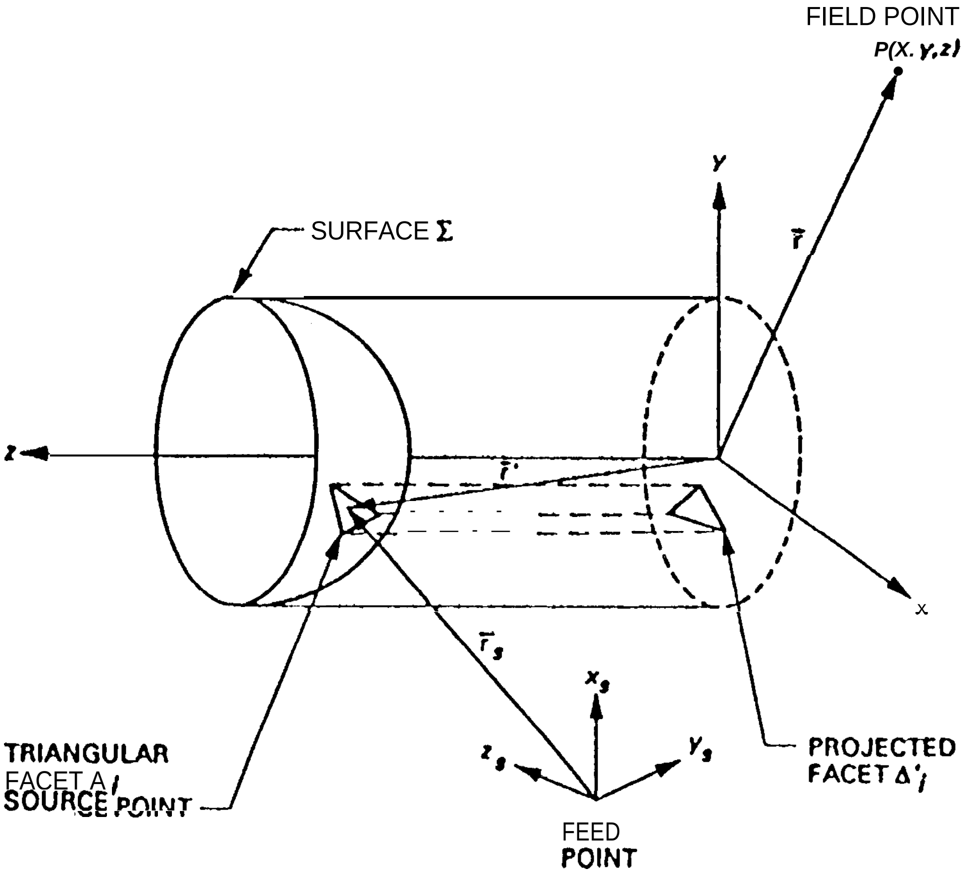

Figure 1. Reflector analysis coordinate systems and a typical triangular facet.

Let (x_s, y_s, z_s) denote the coordinates of the reflector surface. For the purpose of analysis, the reflector surface is subdivided into small triangular regions. Within each triangular region, the actual subreflector surface is approximated by a planar surface, or facet. Now, let R_{k1}, R_{k2}, R_{k3} be vectors directed to the three vertices of the kth facet. ‘1’hen, vectors along the sides of the facet are given by

$$A = R_{k1} - R_{k2}$$

$$B = R_{k2} - R_{k3}$$

$$C = R_{k3} - R_{k1}$$

and a unit normal n_k may be constructed by the following vector operations:

$$n_k = \frac{A \times B}{|A \times B|}$$

The area A_k of the triangle is readily calculated from

$$A_k = \& \$ - A)(S - B)(S - C)$$

where

$$S = \frac{|A| + |B| + |C|}{2}$$

With this triangularization established, the PO surface current is approximated by

$$J_s \approx \sum_{k=1}^{N} J_s^{(k)}$$

where $J_s^{(k)}$ *is* the PO current evaluated at the center of the kth facet. In other words, the PO surface current is assumed to be constant over a facet. Using this expression in Eq. (1) gives the following approximate ion for the PO radiation integral:

$$H \approx \frac{1}{4\pi} \sum_{k=1}^{N} \left(jk + \frac{1}{R_k} \right) \hat{R}_k \times J_s^{(k)} \frac{e^{-jkR_k}}{R_k} A_k \tag{3}$$

Notice that a discrete approximation is used for the current as well as the surface. 1 ‘or aid in convergence, the distance R_k is chosen to be on the surface rather than on the triangular facet.

l)ual-Reflector Calculation

For generality, we will consider the dual-reflector calculation; the single reflector calculation can be done in the same manner, but the time associated with the calculation is considerably less and may not require the capabilities of parallel computing.

Referring to Figure 2, the clual-reflector case consists of a feed, subreflector surface, and main reflector surface. The field scattered from these surfaces is evaluated at a given field point and the calculation can be thought of as three sequential operations: (1) compute the currents on the first reflector using the standard PO approximation; (2) utilizing the currents on the first reflector as the field generator, compute the currents on the second reflector; and (3) compute the required field values by summing the fields from the currents on the second reflector.

Utilizing the method described in the previous section, each surface is subdivided into small triangular regions, with a typical mesh projected into the x-y plane shown in Figure 3. The currents on the first surface (typically called the subreflector) arc evaluated using Eq. (2). For the examples considered in this paper, the incident will be in the form of a cosine to the power Q, although any desired incident field evaluation could be used. Thus the incident field is of the form

$$H_s(r') = \cos^q(\theta)\frac{e^{-jkr'}}{r'}$$

where the feed is assumed to be pointing along the Z_f coordinate and θ is the polar angle. The magnetic field incident on each triangle of the second reflector (typically called the main reflector) is evaluated using Eq. (3). The currents on each triangle arc then obtained by using the physical optics approximation of $J = 2\hat{n} \times H$. Observe that to obtain the current for **each** triangle of the second reflector requires a sum over all the triangles of the first reflector. The field scat tered from the second reflector is then evaluated by another application of Eq. (3) (or a similar form of the equation if the E-field is required).

Parallel Algorithm

Observe that the most time-consuming part of the calculation is the computation of the currents on the second reflector due to the currents on the first, since for N x N triangles on the first reflector, each of the M x M triangles on the second reflector required an N^2 sum over the first. Since each calculation requires the identical number of operations, the N^2 triangles can be evenly distributed over the nodes, and the sum done in parallel for each of the M^2 triangles on the second reflector. However, since the computation of the currents on the first surface is trivial, little (other than storage) is gained by distributing the N^2 triangles of the first reflector over the nodes. Computation of the currents on each of the M2 triangles is evenly distributed over the nodes ant] the computations are done in parallel. Hence each node has a copy of the program for computing the integrand of Eq. (3), and each node computes the H-field and current for its assigned triangles. After each of the nodes computes its M^2/NODES of the currents, the currents are then collected such that each node has all the currents. To compute the field values, each node does M^2/NODES of the sum of Eq. (3) utilizing the currents on the main (second) reflector. The final result is obtained by summing the

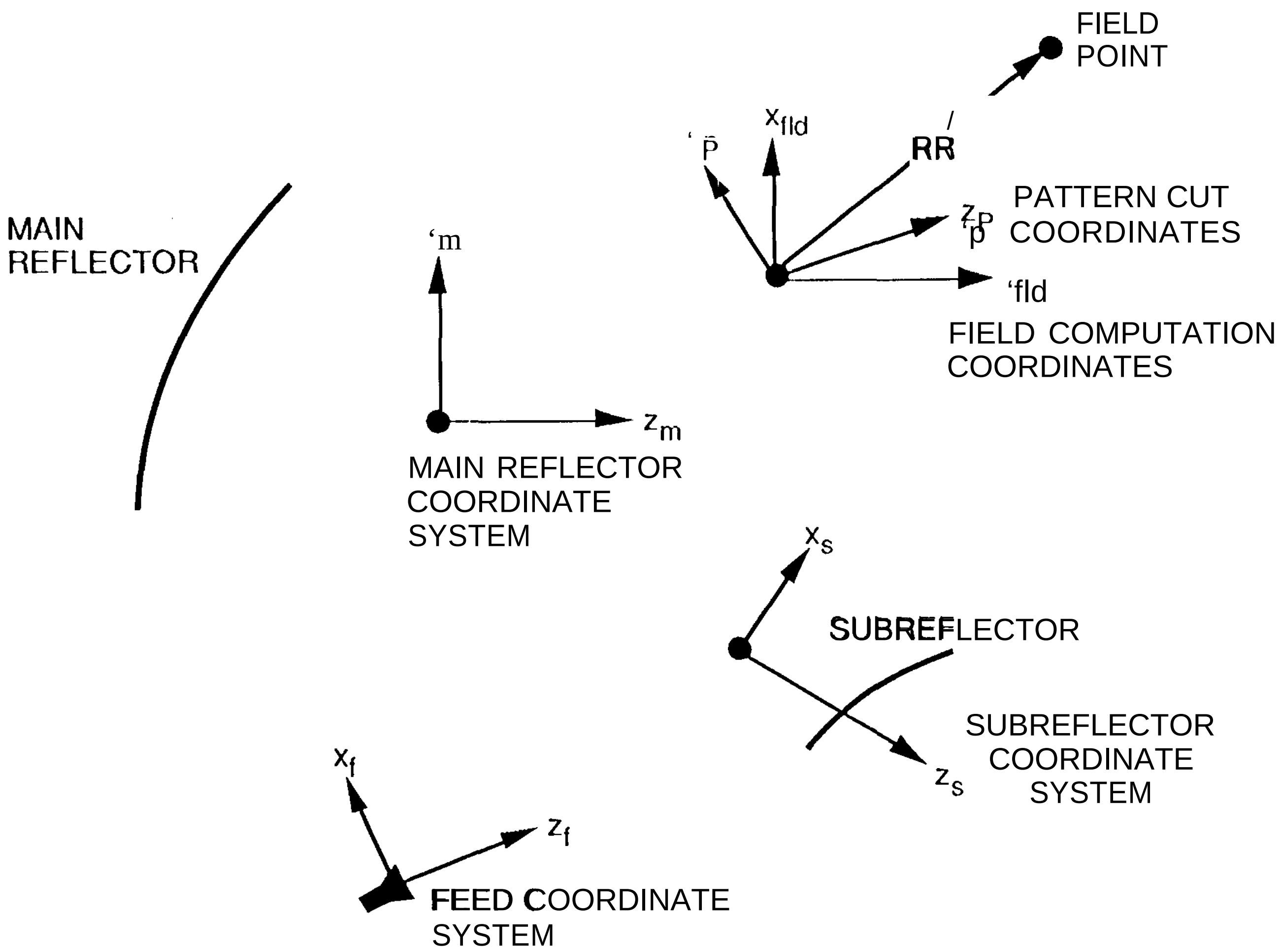

Figure 2. Schematics of the coordinate systems.

fields obtained from each of the nodes. It is possible to improve the algorithm for storage (but only a minor time savings) by not collecting all the currents on the main reflector, since each node only needs its M^2/NODES currents to do the field summation.

Examples and Conclusions

As an example, we consider the two parabola example shown in Figure 4 (Reference 2). This is a portion of JPL beamwaveguide system and is designed to image the input feed pattern to the first parabola at the output focus of the second parabola. The geometry is as shown and for this calculation a $\cos^q(\theta)$ feed pattern with $q = 238.25$ is used as an input. A typical output is shown in Figure 5 with a comparison to measured data included.

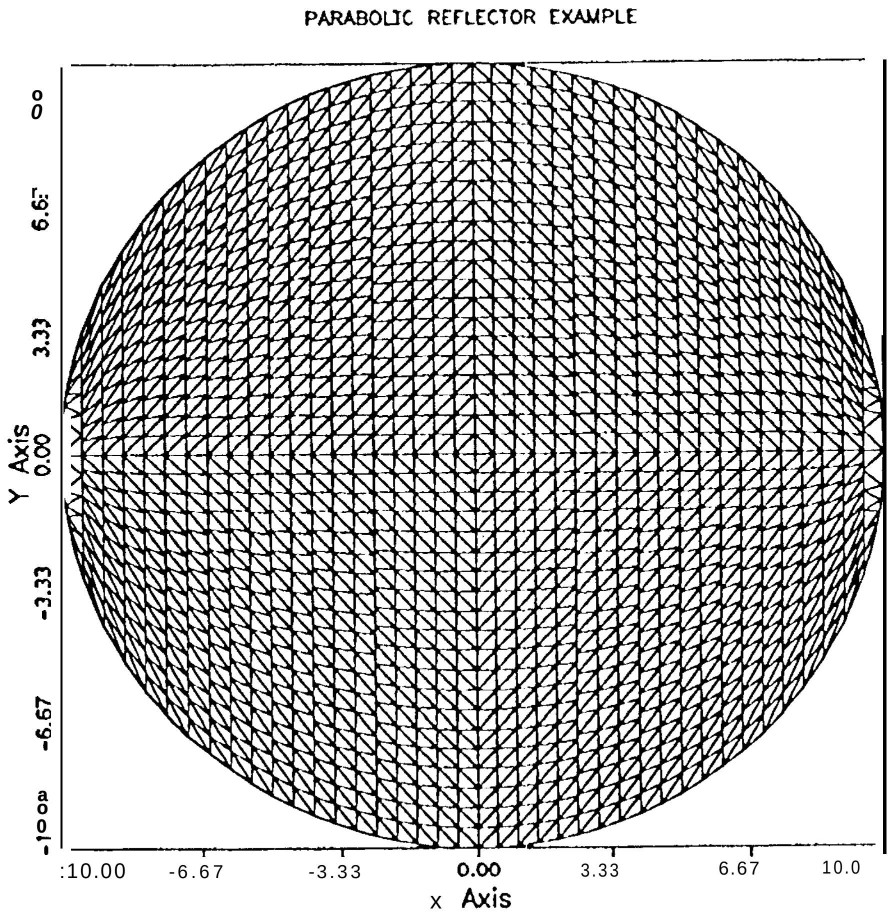

Figure 3. Typical mesh projected in X-Y plane.

The typical time for a small example is shown in Table 1. The only part of the program that was parallelized was the portion which computes the second reflector currents from the first reflector currents. For sufficiently large problems this portion of the code dominates. However, it is possible to parallelize the field evaluation and this will be done in the future. Observe that the efficiency of calculating the currents using 8 nodes is 98.6%.

Table 1. Dual reflector example with 1(1372 triangles on each surface

# Nodes	1	4	8
Time for first reflector currents	11	12	13
Time for second reflector currents	11,339	2849	1437
Field evaluation	191	192	194
Total (seconds)	11,721	3053	1644

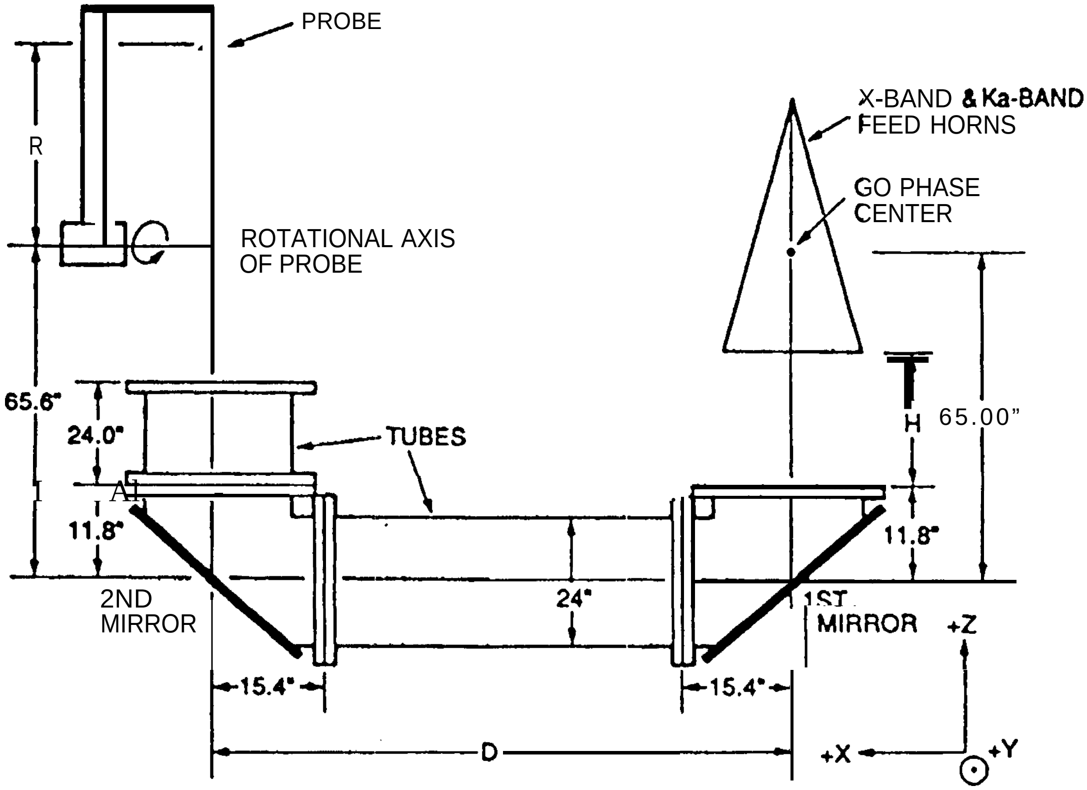

Figure 4. A two-mirror BWG system test setup.

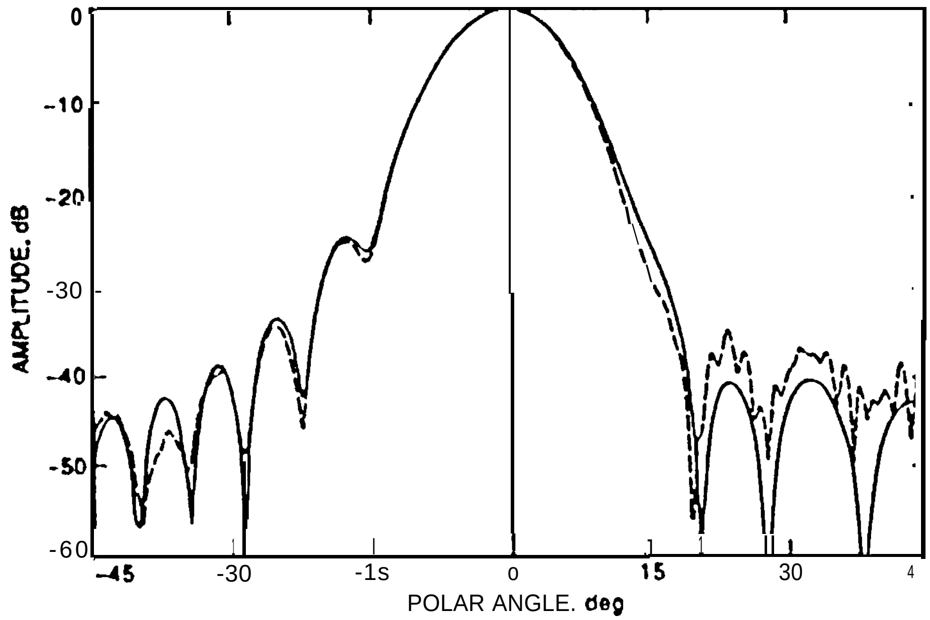

Figure S. Measured (dashed line) and computed (solid line) data for offset plane (X-band).

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References

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