

EFFECTS OF PHASE NOISE FROM LASERS AND OTHER SOURCES ON PHOTONIC RF PHASED-ARRAYS

Ronald T. Logan Jr. and I. L. Maleki

Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Pasadena, California 91109

ABSTRACT

The beam pattern of a linear phased-array antenna system employing a photonic feed network is analyzed using a model for the individual feed element noise including both additive and multiplicative equivalent noise generators. It is shown that uncorrelated multiplicative noise power of the individual feeds is reduced by a factor of N in the output of an N -element linear array. However, the uncorrelated additive noise of the individual feed paths is not mitigated, and therefore will determine the minimum noise floor of a large phased-array antenna.

1. INTRODUCTION

In phased-array antennas, the beam pattern depends critically on the phase control of the signals at the individual antenna elements. The ability to feed and adjust the phase of the microwave signals to the individual radiating elements using optical fiber and photonic components offers obvious advantages in size, weight, mechanical flexibility, and cross-talk, compared to metallic waveguides and phase-shifters. Various phased-array antenna system architectures with photonic feed networks have been proposed, however, the issues of phase stability and signal purity are not typically addressed in these proposals. Therefore, determination of the acceptable phase noise contribution of the individual active feed components has been problematic. In this paper, the general factors contributing to the phase stability of an array feed network are outlined, with particular attention paid to the type of noise encountered in photonic feed elements. It is shown that the analysis of array phase stability must consider both additive and multiplicative noise generation processes, and that the additive noise of the active feed components will limit the phase stability of a large phased-array.

2. PHASED-ARRAY SYSTEM MODEL

The general architecture of a phased-array antenna system comprised of M elements is depicted schematically in Figure 1. In this analysis, the phase noise contribution due to the array feed and antenna elements only is calculated. The

effects of the source phase noise will be common to all elements, and may be treated in the usual manner for a single antenna element.

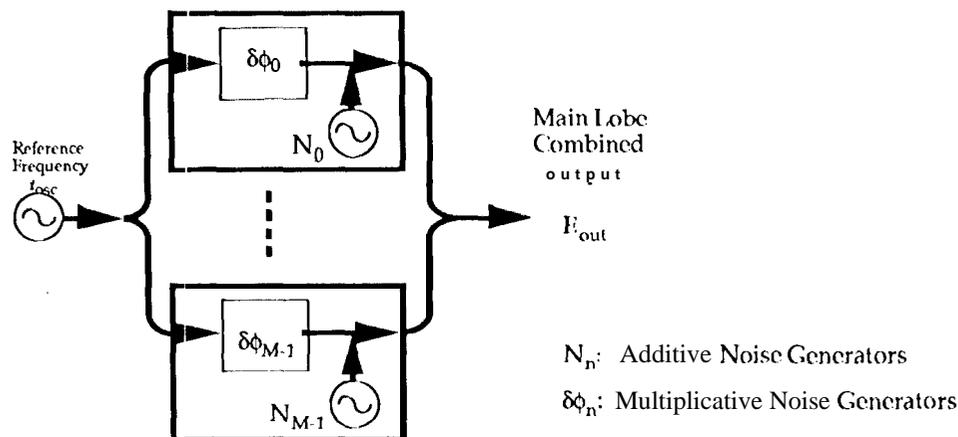


Figure 1. Noise Sources in M-Element Phased-Array Feed System

The feed system is driven by a common source oscillator whose power output is divided M ways. The phase delays required to point the antenna beam are generated in the separate feeds. To study the effect of noise on the elements, we assume a simple case in this model: the signals acquire equal delays in the feeds, and are then recombined in a second M-way power combiner. Thus, the output signal amplitude is scaled to be equal to the input signal amplitude, to facilitate comparison between a single-element antenna system and an array.

3. NOISE PROPERTIES OF PHOTONIC FEEDS

The photonic feed elements contain active components such as laser diodes, photodiodes, and amplifiers. The phase noise contribution of a microwave fiber optic feed system is therefore comprised of an additive noise term and a multiplicative noise term. Laser relative intensity noise ($1/fN$), shot noise, and thermal (Johnson) noise are additive noise sources which are present at all times independent of signal level. Low frequency gain or path-length instabilities that modulate the microwave signal amplitude and phase are multiplicative noise sources that are only observed when a signal is present. As shown in Figure 2, additive noise usually determines the noise floor at higher offsets from the carrier frequency, but multiplicative noise often has a $1/f^\alpha$ power law characteristic, $1 < \alpha < 2$, so is typically dominant close to the microwave carrier frequency [1].

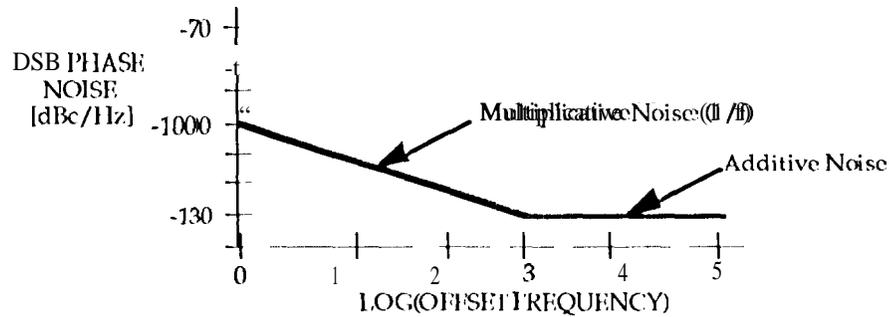


Figure 2. Typical Phase Noise of Photonic Feed System at 10 GHz.

Additive noise sources due to thermal and shot processes or laser RIN are independent random processes and therefore can be assumed uncorrelated between the feed elements, Multiplicative noise may or may not be uncorrelated between elements, depending on its origin. For example, thermal expansion or vibration of all the optical fibers in the feed network may produce a phase modulation that is common to all elements, whereas, laser or amplifier-induced $1/f$ gain fluctuations will be uncorrelated between elements. Noise that is common to all the elements may be referred to the source oscillator and treated as if the array were a single element. In the analysis that follows, all of the noise sources in the individual feed elements are assumed to arise from independent random processes, so can be treated as uncorrelated sources.

It is noted that the multiplicative noise is not detectable by a standard noise figure measurement in the microwave signal frequency band. In fact, it is difficult in practice to predict the amount of multiplicative noise in an amplifier or laser diode, because the noise level may itself be a function of the modulation signal frequency or amplitude. Therefore, the amount of multiplicative phase noise is usually determined empirically.

4. ANALYSIS

We now proceed to calculate the effect of the noise added by the feed elements on the total array performance, Consider first a unity gain single feed element consisting of a fiber optic link and electronic amplifiers, with a microwave input signal $E_{in}(t) = E_0 e^{j\omega_0 t}$ of constant amplitude E_0 at microwave frequency ω_0 . The output signal amplitude for a single feed element may be written

$$E_{out}(t) e^{j\omega_0 t} = E_0 e^{j\omega_0 t} e^{j\delta\phi(t)} + N'(t) \quad (1)$$

where $N'(t)$ represents an additive Gaussian noise process with a white power spectral density from dc to well above ω_0 , and $\delta\phi(t)$ is a small multiplicative phase noise term. The multiplicative noise term can, in general, be complex and thus also represent gain fluctuations. Factoring out the sinusoidal variation at ω_0 yields the slow time-variation of the output field around the microwave carrier

$$\begin{aligned}
 I_{out}(t) &= I_o e^{j\delta\phi(t)} + N'(t) e^{-j\omega_o t} \\
 &= I_o e^{j\delta\phi(t)} + N(t)
 \end{aligned}
 \tag{2}$$

Limiting the analysis to a band of frequencies $\delta\omega$ in width around ω_o , the additive noise term can be written as a random phasor: $N(t) = N'(t) e^{-j\omega_o t} = \delta r(t) + j\delta i(t)$, where $|\delta i(t)| = |\delta r(t)|$, and are assumed to be independent Gaussian noise processes with white power spectral density from dc to $\infty/2$. Note that as the input signal amplitude I_o is decreased to zero in Equation (2), the multiplicative noise term vanishes, but the additive noise term $N(t)$ is unchanged.

The total array noise is now calculated by using the M individual feed element expressions from Equation (2) in the standard calculation [2] of the array output field. For a linear array at a steering angle ϕ , the field distribution in the far field of the array as a function of observation angle θ may be expressed as

$$I_{out}(\theta, \phi, t) = \sum_{n=0}^{M-1} \left(\frac{I_o}{\sqrt{M}} e^{j\delta\phi_n(t)} e^{jn(\pi \cos \theta - \phi)} + N_n(t) \right)
 \tag{3}$$

Figure 3 is the calculated radiation pattern for a linear array of ten antennas vs. observation angle θ when the steering angle $\phi = 1$.

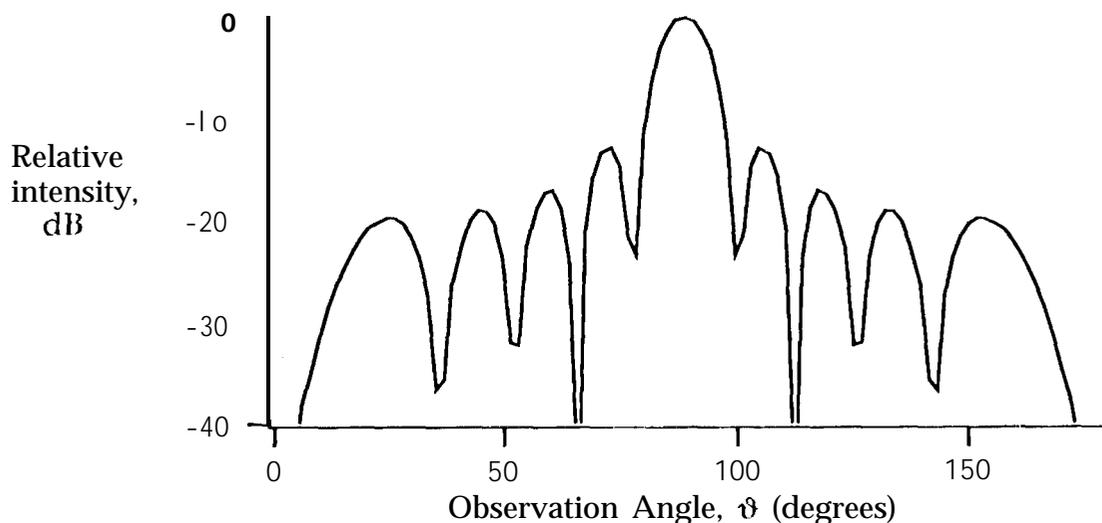


Figure 3. Calculated radiation pattern vs. observation angle for a 10-element linear array.

We wish to investigate the magnitude of the amplitude and phase fluctuations of the main lobe versus the number of elements in the array. If the noise sources in the feeds are uncorrelated, the statistics of the time-variation of the amplitude and phase of the main-lobe peak will be independent of the steering angle ϕ .

Therefore, for the purposes of this noise analysis, the array can be modeled simply as M equal-length feed systems sandwiched between two back-to-back ideal M -way power splitters, as depicted in Figure 1. Now, the output field amplitude is equal to the input field amplitude, independent of the number of elements M .

At the main lobe peak, $\pi \cos \theta = \phi$, and the signal amplitude at the output of the M -way coupler in Figure 1 corresponds to the main lobe peak amplitude of the antenna pattern, divided by \sqrt{M} . For small-angle phase noise $\delta\phi(t) \ll 1$ radian, the time variation of the output field can be written

$$E_{out}(t) = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} \left(\frac{E_o}{\sqrt{M}} (1 + j\delta\phi_n(t)) + N_n(t) \right). \quad (4)$$

First consider the output of an "array" comprised of only one feed element with unity gain. In the above equation, this corresponds to the case of $M=1$. The output field is then given by

$$E_{out}(t) = E_o + j\delta\phi(t)E_o + N(t). \quad (5)$$

which is just the same as the expression for a single feed given earlier in Equation (2), as required. The signal and noise power are proportional to the time-average of the squared-magnitude of the output voltage:

$$P_{out} \propto \left(\begin{aligned} &\langle E_o E_o^* \rangle + \langle E_o (j\delta\phi)^* E_o^* \rangle + \langle E_o N^* \rangle \\ &+ \langle j\delta\phi E_o E_o^* \rangle + \langle j\delta\phi E_o (j\delta\phi)^* E_o^* \rangle + \langle j\delta\phi E_o N^* \rangle \\ &+ \langle N E_o^* \rangle + \langle N (j\delta\phi)^* E_o^* \rangle + \langle N N^* \rangle \end{aligned} \right) \quad (6)$$

where the angle-brackets $\langle \rangle$ denote the time-average of the enclosed quantity, and the explicit time-dependence of the random functions has been dropped for clarity. For independent zero-mean noise processes, the time-average of products of the constant and random terms are zero, because we have assumed zero-mean random noise processes. Also, the average of the product of two uncorrelated terms, such as $\langle N (j\delta\phi)^* \rangle$, is zero. But the time-average of the square of any signal or noise term is non-zero. Thus, the ratio of signal to noise power (SNR) for one feed is

$$SNR|_{1\text{-element}} = \frac{\langle |E_o|^2 \rangle}{\langle |j\delta\phi(t)|^2 |E_o|^2 \rangle + \langle |N(t)|^2 \rangle}. \quad (7)$$

Similarly, for the case of M parallel feeds with independent equal-amplitude multiplicative and additive noise sources, the output field is given by

$$\begin{aligned} E_{out}(t) &= \frac{1}{\sqrt{M}} \left(\frac{E_o}{\sqrt{M}} (1 + j\delta\phi_0(t)) + N_0(t) + \frac{E_o}{\sqrt{M}} (1 + j\delta\phi_1(t)) + N_1(t) + \dots \right) \\ &= E_o \left(\frac{1}{M} (j\delta\phi_0(t) + j\delta\phi_1(t) + \dots) + \frac{N_0(t)}{\sqrt{M}} + \frac{N_1(t)}{\sqrt{M}} + \dots \right) \end{aligned} \quad (8)$$

The individual powers of equal-amplitude, independent (therefore uncorrelated) noise sources may be added linearly. Thus, the equivalent noise voltage due to the sum of M uncorrelated equal-amplitude noise sources is just \sqrt{M} times the amplitude of a single noise source. Since all cross-terms between uncorrelated noise sources average to zero, we can write the output field for an M -element array in terms of a single multiplicative noise source $\delta\phi(t)$ and a single additive noise source $N(t)$

$$I_{out}(t) = I_o + \frac{I_o j \delta\phi(t)}{\sqrt{M}} + N(t). \quad (9)$$

Now, the ratio of signal to noise power in the combined output of an M -element array is

$$SNR|_{M\text{-elements}} = \frac{\langle |I_o|^2 \rangle}{M \langle |j\delta\phi(t)|^2 |I_o|^2 \rangle + \langle |N(t)|^2 \rangle} \quad (10)$$

The multiplicative noise power is thus mitigated by a factor of M in the combined output of an M -element array, whereas the signal power and additive noise power are unchanged from the single-element case. Figure 4 illustrates these results for an array of ten elements compared to a single element.

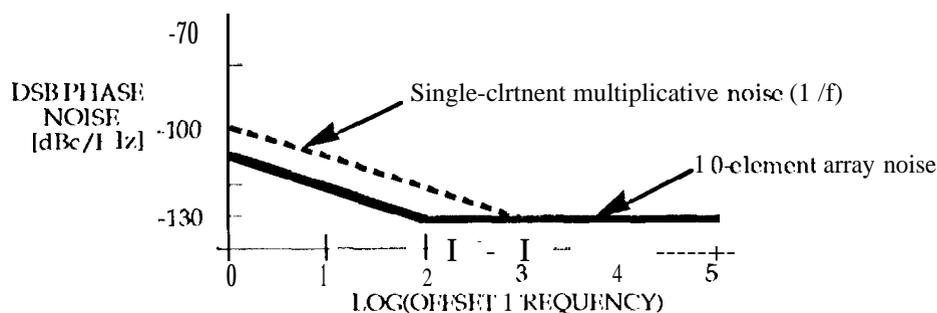


Figure 4. Comparison of Single-element and 10-element Phase Noise

The close-to-carrier $1/f^\alpha$ multiplicative noise of the combined array output is reduced by 10 dB compared to the noise of an individual element. The white additive noise power further from the carrier is unchanged from the single-element case. This behavior is analogous to the improved frequency stability obtained from an ensemble of oscillators, compared to the stability of a single oscillator. This property may therefore make phased-array antennas more desirable than single-element antennas for applications in which high levels of long-term phase stability are required. Alternatively, this property relaxes the requirements on multiplicative phase noise for the elements of a large phase array. It is emphasized that the additive noise requirements are not relaxed, however.

5. SUMMARY

It was shown that as the number of array elements M is increased, the effect of uncorrelated multiplicative phase noise of the feed elements on the total array stability is diminished. However, the uncorrelated additive noise of the feed elements is not diminished, so that the signal-to-noise ratio becomes independent of array size for large enough M . It is therefore important to quantify both the additive and multiplicative noise of the feed elements to correctly predict the total array phase stability.

6. ACKNOWLEDGEMENTS

This work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, and was sponsored by the United States Air Force Rome Laboratory and the National Aeronautics and Space Administration.

7. REFERENCES

1. R. T. Logan, G. F. Lutes, High Stability Microwave Fiber Optic Systems: 1 Demonstrations and Applications, IEEE Frequency Control Symposium Hershey, Pennsylvania, 1992), pp. 310-316.
2. S. Ramo, J. R. Whinnery, T. V. Duzer, *Fields and Waves in Communication Electronics*. (John Wiley and Sons, New York, 1984).