LOW FREQUENCY GUIDED PLATE WAVES PROPAGATION IN FIBER REINFORCED COMPOSITES

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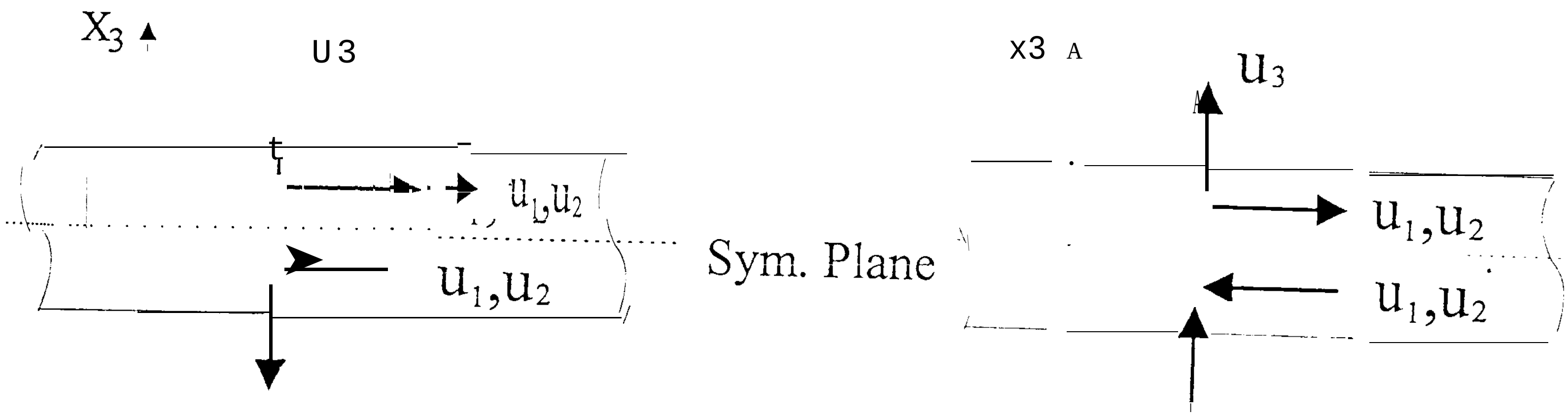


(a) symmetric Mode

(b) Antisymmetric Mode

Symmetric and Antisymetric Mode of Guided Waves

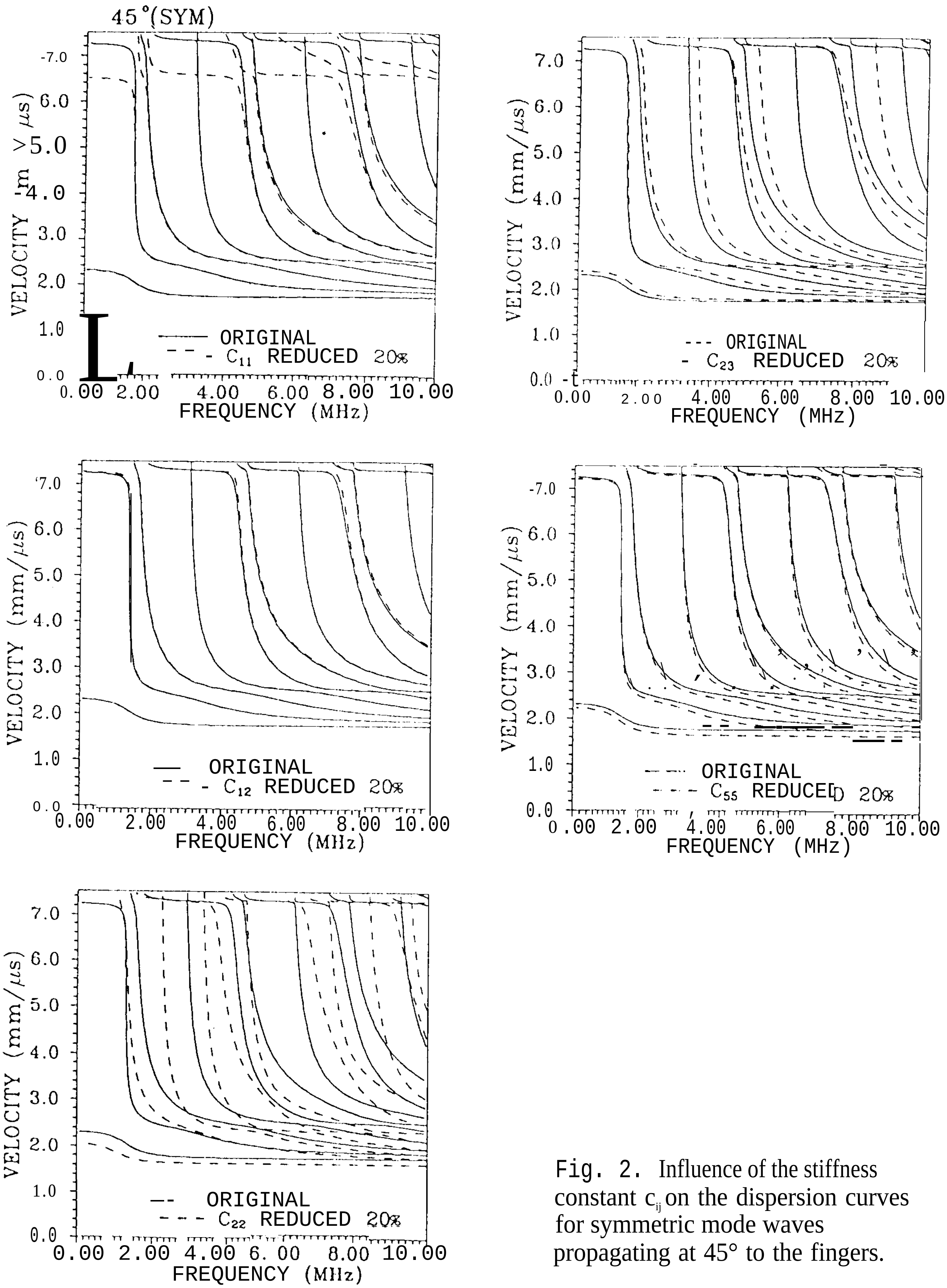
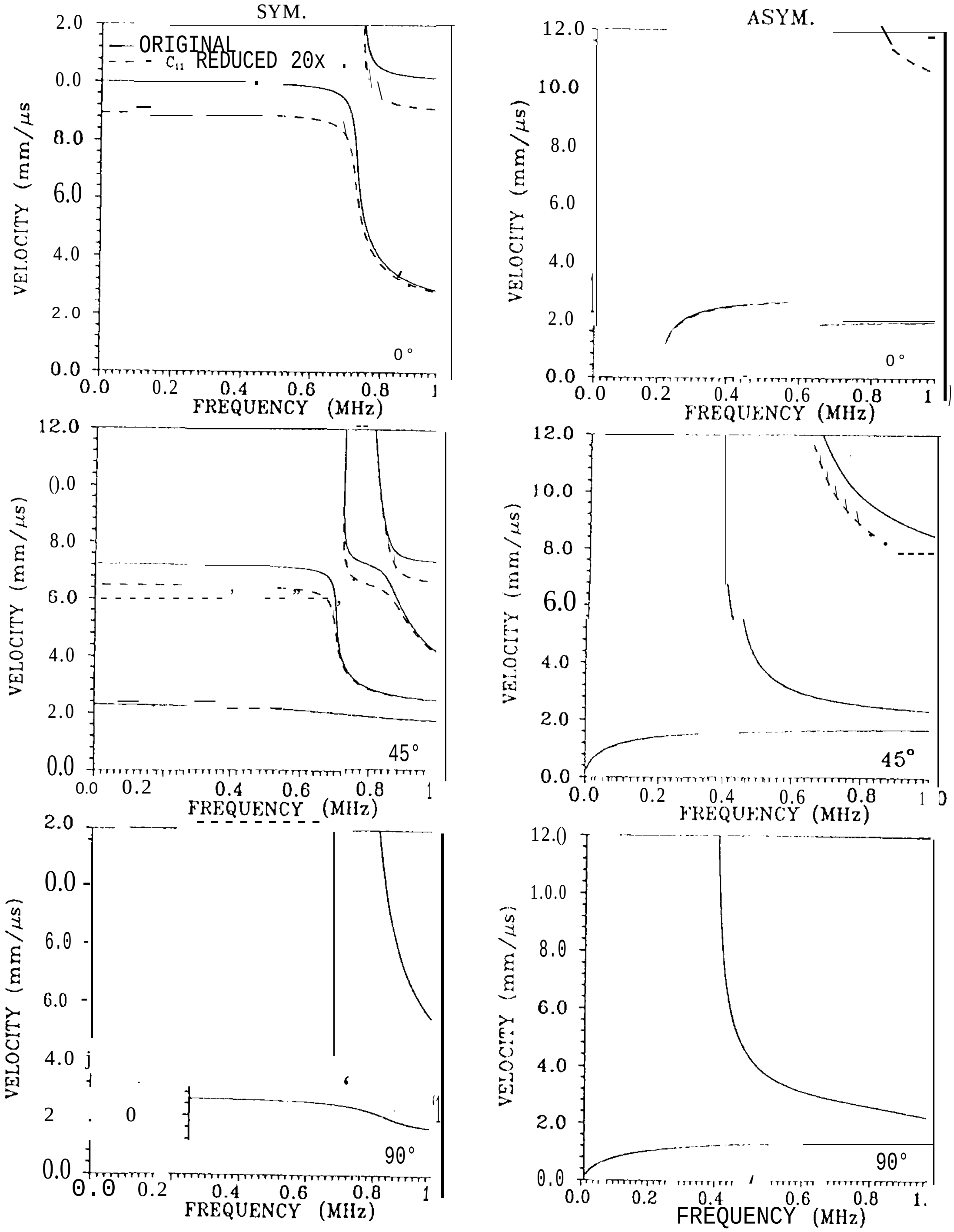
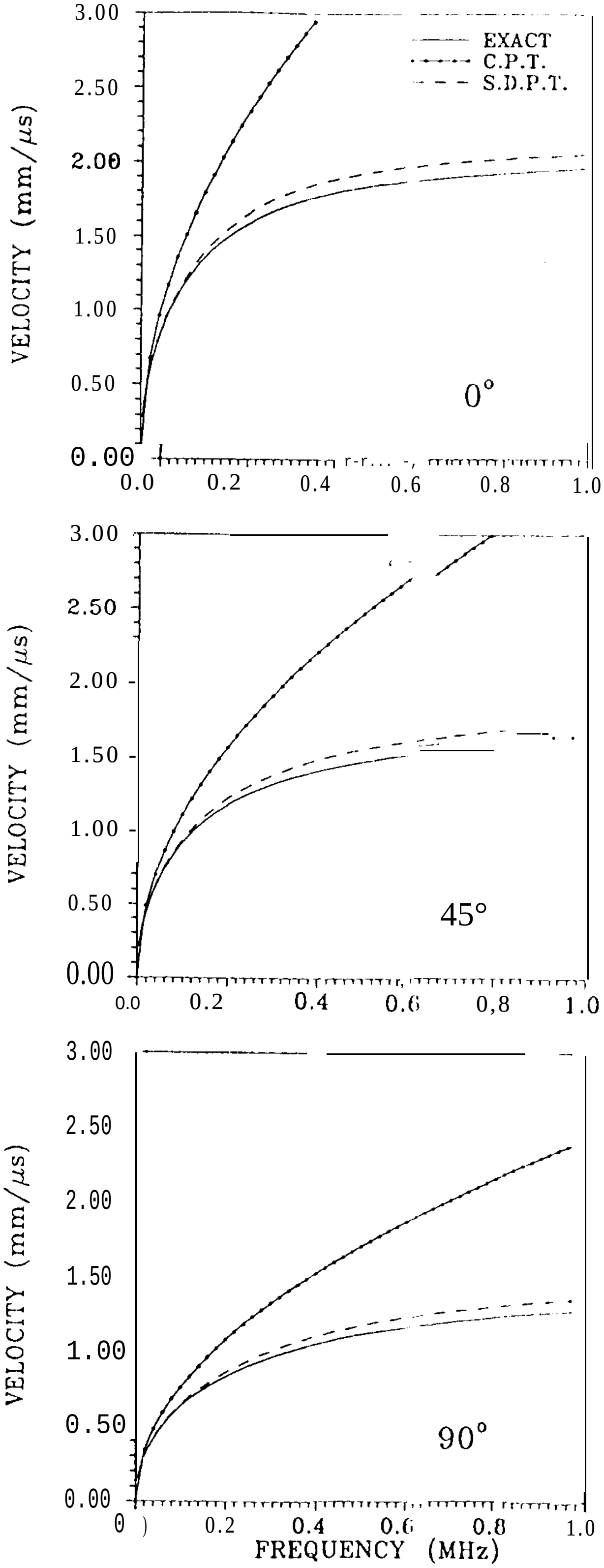


Fig. 2. Influence of the stiffness constant c_{ij} on the dispersion curves for symmetric mode waves propagating at 45° to the fingers.



SYM.
ASYM.
ORIGINAL
C11 REDUCED 20X
VELOCITY (mm/μs)
FREQUENCY (MHz)
0°
45°
90°



EXACT
C.P.T.
S.D.P.T.
VELOCITY (mm/μs)
0°
45°
90°
FREQUENCY (MHz)

SYMMETRIC PLATE WAVE **IN** A FIBER-REINFORCED COMPOSITE **LAMINATE**

a) Exact Linear **Elastic** Solution

$$\Delta_1 \cot(\zeta_1 \omega h) + \Delta_2 \cot(\zeta_2 \omega h) + \Delta_3 \cot(\zeta_3 \omega h) = 0$$

$$\begin{aligned}
\Delta_1 &= \zeta_2[(\xi_2^2 + \zeta_3^2)q_{22} - (\xi_2^2 - \zeta_3^2)q_{12}][(a_5 - a_3)\xi_1^2 q_{11} - (a_1 - 2a_4)\xi_2^2 q_{21} - a_1\zeta_1^2 q_{21}] \\
\Delta_2 &= -\zeta_1[(\xi_2^2 + \zeta_3^2)q_{21} - (\xi_2^2 - \zeta_3^2)q_{11}][(a_5 - a_3)\xi_1^2 q_{12} - (a_1 - 2a_4)\xi_2^2 q_{22} - a_1\zeta_1^2 q_{22}] \\
\Delta_3 &= 4a_4\xi_2^2\zeta_1\zeta_2\zeta_3(q_{11}q_{22} - q_{12}q_{21})
\end{aligned} \qquad \textbf{(2)}$$

when frequency times thickness tends to zero, i. e $\omega H \to 0$

$$\frac{\Delta_1}{\zeta_1} + \frac{\Delta_2}{\zeta_2} + \frac{\Delta_3}{\zeta_3} = 0 \tag{3}$$

or

$$\begin{aligned}
& a_1a_2^2b_3n_1^4n_2^2 - a_2a_3^2b_3n_1^4n_2^2 - 2a_2^2a_4b_3n_1^4n_2^2 + a_2a_3a_5b_3n_1^4n_2^2 - 2a_2a_3a_4b_3n_1^2n_2^4 \\
& + [(2a_1a_2a_3b_1b_2 - 2a_3^3b_1b_2 - 4a_2a_3a_4b_1b_2 + 2a_3^2a_5b_1b_2 - 2a_1a_2b_3 \\
& + a_3^2b_3 + 4a_2a_4b_3 - a_3a_5b_3 + 2a_2a_3a_4b_1b_3 + a_1a_2a_5b_1b_3 - 2a_2a_4a_5b_1b_3 \\
& + 2a_2a_3a_4b_2b_3 + a_1a_2a_5b_2b_3 - 2a_2a_4a_5b_2b_3)n_1^2n_2^2 \\
& + (2a_1a_3a_5b_1b_2 - 4a_3a_4a_5b_1b_2 + 2a_3a_4b_3)n_2^4]\rho V^2 \\
& + [(-a_1a_2a_3b_1b_2b_3 + a_3^3b_1b_2b_3 - 2a_3^2a_5b_1b_2b_3 + a_3a_5^2b_1b_2b_3)n_1^2 \\
& + (-2a_1a_3b_1b_2 + 4a_3a_4b_1b_2 + a_1b_3 - 2a_4b_3 \\
& - 2a_3a_4b_1b_3 - a_1a_5b_1b_3 + 2a_4a_5b_1b_3 - 2a_3a_4b_2b_3 - a_1a_5b_2b_3 \\
& + 2a_4a_5b_2b_3 - a_1a_3a_5b_1b_2b_3 \\
& + 2a_3a_4a_5b_1b_2b_3 + a_1a_5^2b_1b_2b_3 - 2a_4a_5^2b_1b_2b_3)n_2^2]\,(\rho V^2)^2 \\
& + a_1a_3b_1b_2b_3(\rho V^2)^3
\end{aligned} \tag{4}$$

$$
\begin{aligned}
&(b_1 - b_2)(\rho V^2 - c_{55} n_1^2)(\rho V^2 - c_{11} n_1^2)\Omega(c)ij,\ n_1,\ n_2) = 0 \\
&\Omega(c_{ij},\ n_1,\ n_2) = (-c_{12}^2 c_{55} + c_{11} c_{22} c_{55}) n_1^4 \\
&(-2c_{12}^2 c_{22} + c_{11} c_{22}^2 + 2c_{12}^2 c_{23} - c_{11} c_{23}^2 - 2c_{12} c_{22} c_{55} + 2c_{12} c_{23} c_{55}) n_1^2 n_2^2 \\
&+ (c_{22}^2 c_{55} - c_{23}^2 c_{55}) n_2^4 \\
&+ [(c_{12}^2 - c_{11} c_{22} - c_{22} c_{55}) n_1^2 + (-c_{22}^2 + c_{23}^2 - c_{22} c_{55}) n_2^2]\, \rho V^2 + c_{22} \rho V^4
\end{aligned}
\tag{5}
$$

$\Omega(c_{ij}, n_1, n_2) = 0$ represents the dispersion equation of the limit of the lowest symmetric mode.

Wave propagates along 0°

$$\rho V^2 - c_{55})(c_{22}\rho V^2 + c_{12}^2 - c_{11}c_{22}) = 0 \tag{6}$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}},\ \sqrt{\frac{c_{11} - c_{12}^2/c_{22}}{\rho}} \tag{7}$$

Wave propagates along 0°

$$(\rho V^2 - c_{55})(c_{22}\rho V^2 - c_{22}^2 + c_{23}^2) = 0 \tag{8}$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}},\ \sqrt{\frac{c_{22} - c_{23}^2/c_{22}}{\rho}} \tag{9}$$

For isotropic material

$$\rho V^2 - c_{55})\ c_1\ \rho S - 4c\ \ c_{55} + 4c_{55}^2) = 0 \tag{10}$$

$$V_{1,2} = \sqrt{\frac{c_{55}}{\rho}},\ 2\sqrt{\frac{c_{55}}{\rho}}\sqrt{1 - \frac{c_{55}}{c_{11}}} \tag{11}$$

APPROXIMATE PLATE THEORIES

Classical plate theory

$$\begin{vmatrix} \hat{c}_{11}n_1^2 + \hat{c}_{55}n_2^2 - \rho v^2 & (\hat{c}_{12} + \hat{c}_{55})n_1 n_2 \\ (\hat{c}_{12} + \hat{c}_{55})n_1 n_2 & (\hat{c}_{55}n_1^2 + \hat{c}_{22}n_2^2) - \rho v^2 \end{vmatrix} = 0 \tag{12}$$

where

$$\begin{aligned} \hat{c}_{11} &= c_{11} - c_{12}^2/c_{22} \\ \hat{c}_{22} &= c_{22} - c_{23}^2/c_{22} \\ \hat{c}_{12} &= c_{12} - c_{12}c_{23}/c_{22} \\ \hat{c}_{55} &= c_{55} \end{aligned} \tag{13}$$

Shear Deformation **Plate** Theory

$$
\begin{aligned}
u_1 &= u_1^0(x_1,\ x_2,\ t) \\
u_2 &= u_2^0(x_1,\ x_2,\ t) \\
u_3 &= x_3\psi_3(x_1,\ x_2,\ t)
\end{aligned}
\tag{14}
$$

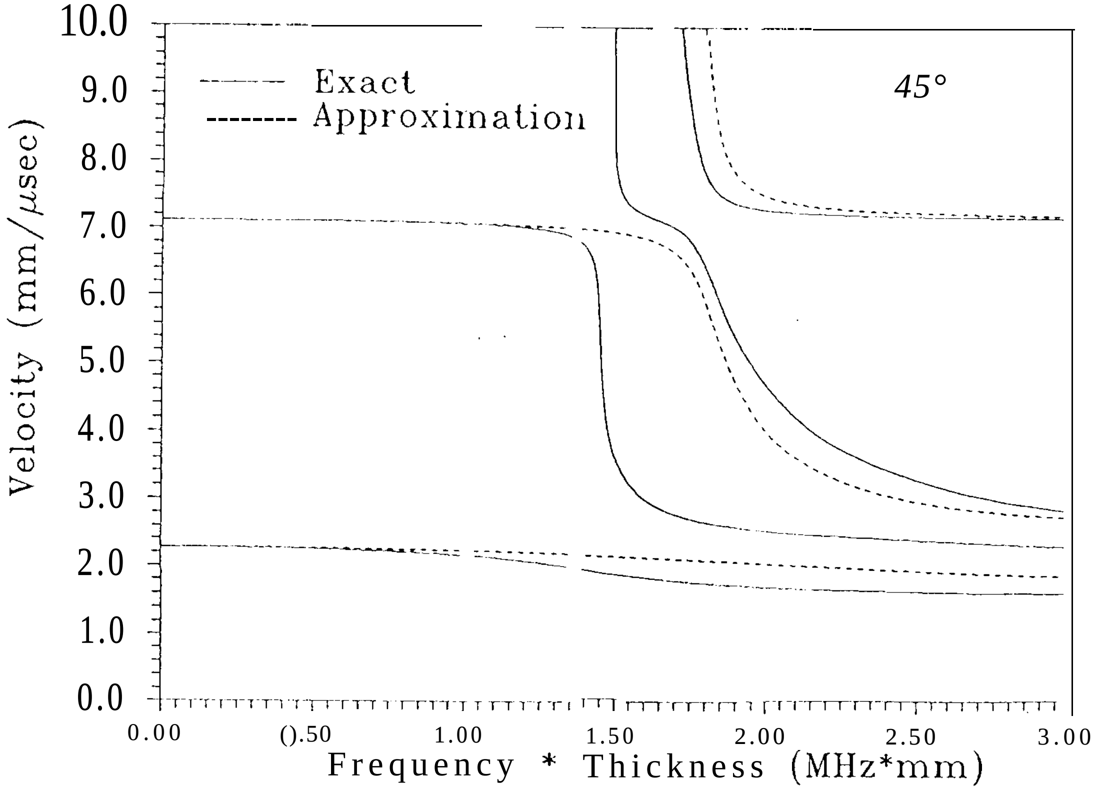
Assume plane wave solutions of **(21)** in the form

$$
\begin{aligned}
u_1^0 &= U_1^0 e^{i(k_1x_1 + k_2x_2 - \omega t)} \\
u_2^0 &= U_2^0 e^{i(k_1x_1 + k_2x_2 - \omega t)} \\
\psi_3 &= \Psi_3 e^{i(k_1x_1 + k_2x_2 - \omega t)}
\end{aligned}
\tag{15}
$$

where k_1, k_2 and k_3 represent the wavenumbers along the x_1, x_2 and x_3 directions, respectively, and ω is the circular frequency.

$$
\begin{bmatrix}
-(A_{11}k_1^2 + A_{55}k_2^2) + I_1\omega^2 & -(A_{12}+A_{55})k_1k_2 & iA_{12}k_1 \\
-(A_{12}+A_{55})k_1k_2 & -(A_{55}k_1^2 + A_{22}k_2^2) + I_1\omega^2 & iA_{23}k_2 \\
iA_{12}k_1 & iA_{23}k_2 & D_{55}k_1^2 + D_{44}k_2^2 + A_{22} + I_3\omega^2
\end{bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}
\tag{16}
$$

where $n_1 = \cos\phi$ and $n_2 = \sin\phi$; ϕ is the wave propagating angle, and $k_1 = \omega/V\ n_1$, $k_2 = \omega/V\ n_2$, V is the phase velocity.



10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Velocity (mm/μsec)
Exact
Approximation
45°
0.00
().50
1.00
1.50
2.00
2.50
3.00
Frequency * Thickness (MHz*mm)

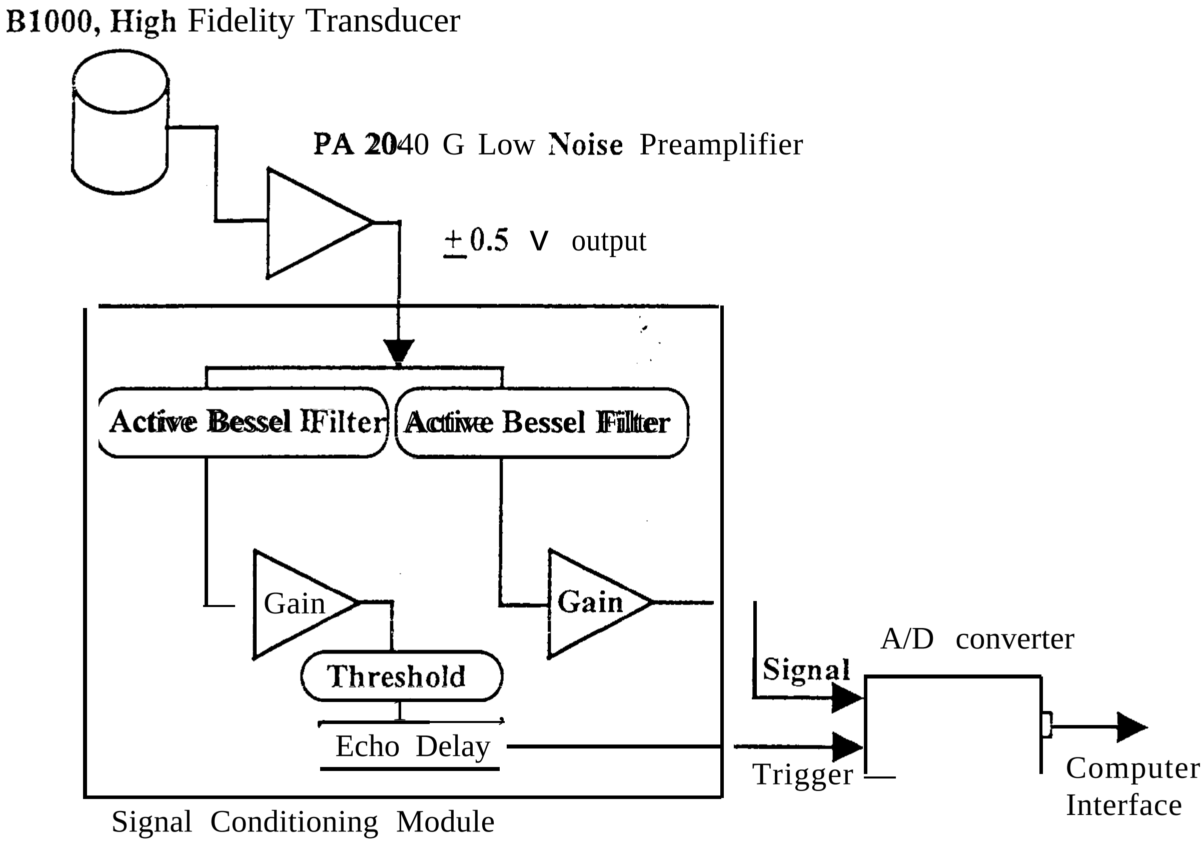
EXPERIMENT & RESULT

A $[0]_{16}$ 12 x 12 cm^2 unidirectional graphite/epoxy plate is used in the experiment.

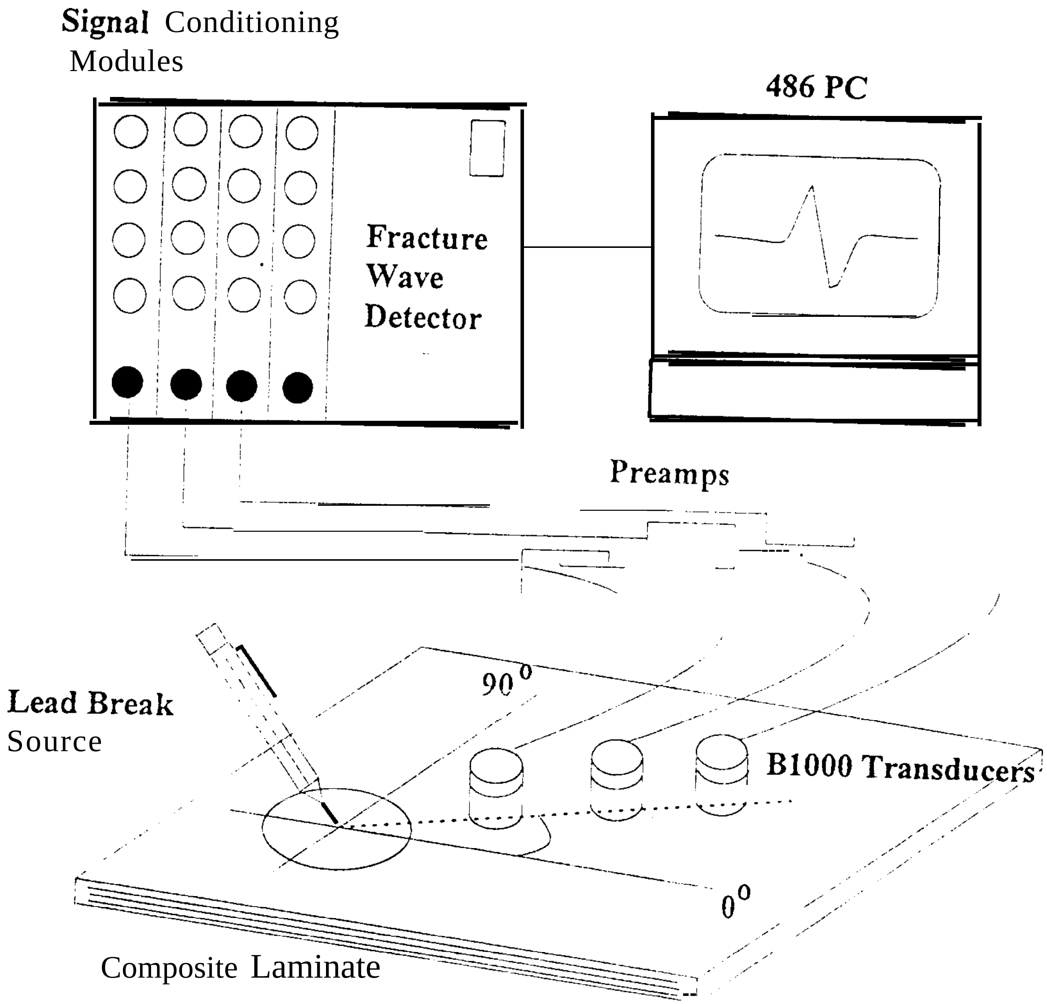
c_{11} = 155.01, c_{12} = 6.44, c_{22} = 15.6, c_{23} = 7.89, c_{55} = 5.00 (Gpa).
ρ = 1.56 g/cm^3.

Group velocity $\mathbf{V_e}$

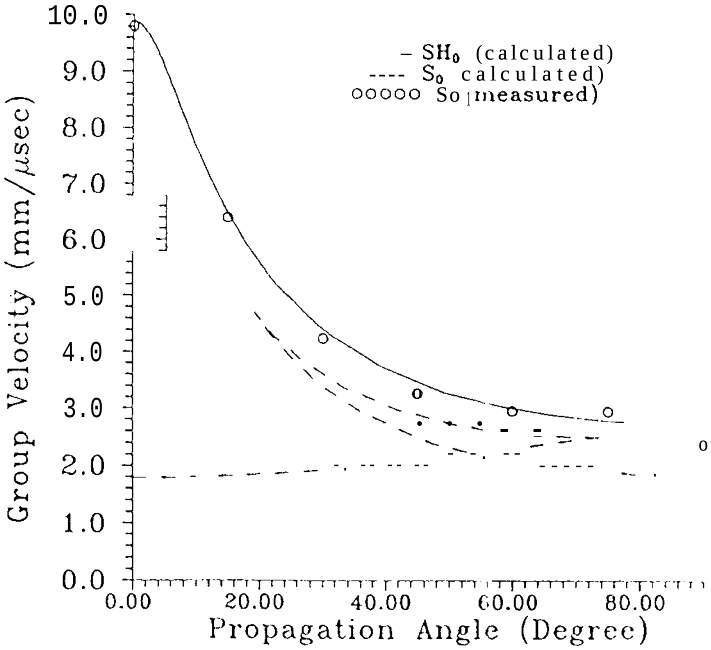
$$V_e = -\frac{\partial\Omega/\partial \boldsymbol{n}}{\partial\Omega/\partial V}$$



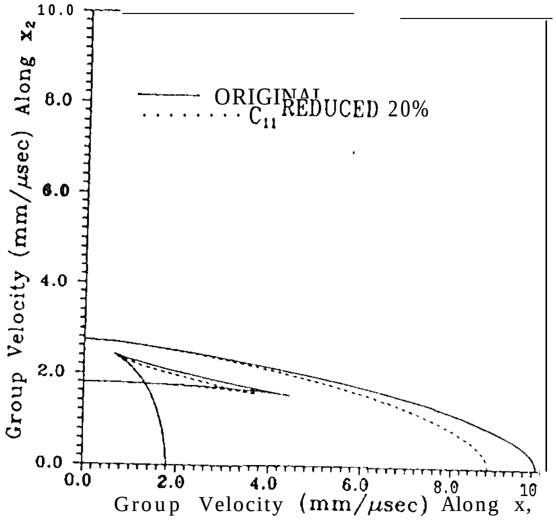
F4000 Fracture Wave Detector Block Diagram

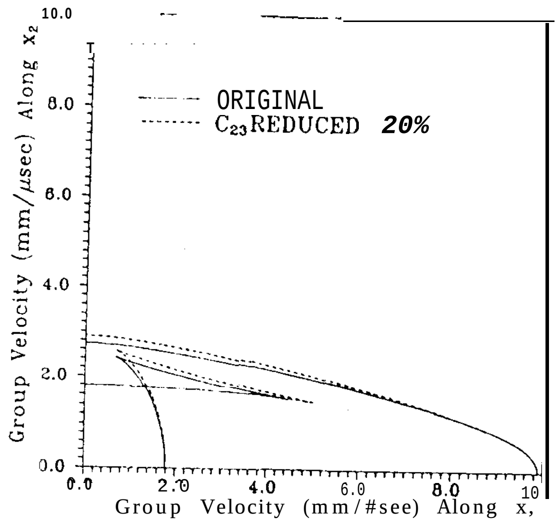


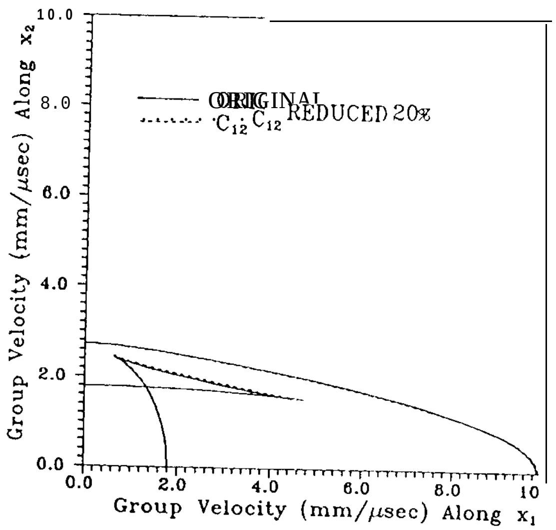
The Experimental Setup.

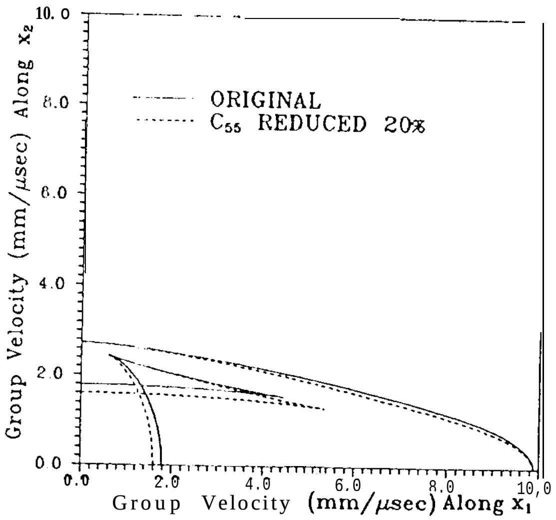


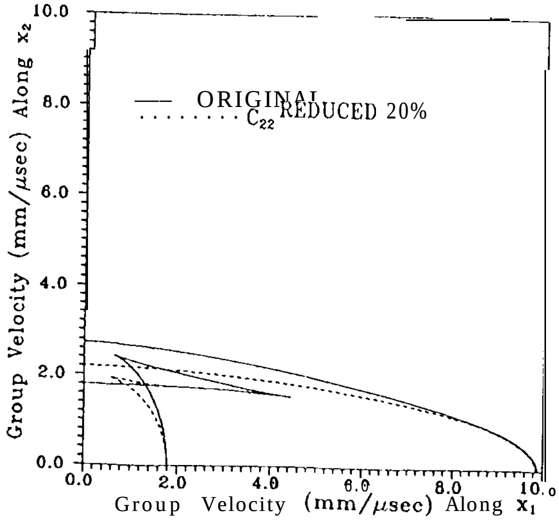
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.00
20.00
40.00
60.00
80.00
Group Velocity (mm/μsec)
Propagation Angle (Degree)
– SH0 (calculated)
---- S0 calculated)
ooooo So measured)


ORIGINAL
C11 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C23 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C12 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C55 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1


ORIGINAL
C22 REDUCED 20%
Group Velocity (mm/μsec) Along x2
Group Velocity (mm/μsec) Along x1

CONCLUDING REMARKS

- The phase and group velocity of symmetric plate waves is analyzed.

- A comparison between measured and calculated group velocity for a unidirectional graphite epoxy is presented.

- The agreement of the results yield the validity of the characterizing the elastic constants from measured low frequency extensional mode group velocity data.

- The influence of all five stiffness constants on the dispersion curves of symmetric plate waves was theoretically investigated. All but C_{12} were found to have an influence on the dispersion curves in this frequency range.