NONLINEAR MULTI-WINDOW CONTROLLERS

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Abstract. In many control designs the error signal is split into parallel channels which are tailored to be significant over distinct frequency ranges and are then recombined. Often at least one channel contains nonlinear elements. The most common example is a PID controller with an anti-windup nonlinearity in the I-channel. A more general example using higher-order compensators is presented, which is a thermal controller for a spacecraft-mounted telescope. Although the linear elements can be designed effectively using frequency response methods, the design of the nonlinear elements is guided by step-response simulations or experiments. The Jolt of the nonlinearities in the parallel channel structure is further examined by introducing the concept of amplitude/frequency windows. Acquisition and tracking systems are considered which use amplitude windowing of the error signal to change control modes. The performance of such "multi-window" control systems is then discussed, and their design is compared to the design of fuzzy-logic controllers.

INTRODUCTION

A common control system design methodology entails splitting the error signal into parallel channels with linear compensation such that the channels are significant over distinct frequency ranges. Nonlinearities are included in some channels, and are most often of the non-dynamic type. Usually the channels are recombined linearly. Such an arrangement allows the designer to restrict the action of a particular nonlinearity to the. frequency range which corresponds to the channel in which it appears. This feature is thought to be a key to the popularity of parallel-channel compensators, since for the purely linear case, the design of cascaded compensation is equally simple. An example of this type of controller which is ubiquitous in industry [1-3] is a PID controller with an anti-windup nonlinearity in the I-channel. The nonlinearity prevents the undesirable overshooting that results from actuator saturation during large transients. The application of a PID controller typically results in a Bode diagram which is sub-optimally shallow at crossover [4]. However, the parallel-channel structure is easily generalized to higher-order systems which can be designed close to the optimum. In the following discussion) the three parallel channels are referred to as low-frequency (LF), medium-frequency (MF), and high-frequency (HF), and they are analogous to the I-channel, the P-channel, and the D-channel, respectively. of the PID controller.

CONTROLLERS WITH ANTI-WIND-UP DEVICES

This investigation was prompted by a few specific design problems, and their discussion provides an introduction Fig. 1 shows an electrical analogy to a thermal control system for a spacecraft-mounted telescope. The heater is used to keep the temperature of the primary and the secondary mirrors within 1.6 K of each other. (Another loop, which is not shown, drives the primary mirror heater which maintains the absolute temperature at 263-298 K). The heaters are pulse-width modulated with a modulation period of 6 s, and a Imlsc-width timing, resolution of 125 msec. The total heater power

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cannot exceed 6 W. The frequency response of the plant transfer function from the heater to the temperature differential is basically that of an integrator; however there are also radiative losses, which are nonlinear.

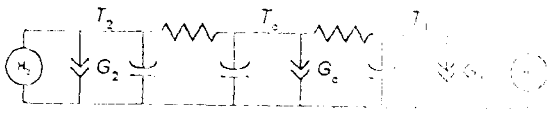


Fig. 1: Thermal controller **electrical analogy**

The compensator is implemented in three parallel channels. The compensator for the high-frequency (HF) channel is a complex pole pair:

$$C^{HF} = \frac{0.1}{s^2 + 0.125s + 0.1}$$

(1)

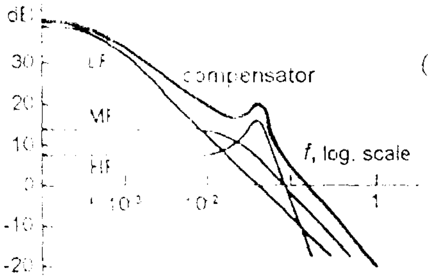


Fig. 2: Parallel-channel compensator for thermal controller

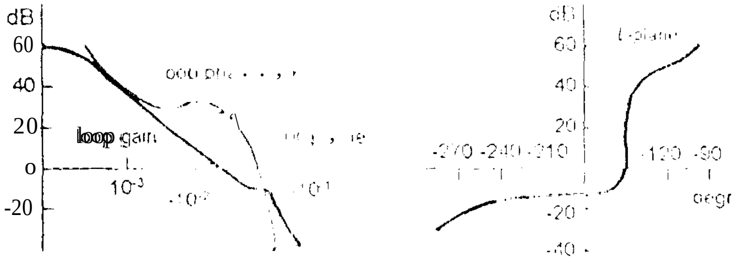


Fig. 3: **Loop frequency response for thermal controller**

The medium-frequency (MF) and low-frequency (LF) channels are first-order:

$$C^{MF} = \frac{0.5}{s + 0.1} \quad and \quad C^{LF} = \frac{0.5}{s + 0.03}$$

and a saturation element precedes the LF compensation. The separate and combined frequency responses of the compensator channels (ignoring the nonlinearity) are shown in Fig. 2, and the loop frequency response is shown in Fig. 3. The parallel connection of the MF and HF channels forms the constant gain region on the Bode diagram near 30 mHz, which is known as the "Bode step," and provides the optimal cut-off.[4] The feedback bandwidth was ultimately limited by the effects of sampling.

The LF compensation steepens the response in the 1-10 mHz range, providing larger feedback at low frequencies. This would result in wind-up, i.e. excessive overshooting, for transients in which the heater saturates, unless an anti-windup device is provided. (For the typical power-on transient, the heater saturates immediately.) The anti-windup device used here is a saturation element preceding the LF path. This prevents the LF path from "integrating up" excessively when the actuator is saturated and the error is large. After observing a few (5-10) step response simulations, the saturation threshold in the LF path was chosen to be 0.8 K. The closed-loop system transient response is notably insensitive to variations in this level, which makes a good level easy to determine. Note that placement of the saturation element *after* the LF compensation results in a transient with a windup error that is small but takes an excessive amount of time to decay. Industrial controllers, which often place the saturation link after the I-path, frequently use integrator reset features to overcome this problem

The power-on step response for the controller is shown in Fig. 4. The heater is saturated most of the time, providing nearly time-optimal performance, and the overshoot is insignificant.

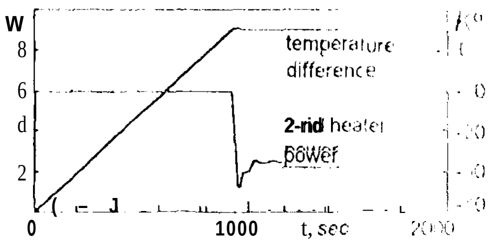


Fig. 4: Step response for thermal controller

PID controllers are also used for servos where the plant is essentially a double integrator. (This idealization is usually complicated by nonlinear friction and/or load characteristics, and sometimes high-frequency structural resonance modes.) At frequencies where the I-channel is dominant, the loop response has a steep slope of 3 integrators, and an anti-windup device is typically necessary to provide an acceptable response to large transients.

A related example are thruster controllers for spacecraft attitude control. The plant is close to a pure double integrator, although there may be flex modes at high frequencies. The thrusters are arranged so that they can be used to provide a positive or negative torque about each axis, but they are not throttled - the torque is either positive or negative a fixed magnitude, or zero (similar to a 3-position relay.) These controllers often do not include an I-channel (low frequency disturbances are almost nonexistent), but do use saturation in the P-channel, which is then considered to be the LF-channel. This avoids windup due to the extreme actuator nonlinearity and the "extra" integrator.

With describing function (DF) method, the explanation for the windup phenomenon goes as follows. Actuator saturation reduces the loop gain such that the crossover frequency is shifted (in\\ n into the steep region which is dominated by the LF channel. The phase lag here is excessive and the result is a sort of process instability. The anti-windup device works by reducing the gain of the LF channel at large signal levels. This makes the frequency response more shallow, and thus restores the phase lead. Interestingly, DF analysis does not suggest anything about the placement of the nonlinearity before or after the linear compensation, despite the curious dependence of the observed transient response on this choice. Nor does DF analysis enable the designer to quantify the trade-offs which must be made in the choice of the saturation level. As mentioned previously, this part of the design is usually carried out using simulations or experiments, both of which can be expensive and time-consuming.

In general, the introduction of nonlinearities greatly expands the design space from the purely linear case, where the Bode integral limitations force trade-offs between, for example, fast response to large errors and minimization of steady-state error. It is not obvious, however, which nonlinearities might be advantageous, and where in the control system architecture they should be placed. The anti-windup devices are placed in the LF-channel, since it is known that the combination of this channel's frequency response and the actuator saturation causes the undesirable overshooting. Many other arrangements can be useful for improving a system's performance beyond that achievable by linear compensation. One of the more intuitively appealing is the acquisition and tracking architecture, where the control law is "switched" as the error signal level changes.

ACQUISITION AND TRACKING SYSTEMS

Acquisition and tracking systems, like those used in homing missiles, are designed to operate in two modes: "acquisition mode" when the error is large and "track mode" when the error is small. Another example of the acquisition/tracking type is a pointing control system for a spacecraft-mounted camera, in which a rapid retargeting maneuver is followed by a slow precise scanning pattern to form a mosaic image. When the error signal is in the acquisition regime, the controller should respond as rapidly as possible, i.e. the feedback bandwidth should be as wide as possible. In the acquisition mode it is not necessary that the feedback be very large, since the errors are large. In the tracking regime, the feedback bandwidth needs to be reduced to reduce the effects of sensor noise, but the value of the feedback should be made rather large to minimize the tracking error. The differing loop frequency responses for the two modes are depicted in Fig. 5.

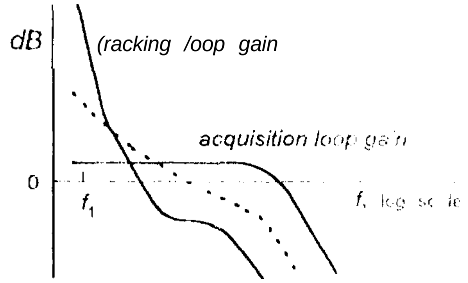


Fig. 5: Acquisition / track loop frequency responses

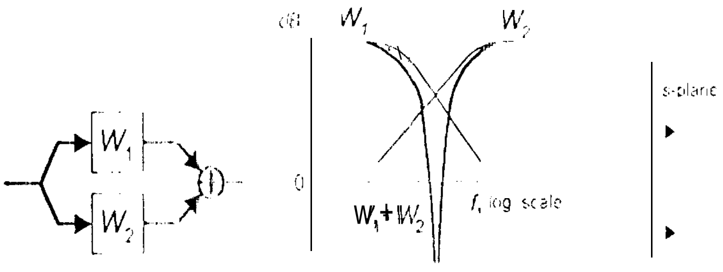
While the determination of the optimal frequency responses for the acquisition mode and for the tracking mode is straightforward, guaranteeing a smooth transition between the two regimes can be difficult. First, a criterion must be established for changing the control law. It is usually sufficient to alter the control law based on the amplitude of the error signal alone, although clearly other information might be factored in. Second, it must be determined whether this change should be done abruptly by using a switch, or gradually somehow, by blending the separate reponses (the saturation element in the LF path of the parallel-channel controllers discussed above is to provide this sort of blending.)

Abrupt switching from acquisition to tracking mode can generate large transients in the output and error signals. The situation may be so bad that the target is de-acquired. A conceptual solution is to smooth the transition so that intermediate frequency responses during the transition will look like the dotted line in Fig. 5. If the frequency responses of the two channels are blended with the relative weight being a function of the error signal amplitude, a smooth transition can be provided. However, care must be taken to ensure that all of the intermediate combined frequency responses of the parallel channels are acceptable. No intermediate response should be allowed which results in an unstable system, or even in a system with small stability margins, since it is known that such systems have poles close to $j\omega$-axis and therefore, produce excessively large transient responses. This is a problem more often than one might think. As an example, let the total loop response T be the weighted sum of the acquisition T_{acq} and tracking T_{tr} responses:

$$T = (1 - k)T_{acq} + kT_{tr} ,$$

and suppose that k smoothly varies from 0 to 1 as the transition from acquisition mode to track mode occurs. For a certain value of k, the gains in the two paths are equal at the frequency f_1 indicated in Fig. 5. At this point the difference in phase between the two transfer functions exceeds 180°, and the result is that a zero of the total transfer function T moves into the right half-plane of s, and the system becomes unstable. The transient generated while the system remains in this state can be big and disruptive, even causing the target to be lost. For this reason care should always be taken to provide a smooth transition.

The conditions for the parallel-channel transfer function $W_1 + W_2$ to become nonminimum-phase (when each of the channels is minimum-phase) is illustrated in Fig. 6. When W_1 is a low-pass, and W_2 is a high pass, and the slopes of the gain responses are increased as shown in Fig. 6(b), then, the phase difference increases between the two channel transfer functions at frequency where the gains are equal. Therefore, from the vector addition of the two output signals, it is clear that zero transmission occurs when the phase difference is π, i.e. the root locus for the transfer function zero crosses the $j\omega$-axis as shown in Fig. 6(c). A general criterion is given in Ref. 4.



(a) (b) (c)

Fig. 6: Parallel channels with non-minimum phase shift

MULTI-WINDOW CONTROLLERS

In a PID controller with an anti-windup device, the error signal is filtered into different frequency channels, but the input to (or output of) the I.F channel is restricted to small amplitudes. In the acquisition and track controllers, the error signal amplitude is used to determine a weighted sum of different frequency responses. A general view is to consider nonlinear compensators which partition the error signal into "windows." The windows divide the frequency spectrum (or, equivalently, time-response behavior) and the amplitude range, as depicted in Fig. 7, This architecture is referred to as "multi-window," and a great number of useful nonlinear control schemes can be cast in this form.

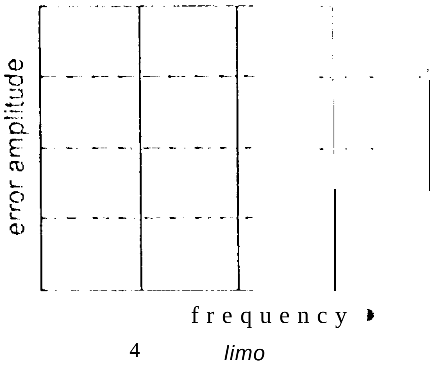
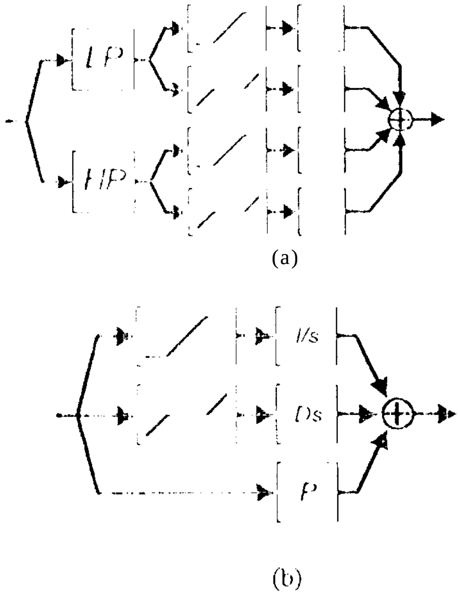


Fig. 7: The multi-window concept

The diagram in Fig. 7 is somewhat ambiguous since it does not indicate whether the frequency selection or the amplitude selection is performed first, and the order does matter (often a particular order is required. In audio systems, for example, the frequency selection must be done first to eliminate intermodulation. On the other hand, when designing nonlinear dynamic compensators to guarantee stability with Popov's method, the frequency selection should be done first so that the nonlinearity can be combined with the actuator nonlinearity.[4] Fig. 8 shows two different types of architectures for multi window systems.



(a)

(b)

Fig. 8: Multi window architectures

For a PID with anti-windup, the best choice depends on the sort of command the controller must respond to and also on the nature of the disturbances When the only command is a step, placing the saturation first will prevent the accumulation of integrated error that causes

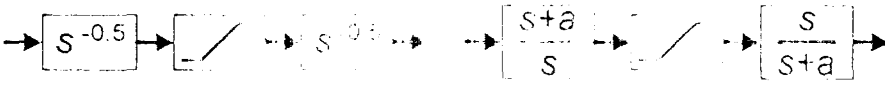
windup. However, if there is large-amplitude short pulse-type random noise, and the low frequency components of this noise are substantial, the integral term needs to be effective to reduce the "hang-off" error in the output - but it is not if the peaks are being clipped by input saturation in the I-channel. In this case it would be better to first smooth the pulses by using a low-pass filter, and only then to clip them.

When the error signal can be arbitrary, the best performance (in a minimax sense) might be provided by a combination of the two strategies, i.e. the saturation link could be sandwiched between two low-pass filters. Here, two options exist: (a) half of an integration can be placed before the saturation, and then another half afterwards; or (b) using two first-order links, one before the nonlinearity cutting off the higher frequencies using a single pole, and one afterwards cutting off low-frequencies with a zero at the same frequency as the pole of the first link. These implementations are indicated in Fig. 9. The pole / zero frequency can be adjusted as a single "knob." But still, counting the saturation level, two extra knobs are required compared to the linear PID controller. (For the half-integrator version, there is only one additional knob.)

COMPARISON TO FUZZY-LOGIC CONTROLLERS

The multi-window architecture is particularly convenient for enhancing a control system by using nonlinearities. The performance is often much better than what could be obtained from a linear controller. The advantages of nonlinear control are also exploited by fuzzy-logic controllers, which are similar to multi-window controllers in that nonlinearities are used to tailor the controller response characteristics for different signal levels. There are several important differences however, which make the multi-window controller design easier than conventional fuzzy-logic systems.

The concept of a linear compensator does not usually appear explicitly in fuzzy-logic controllers. In fact, one of the selling points of such controllers is that the user supposedly does not need to know anything about control



(a) (b)

Fig. 9: Possible I-path implementations for anti-windup

engineering. As an example, one of the rules in a fuzzy controller might be "if the error is large apply maximum torque to correct it." Except for the threshold, the linear information is disregarded. Many fuzzy controllers do compute derivatives of the output variable by back-differencing, and this is recognized as first-order linear compensation. The fuzzy controller uses different laws for different combinations of the output and its rate estimate. Moving further in this direction is the "fuzzified" PID controller[5,6] which uses linear compensation of the error signal to generate three parallel channels, and then applies fuzzy-logic in the low-frequency channel which benefits most from the introduction of nonlinearities. Obviously this sort of approach does require some control engineering knowledge, and also begins to resemble PID controllers with anti-windup devices that have been in use in industry for many years. A common selling point of fuzzy-logic controllers is that they can outperform linear controllers.[5] The comparison is inappropriate since nonlinearities are routinely used with PID's to improve transient response and robustness. A more honest comparison was made in Ref. 6, which considered a fuzzy controller for a batch distillation column and concluded that the performance was about the same as a conventional PID with anti-windup.

There are several important differences between the design of multi-window controllers and the design of fuzzy-logic controllers:

1 For multi-window systems, there are large regimes of operation where the system can be considered to be linear. The classical

Bode integrals can be employed to make trade-offs related to the feedback bandwidth limitation by noise and plant uncertainty and to guarantee the best available performance. In fuzzy logic controllers these trade-offs are not transparent and are difficult to make.

2. For multi-window controllers the number of windows required is not large. As seen in the design examples, 2 or 3 windows result in designs that appear to be close to optimal. For fuzzy controllers, the number of windows required for good performance can be large. If there are several outputs (and their derivatives) and several amplitude windows per measurement, the number of control laws grows in a combinatorial way. This situation is even worse for MIMO systems, since for such systems it is all the more critical that the trade-offs can be made quickly and efficiently.
3. For multi-window controllers, relatively few parameters need to be chosen to design the nonlinear elements. This is important since the choice of nonlinear parameters involves a significant amount of simulation or experiment, and gets out of hand quickly if there are more than a few of them.
4. Implementation in hardware or software is simpler when only a few saturation-type nonlinearities are required.
5. Multi-window controllers do not require any sort of fuzzy-set operations to provide smooth transitions. Instead, the nonlinearities are chosen (like saturation) such that sharp discontinuities are avoided. Minimum-phase conditions can be easily preserved since only adjacent channels need to be considered. (At each signal level only two channels are significant.) Attempts are often made to design fuzzy controllers without paying any attention to this issue, and the results can be confusing and disillusioning.

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