ABSTRACT

Application of Dynamical Systems Theory to Trajectory Design for Libration Point Missions

B.T. Barden and K.C. Howell

School of Aeronautics and Astronautics
GRIS 1282
Purdue University
West Lafayette, Indiana 47907

M.W. Lo and J.K. Miller

Jet Propulsion Laboratory
MS 301/142 California Institute of Technology
4800 Oak Grove Drive
Pasadena, California 91109

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Introduction

In the area of astrodynamics, the complex missions envisioned for the upcoming decades will demand innovative spacecraft trajectory concepts and efficient design tools for analysis and implementation]. It is also increasingly apparent that accomplishment of many short- and long-term science and exploration goals will require a broader view that expands the range of options available. Most recently, for example, the astrophysics community has had a high level of interest in missions to the vicinity of the libration points in the Sun-Earth system. Spacecraft in orbits near libration points offer valuable opportunities for investigations concerning solar and heliospheric effects on planetary environments. Current design capabilities for such missions have significantly improved in the last five years but are still limited. Computational approaches to determine a nominal trajectory are essentially manual numerical searches in a regime where conic approximations are not adequate; standard targeting and optimization strategies based on linear variational methods are very difficult to apply and frequently break down because of the nonlinearities and high sensitivities in the problem. At the same time, recent evidence supports the theory that the rich dynamics in this region of space may lead to previously undiscovered types of trajectory solutions. Conventional tools simply do not incorporate any firm theoretical understanding of the multi-body problem and do not offer the flexibility to take further advantage of the dynamical relationships in producing alternative trajectory designs.

This work has focused on obtaining a clearer understanding of the fundamental, under-lying dynamics associated with the trajectory design problem, with particular emphasis in multi-body regimes, where qualitative information is needed concerning sets of solutions and their evolution. Nonlinear dynamical systems theory is a key component in progress toward that objective. This type of mathematical analysis involves issues such as periodic orbits, stability, instability, quasi periodic motion, asymptotic properties, structure of solution sets, escape, symmetries, and evolution of a dynamical system. Much insight has resulted from an investigation of one family of periodic solutions and its stability characteristics; invariant manifolds associated with these periodic orbits have already served as a guide to generate natural pathways near the libration points. But the primary objective here is to *use* this information for trajectory design. Rather than rely solely on experience and previously determined solutions or propagate, somewhat blindly, until a useful trajectory appears, the goal is to knowledgeably select and compute trajectory arcs in the multi-body problem. The arcs can then be patched together for

optimal results in complex trajectory design. Thus, this investigation also supports development of the necessary capabilities for actual mission planning.

Background

In its most basic form, trajectory design in orbital mechanics is an attempt to find a solution to the n-body problem (subject to various constraints, conditions, and possibly other perturbative forces). However, even in the basic n-body problem, no general solution is known to exist except in the case $n = 2$, i.e., the two-body problem. Never Mess, much success has been achieved in mission design and analysis with traditional approaches including conic analysis, perturbation theory, high order analytical approximations, and extensive simulations. These standard analytical and numerical methods do provide very accurate information, even for multi-body problems (in the Earth's neighborhood or interplanetary), because the present techniques can very accurately represent the evolution of a particular trajectory. The most challenging interplanetary missions involving multiple gravity assists have been designed with such tools. Note, however, that any result based on two-body solutions must exist in the larger context of the n-body problem.

It is now apparent to some that only a limited region of the orbit design space has actually been exploited. In this view, it is necessary to generalize the model and explore larger regions of the solution space in the general n body problem. A logical step to increase the complexity of the model in the basic problem is to examine the cases $n = 3$ and $n = 4$, i.e., the three- and four-body problems. In such cases, two or three or more gravitational fields can be of equal significance. After 200 years, the three-body problem remains unsolved, but, in the last 20 years, substantial progress has been made in recognizing that the unique dynamic characteristics in the problem can be used for mission design. The trigger that refocused interest in the problem was the discovery of new types of particular solutions in the three-body problem: periodic halo orbits and quasi-periodic Lissajous trajectories. These trajectories (three dimensional in configuration space) are not found in the region of the solution space occupied by solutions to the two-body problem. It is significant that Lissajous trajectories exist in the vicinity of the libration points. A number omissions have incorporated Lissajous and/or halo orbits as part of the trajectory design: ISEE-3 (1978 launch), WMAP (currently enroute to arrive 1997), SOHO (1995 launch), ACE (1997 launch), and others currently in development. It is not a coincidence that the trajectories employed in these missions are all similar. Trajectory design is traditionally initiated with a baseline mission concept rooted in the two-body problem and conics. These libration point missions require a baseline concept derived from solutions to the three-body problem. Since no such general solution is available, the mission concepts are founded upon past numerical search results. Thus, the options have been very limited. The few engineers with trajectory design experience in this region are also aware of the high sensitivities in the problem and may expend much time and effort to even slightly modify a previous type of solution through a numerical search process. But this type of mission holds much potential for science and the possibilities expand tremendously should new classes of solutions be discovered in this region.

Dynamical Systems and Invariant Manifolds

Now, however, the best glimpse yet achieved into this region of the solution space is available through the application of nonlinear dynamical systems theory. Some fundamental concepts have already provided critical clues that are used here in generating trajectories. The geometrical theory of dynamical systems (from Poincaré) is based cm the phase portrait of a dynamical system as discussed in various mathematical sources. The geometric model for the set of all possible states of' a system is the phase space or state space, which is assumed to be an open set in 1{". The state space, filled with trajectories, is the phase portrait of the dynamical system Given the. differential equations that govern the system, the goal is to "solve" the equations and produce the phase portrait. Flat, Euclidean spaces will not suffice for all geometric models; in some cases, curved spaces (differential manifolds) are necessary The manifold then becomes the geometrical model for the space of dependent variables. In the phase space, there may also be found special solutions: fixed points (or equilibrium points), periodic orbits, qua$i-periodic motions, homoclinic and heteroclinic motions.

An invariant manifold is a surface (m-dimensional) defined by the following property: orbits starting on the surface remain on the surface throughout the course of their dynamical evolution So, an invariant manifold is a collection of orbits that form a surface, Additionally, the set of orbits that approach or depart an invariant manifold asymptotically are also invariant manifolds (under certain conditions) which arc called stable and unstable manifolds, respectively In reaching toward a complete understanding of the global dynamics, knowledge of the invariant manifolds of a dynamical system as well as the interactions of tile.il respective stable and unstable manifolds is absolutely critical. In the. three-body problem, the 6-dimensional phase space can be envisioned as composed of subspaces of various dimensions. Thus far, the link to these subspaces or manifolds has been the halo orbits (periodic orbits) and Lissajous trajectories (quasi-periodic motions) in the vicinity of the libration points These concepts can be useful in exploring trajectory design issues and offering new trajectory options.

Re.suits/Examples

Earth-to-Halo Transfers

Conceptually, the transfer design process consists of identifying the subspace (or surface) that contains the endpoints of the boundary value problem - the "endpoints" are, in fact, orbits – and then moving on the surface from the. point (orbit) of origin to the destination orbit. Knowledge of the geometrical aspects of the problem near the libration point orbit can lend information that i. valuable in determining the surface, i.e., the transfer path. Specifically, the (near) periodicity of halo orbits has been exploited to calculate invariant manifolds associated with particular points along the nominal halo path. Rather than a targeting problem to reach a specified insertion point on the halo orbit, the transfer design problem becomes one of insertion onto the manifold, directly from the

Earth parking orbit, if possible. Once on the nominal transfer trajectory (stable manifold), the spacecraft approaches the halo orbit asymptotically, with no insertion maneuver; flight time is very reasonable. The resulting algorithm has been very successful at quickly producing insight into the problem, as well as generating Earth-to-halo transfer trajectories. The distinct advantage of this approach over the traditional shooting method is that the designer ultimately has more control over each aspect of the trajectory; and, the transfer path emerges *without* a random search process. The application of this approach can be seen in Fig. 1, in a rotating coordinate frame. The plot represents the x - y projection of a transfer to a halo orbit near L_1, in the Sun-Earth system that is roughly the same size as the halo used for ISEE-3 (The x axis is parallel to the Sun-Earth line.) By understanding the behavior of the stable manifolds along the halo orbit, a point can be identified along the path that is associated with a manifold that passes close to the Earth; the trajectory approaches the halo orbit along a natural pathway that results in a lower cost; time and effort to produce the result is minimal and the process can be automated. The Moon's gravity could also potentially be employed to "bend" the manifold; with sufficient bending, the manifold may pass the Earth at the correct altitude for a wide variety of Lissajous orbits, Bending the three body manifold with a lunar gravity assist, or redefining the manifolds in the four body problem, offers significant advantages in designing transfers to Lissajous trajectories over the complete range of amplitudes.

More Complex Trajectories

These concepts can be useful in exploring other trajectory design issues and offering new trajectory options. Complex trajectories can be created by shifting from one surface to another any number of times. This theoretical view can be demonstrated in the successful design of trajectories between halo/Lissajous orbits around *different* libration points. Experience with the evolution of stable and unstable manifolds has suggested the existence of heteroclinic connections between libration point orbits when they are viewed as asymptotic limit sets. New design options arise if it is possible to take advantage of this dynamical relationship. In the Sun-Earth system, it is now known that there exists a family of heteroclinic-type connections between Lissajous and/or halo orbits associated with L_1 and L_2. Two examples demonstrate the usefulness of this family.

Earth Return. Consider the following mission concept: depart the Earth on a direct transfer to an L_1 Lissajous, after some number of revolutions in the halo/Lissajous orbit, return to a specified Earth site for a dayside landing. A direct return from an L_1 orbit is impossible; a direct return from an L_2 Lissajous, however, will result in a dayside re-entry. Thus, the first trajectory segment includes the transfer (similar to Fig. 1) from Earth to an L_1 Lissajous. The target L_1 Lissajous orbit is selected with an out-of-plane amplitude equal to approximately 300,000 km. The transfer is direct, with no lunar swingby, and is computed using known information concerning the stable manifold associated with this Lissajous orbit; thus, the orbit insertion cost is less than 4 m/s. After some number of revolutions in the Lissajous orbit (here it is 4 revs), a dayside landing requires that the spacecraft must approach the Earth from the far side. To achieve such a

result, two types of trajectory arcs are incorporated. Numerical experiments in the three-body problem suggest a heteroclinic-type connection between L_1 Lissajous trajectories and L_2 orbits. The current L_1 Lissajous orbit is slightly smaller in size than is necessary for the connecting path (as currently known), but it is possible to shift to a heteroclinic-type path and essentally increase the energy to reach the L_2 Lissajous. Thus, the second trajectory segment departs the L_1 Lissajous orbit and, at the first x-axis crossing, a maneuver is implemented and the path moves to the L_2 Lissajous trajectory Subsequently, after less than one quarter revolution in the L_2 orbit, the spacecraft departs along a path that is generally consistent with an unstable manifold (associate.(i with the L_2 orbit) that passes close to the Earth. This third trajectory segment requires only a small final maneuver to hit the appropriate reentry conditions. This final result is computed in a full model and represents a strong baseline solution. (See Fig. 7..) Note that the L_1 Lissajous trajectory was actually designed first unlike conventional approaches -- -since departure (for L_2) from certain types of revolutions was desireable. The transfix from Earth to the L_1 Lissajous is easily able to accommodate this requirement.

"Ballistic Lunar Capture." The second example concerns another member of this same family of heteroclinic-type connections between the Sun-Earth L_1 and L_2 libration point orbits. For the moment, consider only the Sun-Earth three- body system. Experience with the stable and unstable manifolds, and the interactions that produce the heteroclinic paths, reveals that the some members of this family "skim" the lunar orbit as they move between the libration points (in either direction) If a particular connection is identified from L_2 toward L_1, the timing will result in a path that intersects the Moon. The velocities along this path are such that when the spatcecraft passes close to the Moon, the orbital parameters with respect to the Moon result in a lunar capture, i.e., the eccentricity relative to the Moon can be computed to be less than 1.0 (actually, in (his example, e =.81), The resulting trajectory is shown in Fig. 7. Note that this result is produced in the full model that includes solar radiation pressure 'J-his trajectory is familiar to many trajectory designers as a "ballistic lunar capture.," and similar to the trajectory that was employed in the Japanese Hiten mission. It was originally discovered numerically as an isolated integrated solution. It is now known to be a member of an extensive family of natural pathways in the Sun-Earth three-body problem. It is straightforward to locate and compute using invariant manifolds Since the heteroclinic paths also exist from L_1 toward L_2, it is expected that another such solution can be computed as well

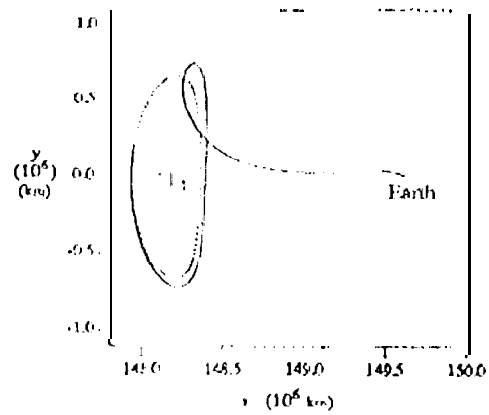

Figure 1: Transfer to L_1 Halo with A_z = 120,000 km; Halo Insertion Cost = 20.3 m/s.

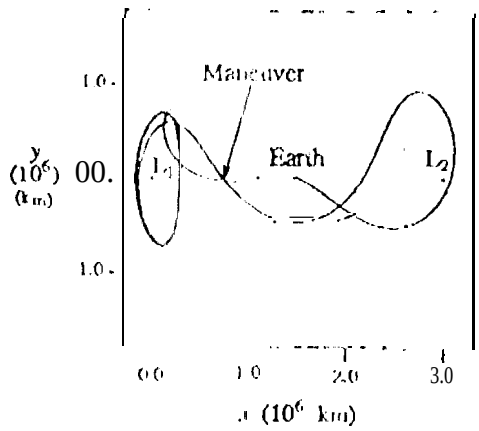

Figure 2: Earth Return Trajectory

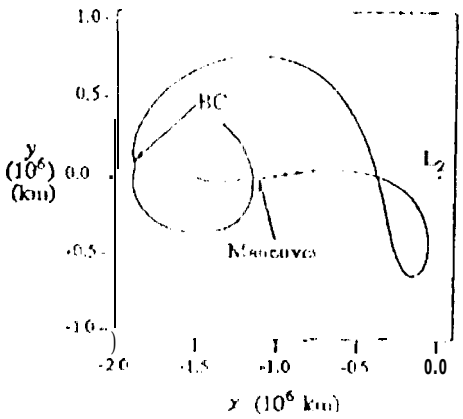

Figure 3: Ballistic Lunar Capture Trajectory