ABSTRACT

OPTIMAL INTERPLANETARY TRAJECTORIES VIA A PARETO GENETIC ALGORITHM

by

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Introduction

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Optimization is an integral aspect of mission planning. In recent years, the drive for faster, more propellant-efficient missions has ~~lead~~ to an increased interest in low-thrust propulsion. Although these propulsion systems result in higher performance, the difficulty in determining the optimal trajectories associated with them is increased. By nature, low-thrust systems are required to operate for a much longer interval during orbit transfer than their impulsive, chemical propulsion counterparts. This results in a notable increase in problem complexity. Sinew the. thrust level and direction must be determined for a significantly longer duration, the continuous nature of the control parameters makes the design of optimization software a particularity challenging task. variations-

Current working methods for determining low-thrust trajectories are calculus-based and require large expenditures of time to produce a single feasible solution. such a problem formulation results in a massively multimodal search space, highly sensitive to user input. Depending upon the complexity of the trajectory, the optimization process may take anywhere from a single day to several weeks. An on-going research venture being conducted by the Jet Propulsion Laboratory (JPL) and the University of Illinois at Urbana-Champaign's Computational Astrodynamics Research Facility (UIUC CARL) is investigating alternative methodologies in an attempt to alleviate these problems. Automation of the optimization process using stochastic search techniques such as simulated annealing and genetic algorithms to drive the existing optimization software is a major topic of research

This study details recent work on implementing a Pareto genetic algorithm in order to perform multiobjective optimization for low-thrust orbit transfers. Development of such a multiobjective optimization algorithm allows for the generation of "families" of optimal trajectories spanning the highly multimodal search space. This is accomplished by ranking and sorting the. population according to individuals' Pareto optimality, and niching over established Pareto fronts. Additionally, it provides increased robustness through its inherent separation of objectives and eliminates the objective conflict [Hans,1988] which arises from the classical technique of scalarizing multiple objectives. Trajectory generation is accomplished through a hybridization of the genetic algorithm with existing JPL trajectory optimization software to produce familiar and usable results.

Pareto Optimization

In order to describe Pareto optimization, it is first necessary to define multiobjective optimization. Multiobjective optimization - as opposed to single-objective optimization - is the optimization of a system with more than one objective. As in single-objective optimization, the objective(s) may have any number of equality or inequality constraints imposed upon them. This can be represented mathematically as (Rae. 1991],

$$
\begin{array}{llll}
\text{Minimize/Maximize} & f_i(\mathbf{x}) & i = 1,2,\ldots,N & \\
 & & & (1) \\
\text{Subject to} & g_j(\mathbf{x}) \leq 0 & j = 1,2,\ldots,F & \\
 & h_k(\mathbf{x}) = 0 & k = 1,2,\ldots,K &
\end{array}
$$

Rather than searching for the solution which yields the globally maximal (or minimal) value for a single objective function, the "best" solution is found by simultaneously optimizing several objectives at once. Optimization problems such as these are particularly relevant in the area of mission design. Trajectory optimization is in fact an inherently multiobjective optimization problem. Although the primary goal of the mission designer in optimizing the spacecraft's trajectory is to achieve the final state defined by the mission requirements, other objectives contribute to what constitutes a "good" trajectory. Flight time and final mass delivered to destination are also issues to be formulated as multiple objectives or constraints.

These types of optimization *problems* have traditionally been dealt with by averaging each objective with a weighting factor. and then combining the objectives into a single scalar objective. Such reduction techniques eliminate the need for a more complex, multiobjective algorithm, but introduce new parameters in the form of the weighting factors. The user must become familiar with the exact relationship between objectives in order to determine the proper weighting values that will yield the desired result. Determination of the weighting factors can, in practice, become an optimization process in and of itself.

Problematic issues such as those mentioned above can be resolved by instituting a search algorithm which performs a Pareto optimization. Pareto optimization is the principle of optimizing multiple competing objectives. A succinct definition of Pareto optimality was provided by Edwin Dean.

> A Pareto optimal solution is not unique, but is a member of a set of such points which are considered equally good in terms of the vector objective. This space may be viewed as a space of compromise solutions in which *each* objective could be improved, but if it was, it would be improved at the expense of at least one other objective.. (Dean, 1995]

Another way of stating this would be to say that a solution is Pareto optimal, or *nondominated*, for a given set of objectives if there is no other existing solution which is superior to that solution in *all* objectives. If a solution exists which is superior in *some* objectives, then that solution would constitute a point on a front of Pareto optimal solutions. Take for example the problem of minimizing both arguments for a set of points, { (0,S).(1,3),(2,4) }. Point 1 is dominated try both points 2 and 3 in its second coordinate; however, it dominates points 2 and 3 with respect to the first coordinate, therefore it is non-dominated. Point 3 is dominated with respect to both coordinates by point 2, therefore it is a dominated individual and not Pareto optimal, For a more thorough discussion of Pareto optimalit y, see Generic *A lgorithms* [Goldberg, 1989].

The *benefits* of incorporating a Pareto search algorithm in trajectory optimization process are twofold: i.) elimination of the problems encountered in classical multiobjective optimization methodologies such as object ive conflict, and ii.) development of a Pareto optimal front of solutions, providing an array of compromise solutions. When applied to the population-based genetic algorithm, these Pareto concepts should enable automatic generation of Pareto optimal solutions. In the context of optimal spacecraft trajectory generation discussed in this study, a Pareto genetic algorithm should provide the mission designer the capability of generating "families"' of optimal orbit transfers, illustrating the trades between defined objectives.

NSGA/SEPTOP Hybridization

The algorithm used in this study is a hybridization of a Pareto genetic algorithm, and classical calculus-of-variations-based optimization software. The Pareto genetic algorithm is one based on the concept of non-dominated sorting originally proposed by Goldberg [Goldberg, 1989], and later developed by Srinivas and Deb [Srinivas and Deb, 1995] as the Non-dominated Sorting Genetic Al gorithm (NSGA). A population of individuals is sorted through and subdivided into Pareto fronts based upon individuals' Pareto optimality, each front being assigned a certain rank. Fitnesses for each individual solution arc then assigned based on rank, and adjusted according to their proximit y (resemblance) to other solutions using a technique known as *niching*. Niching helps to maintain population diversity and serves to counter premature convergence of the population. For further details, see [Srinivas and Deb, 1995].

The Pareto genetic algorithm essentially maintains and evolves a set of possible solutions, making adjustments to individuals depending upon their corresponding fitness.
-objective functions-
Fitness values are obtained by integrating the genetic algorithm with a calculus-based low-thrust trajectory optimization program known as SEPTOP, developed at the Jet Propulsion Laboratory in Pasadena, California. Hybridization is accomplished through a Baldwinian evolution strategy. The objective vector for each individual solution is determined by running SEPTOP for a given set of input parameters determined by the genetic *algorithm*. The procedure is Baldwinian in that the fitness returned to NSGA by SEPTOP corresponds to *the* input values of the parameters, even though those parameters may have been adjusted by SEPTOP's procedures. Baldwinian hybridization strategies are another way of maintaining diversity in a population of solutions and preventing premature convergence.

Results

Results have been obtained which duplicate the performance remonstrated by Srinivas and Deb's algorithm [Srinivas and Deb 1995], as well as those. which provide proof-of-concept for the NSGA/SEPTOP hybrid formulation. Diagnostics were run on the NSGA algorithm alone, using the test functions provided by Srinivas and Deb [1995] *as* well as several devised by the authors to demonstrate efficacy on problems involving more than two objectives and mixed min/max objectives. Test cases were then run to demonstrate the effectiveness of the hybridized methodology for Earth-Mars flyby and rendezvous trajectories, and Earth-Mercury rendezvous trajectories with multiple heliocentric revolutions. All cases were successful in generating Pareto optimal fronts, and thus providing the desired result of generating "families" of optima] trajectories containing arrays of compromise solutions [Figs. 1-3].

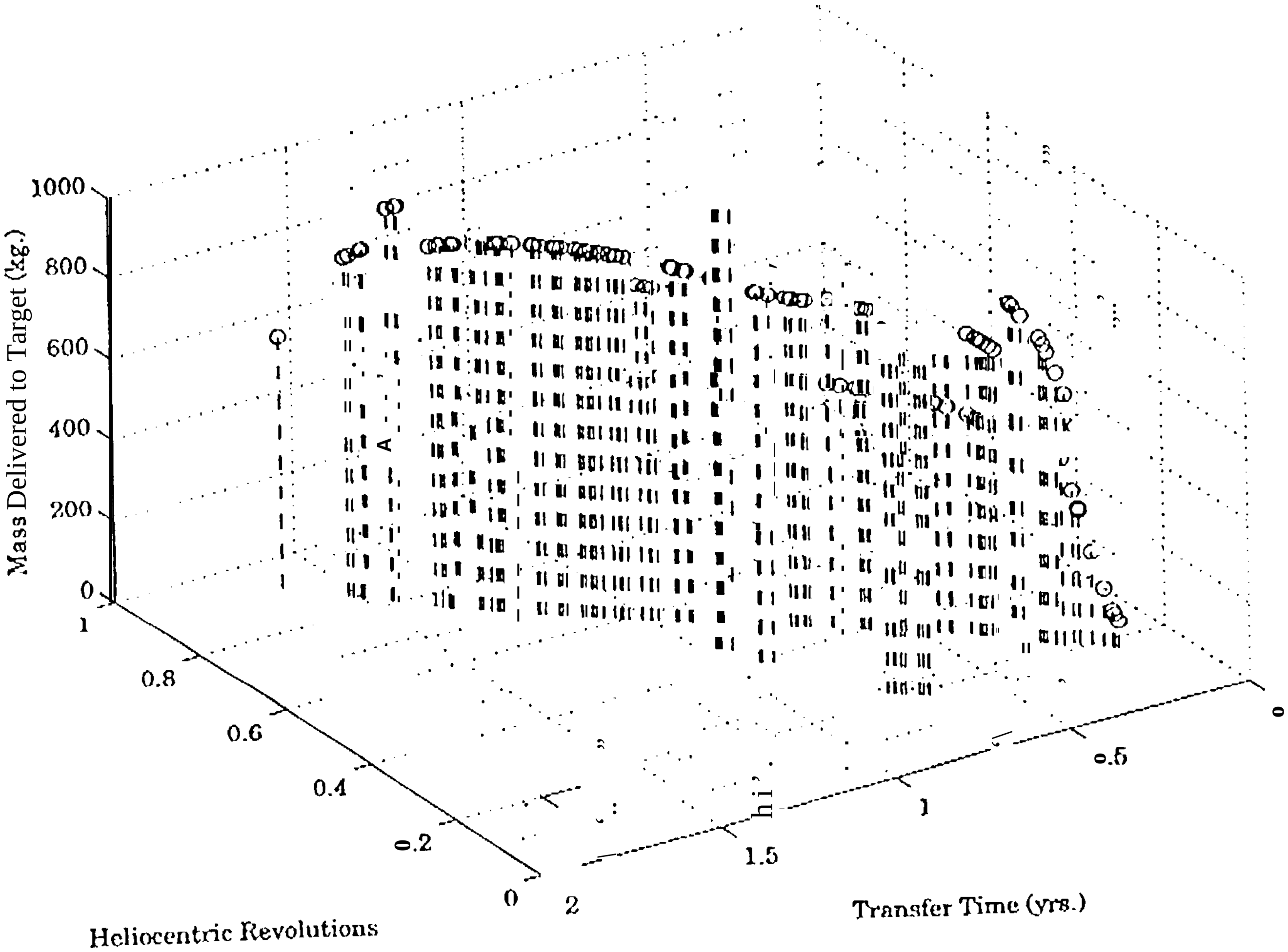

Figure 1. Pareto front for Earth-Mars flyby

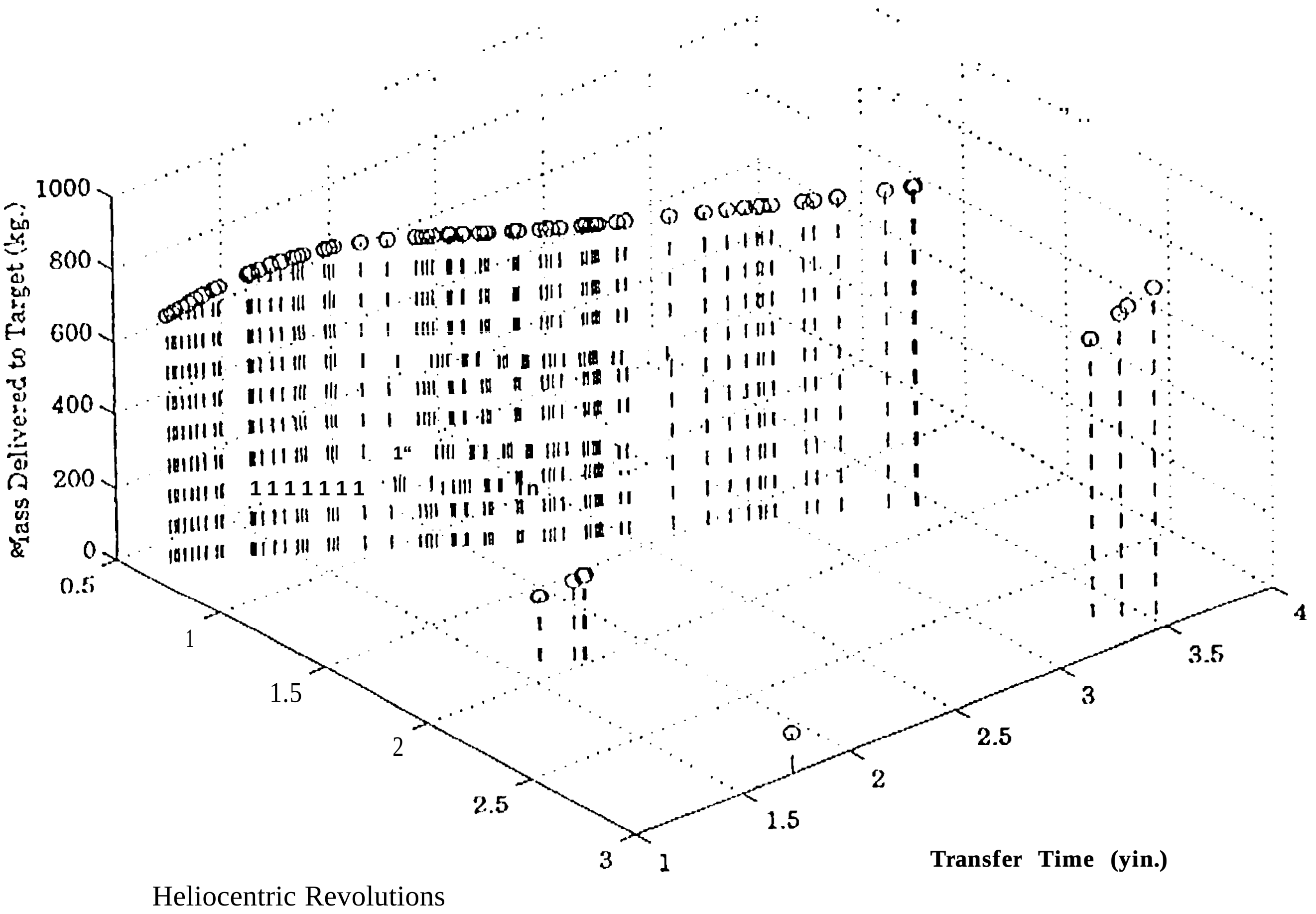

Figure 2. Pareto front for Earth-Mars Rendezvous

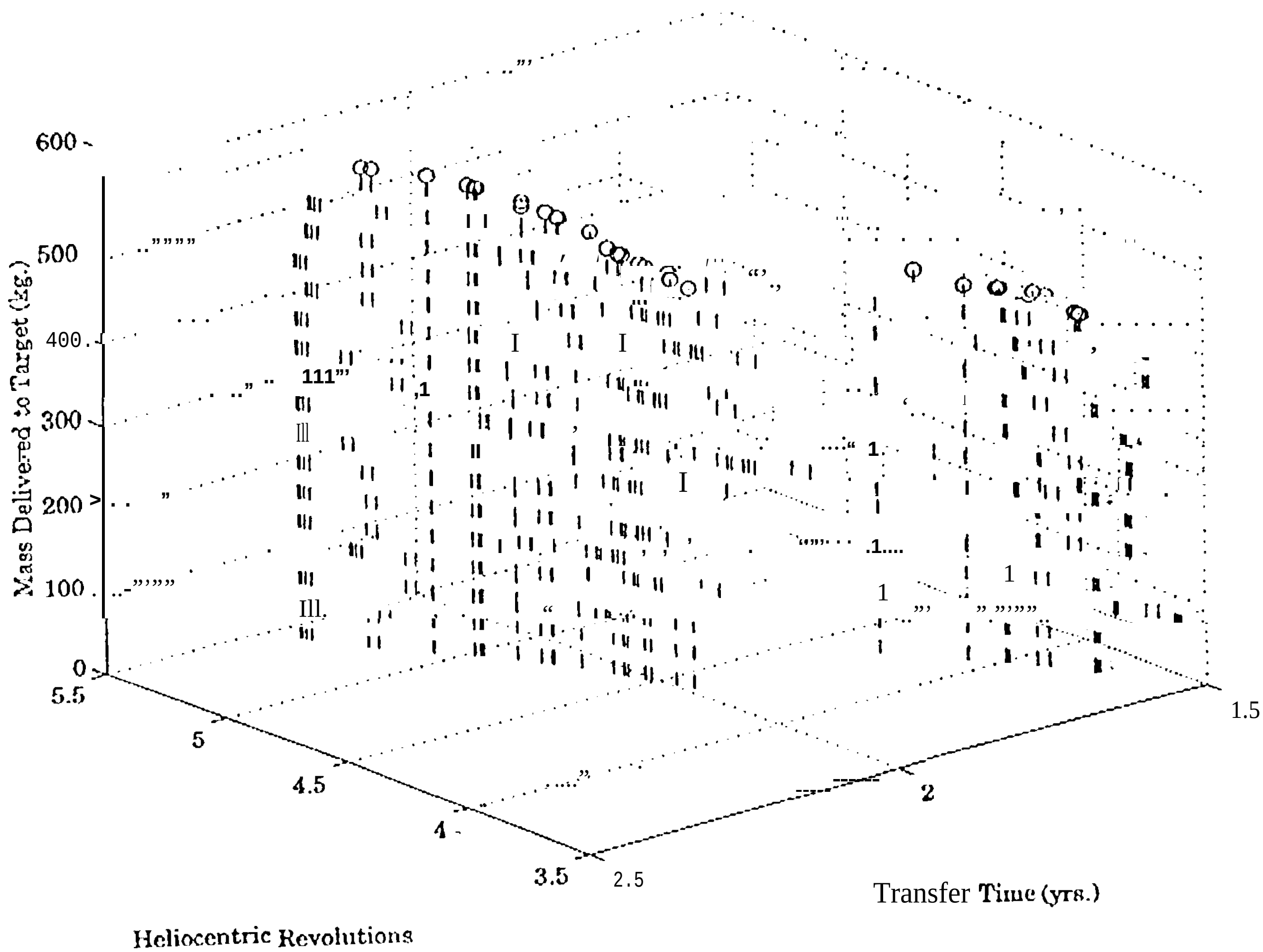

Figure 3. Pareto front for Earth-Mercury Rendezvous

References

Dean, E. B. (1995). Multiobjective optimization from the perspective of competitive advantage. http://mijuno.larc.nasa.gov/dfc/mdo/moo.html

Goldberg, D. E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley

Hans, A. E. (1988). Multicriteria optimization for highly accurate systems. In W. Stadler (Ed.), *Multicriteria Optimization in Engineering and Sciences*, Mathematical concepts and methods in science and engineering, 19.309-352. New York: Plenum Press

Rae, S. S. (1991). *Optimization theory and application*. New Dehli: Wiley Eastern Limited

Srinivas, N. and Deb, K. (1995). Multiobjective optimization using nondominated sorting in genetic algorithms. Involutionary Computation, 2.(3).