# Phase Noise of Optical Interference in Photonic RF Systems

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Close-in optical interferometric noise spectrum and its conversion to RF close-in phase noise are theoretically and experimentally studied. The optical $1/f$ frequency noise is the dominant source of the close-in interferometric phase noise and is converted into the $1/f^\nu$ noise, where $\nu$ is determined by the time delay scaled to the coherence time of the $1/f$ noise. The study also indicates that reflective type devices and optically amplified links enhance the interferometric phase noise degradation.

## I Introduction

RF photonic systems generally require very low phase noise. In particular, it is usually desired that the phase noise of these systems be limited only by the RF components, and that any noise degradation due to optical components be avoided. Close-in (below 1MHz) phase noise is a critical parameter in some of the important applications such as the fiber-fed phased array antenna systems, frequency and timing reference distribution, and ultra stable optoelectronic oscillators [1,2,3]. In these and similar systems, it is desired that the phase noise due to optical components be smaller than −110 dBc/Hz at 1Hz for an X-band carrier, which is typical of a low noise amplifier (LNA). Commercial laser diodes already achieve Relative Intensity Noise (RIN) values less than −150 dBc/Hz, and linewidth from 1MHz to 10 MHz. The phase noise of an X-band signal resulting from the laser RIN, including phase-to-intensity conversion, is negligibly low (<−150 dBc/Hz) even with moderate reflectivity [4]. In this paper, we investigate another kind of noise, due to interferometric optical phase noise converted to close-in RF multiplicative phase noise. We find that this multiplicative noise places more stringent requirements on the optical components. Our study begins with an analysis in Section II which shows that optical $1/f$ noise is the dominant source of the close-in interferometric phase noise and is converted into the $1/f^{\nu}$ noise, where $\nu$ is determined by the time delay of the interference scaled to the coherence time of the $1/f$ noise. In Section III we present results of experiments performed to test our theoretical analysis and further study the effect of this interferometric noise. The analysis indicates that the reflective type devices and optically amplified links enhance the interferometric phase noise degradation.

## II Theoretical Calculation

The interferometric noise in an externally modulated X-band link is schematically shown in Figure 1a. The interferometric noise arises from (i) multiple pairs of reflection from connectors and splices, and (ii) the 'leakage' from circulator-based reflective devices. The detected signal, $I(t)$ can be expressed as [4]:

$$I(t) = 2E_0^2(1 - m\sin(\omega_m t)) + 4rE_0^2\cos(\Theta(t)) + 4rE_0^2 m\cos^2(\omega_m \tau_d / 2) \cdot \cos\Theta \cdot \sin(\omega_m t)$$
$$- 2rE_0^2 m \cdot \sin(\omega_m \tau_d) \cdot \cos\Theta \cdot \cos(\omega_m t)$$

Eq. 1

$$\Theta(t) = \Omega \cdot \tau_d + \Psi(t + \tau_d) - \Psi(t)$$

Eq. 2

where $\tau_d$ and $r$ are the time delay and amplitude ratio between the direct wave and delayed wave respectively. $E_0$, $\Omega$ and $\Psi(t)$ are the amplitde, angular frequency and the phase noise of the

optical field, and m and $\omega_m$ are the modulation index and frequency of the external modulator. The first term in Eq. 1 is the DC plus the RF signal with modulation index m. The second term is the additive noise independent of m. This term has been extensively studied for both digital and analog systems [4,5]. It is negligibly low (~-160 dBc/Hz) at X-band and is subsequently dropped in the analysis. The third and fourth terms are multiplicative noise proportional to m. We use the standard frequency and timing notation by writing an RF signal as [6]:

$$I(t) = A(1 + \varepsilon(t)) \cdot \sin(\omega_m t + \psi(t))$$
Eq. 2

where $\varepsilon(t)$ and $\psi(t)$ are the amplitude noise and phase noise respectively. Combing Eq. 1 and Eq. 2, we obtain:

$$\varepsilon(t) = 2r \cos^2(\omega_m \tau_d / 2) \cdot \cos(\Theta)$$
Eq. 3

$$\psi(t) = r \sin(\omega_m \tau_d) \cdot \cos(\Theta)$$
Eq. 4

From now on, we will focus on the phase noise term $\psi(t)$ because it is an extremely critical parameter in phased array antenna systems and ultra stable optoelectronic oscillators. However, all conclusions regarding the phase noise can be applied to the amplitude noise except for a trivial normalization factor. Eq. 4 also shows that the multiplicative noise of the RF signal is exactly the replica of the optical interferometric noise. The interferometric phase noise is represented by the single side band (SSB) power spectral density ($S(f)$) of $\psi(t)$, or mathematically the Fourier transform of autocorelation function ($R(\tau)$) of the $\psi(t)$ :

$$R(\tau) = <\psi(t) \cdot \psi(t + \tau)> = r^2 \sin^2(\omega_m \tau_d) \cdot R'(\tau)$$
Eq. 5

$$S(f) = \frac{1}{2} F(R(\psi)) = \frac{1}{2} r^2 \sin^2(\omega_m \tau_d) \cdot F(R'(\tau))$$
Eq. 6

where we define $R'(\tau) = <\cos(\Theta(t)) \cdot \cos(\Theta(t + \tau)>$. If we a assume a Gaussian distribution for the phase noise, following the references [7,8], we have:

$$R'(\tau) = 1/2 \cdot \exp[H(\tau)], \quad H(\tau) = -2 \int_0^\infty S_\psi (1 - \cos(2\pi f \tau_d))(1 - \cos(2\pi f \tau)) \cdot df$$
Eq. 7

where $S_\psi(f) = C_1 / f^2 + C_2 / f^3$ is the phase noise of the optical signal consisting of the white frequency noise and $1/f$ flicker noise [7,8]. $H(\tau)$ can be integrated explicitly:

$$H_{1/f}(\tau) = -2\pi^2 C_2 \cdot \tau^2 \cdot \left[\ln|1 + a| \cdot (1 + a)^2 + \ln|1 - a| \cdot (1 - a)^2 - 2a^2 \cdot \ln a\right]$$
Eq. 8

$$H_w(\tau) = -\pi^2 C_1 \tau(|1 + a| - |1 - a|)$$
Eq. 9

where $a = |\tau_d / \tau|$. $H_{1/f}$ and $H_w$ are the $1/f$ noise and white frequency noise contributions to $H(\tau)$ respectively. When $\tau_d$ is very small, the interferometric noise is equivalent to the discriminator frequency noise measurement. The Fourier transform of $R(\tau)$ results in the following [4,7]:

$$S(f) = 2r^2 \sin^2(\omega_m \tau_d) \frac{\sin^2(\pi f \tau_d)}{f^2}(C_1 + C_2/f)$$

Eq. 10

which means that the interferometric RF phase noise is exact replica of the optical frequency noise. Another simplification can be made by observing that at very low frequency, such as 1Hz, the major contribution is coming from very large $\tau$. Under such assumption, we have

$$R'(\tau) = \exp(-2\pi B \tau_d) \cdot \exp(-(\tau_d / \tau_{1/f})^2 \ln(\frac{\tau}{\tau_d})) = \exp(-2\pi B \tau_d)(\frac{\tau}{\tau_d})^{-\upsilon}$$

Eq. 11

where $B = \pi C_1$ and is the linewidth of the laser [7,8]. We also define $\tau_{1/f} = (4\pi^2 C_2)^{-1/2}$ and $\upsilon = (\tau_d / \tau_{1/f})^2$. The Fourier transform of the Eq. 11 gives:

$$S(f) = r^2 \sin^2(\omega_m \tau_d) \cdot \exp(-2\pi B \tau_d) \cdot \tau_d^{\upsilon} \cdot \sin(\frac{1}{2}\upsilon\pi)(2\pi f)^{\upsilon-1}$$

Eq. 12

A Gamma function of $\Gamma(1-\upsilon)$ is dropped to avoid artificial singularity at $\upsilon=1$. The interpretation of the Eq. 12 is that when the time delay $\tau_d$ is very small compared to $\tau_{1/f}$ defined by the $1/f$ noise ($\upsilon<<1$), the spectrum resembles $1/f$ noise at low frequency. However, when $\tau_d$ approaches to $\tau_{1/f}$, the spectrum exhibits a linear dependence of the frequency with slope smaller than 1 on logarithmic scale. Although there are previous works on the optical interference spectrum in the regime of small $\tau_d$ and the regime of large $\tau_d$ [7], to the best of the our knowledge, this is the first analytical result in the intermediate regime closed to $\tau_{1/f}$. Eq. 12 also shows that when $\tau_d >> \tau_{1/f}$, the power spectral density contributed by $R(\tau)$ at large $\tau$ becomes small and will be dominated by the power spectral density of $R(\tau)$ at small $\tau$. Thus when the time delay of the interference exceeds time scale $\tau_{1/f}$, $1/f$ noise 'signature' ($1/f$ spectral dependence) disappears and the interferometric RF phase noise becomes flat. For $\tau_d >> \tau_{1/f}$, we make a small $\tau$ approximation:

$$R'(\tau) = \exp(-2\pi B \tau) \cdot \exp\left[-(\frac{\tau}{\tau_{1/f}})^2 \ln(\frac{\tau_d}{\tau})\right] \cong \exp(-2\pi B \tau) \cdot \exp\left[-(\frac{\tau}{\tau_{1/f}})^2 \ln(\frac{\tau_d}{\tau_{1/f}})\right]$$

Eq. 13

where $B = \pi\Gamma_1$ is the linewidth of the laser [7,8]. From Eq. 13, we see that $\tau_{1/f}$ can be also interpreted as the coherent time of $1/f$ noise because it is the time scale for $\tau$ when $1/f$ noise becomes significant. Since the Fourier transform of $R(\tau)$ is the convolution of the Lorentian and the Gaussian spectrum. we expect the spectral density below B and $\tau_{1/f}^{-1}$ to be flat. and we approximate it as the inverse of the effective net linewidth $B_r$ [7,8]:

$$S(f) = \frac{r^2 \sin^2(\omega_m \tau_d)}{2B_t} = r^2 \sin^2(\omega_m \tau_d)\left( B^2 + \ln(\frac{\tau_d}{\tau_{1/f}})/(\pi^2 \tau_{1/f}^2) \right)^{-1/2} \qquad \text{Eq. 14}$$

## Experimental Results and Discussion

In order to examine the predictions of the above analysis. we devised an experiment to measure the interferometric phase noise in an externally modulated X-band link. The experimental set up is shown in Figure 1b. The interference is caused by splitting and recombining an optical signal. In our set up. we measured the RF phase noise spectra while varying the fiber delay. Figure 2 shows the RF close-in phase noise spectra. Experimental results agree very well with those predicted by the theory [Eq. 8,12,14].

Since evidently the multiplicative interferometric noise can be significantly ($-100$ dB) higher than the additive noise at 1Hz. caution should be taken to avoid this noise. Figure 3 is the calculated interferometric phase noise at 1Hz for various optical component parameters, such as linewidth. $1/f$ coherence time. delay and reflectivity. This is for comparing the RF phase noise using a 'semiconductor laser'. which has high phase noise. and a 'YAG laser'[1]. which has low phase noise. The figure shows that at the shorter delay (<10m). the low phase noise laser ('YAG') is preferred. This is because at short delays. the interferometric noise is proportional to the optical frequency noise. However. the high phase noise laser is preferred at long delays (>100m) because the phase noise is inversely proportional to the effective linewidth of the laser. Also. it can be shown that the maximum phase noise at 1Hz is about $-16$ dB below the level of $r^2$ when the $\tau_d \cong 0.3\, \tau_{1/f}$. In order for the interferometric phase noise to be lower than the level of the X-band LNA of $-110$ dBc/Hz. $r^2$ should be lower than $-95$ dB. Although this is achievable in case of a pair of reflections with the reflectivity lower than $-50$ dB at each end. it is impossible for the reflective type device. The isolation is always much poorer than $-90$ dB and typically

[1] YAG laser here is used only for generalization of low phase noise laser. In fact the YAG laser frequency noise does not very well follow 1/f frequency dependence.

around -50 dB. This implies that the reflective type devices can produce interferometric RF noise as high as -65 dBc/Hz, and so should be avoided in stable RF frequency distribution links. Also the RF photonic link incurs a loss of about 30 dB, suggesting that an optical amplifier with an optical gain (G) about 15 dB may be used to compensate the loss. However, the insertion of an optical amplifier essentially increases the interferometric noise by $G^2$[5]. The maximum noise increases to -80 dBc/Hz at 1Hz when $r^2 = -95$ dB, which is far above the level of LNA phase noise and 80 dB above the RIN level (-160 dBc/Hz). Therefore stringent requirement of the component reflectivity (<-60 dB) should be placed on the optically amplified low phase noise RF links.

In summary, we have examined the contributions of the interferometric noise to the close in phase noise in RF photonic systems, using a theoretical analysis and experimental measurements. We demonstrate that this type of noise places stringent requirements on optical components in photonic systems, where the contribution from the optical phase noise is desired to be lower than the phase noise of electrical components.

**Figure Captions:**

1 (a) Schematic diagrams of optical interference due to (i) a pair of optical reflections , and (ii) a reflective device using optical circulator. (b) Experimental setup for the measurement of the optical close-in phase noise to RF close-in phase noise conversion.

2. The RF interferometric noise with varying fiber delay and at $r^2 = -28$ dB; this is equivalent to a pair of fiber-to-glass interfaces. The smooth curves are the theoretical results using Eq. 8, 12 and 14. The parameters used are $C_1$ of $1.5 \times 10^6$ Hz$^2$/Hz and $C_2$ of $1.0 \times 10^{11}$ Hz$^2$. Also the optical frequency noise of the laser is shown on the right axis.

3. The interferometric RF phase noise at 1Hz as a function of the fiber delay. In the case of interference in a reflective type device [see Fig. 1a], the circulator isolation is assumed to be 50 dB. The parameters used for the simulation are $C_1$ of $6.3 \times 10^6$ Hz$^2$/Hz and $C_2$ of $1.2 \times 10^{12}$ Hz$^2$ for the 'semiconductor laser', and $C_1$ of $1.5 \times 10^3$ Hz$^2$/Hz and $C_2$ of $1.0 \times 10^8$ Hz$^2$ for the 'YAG laser'.

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**(a)**

**(i)**

In    Out    $E_0 e^{i\Omega t + \Phi(t)}$

$r E_0 e^{i\Omega(t+\tau_d) + \Phi(t+\tau_d)}$

**(ii)**

In

$r E_0 e^{i\Omega(t+\tau_d) + \Phi(t+\tau_d)}$    $E_0 e^{i\Omega t + \Phi(t)}$

Out

**(b)**
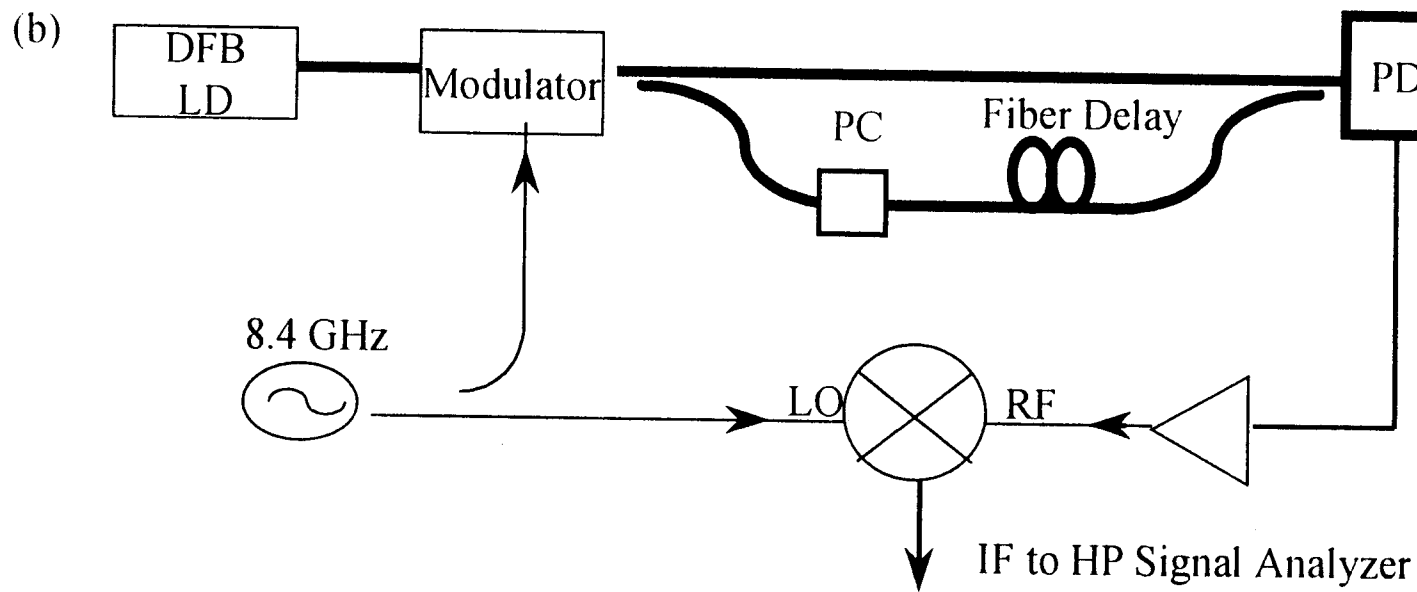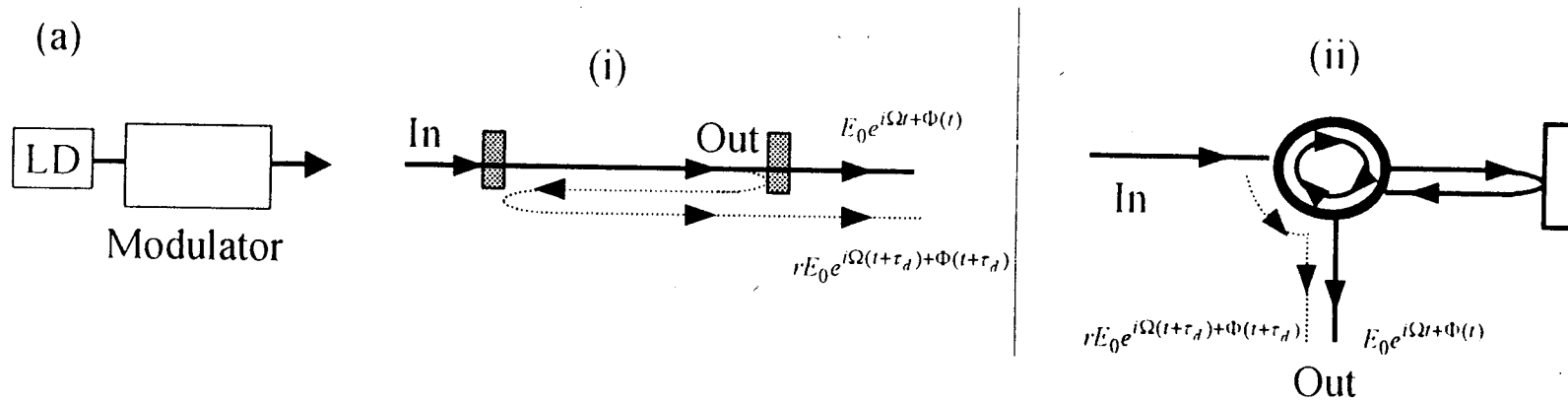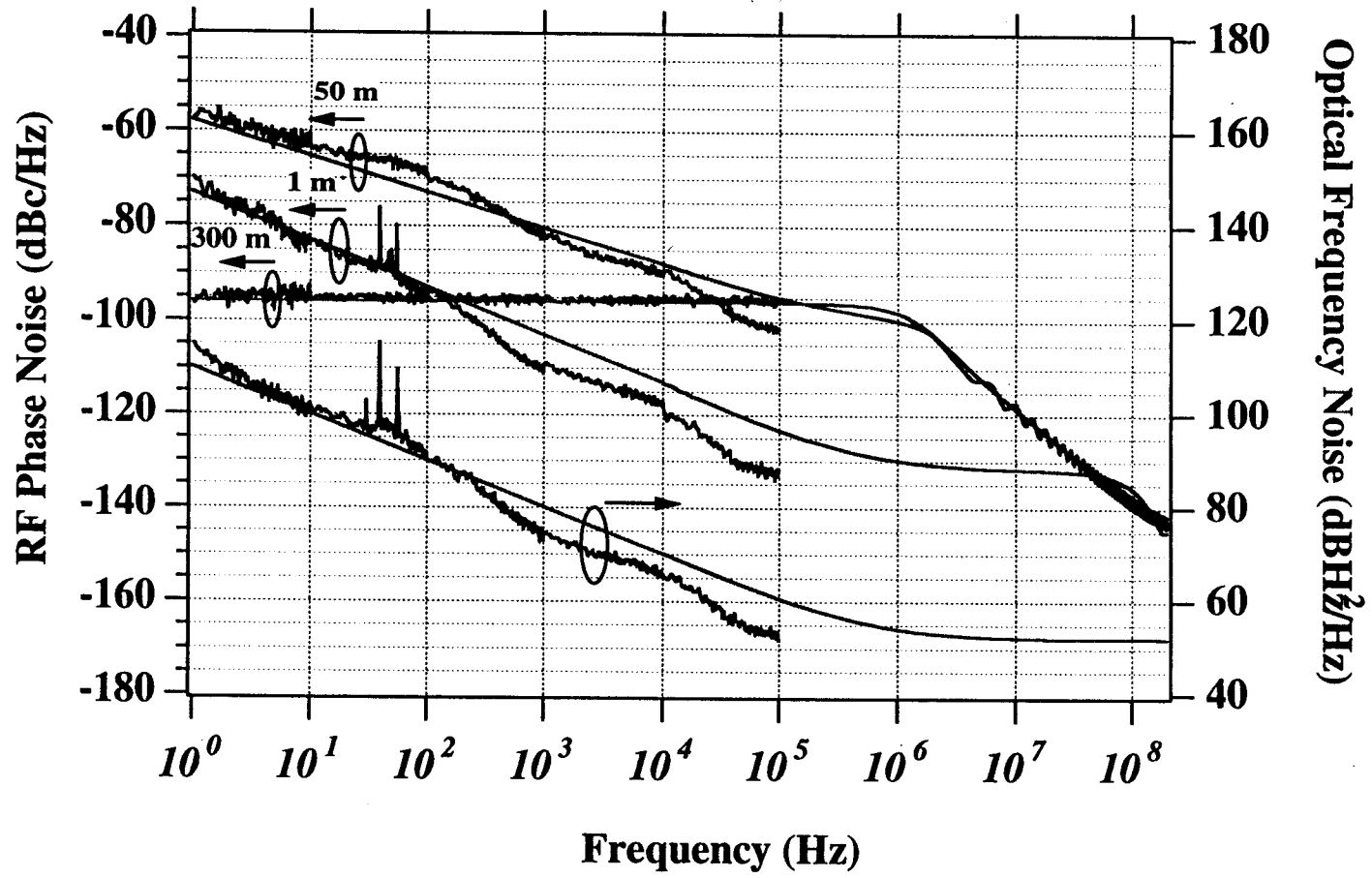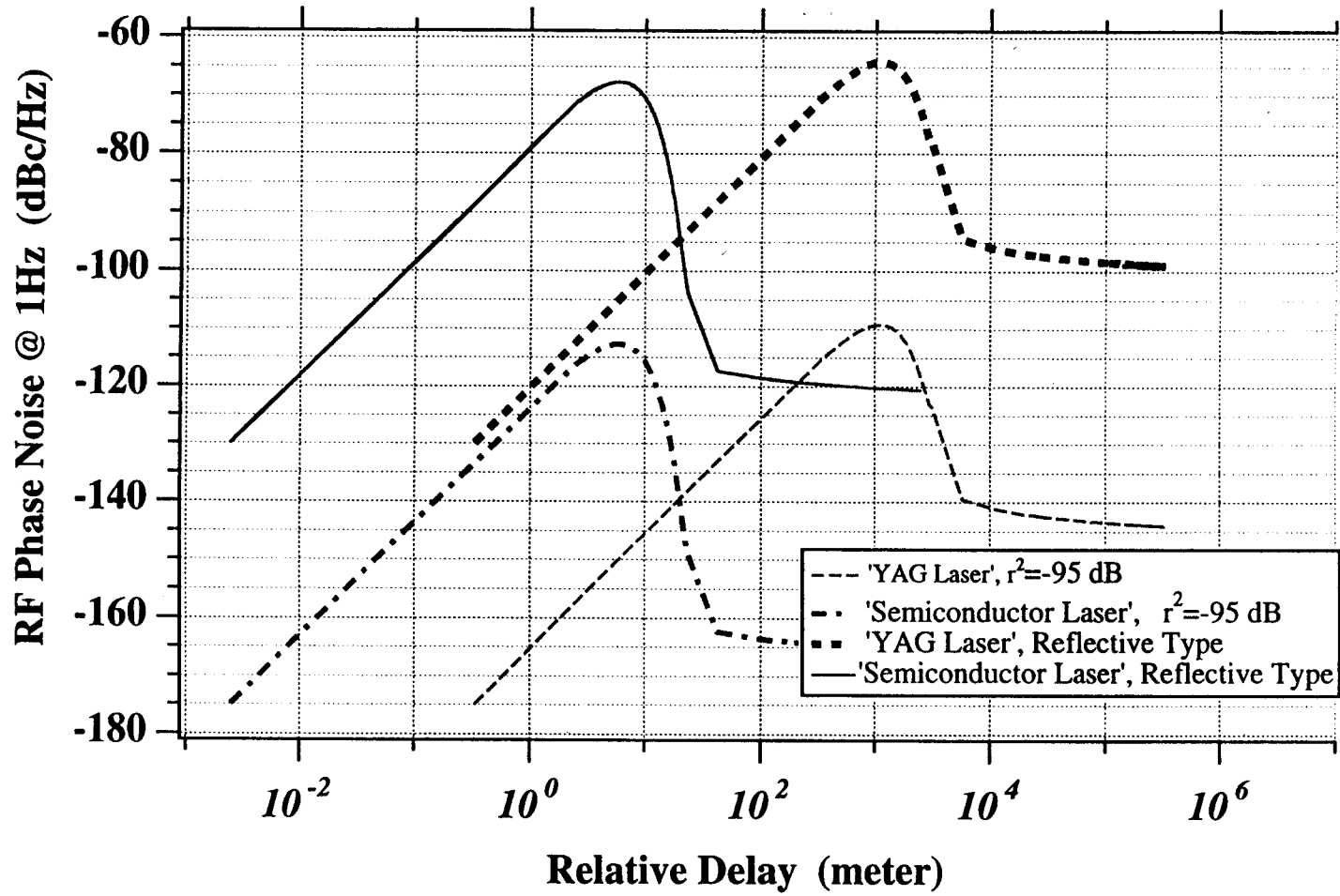
DFB LD   Modulator   PC   Fiber Delay   PD

8.4 GHz

LO   RF

IF to HP Signal Analyzer

Figure 1

Figure 2

Figure 3