A Centralized Optimal Controller for Formation Flying Spacecraft

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Abstract: Formation flying spacecraft is emerging as an enabling technology for the discovery of new type of science for the emerging NASA deep space and Earth science missions. A formation as an integrated unit will make a system to perform a task and to implement a mission objective. The control system of these formations has been the subject of research due to the complexity introduced by number, the distributed nature of the system, and the onboard requirements for reconfiguration and station keeping. This paper presents a centralized controller design for the autonomous control of formation of a set of spacecraft making an optical interferometer named Terrestrial Planet Finder (TPF). The control system will consider realistic assumptions about the space environment, spacecraft dynamics, and the attitude control system.

Keywords: Formation flying, autonomous control, intelligent control, multi-spacecraft, interferometers

1. Introduction

Formation flying spacecraft is emerging as an enabling technology for the discovery of new type of science for the emerging NASA deep space and Earth science missions. A formation as an integrated unit will make a system to perform a task and to implement a mission objective. The mission may be the co-observation of a planetary phenomenon, massive distributed sensing of Earth or another planet atmosphere, or detection of Earth like planets in our galaxy. Formation flying also called "distributed spacecraft," or "separated spacecraft," will allow variability in the baseline, flexibility in deployment, and increased system capability well beyond the scope of a single spacecraft.

The control system of formation flying spacecraft has been the subject of research in the recent years[1-7]. Due to the complexity introduced by the number, the distributed nature of the system, and onboard autonomy requirements for the reconfiguration and station keeping, FF systems pose significant challenges in the area of modeling, estimation and control. This paper presents a centralized controller design for the autonomous control of formation of a set of spacecraft making an optical interferometer named Terrestrial Planet Finder (TPF) as shown in Figure 1. The control system will consider realistic assumptions about the space environment, spacecraft dynamics, and the attitude control system.

Figure 1: Simulation of TPF using centralized control system design

2. TPF Formation Dynamics

Consider a likely representation of TPF formation flying spacecraft consisting of a combiner spacecraft S_0 and four collector spacecraft S_i, $i = 1,2,3,4$, in an Earth trailing heliocentric orbit, as depicted in Figure 2. Suppose that the spacecraft can be modeled as rigid bodies, that is, no flexible

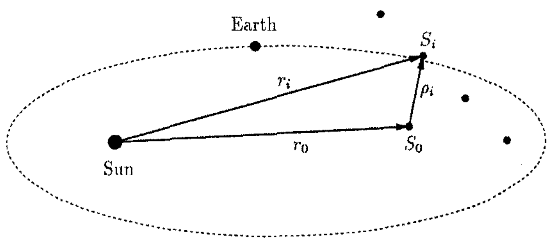

Figure 2: A representative configuration for TPF formation flying spacecraft.

structural modes are present. Assume further that the translational dynamics and the rotational dynamics are uncoupled. It then follows that the translational equation of motion for the i^{th} spacecraft is of the form

$$\ddot{r}_i = \mu_s \frac{r_0}{|r_i|^3} + a_i + b_i + c_i; \qquad i = 0,1,2,3,4,$$

where μ_s is the solar gravitational parameter and is given by $\mu_s = 1.327\times10^{20} m^3/s^2$, a_i is the acceleration produced by the thruster force or control actions on the i^{th} spacecraft, b_i is the acceleration due to solar pressure forces acting on the i^{th} spacecraft, and c_i is the acceleration due to the third body effects from the Earth and the Moon on the i^{th} spacecraft. It can be shown that for $|\rho_i| << |r_i|$, the relative motion of the i^{th} collector and the combiner spacecraft can be described by Hill's equation:

$$\ddot{\rho}_i + \mu_s \frac{r_0}{|r_0|^3}(I_{3x3} - 3r_0r_0^T)\rho_i = a_i - a_0 + b_i + c_i;$$
$$i = 1,2,3,4.$$

The above equation can further be simplified under certain assumptions. Suppose that the maximum TPF baseline is about 1 km. Then the $\rho_{max} = \max|\rho_i| = 0.577$ km. A typical value of r_0 is $|r_0| \approx 1$ AU. In this case,

$$\left|\mu_s \frac{r_0}{|r_0|^3}(I_{3x3} - 3r_0r_0^T)\rho_i\right| \le 4\times10^{-14}|\rho_i| \le 2.3\times10^{-11} m/s^2.$$

Assume that the sun shades in TPF spacecraft are circular and have a diameter of 15 meters. Then the maximum surface reflectivity is about 1400 W/m^2 flux at 1 AU. This results in a solar pressure acceleration of

$$|b_i| \le 5\times10^{-6} m/s^2.$$

The perturbation due to the acceleration c_i depends on the Earth-formation separation. It is possible to devise the formation to be far enough so that the effects of third body perturbations are negligible when compared to those of the solar pressure. Under these assumptions, a relatively simpler model of the translational dynamics for the formation can be described by

$$\ddot{r} = -\mu_s \frac{r_0}{|r_0|^3} + a_0 + b_0$$
$$\ddot{\rho}_i = a_i - a_0 + b_i \qquad i = 1,2,3,4. \qquad (1)$$

3. TPF Geometry and Formation Constraints

The TPF formation flying spacecraft consists of a combiner spacecraft denoted by S_0 and four collector spacecraft denoted by $S_1, S_2, S_3,$ and S_4. The TPF formation geometry depicted in Figure 2 requires that all spacecraft form a planer configuration, adjacent collectors have equal distance from each other along a baseline, and the combiner spacecraft form an equilateral triangle with respect to the two innermost collector spacecraft.

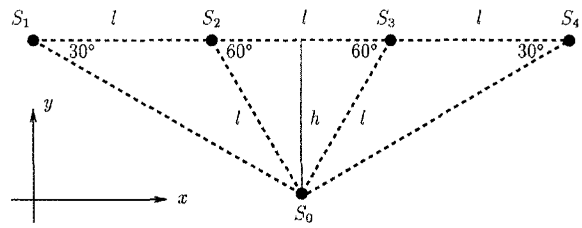

Figure 2: TPF formation geometry.

Define the coordinates of the TPF spacecraft by $S_i(x_i, y_i, z_i)$, where $i = 1,2,3,4$. The collector's coordinates during the TPF formation flying can expressed in terms of the coordinates of the combiner spacecraft by

$$S_0 : (x_0, y_0, z_0)$$
$$S_1 : (x_1, y_1, z_1) = (x_0 - \frac{3}{2}l, y_0 + h, z_0)$$
$$S_2 : (x_2, y_2, z_2) = (x_0 - \frac{1}{2}l, y_0 + h, z_0)$$
$$S_3 : (x_3, y_3, z_3) = (x_0 + \frac{1}{2}l, y_0 + h, z_0)$$
$$S_4 : (x_4, y_4, z_4) = (x_0 + \frac{3}{2}l, y_0 + h, z_0).$$

The above configuration poses a virtual optical truss that must be maintained throughout the formation. This optical truss can be characterized by 7 relative constraints in each axis. Define the relative error in the translational constraints by $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ with $i = 1,2,\ldots,7$, where

$$\begin{cases} \bar{x}_1 = x_1 - x_0 + 3l/2 \\ \bar{x}_2 = x_2 - x_0 + l/2 \\ \bar{x}_3 = x_3 - x_0 - l/2 \\ \bar{x}_4 = x_4 - x_0 - 3l/2 \\ \bar{x}_5 = x_1 - x_2 + l = \bar{x}_1 - \bar{x}_2 \\ \bar{x}_6 = x_2 - x_3 + l = \bar{x}_2 - \bar{x}_3 \\ \bar{x}_7 = x_3 - x_4 + l = \bar{x}_3 - \bar{x}_4 \end{cases}$$

$$\begin{cases} \bar{y}_1 = y_1 - y_0 - h \\ \bar{y}_2 = y_2 - y_0 - h \\ \bar{y}_3 = y_3 - y_0 - h \\ \bar{y}_4 = y_4 - y_0 - h \\ \bar{y}_5 = y_1 - y_2 = \bar{y}_1 - \bar{y}_2 \\ \bar{y}_6 = y_2 - y_3 = \bar{y}_2 - \bar{y}_3 \\ \bar{y}_7 = y_3 - y_4 = \bar{y}_3 - \bar{y}_4 \end{cases}$$

$$\begin{cases} \bar{z}_1 = z_1 - z_0 \\ \bar{z}_2 = z_2 - z_0 \\ \bar{z}_3 = z_3 - z_0 \\ \bar{z}_4 = z_4 - z_0 \\ \bar{z}_5 = z_1 - z_2 = \bar{z}_1 - \bar{z}_2 \\ \bar{z}_6 = z_2 - z_3 = \bar{z}_2 - \bar{z}_3 \\ \bar{z}_7 = z_3 - z_4 = \bar{z}_3 - \bar{z}_4 \end{cases}$$

The above expressions illustrate that only four elements in each axis are linearly independent; therefore, any imposed error constraints can be expressed in terms of the first four elements of each axis. These elements are referred to as the translational error states and are formalized by

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix}, \; \bar{y} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \\ \bar{y}_4 \end{bmatrix}, \; \bar{z} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \\ \bar{z}_3 \\ \bar{z}_4 \end{bmatrix} \qquad (2)$$

4. An Optimal Control Formulation for TPF Formation

The problem of TPF formation flying spacecraft can be cast as an optimal control problem. Suppose that the TPF spacecraft construct a leader-follower approach, where the collectors are followers and the combiner spacecraft is the leader. A representative continuous-time cost function associated with these formation constraints can be expressed by

$$\begin{aligned} J_x &= \int_0^\infty \left(\bar{x}^T Q_x \bar{x} + \dot{\bar{x}}^T N_x \dot{\bar{x}} + u^T R_x u\right) dt \\ J_y &= \int_0^\infty \left(\bar{y}^T Q_y \bar{y} + \dot{\bar{y}}^T N_y \dot{\bar{y}} + v^T R_y v\right) dt \\ J_z &= \int_0^\infty \left(\bar{z}^T Q_z \bar{z} + \dot{\bar{z}}^T N_z \dot{\bar{z}} + w^T R_z w\right) dt, \end{aligned} \qquad (3)$$

where $(\bar{x}, \bar{y}, \bar{z})$ are as in (2), (Q_x, Q_y, Q_z) and (N_x, N_y, N_z) are the weighting matrices associated with the error states to be defined, (u, v, w) are the translational forces, applied to each collector, that drive the error states along the three axes of motion , and (R_x, R_y, R_z) are appropriate input weighting matrices to be determined. The positive definite matrix Q_x can be chosen such that

$$\begin{aligned} &\bar{x}^T Q_x \bar{x} \\ &\quad = \alpha_1 \bar{x}_1^2 + \alpha_2 \bar{x}_2^2 + \alpha_3 \bar{x}_3^2 + \alpha_4 \bar{x}_4^2 + \alpha_5 \bar{x}_5^2 + \\ &\quad \alpha_6 \bar{x}_6^2 + \alpha_7 \bar{x}_7^2; \; \alpha_i > 0; \\ &\quad = \alpha_1 \bar{x}_1^2 + \alpha_2 \bar{x}_2^2 + \alpha_3 \bar{x}_3^2 + \alpha_4 \bar{x}_4^2 + \alpha_5 (\bar{x}_1 - \bar{x}_2)^2 + \\ &\quad \alpha_6 (\bar{x}_2 - \bar{x}_3)^2 + \alpha_7 (\bar{x}_3 - \bar{x}_4)^2 \\ &\quad = [\bar{x}_1 \;\; \bar{x}_2 \;\; \bar{x}_3 \;\; \bar{x}_4] Q_x \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix}. \end{aligned}$$

It then follows that

$$Q_x = \begin{bmatrix} (\alpha_1 + \alpha_5) & -\alpha_5 & 0 & 0 \\ -\alpha_5 & (\alpha_2 + \alpha_5 + \alpha_6) & -\alpha_6 & 0 \\ 0 & -\alpha_6 & (\alpha_3 + \alpha_6 + \alpha_7) & -\alpha_7 \\ 0 & 0 & -\alpha_7 & (\alpha_4 + \alpha_7) \end{bmatrix} = Q_x^T > 0. \qquad (4)$$

Similarly, for $\bar{y}$ and $\bar{z}$ as in (2), Q_y and Q_z satisfy

$$\bar{y}^T Q_y \bar{y} = \sum_{i=1}^{7} \beta_i \bar{y}_i^2; \; \beta_i > 0$$

$$\bar{z}^T Q_z \bar{z} = \sum_{i=1}^{7} \gamma_i \bar{z}_i^2; \; \gamma_i > 0$$

and are given by

$$Q_y = Q_y^T = \begin{bmatrix} (\beta_1 + \beta_5) & -\beta_5 & 0 & 0 \\ -\beta_5 & (\beta_2 + \beta_5 + \beta_6) & -\beta_6 & 0 \\ 0 & -\beta_6 & (\beta_3 + \beta_6 + \beta_7) & -\beta_7 \\ 0 & 0 & -\beta_7 & (\beta_4 + \beta_7) \end{bmatrix} > 0 \qquad (5)$$

$$Q_z = Q_z^T = \begin{bmatrix} (\gamma_1+\gamma_5) & -\gamma_5 & 0 & 0 \\ -\gamma_5 & (\gamma_2+\gamma_5+\gamma_6) & -\gamma_6 & 0 \\ 0 & -\gamma_6 & (\gamma_3+\gamma_6+\gamma_7) & -\gamma_7 \\ 0 & 0 & -\gamma_7 & (\gamma_4+\gamma_7) \end{bmatrix} > 0 \quad (6)$$

The parameters $\alpha_i, \beta_i, \text{and} \gamma_i$ are scaling factors that penalize error state deviations from zero to within specified requirements. Similar expressions can be given for (N_x, N_y, N_z) that penalizes the relative rate error between the neighboring spacecraft. Because of the problem symmetry and formulation simplicity, however, these matrices can be chosen to be identical and proportional to an identity matrix, that is

$$N_x = N_y = N_z = \kappa I_{(4\times4)} \quad \kappa > 0 \quad (7)$$

The control input forces (u_x, u_y, u_z) are defined to drive the error states. By construction, the error states describe the deviation of the constrained relative distance between each collector and the combiner spacecraft. Therefore, the collectors must follow the combiner spacecraft and maintain a pre-specified relative distance in each axis. It is then meaningful to relate (u_x, u_y, u_z) to the control input forces needed for the collectors to drive the error states to zero. That is,

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \; v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \; w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \quad (8)$$

where $u_i, v_i, \text{and } w_i$, are the translational forces applied to the i^{th} collector along x-axis, y-axis, and z-axis, respectively. Finally, because of the ability in scaling other parameters of the objective function (3) and formulation simplicity,

$$R_x = R_y = R_z = I_{(4\times4)}. \quad (9)$$

The time evolution of the cost functions in (3) is constrained to the dynamics of the error states $\overline{x}$ and $\dot{\overline{x}}$. The error states dynamics depend directly on the states of the combiner spacecraft S_0 and the baseline separation l between the adjacent collector spacecraft. In other words, any direct implementation of the state equations associated with the error states involves the coordinates of S_0 and l. As S_0 coordinates and l are subject to change, the parameter matrices of the corresponding state equations are not constant, an undesirable implementation. In addition, the optimal control problem is intended to solve for control input to each spacecraft so that the collector spacecraft are stable and the associated error states are nearly zero. It is therefore meaningful to work with the collector spacecraft dynamics directly, construct the error states accordingly, and compute control laws to each collector respectively. The following describes the implementation details.

The control model for each collector spacecraft can be approximated by a simple rigid mass subject to Newtonian motion. Let M_i denote the mass of the i^{th} collector, then

$$M_i \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}.$$

Using this model, the overall state equations along each axis can be written in the compact form by

$$\begin{aligned} \dot{x} &= A_x x + B_x u = Ax + Bu \\ \dot{y} &= A_y y + B_y v = Ay + Bv \\ \dot{z} &= A_z z + B_z w = Az + Bw, \end{aligned} \quad (10)$$

where

$$x = [x_1 \; x_2 \; x_3 \; x_4 \; \dot{x}_1 \; \dot{x}_2 \; \dot{x}_3 \; \dot{x}_4]^T$$
$$y = [y_1 \; y_2 \; y_3 \; y_4 \; \dot{y}_1 \; \dot{y}_2 \; \dot{y}_3 \; \dot{y}_4]^T$$
$$z = [z_1 \; z_2 \; z_3 \; z_4 \; \dot{z}_1 \; \dot{z}_2 \; \dot{z}_3 \; \dot{z}_4]^T$$

$$A = \begin{bmatrix} 00001000 \\ 00000100 \\ 00000010 \\ 00000001 \\ 00000000 \\ 00000000 \\ 00000000 \\ 00000000 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{-1}{M_1} & 0 & 0 & 0 \\ 0 & \frac{-1}{M_2} & 0 & 0 \\ 0 & 0 & \frac{-1}{M_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{M_4} \end{bmatrix}$$

Further, let the desired trajectories along the three axis be denoted by x_d, y_d, and z_d and given by

$$x_d = \begin{bmatrix} x_0 - 3l/2 \\ x_0 - l/2 \\ x_0 + l/2 \\ x_0 + 3l/2 \\ \dot{x}_0 - 3\dot{l}/2 \\ \dot{x}_0 - \dot{l}/2 \\ \dot{x}_0 + \dot{l}/2 \\ \dot{x}_0 + 3\dot{l}/2 \end{bmatrix} \quad y_d = \begin{bmatrix} y_0 + h \\ y_0 + h \\ y_0 + h \\ y_0 + h \\ \dot{y}_0 + \dot{h} \\ \dot{y}_0 + \dot{h} \\ \dot{y}_0 + \dot{h} \\ \dot{y}_0 + \dot{h} \end{bmatrix} \quad z_d = \begin{bmatrix} z_0 \\ z_0 \\ z_0 \\ z_0 \\ \dot{z}_0 \\ \dot{z}_0 \\ \dot{z}_0 \\ \dot{z}_0 \end{bmatrix} \qquad (11)$$

We can then write

$$\begin{bmatrix} \overline{x} \\ \dot{\overline{x}} \end{bmatrix} = x - x_d \quad \begin{bmatrix} \overline{y} \\ \dot{\overline{y}} \end{bmatrix} = y - y_d \quad \begin{bmatrix} \overline{z} \\ \dot{\overline{z}} \end{bmatrix} = z - z_d \qquad (12)$$

4.1 Continuous-Time LQR Formulation of the TPF Formation

Suppose that the continuous-time cost functions in (3) are equivalently expressed by

$$\begin{aligned} J_x &= \int_0^\infty \left((x - x_d)^T \begin{bmatrix} Q_x & 0 \\ 0 & N_x \end{bmatrix} (x - x_d) + u^T R_x u \right) dt \\ J_y &= \int_0^\infty \left((y - y_d)^T \begin{bmatrix} Q_y & 0 \\ 0 & N_y \end{bmatrix} (y - y_d) + v^T R_y v \right) dt \qquad (13) \\ J_z &= \int_0^\infty \left((z - z_d)^T \begin{bmatrix} Q_z & 0 \\ 0 & N_z \end{bmatrix} (z - z_d) + w^T R_z w \right) dt \end{aligned}$$

and are subject to

$$\begin{aligned} \dot{x} &= Ax + Bu \\ \dot{y} &= Ay + Bv \\ \dot{z} &= Az + Bw \end{aligned}$$

where the parameters are defined by (4) - (12). The continuous-time LQR optimal control laws for each axis of the collector spacecraft is given by

$$\begin{aligned} u &= -R_x^{-1} B_x^T P_x (x^\circ - x_d) \\ v &= -R_y^{-1} B_y^T P_y (y^\circ - y_d) \qquad (14) \\ w &= -R_z^{-1} B_z^T P_z (z^\circ - z_d) \end{aligned}$$

where $(x^\circ, y^\circ, z^\circ)$ are the states along optimal trajectories and (P_x, P_y, P_z) are the solutions to the algebraic Riccati equations

$$\begin{aligned} 0 &= P_x A + A^T P_x - P_x B R_x^{-1} B^T + G_x;\; G_x = \mathrm{diag}(Q_x, N_x) \\ 0 &= P_y A + A^T P_y - P_y B R_y^{-1} B^T + G_y;\; G_y = \mathrm{diag}(Q_y, N_y) \\ 0 &= P_z A + A^T P_z - P_z B R_z^{-1} B^T + G_z;\; G_z = \mathrm{diag}(Q_z, N_z) \end{aligned}$$

The existence and uniqueness of the optimal control solutions are established under the usual controllability and observability conditions. As can be seen in (14), the control forces are constructed based on the relative error in the states. This means that the formulation does not require absolute knowledge of the collector states; instead, it is sufficient to express the collector states in terms of the combiner states. It is therefore required to measure the collector states relative to the combiner states by a relative sensor. It is a common practice that the sensor measurements can only provide discrete information and the control laws are implemented in discrete time. The following describes the discrete-time formulation of the problem.

4.2 Discrete-Time LQR Formulation of the TPF Formation

Define the discrete-time representation of the collector states by

$$\begin{aligned} x[k] &= x(kT_s) \\ y[k] &= y(kT_s) \\ z[k] &= z(kT_s), \end{aligned}$$

where T_s is the sampling period of the discretization and index k refers to the sampled time of kT_s when the continuous states are sampled. Further assume that the input vectors do not change values between the sampling intervals. Then the discrete-time representation of the collector's dynamic equations in (10) is of the form

$$\begin{aligned} x[k+1] &= Fx[k] + Hu[k] \\ y[k+1] &= Fy[k] + Hv[k] \qquad (16) \\ z[k+1] &= Fz[k] + Hw[k] \end{aligned}$$

where $(u[k], v[k], w[k])$ are the samples of $(u(t), v(t), w(t))$ at $t = kT_s$, and the parameter matrices of F and H are

$$F = e^{T_s A} = \sum_{k=0}^{\infty} \frac{(T_s A)^k}{k!} = I_{8\times8} + T_s A = \begin{bmatrix} I_{4\times4} & T_s I_{4\times4} \\ 0_{4\times4} & I_{4\times4} \end{bmatrix}$$

$$G = \left(\int_0^{T_s} e^{\lambda A} d\lambda\right) B = T_s \begin{bmatrix} I_{4\times4} & \frac{1}{2} T_s I_{4\times4} \\ 0_{4\times4} & I_{4\times4} \end{bmatrix} B$$

$$= T_s \begin{bmatrix} \frac{-T_s}{2M_1} & 0 & 0 & 0 \\ 0 & \frac{-T_s}{2M_2} & 0 & 0 \\ 0 & 0 & \frac{-T_s}{2M_3} & 0 \\ 0 & 0 & 0 & \frac{-T_s}{2M_4} \\ \frac{-1}{M_1} & 0 & 0 & 0 \\ 0 & \frac{-1}{M_2} & 0 & 0 \\ 0 & 0 & \frac{-1}{M_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{M_4} \end{bmatrix} .$$

Define the discrete-time representation of the objective functions in (13) by

$$\begin{aligned} J_x &= \sum_{k=0}^{\infty}\left((x[k]-x_d[k])^T \begin{bmatrix} Q_x & 0 \\ 0 & N_x \end{bmatrix} (x[k]-x_d[k]) + u[k]^T R_x u[k] \right) \\ J_y &= \sum_{k=0}^{\infty}\left((y[k]-y_d[k])^T \begin{bmatrix} Q_y & 0 \\ 0 & N_y \end{bmatrix} (y[k]-y_d[k]) + v[k]^T R_x v[k] \right) \\ J_z &= \sum_{k=0}^{\infty}\left((z[k]-z_d[k])^T \begin{bmatrix} Q_z & 0 \\ 0 & N_z \end{bmatrix} (z[k]-z_d[k]) + w[k]^T R_x w[k] \right) \end{aligned} \quad (17)$$

where the states are subject to the discrete-time dynamic equations described in (16). The discrete-time LQR optimal control laws for each axis of the collector spacecraft is of the form

$$\begin{aligned} u[k] &= \left(H'S_x H + R\right)^{-1} H'S_x F\left(x^{\circ}[k] - x_d[k]\right) \\ v[k] &= \left(H'S_y H + R\right)^{-1} H'S_y F\left(y^{\circ}[k] - y_d[k]\right) \\ w[k] &= \left(H'S_z H + R\right)^{-1} H'S_z F\left(z^{\circ}[k] - z_d[k]\right) \end{aligned} \quad (18)$$

where $\left(x^{\circ}[k], y^{\circ}[k], z^{\circ}[k]\right)$ are sampled states along optimal trajectories and $\left(S_x, S_y, S_z\right)$ are the solutions to the discrete-time algebraic Riccati equations

$$\begin{aligned} 0 &= F'S_x F - S_x - F'S_x H\left(R + H'S_x H\right)^{-1} H'S_x F + G_x \\ 0 &= F'S_y F - S_y - F'S_y H\left(R + H'S_y H\right)^{-1} H'S_y F + G_y \\ 0 &= F'S_z F - S_z - F'S_z H\left(R + H'S_z H\right)^{-1} H'S_z F + G_z \end{aligned} \quad (19)$$

5. Conclusion

A centralized control system for a TPF type formation flying of spacecraft with one combiner and four collector spacecraft is presented. Using a leader-follower approach, the optimal control system will maintain a virtual optical truss that is maintained with the desired level of precision for the interferometry applications.

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