Operations Comparison of Deep Space Ranging Types: Sequential Tone vs. Pseudo-Noise

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Abstract

NASA's Deep Space Network currently has a ranging capability that uses a sequence of square wave tones to determine spacecraft distance. This ranging system correlates tones received from the spacecraft with those it transmitted. The phase shift that maximizes the correlation value is then related to the round trip light time distance. A new ranging system is being developed that can be configured to use sequential square wave tones or to use repeating pseudo-noise tones. This allows more flexibility and the option to obtain better performance.

The tradeoffs between the two types of ranging are presented. A detailed derivation of the ranging performance as a function of configuration, signal strength, and other variables is given. Comparison of the ranging performance for the two ranging types is provided, showing a configuration for pseudo-noise ranging that provides better performance. Operability issues and simplifications resulting from using pseudo-noise ranging are also discussed.

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1 INTRODUCTION

NASA's Deep Space Network (DSN) currently has a ranging capability that uses a sequence of squarewave tones to determine spacecraft distance. The tones are modulated onto the uplink carrier frequency, pass through the spacecraft transponder, and are re-modulated on the downlink carrier frequency. The tones are demodulated from the carrier back at the DSN antenna where the ranging system correlates the received and transmitted tones. The phase shift that maximizes the correlation value is then related to the round trip light time distance.

The Jet Propulsion Laboratory (JPL) is developing a replacement ranging system as part of the Network Simplification Project (NSP). This new NSP ranging can be configured to use sequential squarewave tones or to use repeating pseudo-noise (PN) tones to determine spacecraft distance. The DSN has used pseudo-noise tones in previous ranging systems. However, each DSN ranging subsystem has historically supported only one type of ranging, either pseudo-noise tones or sequential tones, but never both. When the current sequential tone ranging system was designed, it provided more configurations that the pseudo-noise tone system it replaced. The new NSP ranging system will be capable of providing either type of ranging. This will allow more flexibility and in some cases, the option to obtain better performance. However, the user community must have a better understanding of the tradeoffs in selecting the ranging type in order to take full advantage of the NSP ranging benefits. A detailed comparison of ranging performance requires examining several variables. These variables include signal strength, uncertainty of the spacecraft distance, required range accuracy, and number of measurements desired. Both types of ranging provide a variety of tone configurations in order to meet the requested ranging performance.

First, we provide an introduction to both ranging types, focusing on the tradeoffs made by the end users. This includes a detailed derivation of the predicted ranging performance as a function of configuration, signal strength, and other variables. Next, a comparison of the predicted ranging performance for the two ranging types is provided to show a pseudo-noise ranging configuration that provides the same, or better, performance as the sequential tone ranging. Details will be provided on the conditions where pseudo-noise ranging can provide better performance

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than sequential ranging. Finally, operability issues and simplifications resulting from using pseudo-noise ranging are discussed.

2 Deep Space Ranging Description

Ranging is the determination of the distance between the Earth and the spacecraft. The measurement is made by modulating a signal (called the ranging code) onto the uplink carrier that is transmitted to the spacecraft. The spacecraft demodulates the signal, filters it, and then modulates it onto the downlink carrier, which is transmitted back to Earth. Upon reception, the signal is then demodulated and correlated against a copy of what was sent. The period of the lowest frequency of the code determines the modulus of the range measurement; for example, if the period is 0.5 seconds, the range measurement is modulo 0.5 seconds. When the ranging modulus is expressed in terms of distance (by multiplying the period by the speed of light, c, approximately $3x10^5$ km/sec), it is also referred to as the code ambiguity.

The resolution of the measurement is determined by the highest frequency of the code: typically, the highest frequency is 1 MHz and the correlation process can measure to one one-thousandth of the period, the resolution of the code is 1 nsec. The overall accuracy of the measurement is determined by the accuracy of the calibration of the ranging measurement equipment.

The ranging modulation power is normally very weak (when compared to the carrier or data power). This is done to avoid reducing the power available for telemetry data modulation. The key metric is the ratio of the ranging power to noise spectral density, $\frac{P_r}{N_0}$. Generally, the ranging signal needs to be integrated for a period on the order of minutes to accumulate enough signal-to-noise ratio (SNR) to accurately determine the range.

Due to the noise, the range measurement has a certain variance, which is represented by σ^2. The variance can be expressed in meters, or in seconds; the relationship between the two forms is:

$$\sigma^2_{meters} = c^2\sigma^2_{sec} \tag{1}$$

There are two types of ranging codes: sequential and pseudo-noise (PN). We will now discuss and compare the two.

2.1 Sequential Ranging

The sequential ranging sequence consists of a sequence of square waves (tones) being sent, each one a factor of two lower in frequency than the previous one. The first square wave is referred to as the clock component. Since this component is the highest frequency sent, it determines the precision of the ranging measurement. However, since the correlation can only resolve to one period of the tone, the measurement has a large ambiguity. This ambiguity is resolved by the subsequent tones, each one reducing the ambiguity by a factor of two. Thus, the final tone (or component) in sequence determines the ambiguity resolution capability of the code. The range measurement is broken into two phases: first the phase of the clock component must be determined (relative to a reference clock) and then the polarity of each of the subsequent components (relative to the reference) must be detected.

The clock component is transmitted for T_1 seconds. Each of the following tones are transmitted for T_2 seconds. T_1 is normally much larger that T_2, since it requires much more time to correlate the clock component than it does to detect each subsequent component. Each time must be an integer value, with one second being the minimum. Between each of the components, there is a dead time (of length one second) to allow for transitioning from one component to the next. When this is all factored in, the time of the sequence (referred to as the cycle time or T_{cyc}) is [1]:

$$T_{cyc} = T_1 + T_2(n-1) + (n+2) \tag{2}$$

Where n is the total number of square wave tones sent.

The big advantages with sequential ranging is that only one correlation operation is needed at a time and at any time, all of the ranging power is in the component being correlated. The disadvantage is that due to the sequential nature of the code, the correlation can only start at the beginning of the code, which requires coordination between the uplink transmission and the downlink reception, including an accurate estimate of the round trip light time (RTLT).

Now, the general case of the integration time required for clock component measurement is derived in Appendix A. For sequential ranging, T is T_1, the component correlation value, C, is unity, and σ is σ_s (σ_s is expressed in seconds). Thus, (A-30) becomes:

$$T_1 = \left(64\sigma_s^2 F^2 \frac{P_r}{N_0}\right)^{-1} \tag{3}$$

For the non-clock components, we correlate against a reference square wave and only determine the sign of the result. We successfully acquire the component if the received signal plus noise is greater than zero (assuming that a positive value was sent) [2]. Given a gaussian channel, with noise $n(t)$, we have:

$$\begin{aligned} P_a &= \text{Prob}\left(\left(\sqrt{P_r} + n(t)\right) > 0\right) && (4) \\ &= \text{Prob}\left(\left(\frac{n(t)}{\sigma_n}\right) > -\frac{\sqrt{P_r}}{\sigma_n}\right) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\sqrt{P_r}}{\sigma_n}} e^{-\frac{x^2}{2}} dx && (5) \end{aligned}$$

σ_n is defined as:

$$\sigma_n = \sqrt{N_0 \frac{T_2}{2}} \tag{6}$$

Thus:

$$\begin{aligned} P_a &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\sqrt{P_r}}{\sqrt{N_0 \frac{T_2}{2}}}} e^{-\frac{x^2}{2}} dx && (7) \\ &= \frac{1}{2}\left(1 + \text{erf}\left(\sqrt{2T_2 \frac{P_r}{N_0}} \frac{1}{\sqrt{2}}\right)\right) \\ &= \frac{1}{2}\left(1 + \text{erf}\left(\sqrt{\frac{P_r}{N_0} T_2}\right)\right) && (8) \end{aligned}$$

The probability of acquiring all $n-1$ components, each of which is transmitted for T_2 seconds, is the product of the individual component probabilities of acquisition, or:

$$P_{acq} = P_a^{n-1} \tag{9}$$

So:

$$P_{acq} = \left\{ \frac{1 + \text{erf}\left(\sqrt{\frac{P_r}{N_0} T_2}\right)}{2} \right\}^{n-1} \tag{10}$$

Solving for T_2, we finally get:

$$T_2 = \left\{ \text{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2 \left(\frac{P_r}{N_0}\right)^{-1} \tag{11}$$

With the equations for T_1 and T_2, we can now restate the cycle time:

$$T_{cyc} = \left(\frac{P_r}{N_0}\right)^{-1} \left[(n+2)\frac{P_r}{N_0} + \frac{1}{64F^2\sigma_s^2} + (n-1)\left\{\text{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2\right] \tag{12}$$

Also, we solve (12) for σ_s^2:

$$\sigma_s^2 = \frac{1}{64F^2} \left\{ \frac{P_r}{N_0}\left[T_{cyc} - (n+2)\right] - (n-1)\left\{\text{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2 \right\}^{-1} \tag{13}$$

2.2 PN Ranging

The PN ranging code is a sequence that is made up of several relatively short PN subsequences that are logically combined to make a larger sequence. The logical operations include and'ing (AND), or'ing (OR), exclusive-or'ing (XOR), and majority summing (MAJ). Regardless of the operation, the resulting sequence has a length equal to the product of the lengths of the subsequences. This length sets the ambiguity resolution of the code. All potential code combinations include the length 2 sequence, which is the equivalent of the clock component in the sequential case.

The implementation of PN ranging has the choice of correlating against the complete sequence or each of the subsequences. Since the sum of the correlations of the subsequences is much smaller than the correlation of the entire sequence, the code measurement is done using the subsequences. Previously, hardware limitations required only one correlation at a time, but current Digital Signal Processor (DSP) capability now allows us to correlate all of the subsequences in parallel [3], [?]. This means that the total integration time required is the maximum time required by the subsequences, both for accuracy determination (clock acquisition) and ambiguity resolution (other subsequences).

Once again, we use the results for the general case of the integration time required for clock component measurement that is derived in Appendix A. For PN ranging, T is T_{int}, the component correlation value, C, is the correlation of the sequence with the length 2 subsequence (C_2, the value of which depends on the construction of the code) , and σ is σ_p (expressed in seconds). Thus, (A-30) becomes:

$$T_{int} = \left(64\sigma_p^2 F^2 C_2 \frac{P_r}{N_0}\right)^{-1} \tag{14}$$

From Appendix B, we have (B-13), which gives us the probability of acquiring an individual component as a function of (among others) β. And, (B-8) defines β as:

$$\beta_i = \Delta C_i \sqrt{\frac{P_r}{N_0} T_{int,i}} \tag{15}$$

Or:

$$T_{int,i} = \left(\frac{P_r}{N_0}\right)^{-1} \left(\frac{\beta_i}{\Delta C_i}\right)^2 \tag{16}$$

Where ΔC_i is the difference between the maximum and minimum correlation values for the ith subsequence, β_i is the β for the ith subsequence, and $T_{int,i}$ is the integration time for the ith subsequence. Again, the probability of acquisition of the entire code, P_{acq} is the product of the probability of acquisition for each of the subsequences. Using (B-13), we have:

$$\begin{aligned} P_a &= (P_{acq})^{\frac{1}{M}} && (17) \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \left(\frac{1+\operatorname{erf}(x+\beta)}{2}\right)^{n_i - 1} dx && (18) \end{aligned}$$

Where n_i is the length of the ith subsequence and M is the number of subsequences. The integration time required for the code is the i that gives the maximum $T_{int,i}$. So, we define m to be the i that gives the maximum integration time, T_{int}:

$$T_{int} = \left(\frac{\beta_m}{\Delta C_m}\right)^2 \left(\frac{P_r}{N_0}\right)^{-1} \tag{19}$$

Substituting this result into (14) and solving for σ_p^2 gives us:

$$\sigma_p^2 = \frac{1}{64F^2C_2} \left(\frac{\Delta C_m}{\beta_m}\right)^2 \tag{20}$$

3 Comparison of Sequential and PN Ranging

We wish to compare the two codes. To do this, we must define the metrics to be used. For this paper, we will use two metrics, the ratio of the variances (σ^2) and the ratio of the integration times. Both will be compared as functions of the $\frac{P_r}{N_0}$. For the ratio of the variances, the probability of acquisition, P_{acq}, code ambiguity resolution, and integration time will be equal for both codes. For the ratio of the integration times, the probability of acquisition, P_{acq}, code ambiguity resolution, and variances will be equal for both codes. Finally, we will compute the crossover $\frac{P_r}{N_0}$, where the integration times are equal, as are the variances.

3.1 *Ratio of* σ

To compute the ratio of the σs, R_σ, we assume that the ambiguity resolution capabilities are equal, the acquisition probabilities are equal, and T_{int} and T_{cyc} are equal. Using (13) and (20), we have:

$$R_\sigma = \frac{\sigma_s^2}{\sigma_p^2} \tag{21}$$

$$= \frac{1}{64F^2}\left\{\frac{P_r}{N_0}\left[T_{cyc}-(n+2)\right]-(n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}}-1\right)\right\}^2\right\}^{-1}\left(\frac{1}{64F^2C_2}\left(\frac{\Delta C_m}{\beta_m}\right)^2\right)^{-1}$$

$$= \frac{C_2\left(\frac{\beta_m}{\Delta C_m}\right)^2}{\frac{P_r}{N_0}\left[T_{cyc}-(n+2)\right]-(n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}}-1\right)\right\}^2} \tag{22}$$

Making use of the equality of the integration times, and of (19):

$$R_\sigma = \frac{C_2\left(\frac{\beta_m}{\Delta C_m}\right)^2}{\frac{P_r}{N_0}\left[\left(\frac{\beta_m}{\Delta C_m}\right)^2\left(\frac{P_r}{N_0}\right)^{-1}-(n+2)\right]-(n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}}-1\right)\right\}^2}$$

$$R_\sigma = \frac{C_2\left(\frac{\beta_m}{\Delta C_m}\right)^2}{\left[\left(\frac{\beta_m}{\Delta C_m}\right)^2-(n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}}-1\right)\right\}^2\right]-\frac{P_r}{N_0}(n+2)} \tag{23}$$

If R_σ is greater than 1, the sequential range code has a larger variance then does the PN code. If it is less than 1, then the opposite is true. In other words, if the ratio is greater than one, the PN code performs better; if it is less than 1, the sequential code performs better.

One other thing to note from the equation for R_σ: it is possible for the denominator to go negative. This happens because one of our assumptions becomes untrue; namely, there are situations when T_{int} and T_{cyc} cannot be equal. Looking at (2), if we allow T_1 and T_2 to go to their minimum values of 1, we get:

$$T_{cyc,min} = 1+(n-1)+(n+2) \tag{24}$$

$$= 2(n+1) \tag{25}$$

This limiting value is due to the one second minimum times for each tone, and by the one second dead bands between each tone. But, T_{int} is an inverse function of $\frac{P_r}{N_0}$, so it is possible that it will be smaller than the T_{cyc} limit:

$$T_{int} \le T_{cyc,min} \tag{26}$$

$$\left(\frac{\beta_m}{\Delta C_m}\right)^2\left(\frac{P_r}{N_0}\right)^{-1} \le 2(n+1)$$

$$\frac{P_r}{N_0} \ge \frac{1}{2(n+2)}\left(\frac{\beta_m}{\Delta C_m}\right)^2 \tag{27}$$

When $\frac{P_r}{N_0}$ goes beyond the equality, the assumption that T_{cyc} equals T_{int} can no longer be met, and the ratio becomes negative. What this means physically is that the PN code's integration time is less than the minimum cycle time of the sequential code, and the PN code's variance is less than the sequential code's variance.

3.2 *Ratio of Integration Time*

To compute the ratio of the integration times, R_T, we assume that the ambiguity resolution capabilities are equal, the acquisition probabilities are equal, and σ_p and σ_s are equal. Using (12) and (19), we have:

$$R_T = \frac{T_{cyc}}{T_{int}} \tag{28}$$

$$= \left(\frac{P_r}{N_0}\right)^{-1}\left[(n+2)\frac{P_r}{N_0} + \frac{1}{64F^2\sigma_s^2} + (n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2\right]\left(\left(\frac{\beta_m}{\Delta C_m}\right)^2\left(\frac{P_r}{N_0}\right)^{-1}\right)^{-1} \tag{29}$$

Making use of the equality of σ_p and σ_s, and (20), we get the final result:

$$R_T = \frac{P_r}{N_0}(n+2)\left(\frac{\Delta C_m}{\beta_m}\right)^2 + \left[C_2 + (n-1)\left(\frac{\Delta C_m}{\beta_m}\right)^2\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2\right] \tag{30}$$

If R_T is greater than 1, the sequential range code requires a longer integration time to get the same results as the PN code. If it is less than 1, then the opposite is true. In other words, if the ratio is greater than one, the PN code performs better; if it is less than 1, the sequential code performs better.

3.3 *Crossover* $\frac{P_r}{N_0}$

The crossover $\frac{P_r}{N_0}$ is the $\frac{P_r}{N_0}$ where the two types of ranging codes are equal. This is the point were both R_T and R_σ are unity. To find this point, we use (30), set R_T to 1 and solve for $\frac{P_r}{N_0}$. This gives us the following:

$$\frac{P_r}{N_0} = \frac{(1-C_2)}{(n+2)}\left(\frac{\beta_m}{\Delta C_m}\right)^2 - \frac{(n-1)}{(n+2)}\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2 \tag{31}$$

3.4 *"Good" PN Codes*

We now want to find a set of PN codes that perform as well or better than the equivalent sequential codes (equivalent in terms of P_{acq} and ambiguity resolution). For this to be true, we need to find a set of PN codes whose crossover $\frac{P_r}{N_0}$ is at or below the threshold of the DSN ranging system, which is around -10 dB-Hz $\frac{P_r}{N_0}$. Using (31), we have:

$$0.1 \geq \frac{P_r}{N_0} \tag{32}$$

$$0.1 \geq \frac{(1-C_2)}{(n+2)}\left(\frac{\beta_m}{\Delta C_m}\right)^2 - \frac{(n-1)}{(n+2)}\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2$$

$$(1-C_2)\left(\frac{\beta_m}{\Delta C_m}\right)^2 \leq \frac{(n+2)}{10} + (n-1)\left\{\mathrm{erf}^{-1}\left(2\left(P_{acq}\right)^{\frac{1}{n-1}} - 1\right)\right\}^2 \tag{33}$$

For simplicity, we define γ:

$$\gamma = (1-C_2)\left(\frac{\beta_m}{\Delta C_m}\right)^2 \tag{34}$$

For the remainder of this paper, we will assume that the probability of acquisition is 0.999 and that the code clock rates are 1 MHz. Table 1 provides the maximum values of γ for the each of the sequential codes, as defined by the number of components, along with the one-way ambiguity resolution (which means we use $\frac{c}{2}$ instead of c in converting time to distance) capability of each code:

3.5 *Majority Vote PN Codes*

As was pointed out earlier, there are many different operations that can be applied to the subsequences to generate the composite PN code. However, [5], [?] point out that the optimal codes (in the sense of maximizing the difference between the maximum and minimum correlation values, ΔC) are generated by using a majority vote operation. For m subsequences, this operation outputs a +1 if there are greater than $\frac{m}{2}$ +1 values and -1 if the number of +1's are less than $\frac{m}{2}$. For the cases considered here, we only allowed m to be odd, so we did not have to worry about the case where there were the same number of positive and negative values. With this constraint, we considered sets of three and five subsequences. The actual subsequences used are from [7] and are provided in Table 2.

Using 0.999 for the probability of acquistion, we get a value of 0.9997 for P_a when m is equal to 3 and 0.9998 for when m equals 5. The corresponding βs (derived using Mathematica) are given in Table 3.

Table 1 - Code Limits

Number of Components	Ambiguity Resolution (km)	γ Maximum
7	9.6	39.12
8	19.2	47.06
9	38.4	54.75
10	76.8	62.55
11	153.6	70.45
12	307.2	78.45
13	614.4	86.54
14	1,228.8	94.70
15	2,457.6	102.94
16	4,915.2	111.25
17	9,830.4	119.62
18	19,660.8	128.05
19	39,329.6	136.53
20	78,643.2	145.07

Table 2 - PN Subsequences

Code Length	Subsequence
2	+1, -1
3	+1, -1, 1
7	+1, +1, +1, -1, -1, +1, -1
11	+1, +1, +1, -1, -1, -1, +1, -1, +1, +1, -1
15	+1, +1, +1, +1, -1, -1, -1, +1, -1, -1, +1, +1, -1, +1, -1
19	+1, +1, +1, +1, -1, +1, -1, +1, -1, -1, -1, -1, +1, +1, -1, +1, +1, -1, -1
23	+1, +1, +1, +1, +1, -1, +1, -1, +1, +1, -1, -1, +1, +1, -1, -1, +1, -1, +1, -1, -1, -1, -1
31	-1, -1, -1, -1, +1, -1, +1, -1, +1, +1, +1, -1, +1, +1, -1, -1, -1, +1, +1, +1, +1, +1, -1, -1, +1, +1, -1, +1, -1, -1, +1
35	-1, +1, -1, +1, +1, +1, -1, -1, -1, +1, +1, +1, +1, +1, -1, +1, +1, +1, -1, -1, +1, -1, -1, -1, -1, +1, -1, +1, -1, +1, +1, -1, -1, +1, -1
43	+1, +1, -1, -1, +1, -1, +1, -1, -1, +1, +1, +1, -1, +1, +1, +1, +1, +1, -1, -1, -1, +1, -1, +1, +1, +1, -1, -1, -1, -1, -1, +1, -1, -1, -1, +1, +1, -1, +1, -1, +1, +1, -1
59	-1, +1, -1, +1, +1, +1, -1, +1, -1, +1, -1, -1, +1, -1, -1, +1, +1, +1, -1, +1, +1, +1, +1, -1, -1, +1, +1, +1, +1, +1, -1, -1, -1, -1, -1, +1, +1, -1, -1, -1, -1, +1, -1, -1, -1, +1, +1, -1, +1, +1, -1, +1, -1, +1, -1, -1, -1, +1, -1

A search was done over the range of PN subsequences of length 63 or less to find a set of PN codes that have the same (or slightly better) ambiguity resolution as for each of the sequential sequences given in Table 1, and have a γ less than the maximum value specified by the sequential sequence. These codes are listed in Table 4.

The results from Tables 1 and 4 are compared in Table 5. As can be seen, for each sequential code, the corresponding PN code's γ is less than the limit, which means that the PN code performs better. Figures 1 and 2 plot R_T and R_σ versus $\frac{P_r}{N_0}$, respectively. As can be seen in Table 5, the entire ambiguity space can be covered with just two PN codes, the length 1165042, which covers sequential component numbers from 13 to 20, and the length 4130 code, which covers sequential component numbers from 7 to 12. Figures 3 and 4 plot R_T and R_σ versus $\frac{P_r}{N_0}$, respectively, for the length 1165042 code, and Figures 5 and 6 plot R_T and R_σ versus $\frac{P_r}{N_0}$, respectively, for the length 4130 code, each for the appropriate number of sequential components.

One thing to note: the research on these PN codes is still on-going. As Table 5 shows, in terms of the parameters discussed in this paper, these codes perform better than their sequential code equivalents. However, there may be other issues (such as the number of +1's and -1's in a time period) that may preclude their usage. The total system effects of using these codes still needs to be analyzed.

Table 3 - β Values

Length	$m=3$	$m=5$
2	3.45	3.54
3	3.63	3.72
7	3.89	3.98
11	4.00	4.09
15	4.08	4.17
19	4.13	4.23
23	4.18	4.27
31	4.24	4.335
35	4.27	4.36
43	4.32	4.404
59	4.38	4.47

Table 4 - "Good" PN Codes

Number of Subsequences	Subsequence Lengths	Code Length	Ambiguity Resolution	γ Maximum
3	2, 3, 23	138	10.4	33.98
3	2, 3, 43	258	19.4	37.33
3	2, 15, 19	570	42.8	32.41
3	2, 15, 35	1050	78.8	25.20
3	2, 31, 35	2170	162.8	35.97
3	2, 35, 59	4130	309.8	38.62
5	2, 3, 7, 11, 19	8778	658.4	78.16
5	2, 3, 7, 19, 23	18354	1,376.6	79.65
5	2, 3, 7, 23, 35	33810	2,535.8	77.52
5	2, 7, 11, 19, 23	67298	5,047.4	76.26
5	2, 3, 15, 35, 43	135450	10,158.8	65.36
5	2, 7, 19, 23, 43	263074	19,730.6	83.57
5	2, 11, 23, 31, 35	549010	41,175.8	80.51
5	2, 19, 23, 31, 43	1165042	87,378.2	82.76

3.6 The Regenerative Ranging Code

In [9], the problem of regenerative ranging was discussed. Regeneration involves tracking the ranging code on the spacecraft and then re-modulating it onto the downlink. The advantage of this is that there is the potential for increasing the $\frac{P_r}{N_0}$ received on the ground by up to 30 dB. To accomplish that, a PN code of length 1,009,470 was developed, using the subsequences of length 2, 7, 11, 15, 19, and 23. The code was constructed by AND'ing the last five components and then OR'ing the result with the length 2 component. This formed a PN code whose C_2 is very high (0.954), but whose ΔC_i's are very low (0.0456). This code has the nice property that the correlation process is very easy to implement in discrete logic, which is very important for circuitry that must go on a spacecraft.

However, this code is not good when spacecraft regeneration is not used. The γ of this code is 403.35; its ambiguity is 75,710.4 km, which means its γ maximum is 145.07. This is not a problem for the spacecraft regeneration case, since the minimum $\frac{P_r}{N_0}$ the spacecraft will see is 27 dB-Hz. But, for the case where the $\frac{P_r}{N_0}$ is -10 dB-Hz, the code is not acceptable.

4 Operational Considerations

One of the biggest advantages of PN ranging over sequential ranging is that PN ranging configuration is much easier to maintain from a spacecraft operations view. Each spacecraft mission has an operations team that decides the activities, power budget, and required radiometric data for the different phases of the mission. They perform the ranging link margin analysis and instruct the DSN to use particular ranging settings in order to get the ranging measurement performance they desire.

In general, the goal is to get as many range measurements as possible with as small a variance as possible. There is a fundamental tradeoff between integrating each range point for a longer time (to obtain a smaller variance) and the number of range points obtained during a spacecraft pass. As can be seen in the previous sections, for a required variance, the amount of time to integrate each range point varies inversely with the ranging SNR. Determining the integration time is also affected by the need to keep a low probability of error in the determination of the components that resolve the ranging ambiguity.

Table 5 - Code Comparison

Number of Sequential Components	Sequential Ambiguity Resolution	PN Length	PN Ambiguity Resolution	Sequential γ Maximum	PN γ
7	9.6	138	10.4	39.12	33.98
8	19.2	258	19.4	47.06	37.33
9	38.4	570	42.8	54.75	32.41
10	76.8	1050	78.8	62.55	25.20
11	153.6	2170	162.8	70.45	35.97
12	307.2	4130	309.8	78.45	38.62
13	614.4	8778	658.4	86.54	78.16
14	1,228.8	18354	1,376.6	94.70	79.65
15	2,457.6	33810	2,535.8	102.94	77.52
16	4,915.2	67298	5,047.4	111.25	76.26
17	9,830.4	135450	10,158.8	119.62	65.36
18	19,660.8	263074	19,730.6	128.05	83.57
19	39,329.6	549010	41,175.8	136.53	80.51
20	78,643.2	1165042	87,378.2	145.07	82.76

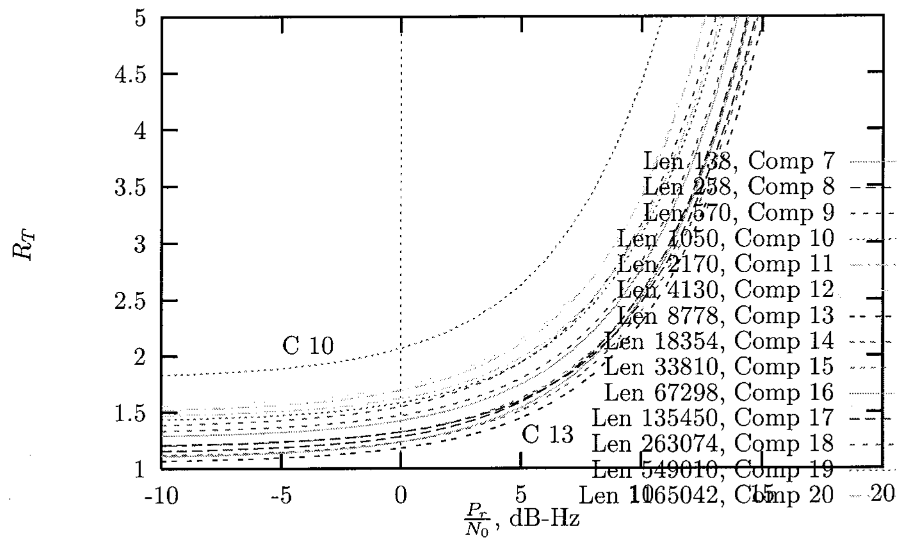

Figure 1. R_T versus $\frac{P_r}{N_0}$, all PN codes

4.1 Sequential Tone Ranging

Using sequential ranging requires the spacecraft operations team to specify the inputs T_1, T_2, Clock Component, and Last Component; specification of the Clock and Last Components provides the number of components, n. As discussed previously, the Clock Component is a number that specifies the highest frequency squarewave tone. Most missions use Clock Component of 4, which corresponds to a 1 MHz tone. Since the tones are sent in sequence, the correlation of the received signal must start at the beginning of the sequence, requiring a priori knowledge of the round trip light time (RTLT) and of the start time of the code transmission. Also, the integration time is fixed by the code definition; for example, it is not possible to integrate the Clock Component longer than T_1.

The steps for determining the sequential ranging settings are:

1. Perform the ranging link margin analysis to determine the $\frac{P_r}{N_0}$ at the receiving ground antenna.
2. Look up the required ranging ambiguity, allowable probability of error (1 - P_{acq}), and desired range variance for this phase of the spacecraft mission.
3. Compute the Last Component from the required ranging ambiguity.
4. Compute the number of components, n, using the Clock and Last Components.
5. Using (11), compute T2 with inputs n and $\frac{P_r}{N_0}$.
6. Using (3), compute T1 with inputs $\frac{P_r}{N_0}$ and the desired range variance.
7. Compute the cycle time, T_{cyc}, using (2), to see if this produces an acceptable number of range points during the

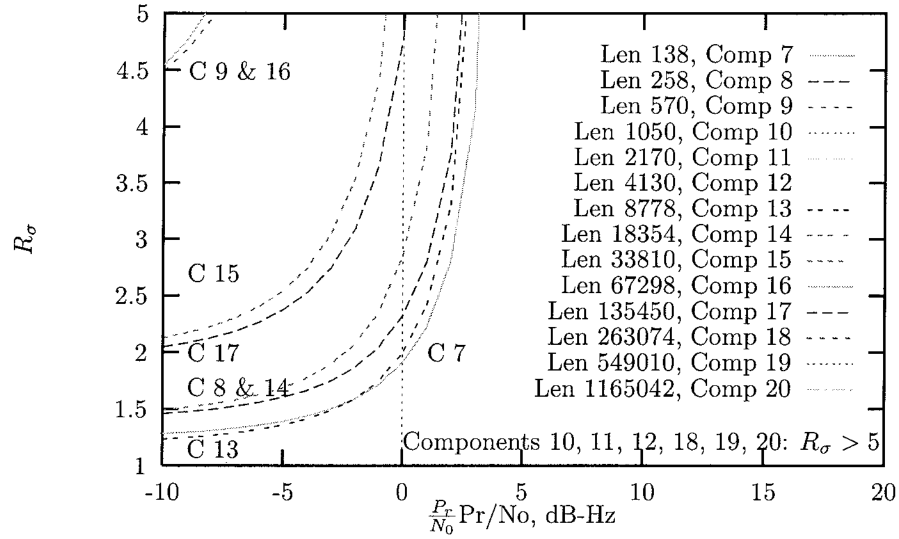

Figure 2. R_σ versus $\frac{P_r}{N_0}$, all PN codes

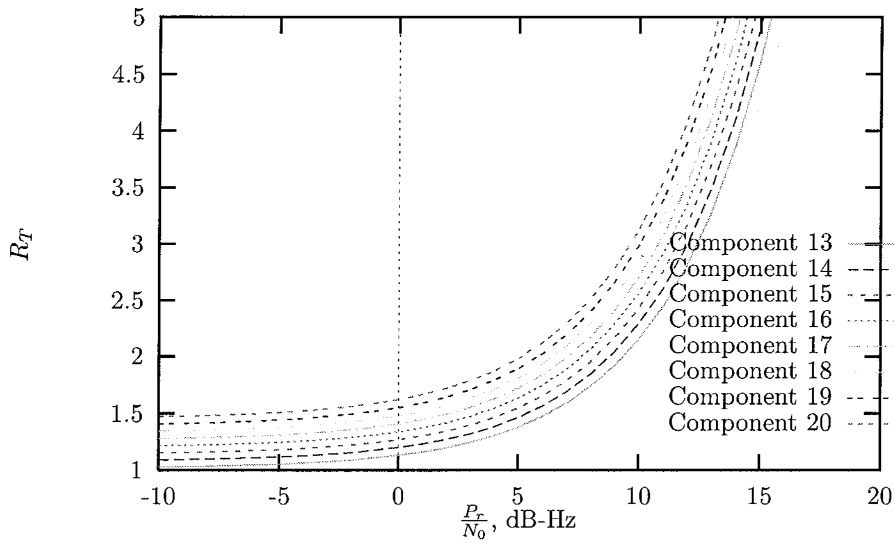

Figure 3. R_T versus $\frac{P_r}{N_0}$, Length 1165042 PN code

pass. Iterate using a larger desired range variance if a shorter cycle time is necessary.

4.2 PN Tone Ranging

For the PN ranging, the ambiguity resolution is fixed by the PN tones selected. The required integration time of the PN code is independent of the actual code construction. This means that the integration time can be modified in real time (if the signal strength assumptions were overly optimistic). Also, since the PN code is repeating with a short period (the longest code discussed here takes 0.5 seconds to cycle through) and the correlation process can start in any phase orientation, there is no need to know the transmission start time and RTLT.

The steps for determining the PN ranging settings are:

1. Perform the ranging link margin analysis to determine the $\frac{P_r}{N_0}$ at the receiving ground antenna.
2. Select the PN code subsequences, based on the desired ambiguity resolution.
3. Using (A-30), compute the range point integration time, T_{int}, with inputs $\frac{P_r}{N_0}$ and desired range variance.

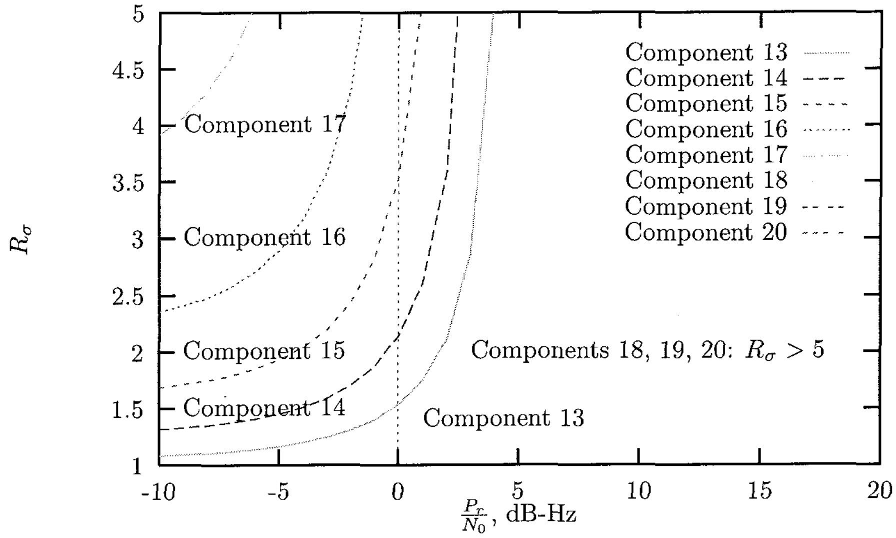

Figure 4. R_σ versus $\frac{P_r}{N_0}$, Length 1165042 PN code

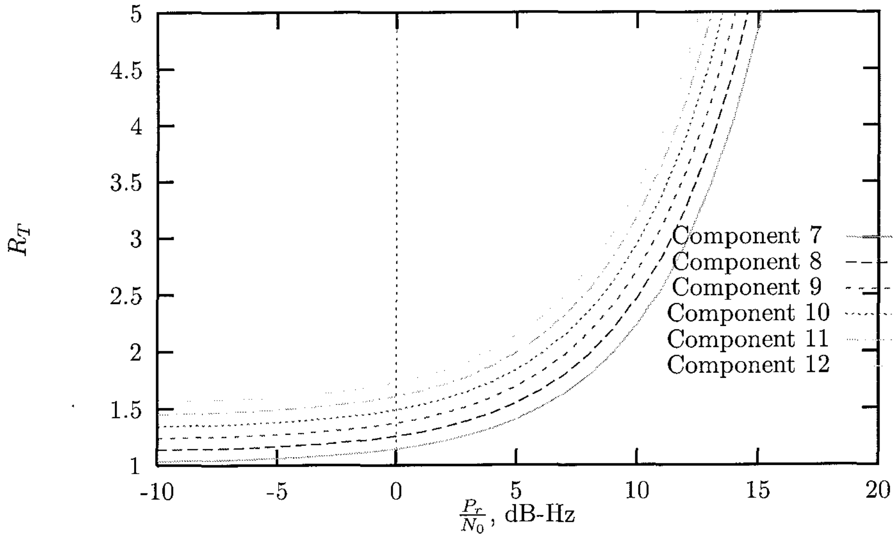

Figure 5. R_T versus $\frac{P_r}{N_0}$, Length 4130 PN code

4. Check the probability of error (1 - P_{acq}) for this T_{int} and $\frac{P_r}{N_0}$ to make sure it is below the allowable probability of error. Increase T_{int} if necessary.

4.3 Operational Details

The range type (sequential or PN) and the ranging parameters are supplied to the DSN stations in a spacecraft ranging configuration file. There is a separate spacecraft ranging configuration file for each spacecraft. This file is updated periodically by the spacecraft operations team and transmitted to the DSN for distribution to all stations.

If sequential ranging is specified, the file must contain up-to-date values for T_1, T_2, Clock Component, and Last Component. When signal strength changes over the course of the mission, the parameters must be updated.

If PN ranging is specified, there are two options for specifying the integration time:

1. The file has an up-to-date value for T_{int} which will be used for the ranging configuration.
2. The file has a target range sigma value, which is the square root of the desired range variance. For this option, the

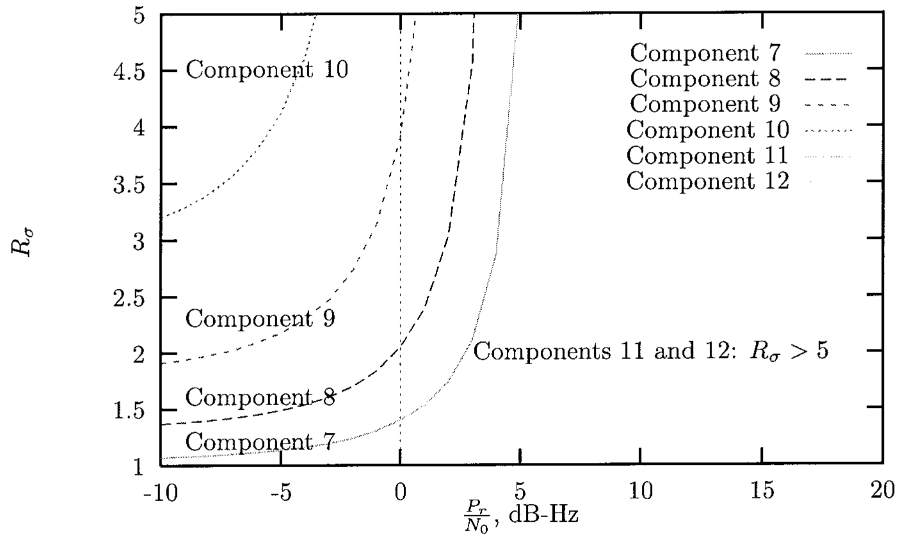

Figure 6. R_σ versus $\frac{P_r}{N_0}$, Length 4130 PN code

system will compute the T_{int} from this desired sigma and the $\frac{P_r}{N_0}$ supplied with the per-pass signal level prediction files. The system will also check that the computed T_{int} yields a P_{acq} greater than 0.999.

Using the sigma specification option means that the spacecraft operations team does not have to update the spacecraft ranging configuration file to adjust for increasing spacecraft distance. They only need to update the file if the target range variance has changed for the different phases of the mission. PN tone ranging will be much simpler to use and maintain from a spacecraft operations teams standpoint.

5 Conclusions

The performance metrics for comparing sequential and pseudo-noise ranging codes have been presented. A set of pseudo-noise ranging codes has been demonstrated to provide better performance (in terms of integration time and variance) than sequential codes, for the same measurement ambiguity and probability of acquisition. These codes are operationally easier to configure and use than the current sequential codes. While there is still research on the system aspects of these codes that needs to be done, the proposed codes provide a rare convergence - improving performance and decreasing operations complexity.

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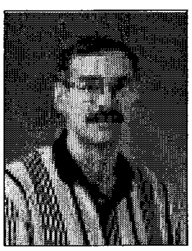

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Appendix A: Integration Time for Sinewave Correlation

Due to filtering on the spacecraft, the clock component of the ranging code is a sine wave when it is received on the ground. The equation for finding the phase as a fraction of a cycle for sine wave ranging is:

$$x = \frac{1}{2\pi}\tan^{-1}\left(\frac{Q}{I}\right) \tag{A-1}$$

Where I is the inphase signal, Q is the quadrature signal, and x is in the range $\left[-\frac{1}{2},\frac{1}{2}\right]$. The correlation of a sine wave with square wave (the replica of what was sent) gives the following result:

$$R(\tau) = A_1 A_2 \frac{2}{\pi}\cos\left(\frac{2\pi}{T_s}\tau\right) \tag{A-2}$$

Where A_1 is the peak amplitude of the sine wave, A_2 is the amplitude of the square wave, and T_s is the period of the two signals.

After the integration of I and Q, we have the signal plus noise terms, n_i and n_q, respectively. The noise terms have equal variances (σ_n^2), which are equal to:

$$\sigma_n^2 = \frac{N_0}{2T} \tag{A-3}$$

$$= \frac{P_r}{2T\frac{P_r}{N_0}} \tag{A-4}$$

Where T is the integration time. Thus:

$$x = \frac{1}{2\pi}\tan^{-1}\left(\frac{Q+n_q}{I+n_i}\right) \tag{A-5}$$

We let Y and Z be random variables:

$$Y = Q + n_q \tag{A-6}$$

$$Z = I + n_i \tag{A-7}$$

Y has mean Q, Z has mean I, and the variance of both is σ_n^2. From [10], equation 7-77, we have the following definition for a function, g, of two random variables:

$$\sigma_{g(Y,Z)}^2 \approx \left(\frac{\alpha g}{\alpha Y}\right)^2 \mu_{20} + \left(\frac{\alpha g}{\alpha Z}\right)^2 \mu_{02} + 2\frac{\alpha g}{\alpha Y}\frac{\alpha g}{\alpha Z}\mu_{11} \tag{A-8}$$

Where:

$$\mu_{rk} = E\left\{\left(Y-\bar{Y}\right)^r\left(Z-\bar{Z}\right)^k\right\} \tag{A-9}$$

For our case:

$$\mu_{20} = E\left(\left(Y-\bar{Y}\right)^2\right) \tag{A-10}$$

$$= \sigma_n^2 \tag{A-11}$$

$$\mu_{02} = E\left(\left(Z-\bar{Z}\right)^2\right) \tag{A-12}$$

$$= \sigma_n^2 \tag{A-13}$$

$$\mu_{11} = E\left(\left(Y-\bar{Y}\right)\left(Z-\bar{Z}\right)\right) \tag{A-14}$$

$$= E\left(\left(Y-\bar{Y}\right)\right)E\left(\left(Z-\bar{Z}\right)\right)$$

$$= 0 \tag{A-15}$$

$$g(Y,Z) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Y}{Z}\right) \tag{A-16}$$

This gives us:

$$\frac{\alpha g}{\alpha Y} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{1+\left(\frac{Y}{Z}\right)^2}\right)\left(\frac{1}{Z}\right) \tag{A-17}$$

$$\frac{\alpha g}{\alpha Z} = \left(\frac{1}{2\pi}\right)\left(\frac{1}{1+\left(\frac{Y}{Z}\right)^2}\right)\left(\frac{-Y}{Z^2}\right) \tag{A-18}$$

(A-19)

Which then leads to:

$$\sigma_{g(Y,Z)}^2 = \frac{1}{4\pi^2}\left[\frac{1}{Z^2}\left(\frac{1}{1+\left(\frac{Y}{Z}\right)^2}\right)^2\sigma_n^2 + \frac{Y^2}{Z^4}\left(\frac{1}{1+\left(\frac{Y}{Z}\right)^2}\right)^2\sigma_n^2\right] \tag{A-20}$$

$$= \frac{\sigma_n^2}{4\pi^2}\left(\frac{Z^2}{Z^2+Y^2}\right)^2\left[\frac{1}{Z^2}+\frac{Y^2}{Z^4}\right] \tag{A-21}$$

$$= \frac{\sigma_n^2}{4\pi^2}\frac{1}{Z^2+Y^2} \tag{A-22}$$

Evaluating at $\bar{Y}$ and $\bar{Z}$, we get:

$$\sigma^2_{g(\bar{Y},\bar{Z})} = \frac{\sigma_n^2}{4\pi^2}\frac{1}{I^2+Q^2} \tag{A-23}$$

For our case, A_1 is equal to $\sqrt{2P_rC}$ (where C is the correlation factor of the ranging code) and A_2 is equal to 1. this gives us:

$$\begin{aligned}
I &= \sqrt{2P_rC}\left(\frac{2}{\pi}\right)\cos\left(\frac{2\pi}{T_s}t\right) && \text{(A-24)}\\
Q &= \sqrt{2P_rC}\left(\frac{2}{\pi}\right)\sin\left(\frac{2\pi}{T_s}t\right) && \text{(A-25)}\\
\sigma_x^2 &= \sigma^2_{g(Y,Z)} && \text{(A-26)}\\
&= \frac{\sigma_n^2}{4\pi^2}\frac{1}{2P_rC\left(\frac{4}{\pi^2}\right)\left(\cos^2\left(\frac{2\pi}{T_s}t\right)+\sin^2\left(\frac{2\pi}{T}t\right)\right)}\\
&= \frac{P_r}{2T\frac{P_r}{N_0}}\frac{1}{4\pi^2}\frac{1}{\frac{8}{\pi^2}P_rC}\\
\sigma_x^2 &= \left(64T\frac{P_r}{N_0}C\right)^{-1} && \text{(A-27)}
\end{aligned}$$

The units of σ_x^2 are cycles^2. To convert to units of seconds, we normalize by the sine wave frequency, F (which is the inverse of T_s):

$$\begin{aligned}
\sigma^2 &= \frac{\sigma_x^2}{F^2} && \text{(A-28)}\\
&= \left(64T\frac{P_r}{N_0}F^2C\right)^{-1} && \text{(A-29)}
\end{aligned}$$

Finally, to get the integration time, we solve for T:

$$T = \left(64\sigma^2\frac{P_r}{N_0}F^2C\right)^{-1} \tag{A-30}$$

Appendix B: Probability of Acquiring a PN Subsequence

When we correlate the PN ranging code against a PN subsequence of length m, we get either C_{max} if the codes are aligned, or C_{min} if they are not aligned, plus noise (which we assume is gaussian, with variance σ_n^2). For the correct position to be detected (or acquired), the result of the correlation when the sequences are aligned must be greater than the $m-1$ results when the sequences are not aligned. For a value x, and a received ranging power of P_r, we get the following:

$$\begin{aligned}
\text{Prob}\,(\text{acquisition}|x) &= \text{Prob}\left(\left(C_{max}\sqrt{P_r}+n(t)\right)>x\right)\left[\text{Prob}\left(\left(C_{min}\sqrt{P_r}+n(t)\right)<x\right)\right]^{m-1} && \text{(B-1)}\\
&= \text{Prob}\left(\frac{n(t)}{\sigma_n}>\left(\frac{x-C_{max}\sqrt{P_r}}{\sigma_n}\right)\right)\left[\text{Prob}\left(\frac{n(t)}{\sigma_n}<\left(\frac{x-C_{min}\sqrt{P_r}}{\sigma_n}\right)\right)\right]^{m-1} && \text{(B-2)}
\end{aligned}$$

The probability of acquisition, P_a, is then:

$$P_a = \int_{-\infty}^{\infty}\text{Prob}\,(\text{acquisition}|x) \tag{B-3}$$

We make the follwing substitution:

$$u = \frac{x-C_{max}\sqrt{P_r}}{\sigma_n} \tag{B-4}$$

Using the substitution and the fact that the noise is gaussian, we get:

$$P_a = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \left[\int_{-\infty}^{\frac{(C_{max}-C_{min})\sqrt{P_r}}{\sigma_n}+u} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv \right]^{m-1} du \tag{B-5}$$

Now, σ_n is defined as:

$$\sigma_n = \sqrt{N_0 \frac{T_{int}}{2}} \tag{B-6}$$

We make the following definitions:

$$\Delta C = C_{max} - C_{min} \tag{B-7}$$

$$\beta = \Delta C \sqrt{\frac{P_r}{N_0} T_{int}} \tag{B-8}$$

Which gives us:

$$P_a = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \left[\int_{-\infty}^{\sqrt{2}\beta+u} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} dt \right]^{m-1} dx \tag{B-9}$$

Defining x and v:

$$x = \sqrt{2}u \tag{B-10}$$

$$v = \sqrt{2}t \tag{B-11}$$

Substituting, we get:

$$P_a = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \left[\int_{-\infty}^{\beta+x} \frac{1}{\sqrt{\pi}} e^{-t^2} dt \right]^{m-1} dx \tag{B-12}$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \left(\frac{1+\mathrm{erf}(x+\beta)}{2} \right)^{m-1} dx \tag{B-13}$$

Where erf $(\cdot)$ is defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{B-14}$$