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TEST OF ϕ^4 MODEL PREDICTIONS NEAR THE ^{3}He LIQUID-GAS CRITICAL POINT

M. Barmatz*, Fang Zhong, and Inseob Hahn

Jet Propulsion Laboratory, California Institute of Technology
Pasadena, CA, USA

ABSTRACT

NASA is supporting the development of an experiment called MISTE (Microgravity Scaling Theory Experiment) for a future International Space Station mission. The main objective of this flight experiment is to perform in-situ *PVT*, heat capacity at constant volume, C_V, and isothermal susceptibility, χ_T, measurements in the asymptotic region near the ^{3}He liquid-gas critical point. On the ground, gravity induces a measurable density gradient for reduced temperatures $|T/T_c - 1| < 10^{-4}$. An accurate test of theoretical predictions within the asymptotic region close to the critical point is limited because of this gravity effect. Precision ground-based measurements are now being performed in the crossover region away from the critical point in preparation for this flight experiment. The ϕ^4 model, applied to the $O(1)$ universality class, was tested using χ_T measurements in the crossover region of ^{3}He. The susceptibility measurements were performed in a 0.05 cm high sample cell along the critical isochore and coexistence curve over the range $10^{-5} < |T/T_c - 1| < 10^{-1}$. This renormalization-group based ϕ^4 model with a minimal set of *three* adjustable parameters provided an good fit to the χ_T data in the gravity-free crossover region. The agreement between the ϕ^4 model calculations and the experimental χ_T measurements extended beyond the theoretically predicted crossover range.

INTRODUCTION

The Microgravity Scaling Theory Experiment (MISTE) will perform static measurements along various thermodynamic paths near the liquid-gas critical point of ^{3}He in a microgravity environment. Due to the strong divergence in compressibility of a fluid near its critical point, the quality of Earth-bound experiments near the transition is hampered by the gravity rounding effect. The result of a successful microgravity flight experiment will provide high quality data close to the critical point that can be used to test asymptotic and crossover theories. Historically, similar low temperature experiments studying the superfluid transition of helium have been successfully performed on the Space Shuttle [1,2].

The main objective of the MISTE experiment is to the test theoretical predictions of the behavior of thermodynamic quantities near the ^{3}He liquid-gas critical point. In this paper, we test the ϕ^4 minimal renormalization scheme as applied to the $O(1)$ universality class that includes the liquid-gas critical point. This minimal renormalization approach, developed by V. Dohm and co-

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* corresponding author Martin.Barmatz@squid.jpl.nasa.gov

workers [3,4], was first applied to measurements near the superfluid transition [5] that is in the $O(2)$ universality class. Previously, ^{3}He measurements in the crossover region along the critical isochore were analyzed using this ϕ^4 model as well as a crossover parametric model (CPM) [6,7]. In that paper, measurements of the heat capacity at constant volume and the isothermal susceptibility (above the transition only) were fit well by both the first-principal ϕ^4 model and the phenomenological CPM model. Isothermal susceptibility measurements above the ^{3}He transition were also used to demonstrate universal crossover behavior using this ϕ^4 model [8]. The present study goes further in applying the ϕ^4 model to susceptibility measurements along the critical isochore above the transition and along the coexistence curve below the transition. The model parameters obtained from this susceptibility fit are also used to predict the behavior of the correlation length along the critical isochore above the ^{3}He transition.

THEORETICAL DEVELOPMENT

There are several theoretical approaches to analyze measurements in the crossover region. The theoretical expression for the isothermal susceptibility that takes into account the asymptotic and correction-to-scaling behavior is given by

$$\chi_T^{\pm *} = \left(P_c/\rho_c^2\right)\chi_T^{\pm} = \Gamma_0^{\pm}|t|^{-\gamma}\left[1+\Gamma_1^{\pm}|t|^{\Delta_s}+\ldots\right], \tag{1}$$

where $\gamma \cong 1.24$ is a universal critical amplitude and $\Gamma_0^{\pm}$ are system-dependent critical amplitudes. The "+" and "–" superscripts correspond to positive and negative reduced temperatures $t = (T - T_c)/T_c$, respectively. The critical parameters for the ^{3}He critical point are $P_c = 114.6$ kPa, $\rho_c = 0.04145$ g/cm^3 and $T_c = 3.31556$ K. The expansion in the brackets is commonly known as the Wegner expansion and contains the leading system-independent amplitudes $\Gamma_1^{\pm}$, and an additional universal correction-to-scaling exponent [9], $\Delta_s \cong 0.52$. One difficulty in applying Eq. (1) to test experimental measurements in the crossover region is the large number of Wegner adjustable parameters required and the slow convergence of this expansion. The motivation for using the ϕ^4 model approach is that only two model parameters are needed to describe the susceptibility and all asymptotic and crossover critical amplitude ratios.

A brief description of the ϕ^4 model used for this study will now be presented. The Hamiltonian associated with the ϕ^4 model is given by

$$H_\phi = \int d^3x\left\{\frac{1}{2}r_0\phi_0^2 + \frac{1}{2}\left(\nabla\phi_0\right)^2 + u_0\phi_0^4\right\}. \tag{2}$$

The parameter r_0 is related to the reduced temperature by $r_0 = a_0 t$, where a_0 is a nonuniversal constant. The total Hamiltonian includes an additional analytical background-free energy term H_0. The bare order parameter ϕ_0 and the nonuniversal, bare coupling parameters u_0 and r_0 are renormalized to u and r using field theoretical techniques. Effective coupling functions, $\bar{u}(\ell)$ and $\bar{r}(\ell)$ are defined in terms of a flow parameter, ℓ, that describes the behavior of the system as it moves away from the critical point. The value $\ell = 0$ corresponds to the critical point (fixed point defined by u^*). The renormalized bare parameters, u and r, are defined at the reference point, $\ell = 1$, as $u \equiv \bar{u}(\ell = 1)$ and $r \equiv \bar{r}(\ell = 1)$. The reduced temperature corresponding to $\ell = 1$

depends on the relative distance from the critical point defined by $u - u^*$. The flow parameter is simply related to the correlation length through the expression, $\ell = (\mu\xi)^{-1}$, where μ^{-1} is an arbitrary reference length. The ϕ^4 model self-consistently determines all critical exponents and critical amplitude ratios. The critical amplitudes themselves are system-dependent and must be determined from fits to experiment.

Within this model, the temperature and susceptibility can be written as

$$|t| = t_0 b_\pm(l) l^{1/\nu} \exp[-F_r(l)], \tag{3}$$

$$\chi_T^{\pm *} = \chi_0 l^{-\gamma/\nu} \exp[-F_\phi(l)] / f_\pm(l). \tag{4}$$

The functions $b_\pm$, F_r, F_ϕ, and $f_\pm$ are evaluated from Borel resummations of higher-order perturbation series. There are two system dependent parameters, t_0 and χ_0. These parameters can be expressed in terms of the more fundamental ϕ^4 model parameters a, μ and u. Two out of the three fundamental parameters are independent. We choose a and u as independent parameters and fixed $u = 0.999u^*$.

Baguls and Bervillier [10] have concluded that the upper bound of the range of validity of the ϕ^4 model corresponds to the Wegner expansion up to the first order term. Other corrections neglected by this ϕ^4 model could influence theoretical predictions for reduced temperatures beyond this range. One of the objectives of this study is to test the ϕ^4-model range of validity within the ^{3}He crossover region.

COMPARISON BETWEEN THEORY AND EXPERIMENT

The isothermal susceptibility, $\chi_T = \rho(\partial\rho/\partial P)_T$, was determined from P,ρ measurements taken along isotherms both above and below the transition [6,11]. The susceptibility along the critical isochore was obtained from isotherm measurements above the transition. The susceptibility in the liquid and gas phases below the transition was obtained from the intersection of the P,ρ isotherm measurements at the coexistence curve.

The log-log plot in Fig. 1 shows the result of a simultaneous fit of the susceptibility data for both $T > T_c$ and $T < T_c$. The data had a weight of $w = 1/\sigma = 1/(0.02\chi_T^*)$ that represented a standard deviation of 2% in the susceptibility measurements. Since χ_T $(T > T_c)$ has higher precision than χ_T $(T < T_c)$, its weight was increased by a factor of 4.94 that corresponded to the theoretically predicted amplitude ratio Γ_0^+/Γ_0^-. The critical temperature was also included as an adjustable parameter in this fit. The ϕ^4 parameters $a = 0.1375$ and $u = 2.115\times10^{-4}$ and the critical temperature $T_c = 3.315533$ obtained from this fit are in good agreement with the earlier analysis of only the $T > T_c$ susceptibility data [8]. The asymptotic and first Wegner correction-to-scaling amplitudes obtained from this fit are $\Gamma_0^+ = 0.1495$, $\Gamma_0^- = 0.03028$, $\Gamma_1^+ = 0.9909$, and $\Gamma_1^- = 4.34$. A more sensitive test of the fit can be obtained by plotting the data on a linear rather than a log scale by removing the power law divergence. Figure 2 shows such a plot. The dot-dashed lines represent the expected asymptotic behavior and thus define the value of the asymptotic critical

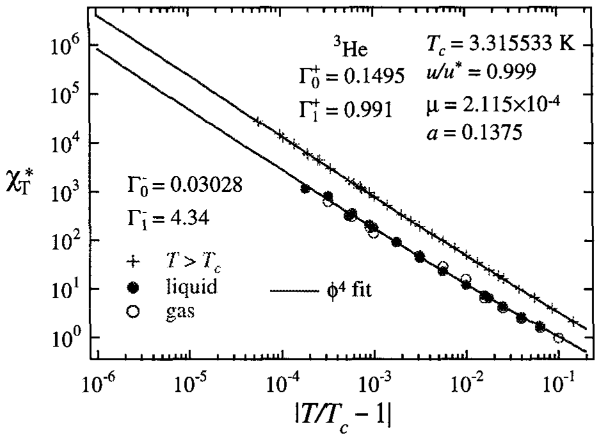

Fig. 1. Log-log plot of isothermal susceptibility versus reduced temperature. The solid lines correspond to the best fit to the ϕ^4 model. The asymptotic and first Wegner correction critical amplitudes calculated from the fit are also shown.

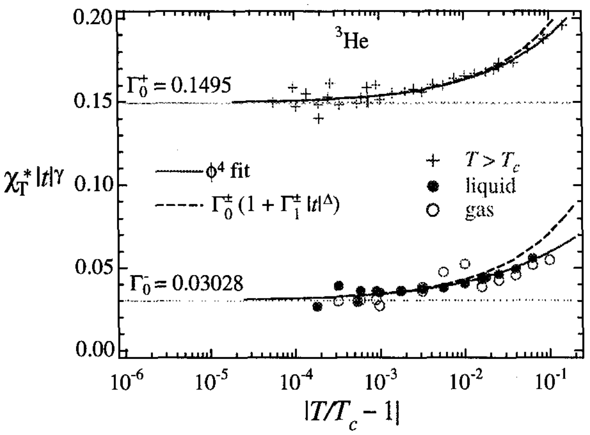

Fig. 2. Linear-log plot of the susceptibility after removing the power law divergence. Solid lines show best fit to the ϕ^4 model. The dot-dashed lines correspond to the predicted asymptotic behavior. The dashed lines are calculated with the Wegner expansion to first order.

amplitudes, $\Gamma_0^\pm$. The experimental measurements deviate from the asymptotic behavior for reduced temperatures $|t| \geq 10^{-4}$. This indicates that almost all of the ground-based measurements, unaffected by gravity, are in the crossover (or correction-to-scaling) region. The dashed lines are calculated using the Wegner expansion to first order only. It is seen that these dashed lines begin to deviate from the ϕ^4 prediction (solid lines) for reduced temperatures $|t| \geq 10^{-2}$. Thus, this implies that higher order Wegner correction terms become important for reduced temperatures above $|t| \cong 10^{-2}$. However, Figs. 1 and 2 show that the ϕ^4 model fits the data farther out to a reduced temperature $|t| \cong 10^{-1}$. To verify that the resultant model parameters and calculated $\Gamma_1^\pm$ are not significantly affected by the fitting range of the data, we removed measurements with temperatures greater than $|t| = 10^{-2}$ and reanalyzed the data. For this smaller temperature range, $\Gamma_1^\pm$ both increased by 19%, however this small increase did not significantly change the reduced temperatures where the Wegner expansion to first order (dashed lines) begins to deviate from the complete ϕ^4 fit (solid lines) in Figs. 1 and 2.

Once μ is determined from the χ_T fit, the correlation length can be calculated using $\xi = (\mu\ell)^{-1}$. Figure 3 shows the ϕ^4-model prediction for the dimensionless correlation length in ^{3}He along the critical isochore above the transition. Again, we have removed the power law divergence by plotting $\xi^* t^\nu$ against reduced temperature. The length scale, l_0, used to normalize the correlation length is $l_0 = (k_B T_c / P_c)^{1/3}$, where k_B is Boltzmann's constant. The dashed line in Fig. 3 represents the theoretically predicted asymptotic behavior and defines the calculated correlation length critical amplitude $\xi_0^* = 0.373$. This critical amplitude is consistent with the experimentally determined value $\xi_0^* = 0.35$ [12].

CONCLUSION

We have fit the ϕ^4 model to ^{3}He measurements of the isothermal susceptibility both above and below the transition. A good fit between theory and experiment was obtained for the reduced temperature range $10^{-4}<|t|<10^{-1}$. This ϕ^4-model approach appears to fit the ^{3}He susceptibility measurements beyond the theoretically predicted range of validity that is $|t|\leq 10^{-2}$. This conclusion was also recently reached in a study of universal crossover behavior in both ^{3}He and Xenon [8]. One possible explanation for such a good fit beyond the theoretical range of validity is that higher order correction terms neglected in the presently developed ϕ^4-model are small. Further analysis of crossover measurements near the liquid-gas critical points in other fluids may shed additional light on this unexpected result. It should also be noted that the determination of $\Gamma_1^\pm$ is strongly dependent on the values of $\Gamma_0^\pm$ that cannot be accurately determined from ground-based measurements. The MISTE flight experiment plans to precisely determine the critical temperature and the leading susceptibility critical amplitudes, which should significantly reduce the uncertainty in determine the crossover critical amplitudes.

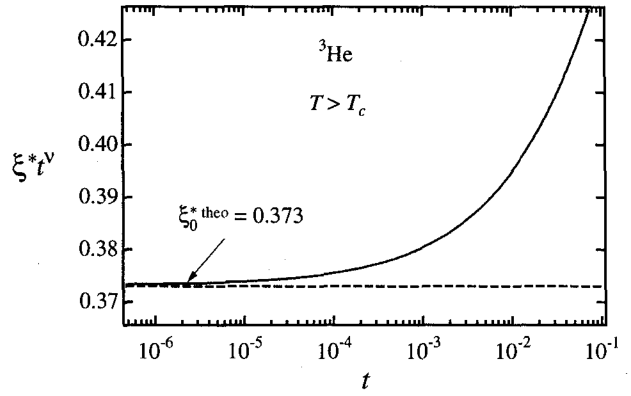

Fig. 3. Dimensionless correlation length above the transition versus reduced temperature. The solid line was calculated from the best-fit ϕ^4 parameters. The dashed line corresponds to the predicted asymptotic critical amplitude ξ_0^*.

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