Abstract

In recent years the technological advances of adaptive optics have enabled a great deal of innovative science. In this lecture I review the system-level design of modern astronomical AO instruments, and discuss their current capabilities. I start by reviewing the basic optics needed to understand the first-order properties of astronomical telescopes, and then introduce the powerful techniques of Fourier optics. I briefly discuss the effects of the turbulent atmosphere on imaging, and the extent to which AO can correct those effects. Specific implementations of astronomical AO, including those based on curvature and Shack-Hartmann wavefront sensors, are discussed, with detailed examples of design tradeoffs and scientific results drawn from the Palomar AO system.
Infrared Wavefront Sensors

- it would also be nice, in many observing projects, to have an infrared WFS...
- often, especially in star-formation regions, the bright stars are deeply embedded, and much brighter in the NIR than at visible...

Deformable mirrors come in many sizes...

- from 13 to > 900 actuators (degrees of freedom)
Outline of the talk:

- Basic Telescope Optics
  - telescope model: it's just like a big lens
  - "first order" quantities
- Fourier Optics
  - pupil and image planes are (essentially) Fourier conjugates
- The Atmosphere Intervenes...
  - seeing is complex...
  - …but let’s just describe it by r0
- Adaptive Optics
  - AO = WFS + DM:
    - measure phase distortions; correct them
- Specific Implementations in Astronomy
  - Shack-Hartmann wavefront sensing
  - curvature wavefront sensing

AO increases spatial resolution

Lick Hα 234 - a young star cluster

Without adaptive optics

With Lick adaptive optics

5 or 6 separate stars
AO increases peak point source intensity

A Cassegrain telescope... is like a big lens:

Ancient optical jargon: f/# ["f-number"] = f/D = 16, say;

Palomar Cass focus is f/16 (this sketch is f/4 or so)

NB: the f/# of the final beam determines most of the basic optical parameters...
Some “first-order” quantities:

1. “Plate scale”: a slightly different angle on sky, $d\theta$, is offset in the focal plane $dx = f d\theta$ (see diagram). 
so $\text{arcsec/mm} = 1/f = 1/[f(\#)D]$ (2E5 "/rad)

2. Diffraction-limited spot size: the optics-lab rule of thumb is $\theta(\#)$. Notice this is the same as

$$\lambda \quad \theta(\#) = \theta(D) f$$

the usual diffraction limit in angular units (cf. the Rayleigh criterion, 1.22 $\theta/D$) 
as above, the factor to convert angle to focal-plane distance

3. “Throughput” (a.k.a. étendue, Helmholtz invariant).... 
AO...in one dimension this is $Dd\theta = dx(\tan^{-1}(D/f))$

The telescope is now just a pupil plane...

and a focal plane...

...and these are related by a Fourier transform: 
(cf. the “Fourier transforming properties of a lens”)
The basic equation of Fourier optics gives the image (in the focal plane) as a function of the shape of the telescope aperture (in the pupil plane) and of the phase distortions above it:

\[ \text{Image}(x, y) = \left| F.T.\{A(\xi, \eta) e^{i\phi(\xi, \eta)}\} \right|^2 \]

Aperture fn. A \quad Phase fn. F

A and F are both real functions in the pupil plane (a plane optically conjugate to the primary mirror, or "aperture").

**Special case:** plane incident wavefronts... \( F = \text{const} \) i.e. point source seen by a telescope in space, or with a perfect AO system that completely correct the phase-scrambling effects of atmospheric turbulence...

Then the image is the PSF (Point Spread Function):

\[ \text{PSF} = \left| F.T.\{A\} \right|^2 \]

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**Applying Fourier Optics to Telescopes:**

The PSF from a circular aperture...

is the familiar Airy pattern:

\[ \text{Airy}(x, y) = \left| 2 \frac{J_1(\pi \theta)}{\pi \theta} \right|^2 \]

(\( \theta \) in units of \( \lambda / D \))
If you can remember only one thing...

- about Fourier transforms, remember this reciprocal-width property:

  \[ \text{F.T. \{broad Gaussian\} = a narrow Gaussian} \]
  
  (if you can remember two things, remember this one twice…)

- To illustrate, recall the complexities of atmospheric turbulence may be summarized in \( r_0 \), the transverse coherence length (a.k.a. Fried parameter)…the phase of the wavefront is roughly preserved over \( r_0 \)...
the classical, heuristic speckle model...

- The aperture $D$ is divided into $r_0$-size "seeing cells", each with random, turbulence-induced phase;
- A few cells may by chance combine coherently to produce an image (a "speckle") in the focal plane;
- Since these cells may range up to $D$ in separation, they have diffraction-limited ($\sim \lambda/D$) size (e.g. 0.1")
- But they move within an envelope $\sim \lambda/r_0$ in size (maybe 1" or so at visible)

- [there are, in this model, $N_s \sim 0.342 (D/r_0)^2$ speckles]

This is a lot like aperture synthesis, b.t.w...

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**Observations of Speckles**

Lick Observatory, 1 m telescope

<table>
<thead>
<tr>
<th>Long exposure image</th>
<th>Short exposure image</th>
<th>Image with adaptive optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta \sim 1$ arc sec</td>
<td>Speckles (each is at diffraction limit of telescope)</td>
<td></td>
</tr>
</tbody>
</table>

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Model of speckle formation:
an application of the reciprocal-width property of Fourier conjugates

\[
\text{Image} = |F.T.(A e^{i\phi})|^2
\]

Atmospheric Turbulence Summary
in slightly more detail

- recall, there is a transverse coherence scale over which the mean-square wavefront variance is about 1 rad²...

\[
r_0 \sim \lambda^{6/5} (\sec \zeta)^{-3/5} \left[ \int C_N^2(z) dz \right]
\]

... (large \( r_0 \) means good seeing...could be 10 cm at vis)
- equivalently, \( r_0 \) is the largest telescope aperture that will be diffraction limited under those seeing conditions
• if one assumes the "Taylor hypothesis", that a frozen pattern of phase flows by the telescope, the timescale of turbulence is related to \( r_0 \) by the wind velocity, \( v \):

\[
t_0 \sim \frac{r_0}{v} \sim \frac{6}{5}, \text{ inherited from } r_0;
\]

• a simple expression for the isoplanatic angle is:

\[
\theta_0 \sim 0.314 (\cos \xi) \frac{r_0}{h_{\text{eff}}} \quad (\text{Strehl drops to } 1/e=37\% \text{ at } \theta_0)
\]

(limits guide-star distance)

• here \( h_{\text{eff}} \) is an effective height of the turbulence, strongly weighted by high altitudes (this is sensible...e.g., a layer of turbulence right on primary mirror will affect lines of sight to target and guide star in the same way)

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**Fourier Optics of Real Telescopes in more detail**

A central obscuration (due to e.g. a secondary mirror) changes the PSF slightly in a couple of ways:

• the central peak is narrowed (slightly)
• the secondary maxima (Airy rings) are increased
• the secondary maxima are no longer monotonic

\[
|F.T\{A\}|^2
\]
Obscuration of the pupil due to “spiders” (supports for the secondary mirror) adds characteristic features to the PSF:

- note the four-fold symmetry
- “notches” in the Airy rings
- sharp features in pupil plane give broad features in Fourier space (the image plane)

That last version was a little exaggerated...

the spiders at Palomar look more like:

- narrower spiders give more subtle features in the image plane
Many of these features of the ideal case are seen amazingly well in the adaptively-corrected PSF:

Schematic of adaptive optics system

Feedback loop:
next cycle corrects the (small) errors of the last cycle
Classification of AO Systems

- NGS (Natural) vs. LGS (Laser) Guide Star
- Type of wavefront sensor: Shack-Hartmann; curvature
  (some other possibilities, less commonly used in facility instruments for astronomy, include shearing interferometers, phase diversity, even the classic “knife-edge” test...)

To date, curvature systems have typically achieved lock on fainter guide stars, while higher numbers of actuators have been achieved by S-H systems.

Traditionally, S-H WFS coupled to a piston-type DM, while curvature WFS is coupled to a bimorph mirror.

Astronomical observatories with AO on 3-5 m telescopes

- ESO 3.6 m telescope, Chile
- University of Hawaii
- Canada France Hawaii
- "Curvature sensing" systems
- Mt. Wilson, CA
- Lick Observatory, CA
- Mt. Palomar, CA
- Calar Alto, Spain

> 80 journal articles on AO astronomy, to date
Astronomical observatories with AO on 5-10 m telescopes

- Keck I and II, 2x10 m, Mauna Kea
- Gemini North (MKO) and South (Chile)
- Subaru, (MKO)
- ESO/VLT (4xxxm)

AO = WFS + DM
Design Features to Note:

- the FSM is conceptually part of the DM, but there's so much tip/tilt DM can't remove it
- DM works in collimated space
- DM (optically) conjugate to telescope primary (i.e. \(1/s_1+1/s_3=1/f\) thru intervening optics)
- beam is collimated and later refocussed by OAPs
- a dichroic sends visible light to WFS, infrared light to science camera

PALAO (Palomar): Shack-Hartmann WFS
A Few Design Details:

- after the dichroic, beams to WFS and to science camera travel "non-common" paths...
- these are particularly susceptible to flexure-induced aberrations (which won't be sensed, therefore won't be corrected)
- so on PALAO, WFS components are mounted on a special, compact breadboard
- FS pickoff mirror (4" patch of metallization on a transparent substrate) sends GS light to WFS; rest goes to a handy acquisition camera

Shack-Hartmann wavefront sensor concept - measure subaperture tilts
Promising variant on S-H WFS: correlations among sub-apertures

- guide star (pt. source)
- guide scene (extended)

Correlation variant on S-H (cont’d):

- guide star may now be an extended object
- each subaperture is an image on a sub-CCD
- subaps are cross-correlated (w.r.t. one that acts as a reference) to get relative tip/tilts (these are the desired phase gradients)
- this scheme has been used effectively in solar AO, where they guide on features on the sun...one has lots of photons, but the contrast is not great...
How a deformable mirror works (idealization)

**Principle of curvature WF Sensing:**

image that the phase here has "a little extra" curvature...

which gives a bright patch on the annular pupil-like image in this o.o.f. plane...

...and a corresponding faint patch in the other, complementary o.o.f. plane

So the out-of-focus pupil images contain information about the phase curvature, or second derivative (cf. the Shack-Hartmann pieces together a grid of local measurements of the first derivative, or phase slope)
DM for curvature systems

- The "bimorph" mirror is well suited to correcting phase sensed by the curvature wfs...
- The bimorph consists of two layers (electrodes) separated by an insulator
- The curvature of a section of bimorph is directly proportional to the applied voltage
- Natural match: curvature sensing/correcting
- Bimorph "sheet stock" is conveniently patterned into subapertures with photolithographic etching

Architecture of Curvature Systems

- In practice, the two defocus settings are reached by vibrating a diaphragm in the optical path
- There is a lenslet array (as with S-H), but geometry has radial symmetry matching that of the WFS and bimorph mirror
- Detection is by APD (avalanche photodiode), one detector per WFS subaperture…v. low read noise
Layout of a Curvature AO System

A Curvature WFS System:
Laser is operating at Lick Observatory, being tested at Keck

Laser guide star at Lick Observatory is working well

Images of a 15th magnitude star, \( \lambda = 2.2 \) microns
AO Performance Measures

- main metric is Strehl ratio, S; working def'n...
  \[ S = \frac{\text{PSF peak (msd)}}{\text{PSF pk (no aberr'ns)}} \]
- image FWHM is of interest, but it turns out that FWHM quickly become diffraction-limited as \( S \) rises (you quickly get core-halo structure)
- alternative def'n:
  \[ S = \frac{\text{flux in diff'n ltd core}}{\text{total image flux}} = \frac{\text{core flux}}{\text{core flux + halo flux}} \]

AO Performance Measures

A completely uncorrected telescope has \( S < 1\% \).

A very high value of Strehl would be 60\% or so (NIR).

When the Strehl ratio is decently high (g.t. 10\% or so) one may write a simple equation (the Marechal approx) relating it to the variance of the phase of the wavefront over the pupil plane:

\[ S \sim \exp(-s_F^2) \]

Zernike polynomials are convenient for describing the pupil-plane phase (they are a complete set, cf. \( Y_{\text{lm}s} \))
Strehl Prediction/Error Budget

The errors limiting pupil-plane wavefront quality are fairly well understood...i.e., one can tabulate:

\[ s^2_F = s^2_{DM} \] fitting error (finite actuator spcg.)
\[ + s^2_{BW} \] finite control-loop bandwidth
\[ + s^2_{WFS} \] noise on WFS (photon stats)
\[ + s^2_{cal} \] calibration
\[ + s^2_{iso} \] isoplanatic (off guide star axis)
\[ + s^2_{cone} \] cone effect (LGS case)
\[ + s^2_{spot} \] elongated LGS spot

Each may be related to basic experimental #'s...

e.g.  \[ s^2_{DM} = k \left( \frac{D}{r_0} \right)^{5/3} \] \( r_0 \) is the seeing cell size
\[ s^2_{BW} = (f_g/f_c)^{5/3} \] \( f_g \) is the Greenwood freq.
\[ s^2_{iso} = (\tau/\tau_0)^{5/3} \] \( \tau_0 \) is the isoplanatic angle

(see Hardy’s book for a detailed discussion)

The errors limiting pupil-plane wavefront quality are fairly well understood...i.e., one can tabulate and compare with observed Strehl ratio:
Detailed Palomar Error Budget
(Troy et al. 2000, SPIE)

- Computed theoretical expressions for the major on-axis errors of NGS system, for a bright and a faint guide star
- Found good agreement with measured S
- In poor seeing: time delays are the limit
- In good seeing: calibration errors

Some Design Trade-Offs

- Subaperture size (DM actuator spacing) is chosen to balance DM-fitting and WFS-measurement errors [this sets NGS mag. limit]
- Maximize bandwidth in control loop, but balanced by WFS-msmt error for a given GS brightness
- In LGS case, minimize WFS measurement error by using brightest feasible laser (cost limits)
- On the sky: trade off NGS brightness (WFS measurement error) vs. proximity (isoplanatic error)
Additional Performance Issues

- Guide star sensitivity limits ("sky coverage" is only ~5% for 13th mag. natural guide stars)
- Temporal stability
- Science camera capabilities
  (spectroscopy, coronagraphy, etc.)
- Ease of use, field ruggedness of AO system

STABILITY of AO SYSTEMS:
1. The Main Lobe of the PSF

...this data set has very high correction (S~60%)...

![Graph showing FWHM vs. time](image)

cf. Palomar 5 m has FWHM ~ 1.02 \( \lambda / D \) ~ 92 mas in the near-infrared
Stability of PSF Main Lobe (cont’d)

...corresponding Strehl ratios measured from the images...

![Graph showing Strehl ratios vs time](image1)

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Stability of PSF Main Lobe (cont’d)

...this is a different data set, with lower degree of correction (S~30%)...

![Graph showing FWHM vs time](image2)

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Stability of PSF Main Lobe (cont’d)

...and the corresponding Strehl ratios measured from the images...

2. Stability of the Inner Airy Rings

$t = 0, 4, 8$ seconds...
(fairly repeatable)

$t = 7$ minutes $+ 0, 4, 8$ s...
(a new pattern, but again fairly repeatable)

$t = 16$ minutes $+ 0, 4, 8$ s...
(back to 3-fold symmetry on the first Airy ring)
3. Stability of the Outer Halo

$t = 0, 2, 4, 6,$
8, 10, 12, 14 seconds… these are remnant (post-correction) speckles
Anisoplanatism study at Palomar
image quality falls off away from guide star

Lagoon Nebula imaged with PALAO / PHARO

Composite J, H, K band image
Field of view 40"x40" (at 0.04 arc sec /pixel)
On-axis K-band Strehl ~ 40%, falls to 25% at corner
Keck I AO image in H band taken during the first Keck I AO night (Dec. 12, 2000).

Io angular size: 1.23 arcsecond
Spatial resolution: 120 km
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References, Further Reading: Textbooks

- Born and Wolf, “Optics”...the definitive reference, will impress your friends
- Hecht, “Optics”...more readable, for everyday...
- Bracewell, “The Fourier Transform and its Applications”
- Goodman, “Fourier Optics”
- Hardy, “AO for Astronomical Telescopes”
References, Further Reading: Papers

- Beckers 1993, ARAA, 31, 13, "AO for Astronomy…"
- Sandler et al. 1993, JOSA A, 11, 925, "AO for Dif-Ltd IR Imaging w. 8-m Telescopes"

References, Further Surfing: Web Sites

- http://cfao.ucolick.org/links/
  Center for Adaptive Optics’ collection of links to projects around the world