Lunar Sample Return via the InterPlanetary Superhighway

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ABSTRACT
The Lunar Sample Return mission consists of two spacecraft, a communications module, and a lander/sample return module carried to the Moon by a mother ship. The desired landing site in this case is on the backside of Moon where a communications module is required. Knowledge of the InterPlanetary Superhighway tunnels and their dynamics provided a quick back-of-the-envelope estimation of the timing and costing of such libration missions which compared well with fully integrated solutions. It also provided good initial guess solutions for the more accurate integrated solutions. The exploration of the design trade space was facilitated by JPL’s LToo12001 mission design tool.

1. INTRODUCTION
It has been decades since the last of the Moon rocks were gathered by astronauts and returned to Earth by the Apollo Program. There is now renewed interest in returning to the Moon. Where humans are involved, the roundtrip flight time must be minimized. However, in the case of a robotic sample return mission, the flight time is not a critical parameter. It may be relaxed and lengthened to minimize the energy required to return samples from the Moon.

In particular, we can take our cue from comets and asteroids and exploit the low energy natural dynamics of the InterPlanetary Superhighway (IPS) in the Earth’s Neighborhood as shown in Figure 2. The Earth’s Neighborhood is the spherical region of space around the Earth with a radius of roughly 1.5 million km. See Lo and Ross [1], Lo [2] for more details on the IPS.

2. THE IPS IN THE EARTH-MOON ENVIRONMENT
The InterPlanetary Superhighway is a vast network of tunnels and passageways that connects the entire Solar System. Loosely speaking, it is generated by the invariant manifolds of the Lagrange Points of all of the planets and moons in the Solar System. In the Earth’s Neighborhood, this complex web of passageways provide many interesting ultra-low-energy trajectories we used to design a lunar sample return mission using libration orbits about LL_1 (Lunar L1), LL_2, and EL_2 (Earth L2) as shown in Figure 3 below. Mathematically, these tunnels are formed by the invariant manifolds of unstable periodic orbits such as halo orbits around the Lagrange Points. See Koon, Lo, Marsden, Ross [3] for a mathematical description of the IPS.

MISSION DESCRIPTION
The Lunar Sample Return mission consists of two spacecraft, a communications module, and a lander/sample return module carried to the Moon by a single Mother Ship. When the Mother Ship reaches the Moon, the two modules separate. Several different scenarios are studied and described below. The landing site in this case is at 180 deg. longitude, -57 deg. latitude, on the backside of Moon which is why a communications module is required for communications with Earth. We exploit the heteroclinic dynamics that connects the LL_1 and LL_2 regions to provide flexibility in various design options. This is the same dynamics used to design the Earth return trajectory of the Genesis mission which just launched in August 8, 2002. Knowledge of the IPS tunnels and their dynamics also provide a quick back-of-the-envelope estimation of the timing and costing of such libration missions which compares well with a fully integrated solution. It also supplies good initial guess solutions for the more accurate
integrated solutions. The exploration of the design trade space was facilitated by JPL's LTool2001 mission design tool.

MISSION DESIGN WITH IPS SEGMENTS

In this paper, we describe several scenarios for a Lunar Sample Return mission using the tubes of the InterPlanetary Superhighway in the Earth's Neighborhood provided by dynamical systems theory. The trajectory segments within the InterPlanetary Superhighway in the Earth's Neighborhood provide some of the lowest energy pathways within the Earth-Moon system. Dynamical systems theory provides the means to compute and visualize these pathways through the invariant manifolds of unstable periodic and quasiperiodic orbits such as halo and lissajous orbits, what we frequently refer to as libration orbits. Thus, we see that libration orbits play a much greater role than as venues for solar and astrophysical space observatories. They are the generators and the portals to this vast system of low energy trajectories.

One of the key setbacks for mission design in the libration regime has been the lost of orbital elements. Since libration orbits are nonlinear trajectories in the three body problem, the Jacobi constant is the only "integral" available and then only in the Restricted Three Body Problem formulation. This means one is unable to characterize libration orbits by parameters such as semimajor axis, eccentricity, inclination, etc. as for conic orbits, since orbital elements are "integral" quantities in the two body problem. One of the chief manifestation of the lack of integrals in the three body problem is the fact that we cannot characterize orbits in this regime by parameters such as orbital elements. In its place, the invariant manifold tubes, and knowledge of the distribution of libration orbits in space, provide a "replacement structure" for us to handle mission design using libration orbits.

Our knowledge of the libration orbit design space has advanced to the point, where some rudimentary standard orbital segments may be easily constructed and used in "tinker-toy" fashion to provide a modular approach to designing such missions. Some of these standard components are halo orbits around L1 and L2, orbits connecting halo orbits between L1 and L2, tubes leaving the planet to approach the halo orbit, tubes leaving the halo orbit to approach the planet, tubes leaving the halo orbit to escape the planet, tubes from one the planet intersecting the tubes of another planet or satellite. These basic components can be combined with traditional planetary flybys and low thrust segments to further expand the mission design space. For the basic 'libration components' listed above, estimates of time and energy requirements are available in some instances (such as in the case of the Earth's Neighborhood) to provide quick back of the envelope estimates such as was possible with conic orbits. A mission designer can quickly string these libration components together to provide a preliminary mission design. This design can then be validated using a tool like LTool where the components may be integrated using a more accurate model of the Solar System.

This approach allows the designer to select the orbital components in the mission design prior to the trajectory optimization process. Not only does this provide precise control of the mission scenarios to the mission designer, it also allows for a quick estimate of the performance of the design, and more over provides the all important initial guess trajectories which are crucial for starting the trajectory optimization process. As we understand more about the design process outlined above, obtain additional theoretical understanding and empirical data on the InterPlanetary Superhighway, additional automation and faster algorithms may be achieved through this approach.

3. MISSION SCENARIOS

The following describes a few different mission scenarios using libration point orbit: going via LL2, going via LLi, and going via EL1. Also a conic orbit around the moon is considered for comparison.

GOING VIA LL2

This case is where the combined spacecraft (both the lander and the communication orbiter) is sent to a LL2 Lissajous directly via a heteroclinic connection. It is summarized in detail in Table 1.

<table>
<thead>
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<th>[Table 1]</th>
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<td>The Earth launch is assumed to be from a Shuttle-like (200-km altitude, 28.5-degree inclination, circular) orbit. The combined spacecraft is launched on 14-Jun-09 11:18:12.542 with 3122 m/s. Refer to Figure 4 for the earth launch orbit.</td>
</tr>
<tr>
<td>[Figure 4]</td>
</tr>
</tbody>
</table>
It is inserted into a point on a heteroclinic connection from LL₁ to LL₂ or a stable manifold of an LL₂ Lissajous on 18-Jun-09 16:16:17.274 with 570 m/s. Refer to Figure 5 for the LL₂ stable manifold insertion point.  

[Figure 5]

This delta-V places the combined spacecraft in a LL₂ Lissajous orbit approximately on 25-Jun-09. Refer to Figure 5 again for that orbit. The lander is separated from the communication orbiter on 7-Jul-09 00:00:00 with 35 m/s at the closest point from the moon when it crosses near the Y-zero point. Refer to Figure 5 or Figure 6 for the lander separation point.

[Figure 5]

The lander lands on the far side of the moon (180-degree longitude and -57-degrees latitude) on 17-Jul-09 02:13:57.445 with a deceleration of 2424 m/s. Refer back to Figure 5 for the lander orbit. After the sample collection, it lifts off from the moon in the direction of EL₁ on 28-Jul-09 00:00:00 with 2424 m/s. Refer to Figure 5 for the initial lander return trajectory. Refer to Figure 7 (inertial frame), Figure 8 (Earth-Moon rotating frame), or Figure 9 (Sun-Earth rotating frame) for the complete lander return trajectory. Note that the plot is the most apparent in the Sun-Earth rotating frame as in Figure 9.

[Figure 7]
[Figure 8]
[Figure 9]

It returns to the earth on 7-Nov-09 10:00:00. The communication orbiter continues its Lissajous orbit around LL₂ until 13-Aug-09 12:39:56.141. The whole end-to-end trajectories are captured in SPK files.

GOING VIA LL₁

The trajectories in this case have not been differentially corrected. Thus, the delta-V’s and dates represented in the Table 2 are reasonable estimates.

[Table 2]

The combined spacecraft is launched onto a point in any stable manifold trajectory of the LL₁ Lissajous. Refer to Figure 10 for various LL₁ stable manifold trajectories.

[Figure 10]

The estimated earth launch is on 9-Jun-09 with 3100 m/s. It is inserted into a point on a stable manifold on 14-Jun-09 with approximately 600 m/s. This delta-V places the combined spacecraft on the LL₁ Lissajous. First, the communication orbiter is separated from the lander to the LL₂ Lissajous. A small delta-V on the order of 10 m/s at the Y-zero point closest to the earth usually places the spacecraft to the LL₂ Lissajous via a heteroclinic connection. Refer to Figure 11 for a heteroclinic connection orbit.

[Figure 11]

Second, one rev after the comm orbiter separation the lander is inserted on the way to the moon approximately on 10-Jul-09. An estimated 20 m/s places the lander somewhere on the moon; it is expected to cost more to target a specific place. The touch down deceleration will be approximately 2300 m/s. The rest of the mission scenario is similar to the LL₂ direct case.

GOING VIA EL₁

In order to lower the cost of delta-V to get to LL₂ Lissajous, an EL₁ Lissajous may be used. The resulting orbit is quite similar to the Genesis orbit in its starting phase. The earth launch is 3193 m/s. The EL₁ Lissajous insertion is 60 m/s. The duration between the earth launch and the EL₁ insertion is approximately 91 days. This insertion nearly automatically leads the spacecraft to the LL₂ Lissajous in approximately 300 days later. The insertion into LL₂ Lissajous is approximately 13 m/s. The rest of the mission scenario is exactly the same as LL₂ direct case.

CONIC ORBIT AROUND THE MOON

This analysis is done by Steve Williams with conic approximation. This is not a libration point orbit. It is written here for comparison to the libration point orbits above. The combined spacecraft is sent directly to the orbit around the moon on 16-Jul-09 with 3100 m/s. One day later they are separated. The communication orbiter goes on an elliptic orbit with periapse facing the far side of the moon on 20-Jul-09 with 481 m/s. The lander is inserted into 100-km circular orbit on 20-Jul-09 with 979 m/s. It is sent to the moon with 23 m/s. The lander deceleration is 1703 m/s. After collecting samples it lifts off on 3-Aug-09 with approximately 3220 m/s for a direct return to earth on 8-Aug-09.
MISSION PERFORMANCE

The mission performance for each of the scenarios considered above is summarized in Table 3 for delta-V’s for the combined, lander, and communication orbiter.
[Table 3]

Note that, since LL₂ Lissajous is always facing the far side of the moon, the lander is always in view by the communication orbiter for all libration point orbits. This is a slight advantage over the conic trajectory around the moon. The delta-V saving is not as apparent for sending the spacecraft via either LL₁ or LL₂ Lissajous in comparison to the conic estimate; however, there is a considerable delta-V saving of more than 400 m/s in sending the combined spacecraft via EL₁ than via either LL₁ or LL₂. There is also a considerable delta-V saving by returning to earth via EL₂ rather than returning directly. The delta-V for returning via EL₂ is 2424 m/s. According to Ted Sweetser, the Soviet’s Lunar series used approximately 2.7 km/s to return to earth directly from the near side of the moon. There is a saving of 276 m/s. Besides, it is not apparent whether there can be a direct transfer trajectory with only a single lift from the far side of the moon to earth. Steve William’s conservative estimate of 3220 m/s was obtained by adding the moon’s hyperbolic escape velocity and the conic return trajectory to earth.

4. MISSION DESIGN WITH LTOOL

LTool2001 is a Problem Solving Environment (PSE) based on the object oriented Python language for trajectory and mission design. Although it has specialized functions for libration mission design, it is a completely general mission design tool which may be used for designing conic interplanetary transfers and planetary flybys.

Its class design paradigm is quite unique. Whereas in most software classes are usually designed according to its use cases in the application domain, many LTool’s classes are designed to represent the physical and mathematical objects in the problem domain (Newtonian Space-Time). For example, LTool provides some physical objects such as covariant objects and some mathematical objects such as functions of an object. For example, an Event is a physical class that defines a point in Newtonian Space-Time. It is a covariant object in that it is coordinate-free.
Given an Event class, LTool defines a mathematical object, an Event function of TIME where TIME is another time-keeping class. Thus, any trajectory can be represented by an Event function of TIME. Thus, if Earth and Moon are Event function’s of TIME variables (respective planetary trajectories), LL₁ (Earth-Moon Libration Point 1 trajectory) is represented simply by “Moon - gamma * (Moon - Earth)” where gamma is a three-body constant. Note that the difference (Event - Event) is another physical class, Disp, that defines a covariant segment between two Event’s. Numeric values with respect to a particular coordinate frame can be evaluated simply by applying a CoordBasis, another class that represents any coordinate frame including a rotating coordinate frame as well as an inertial. Thus, if the is a Lissajous spacecraft trajectory around LL₁, its velocity at a given time t₀ with respect to LL₁ in the Earth-Moon rotating coordinate frame is “(sc - LL₁)(t₀)(EarthMoonRotatingCB)(VELOCITY)” where EarthMoonRotatingCB is the Earth-Moon rotating coordinate frame (CoordBasis) variable and VELOCITY is a key word. Thus, once this concept is familiarized, coding an astrodynamics formulation becomes very close to writing down the mathematical expression on a piece of paper. Also its design philosophy of units allows an early detection of errors in the formulation either by compile-time error in C++ or by an expression error in Python environment (rather than returning invalid values). These classes are exported to an interactive Python environment with Qt GUI I/F. Thus, this interactive programming environment is well-suited for most mission design problems.

LISSAJOUS AND MANIFOLD

Given a three-body system, a GUI object can be set up for producing a Lissajous orbit and its stable and unstable manifold trajectories. Figure 12 shows the layout of the GUI object.
[Figure 12]
A Richardson-Cary expansion is set up by specifying the libration point index (either 1 or 2), Z- and Y-amplitudes, phase angles phi and psi, the epoch, and the duration in number of rev’s. By clicking the “plot Lissajous” button, the corresponding Lissajous orbit is generated by differentially correcting the expansion. The differential correction used in LTool is the two-level correction by Roby Wilson. First the position continuity is established, then the velocity continuity is sought. This process is repeated until the velocity norm is smaller than a specified limit. The GUI is connected to a plotter called QTPlotter. The original expansion as well as the intermediate orbits during the differential correction process are displayed in the plotter. Once a Lissajous orbit is generated, its stable and
unstable direction can be computed by clicking on the “compute Direction” button. The stable/unstable directions are computed from the monodromy matrix at a specified reference time which determines a point in the orbit. The rest of the GUI specifies the manifold trajectories of the Lissajous orbit. Either a stable or an unstable manifold can be selected. A time offset limit from the reference time can be specified with a delta time as well as an alpha limit with a delta. The duration of the manifold trajectories is also specified. By clicking on the “plot Manifold” button, the specified manifold trajectories are generated and displayed in the QtPlotter. Each manifold trajectory generated can be accessed analytically with its start and end times and its alpha value.

DELTA-V MAGNITUDE TRAJECTORY

It is often necessary to extend a trajectory of a certain body in the direction of its velocity with a “small” delta-V magnitude to observe where the body ends up. Such are the cases when a heteroclinic connection is to be found between LL₁ and LL₂, when an initial lander trajectory is to be found, when a transfer between EL₄ and LL₂ is to be found, and even when the lander return trajectory is to be found via EL₂. A GUI object is set up to do this task interactively. Refer to Figure 13 and Figure 14 for the GUI.

A particular trajectory from the QtPlotter may be selected by clicking “Define Trajectory” button. The center of the delta-V may be chosen appropriately to each problem. The time of the selected trajectory determines the exact point on the trajectory where the specified delta-V magnitude is to be applied. The trajectory is extended and displayed on the QtPlotter for the specified duration. For example, the values in the GUI setting of Figure 13 are used to obtain the heteroclinic connection from LL₁ to LL₂, and those of Figure 14 are used to find the lander return trajectory.

DETERMINING AN APPROPRIATE LL₂ LISSAJOUS

1. Determine the approximate date of landing on the moon. Since sun will shine only half of its period around the earth, approximately 14 days are available. The date should be determined in the beginning phase of this 14-day window. This can be done relatively easily by noting interactively the rise of the sun or LL₁ with respect to the far side of the moon.

2. Based on the approximate date of landing a LL₂ Lissajous orbit is constructed to find an approximate duration of the landing insertion. The insertion is best performed from the point on the Lunar L₂ Lissajous closest to the far side of the moon. A small delta-V of 10-20 m/s in the opposite direction to a L₂ Lissajous orbit at that point usually moves the spacecraft to the vicinity of the secondary body in the CRTBP. By subtracting the time of separation of the lander from the orbiter from the approximate time of arrival on the moon, the approximate duration of the landing can be found.

3. Once the approximate duration of the landing duration is found, it is subtracted from the date of landing to determine the beginning time of the LL₂ Lissajous orbit. The final LL₂ Lissajous is constructed for at least three rev’s based on this beginning time.

EARTH LAUNCH

In this problem the earth launch is from a 200-km, 28.5-degree-inclination Shuttle-like orbit. Many different points on each stable manifold trajectory can be considered for the least delta-V.

1. Various stable manifold trajectories are computed for the either LL₁ or LL₂ Lissajous orbit. One particular stable manifold trajectory can be computed for each point on the Lissajous orbit with a given alpha value.

2. For each point on a stable manifold trajectory, a conic estimate can be used to compute what the earth launch delta-V and the stable manifold insertion delta-V would be to transfer from the circular earth launch orbit to the point on the stable manifold. In reality the conic estimate fails miserably closer to the moon. However, it provides a quick estimate over many possible such trajectories. This is one area where some improvement can be made to optimize the search.

3. One point on a particular stable manifold trajectory is chosen. A Hohmann-like transfer from the circular earth launch orbit to the point on the stable manifold provides some possible intermediate points along the trajectory. These intermediate points along with those of the Lissajous orbit are differentially corrected to construct the resulting trajectory.
4. A 28.5-degree inclination constraint and a 200-km radius constraint are used on the initial earth launch point. At the point of insertion to the stable manifold, the delta-V is left discontinuous to be determined by the differential correction process. At later stage a maximum value can be specified to press the resulting trajectory more optimally.

LANDER SEPARATION

Given the differentially corrected trajectory from the earth launch to the LL$_2$ Lissajous, the actual orbiter insertion trajectory can be computed as follows:
1. First, an approximate trajectory is determined by propagating the Lissajous above at the separation point as defined above. The target point on the moon has been specified as 180-degree longitude and -57-degree latitude. Usually it is impossible to target such a specific point on the moon by applying an opposite delta-V interactively on the trajectory at the point. Thus, this must be also differentially corrected.
2. A set of patched points can be constructed from the estimated separation trajectory. The target point on the moon is added as the last patch point. The delta-V at the separation point is left unspecified to be automatically determined by the differential correction process. Usually this will be a little more than the estimated value.
3. Since the duration has been already estimated and the LL$_2$ Lissajous has been computed accordingly, the date of landing should be nearly at the beginning of the rise of sun or the EL$_1$ from the landing site.

LANDER RETURN

Since the lander is on the far side of the moon, it is easier to send the lander via the EL$_2$ before it falls back to earth. This can be achieved interactively by updating the delta-V value until the lander goes out toward EL$_2$ and loops back to earth. The specific of the return condition on earth is not worked out. Refer to Figure 9 for the lander return trajectory.

5. CONCLUSIONS

We described two scenarios for a Lunar Sample Return mission using the tubes of the InterPlanetary Superhighway in the Earth’s Neighborhood provided by dynamical systems theory. The trajectory segments within the InterPlanetary Superhighway in the Earth’s Neighborhood provide some of the lowest energy pathways within the Earth-Moon system. The InterPlanetary Superhighway provided a modular approach to mission design in libration space. The resulting missions require less propulsion than a mission using standard conic arcs only for its trajectory design. Although, in general, the use of the low-energy InterPlanetary Superhighway usually requires longer travel time than conventional high-energy hyperbolic transfers. Moreover, LTool was able to provide a fully integrated trajectory where as within the same time, the standard conic-based trajectory tools could not respond as quickly.

The InterPlanetary Superhighway requires development, just as any other natural resource must be developed in order to be fully utilized. One of the key areas for further study is the role of continuous thrust in this regime. Preliminary work has demonstrated that there is a close connection between low-thrust trajectories and those within the InterPlanetary Superhighway. The most obvious examples are cometary orbits which are a sort-of ‘continuous-thrust’ object in space which we know follows the InterPlanetary Superhighway (see Howell, Marchard, and Lo [9]). Another area where development is needed is to understand the relation between the libration regime with conic regimes, particularly hyperbolic flybys. Finally, the InterPlanetary Superhighway itself needs to be mapped, and additional tools need to be developed to explore its structure in order to provide new algorithms and orbits for mission design in this rich regime.

ACKNOWLEDGEMENTS

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REFERENCES


<table>
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<th>Mission Sequence</th>
<th>Date</th>
<th>Elapsed Time (days)</th>
<th>Mothership ΔV (m/s)</th>
<th>Lander ΔV (m/s)</th>
<th>Orbiter ΔV (m/s)</th>
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<td>11:18</td>
<td>3122</td>
<td>4570</td>
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<td>7/17/2009</td>
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Table 1. The values in this table reflect the "Going Directly to LL₂" case. The combined spacecraft is sent directly to LL₂ via a stable manifold of the LL₂ Lissajous. The lander is separated from the orbiter at the separation point. The sample is returned to earth via EL₂. All the trajectory segments are differentially corrected; thus, the values are not estimated but computed.

<table>
<thead>
<tr>
<th>Mission Sequence</th>
<th>Date</th>
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<th>Mothership ΔV (m/s)</th>
<th>Lander ΔV (m/s)</th>
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Table 2. The values in this table reflects the "Going Via LL₁" scenario. The combined spacecraft is first sent to the LL₁ Lissajous via its stable manifold. Then, the orbiter is sent to the LL₂ Lissajous via a heteroclinic connection. The lander is inserted to the landing site on the moon. The values are estimated.

<table>
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<tr>
<th>Scenario</th>
<th>Combined (m/s)</th>
<th>Lander (m/s)</th>
<th>Comm (m/s)</th>
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<td>5925</td>
<td>481</td>
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<td>4750</td>
<td>14</td>
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<tr>
<td>Going via LL₁</td>
<td>3692</td>
<td>4800</td>
<td>0</td>
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<tr>
<td>Going via EL₂</td>
<td>3266</td>
<td>4800</td>
<td>0</td>
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Table 3. The values in this table are the summarized over the combined, the lander, and the communication orbiter. Conic case and Going via EL₂ are added for comparison.
Figure 1. The Lunar Sample Return Mission plotted in Lunar Rotating Frame, centered on LL$_2$. The Figure on the left shows the entire mission. The Transfer Trajectory is the segment from the Earth just above the X-axis to the lunar region in the small square. The return trajectory is the rest of the trajectory departing from the small square around the lunar region. Note the return trajectory passes by the EL$_1$ region and actually uses the dynamics of the Sun-Earth halo orbit for the return.

The figure on the right is the exploded view of the lunar region within the small square region on the left. It shows the use of the LL$_1$ to LL$_2$ 'heteroclinic connection' where the mother ship is conveyed from the Earth to an LL$_2$ halo orbit. The Lander trajectory departs from the LL$_2$ halo orbit to the backside of the Moon.
Figure 2. Artist's conception of portions of the InterPlanetary Superhighway (IPS, tubes) of the Sun-Earth-Moon System generated by the halo orbits (large periodic orbits around the unstable Lagrange Points \( L_1 \), \( L_2 \), and \( L_3 \)). Orbits on the blue-green tubes approach the halo orbits, while those on the red tubes go away from the halo orbits. Thus, the halo orbits are the portals, the literal "Highway Interchanges" to the Interplanetary Superhighway. The exploded view on the right is the Lunar portion of the Interplanetary Superhighway. Arrows indicate the direction of transport.

Figure 3. The Lagrange Points of the Moon (\( LL_1 \ldots LL_5 \)) and the Earth (\( EL_1 \), \( EL_2 \)) in Earth's Neighborhood in Earth rotating coordinates where the X-axis is the line containing the Sun and the Earth.
Figure 4. The earth launch orbit in the Earth-Moon rotating frame is shown in brown. The plot is centered at LL₂ to make the Lissajous appear nicely. In this plot the lander returns to where the earth will be at its return date.
Figure 5. The insertion into the LL$_2$ stable manifold or the heteroclinic connection from LL$_1$ to LL$_2$ shown (from brown to blue) in Earth-Moon rotating frame, centered at LL$_2$. Also the complete lander orbit from its separation point to the touchdown on the moon is shown in red.
Figure 6. The LL$_2$ Lissajous is zoomed, centered at LL$_2$ in Earth-Moon rotating frame. The lander separation point from the communication orbiter is near Y-zero point closest to the moon with respect to LL$_2$ (from blue to red). Refer to Figure 5 also.
Figure 7. The entire trajectories are displayed in an inertial frame, centered at earth. The earth launch leg is in brown, LL\textsubscript{2} Lissajous in blue, the lander insertion in red, and the lander return in purple. The moon's orbit is in gray. LL\textsubscript{1} and LL\textsubscript{2} are snapshots at the lander return lift-off time, they move counterclockwise with respect to earth. Note that LL\textsubscript{2} Lissajous orbit in blue does not appear meaningful in this frame. Also note that the lander return leg in purple does not appear as a conic with respect to the earth in this frame.
Figure 8. The entire trajectories are displayed in Earth-Moon rotating frame, centered at earth. The color scheme is similar. In this frame the LL$_2$ Lissajous orbit in blue appears better. Refer to Figure 5 for the close-up view of LL$_2$ Lissajous. However, note that the lander return trajectory is not apparent. EL$_1$ and EL$_2$ move clockwise about the earth.
Figure 9. The entire trajectories are displayed in Sun-Earth rotating frame, centered at earth. The color scheme is similar. In this frame the LL Lissajous orbit in blue is not apparent. However, the lander return trajectory is more meaningful; it comes close to making a Lissajous orbit around \( \text{EL}_2 \). LL\textsubscript{1} and LL\textsubscript{2} move counterclockwise about the earth.
Figure 10. $L_{L1}$ stable manifold trajectories in dark green are displayed in the Earth-Moon rotating frame, centered at $L_{L1}$. The $L_{L1}$ Lissajous orbit is in brown. Refer to 4. Mission Design with LTool for generating these stable manifold trajectories.
Figure 11. A heteroclinic connection or a stable manifold of the LL₁ Lissajous is generated from LL₁ Lissajous to LL₂ in the Earth-Moon rotating frame, centered at the moon. The Lissajous orbits appear a bit scattered because they are displayed centered at the moon. Refer to 4. Mission Design with LTool for generating a heteroclinic connection.
Figure 12. This Qt GUI shows input data necessary for generating a Lissajous orbit and its stable or unstable manifold trajectories. The GUI object was instantiated for Earth-Moon system.
Figure 13. This Qt GUI was used to interactively find the heteroclinic connection from LL₁ to LL₂ Lissajous. A delta-V magnitude value is updated until the orbit stays around LL₂ in the QtPlotter.
Figure 14. This Qt GUI was used to interactively find the lander return trajectory. A delta-V magnitude value is updated until the orbit returns to the earth via EL$_2$ in the QtPlotter.