Time-Delay Interferometry

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- Reminder of what LISA is and what the main noise sources are
- Why conventional laser noise cancellation methods won't work for LISA
- Time-Delay-Interferometry
  - The main idea: view unequal arm LISA as symmetric system of one-way links
  - Exploit transfer function of signals and noises to cancel the noises while keeping GW signal to construct the TDI observables (with differing GW sensitivities depending on the TDI combination used)
  - Derive basis functions for cancellation of principal noises (X, Y, Z), (α,β,γ), ζ, etc.
- Applications of TDI
  - LISA sensitivities to periodic waves
  - On-orbit instrumental noise calibration and Isolation of GW background
- Practical Problems

References

For LISA configuration, orbits, and raw noises:

LISA Pre-Phase A Report http://lisa.jpl.nasa.gov/documents.html

Folkner, W. et al. 1997 *Classical Quantum Grav.* 14, 1543

For TDI:

Tinto, M., and Armstrong, J. W. 1999 "Cancellation of Laser Noise in an Unequal-Arm Interferometer Detector of Gravitational Radiation", *Phys. Rev. D*, 59, 102003.

Armstrong, J. W., Estabrook, F. B., and Tinto, M. 1999 "Time-Delay Interferometry for Space-Based Gravitational Wave Searches", ApJ, 527, 814.

Estabrook, F. B., Tinto, Massimo, and Armstrong, J. W. "Time-Delay Analysis of LISA Gravitational Wave Data: Elimination of Spacecraft Motion Effects", *Phys. Rev. D.* 62, 042002.

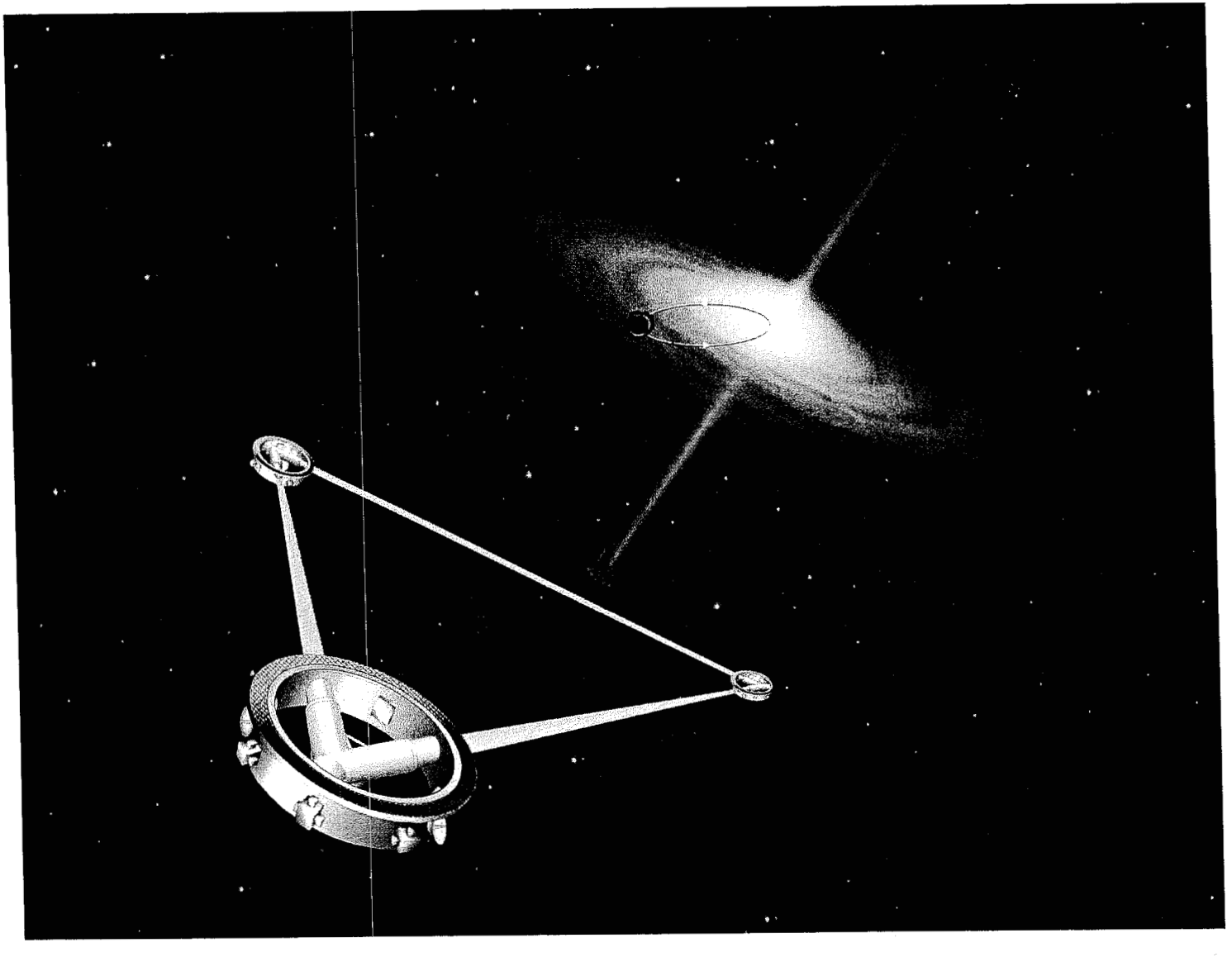
Tinto, Massimo, Armstrong, J. W. and Estabrook, F. B. "Discriminating a Gravitational Wave Background from Instrumental Noise in the LISA Detector", *Phys Rev. D.* 63, 021101(R) (2000)

Tinto, M., Estabrook, F. B. and Armstrong, J. W. " Time-Delay Interferometry for LISA", *Phys. Rev. D*, 65, 082003 (2002).

Tinto, M. "Time Delay Interferometry", CaJAGWR lecture http://www.cco.caltech.edu/~cajagwr/scripts/seminars.html

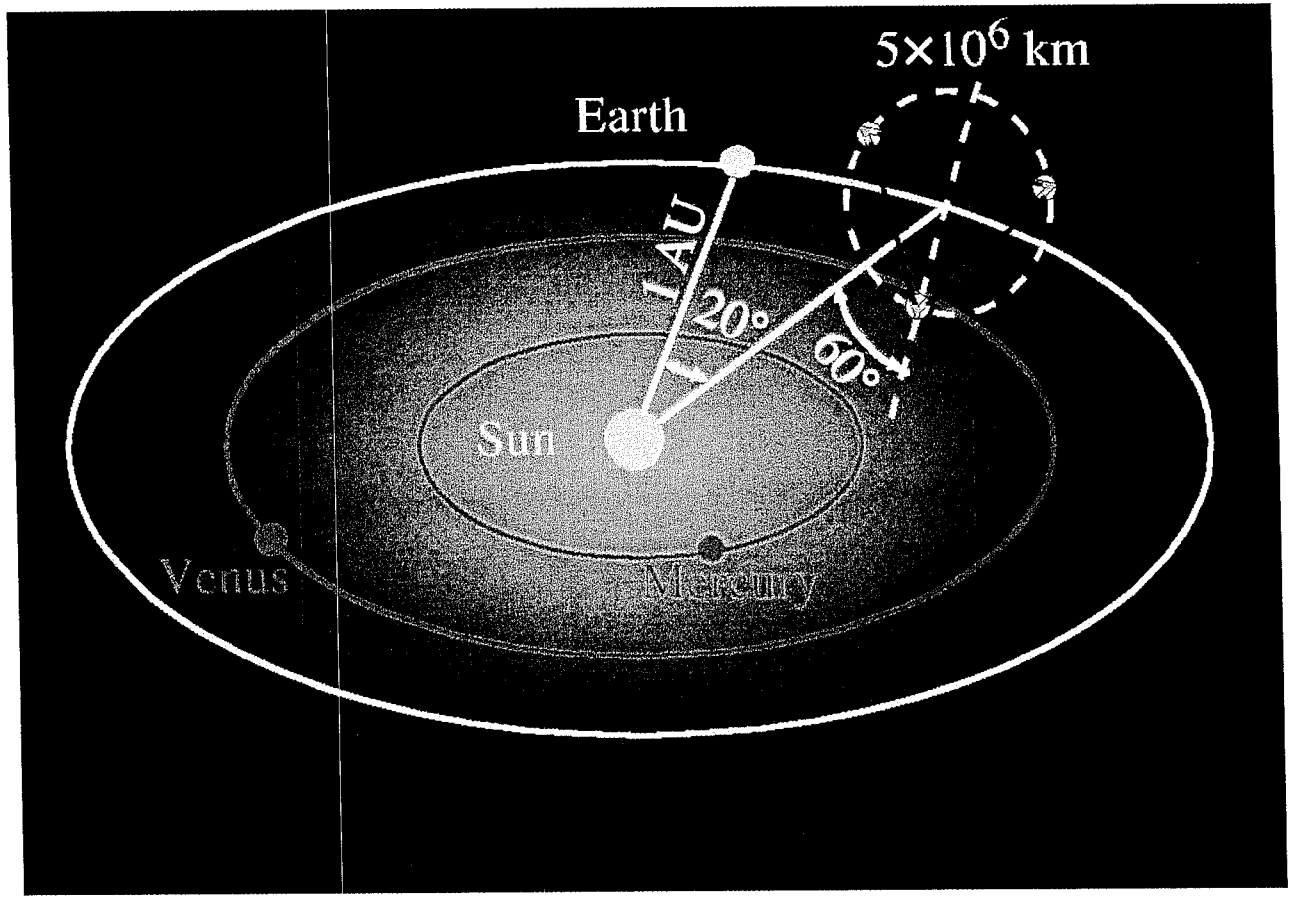
JWA 5/22/02

Mission Concept



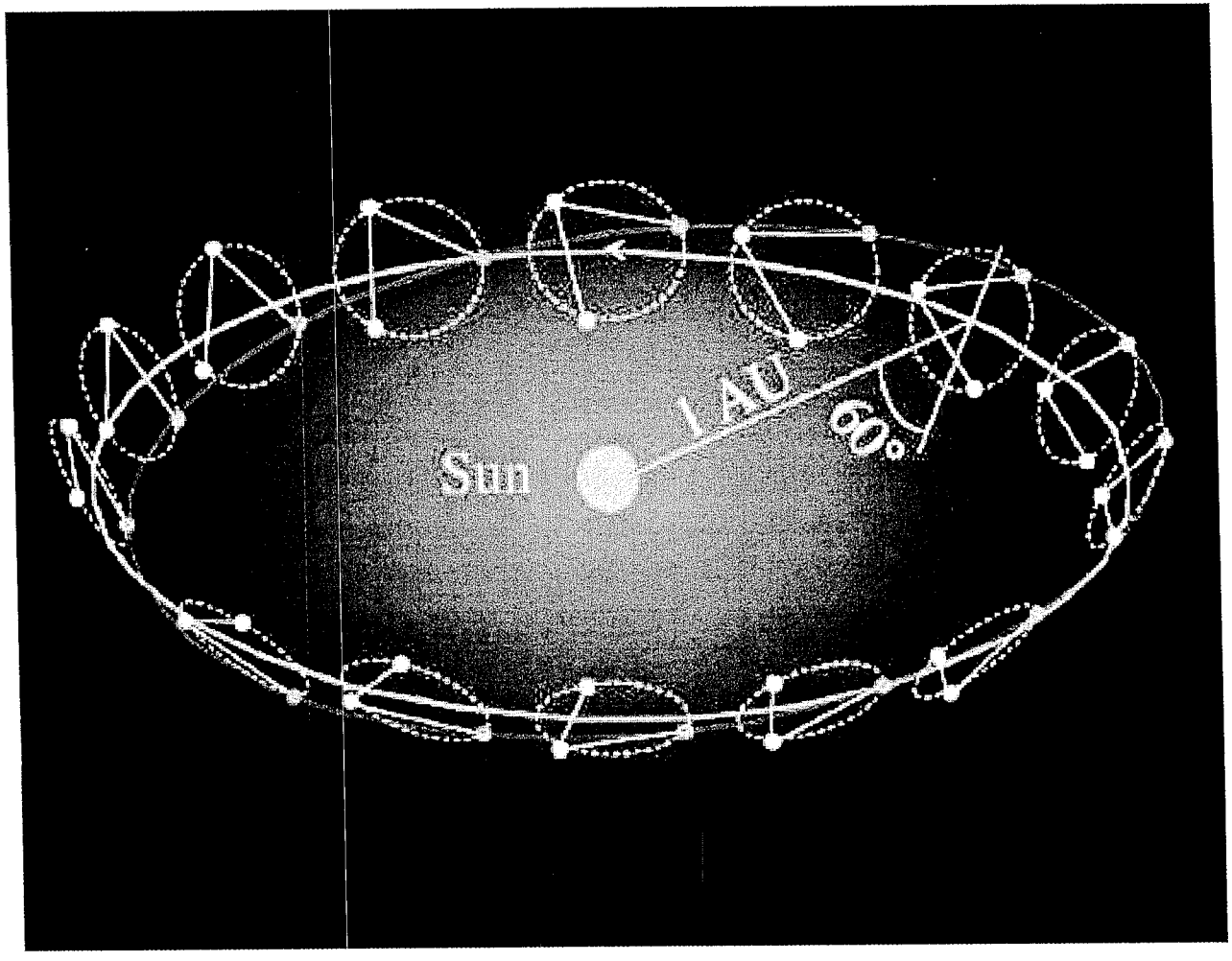
Spacecraft Formation

- Three spacecraft in triangular formation; separated by 5 million km
- Spacecraft have constant solar illumination; payload shielded from sunlight
- Formation trails Earth by 20°; compromise constant arm-lengths vs cost



Spacecraft Orbits

- Spacecraft orbits evolve under gravitational forces only
- Spacecraft fly "drag-free" to shield proof masses from non-gravitational forces



LISA and Its Main Noise Sources

- **GW detector in the LF (~millihertz) band with expected sensitivity to periodic waves ~10^{-23} in a one year integration**
- **3 spacecraft, nominally equilateral triangle (time-variable arm lengths with $\delta L/L \sim 1\%$ and $L \approx 5 \times 10^9$ meters), six lasers, six optical benches, 3 USOs**
- **Laser signals exchanged between spacecraft pairs**
- **Nominal noise spectra**
  - **Raw laser phase noise: $S_y(f) = 10^{-26}\ \mathrm{Hz}^{-1}$**
  - **Optical path noise: $20 \times 10^{-12}\ \mathrm{m\ Hz}^{-1/2} \rightarrow S_y(f) = 1.8 \times 10^{-37}\ (f/1\ \mathrm{Hz})^2\ \mathrm{Hz}^{-1}$**
  - **Optical bench noise: $10 \times 10^{-9}\ \mathrm{m\ Hz}^{-1/2}$**
  - **Proof-mass noise: $3 \times 10^{-15}\ \mathrm{m/sec}^2\ \mathrm{Hz}^{-1/2} \rightarrow S_y(f) = 2.5 \times 10^{-48}\ (f/1\ \mathrm{Hz})^{-2}\ \mathrm{Hz}^{-1}$**

Digression on Transfer Functions

- **Linear system: input = x(t), output = y(t)**
- **Characterized by a time-domain transfer function g(t)**
- **y(t) = x(t) * g(t), where "*" means convolution**
- **Thus Fourier transform of y is product of Fourier transform of x and Fourier transform of g**
- **If g(t) is a time-shift operation, $g(t) = \delta(t - T_1)$, then the Fourier transforms of the input and output are related by**

$$y(f) = x(f)\ \mathrm{Exp}[-2\,\pi\, i\, f\, T_1]$$

Why Consider Alternatives to Michelson Interferometry?

- **Because of orbital dynamics, LISA will have unequal and time-variable arms lengths**

- **In unequal arm length cases, laser noise will not cancel exactly**

$$C(t - 2\,L_1) - C(t - 2\,L_2) \approx 2\,\varepsilon\,L_1\,dC/dt$$

- **Magnitude of the effect is huge: for $\varepsilon\,L_1 = 3\%$ arm length difference and nominal stability of lasers, imprecise phase noise cancellation introduces residual noise orders of magnitude larger than the desired GW signal sensitivity**

Unequal Arm Michelson Interferometry

- **Consider a one-bounce, unequal arm transponding Michelson interferometer**

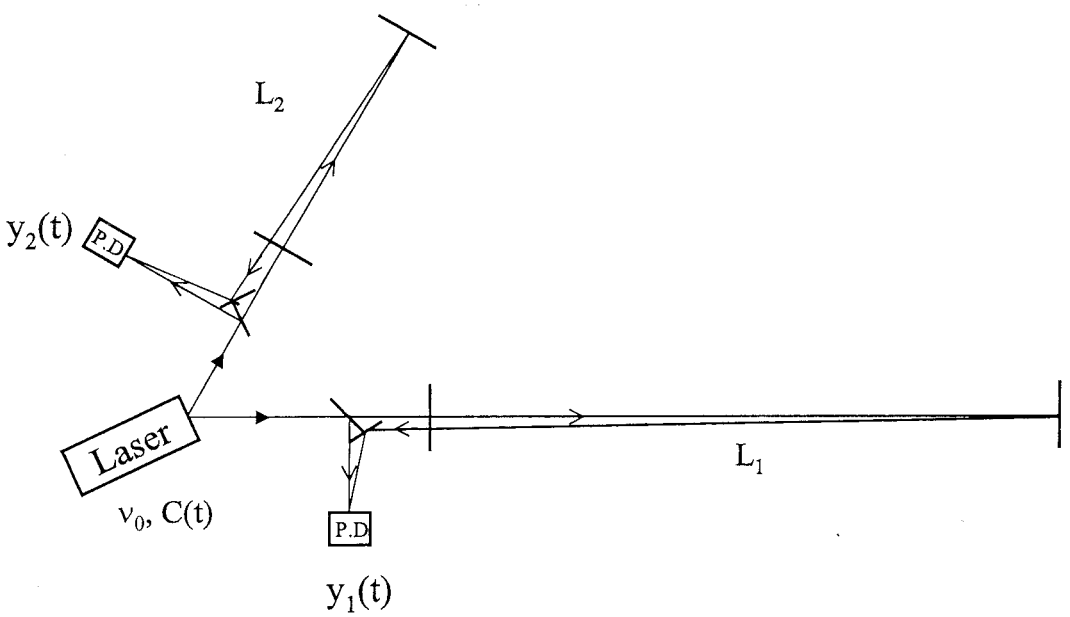
- **Measure phase difference between transmitted and received signals ineach arm separately, then combine data in post-processing to cancel the laser noise**

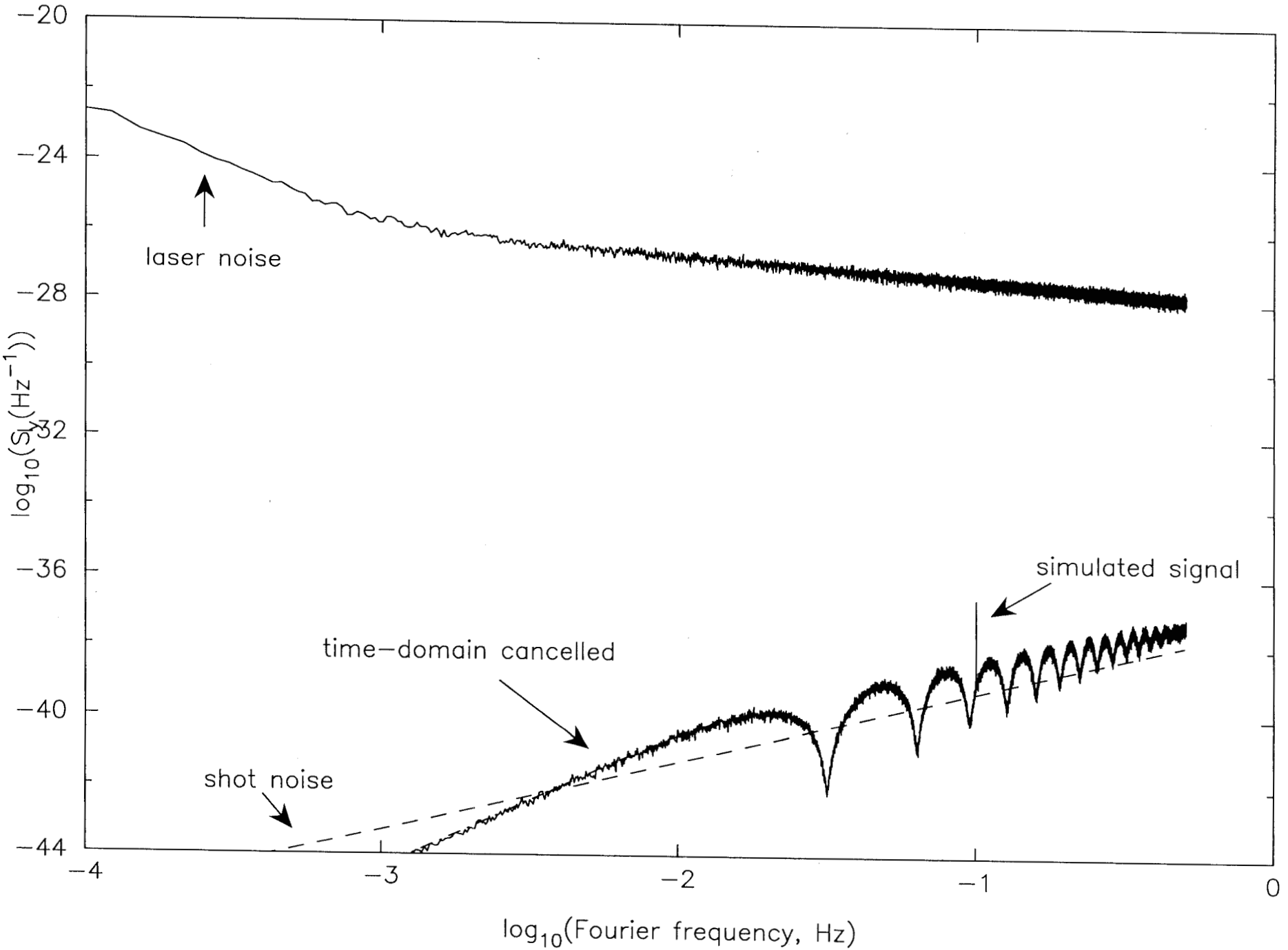
- **If ϕ_i is the phase difference in the arm i (i= 1,2) then the data combination**

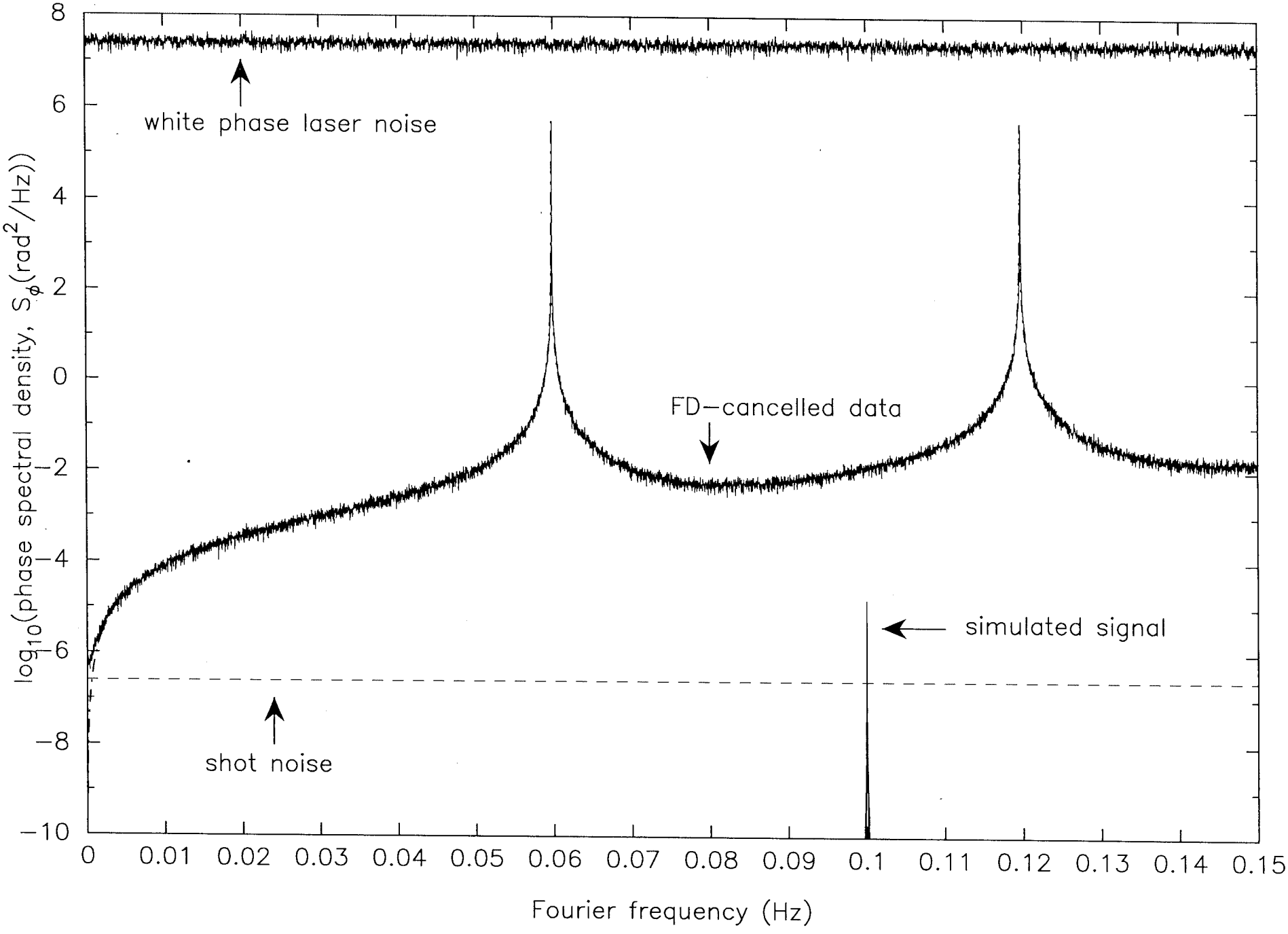
$$[\ \phi_1(t - 2L_2/c) - \phi_1(t)\ ] - [\phi_2(t - 2\ L_1/c) - \phi_2(t)\ ]$$

cancels the phase noise, retaining the gravity wave

- **Tolerance on arm length knowledge can be determined by requirement that imperfection in laser noise cancellation be no worse than the uncancelled noises (e.g. shot noise). For nominal LISA parameters, this means the arms must be known to about 30 meters. See Tinto &Armstrong 1999 *Phys. Rev. D*, 59, 102003 and Tinto, M. "Time Delay Interferometry", CaJAGWR lecture http://www.cco.caltech.edu/~cajagwr/scripts/seminars.html**


L2
y2(t)
P.D
Laser
ν0, C(t)
P.D
y1(t)
L1


−20
−24
−28
−32
−36
−40
−44
−4
−3
−2
−1
0
log10(Sy(Hz−1))
log10(Fourier frequency, Hz)
laser noise
shot noise
time−domain cancelled
simulated signal


white phase laser noise
FD−cancelled data
simulated signal
shot noise
log10(phase spectral density, Sφ(rad2/Hz))
Fourier frequency (Hz)
8
6
4
2
0
−2
−4
−6
−8
−10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15

The Main Ideas

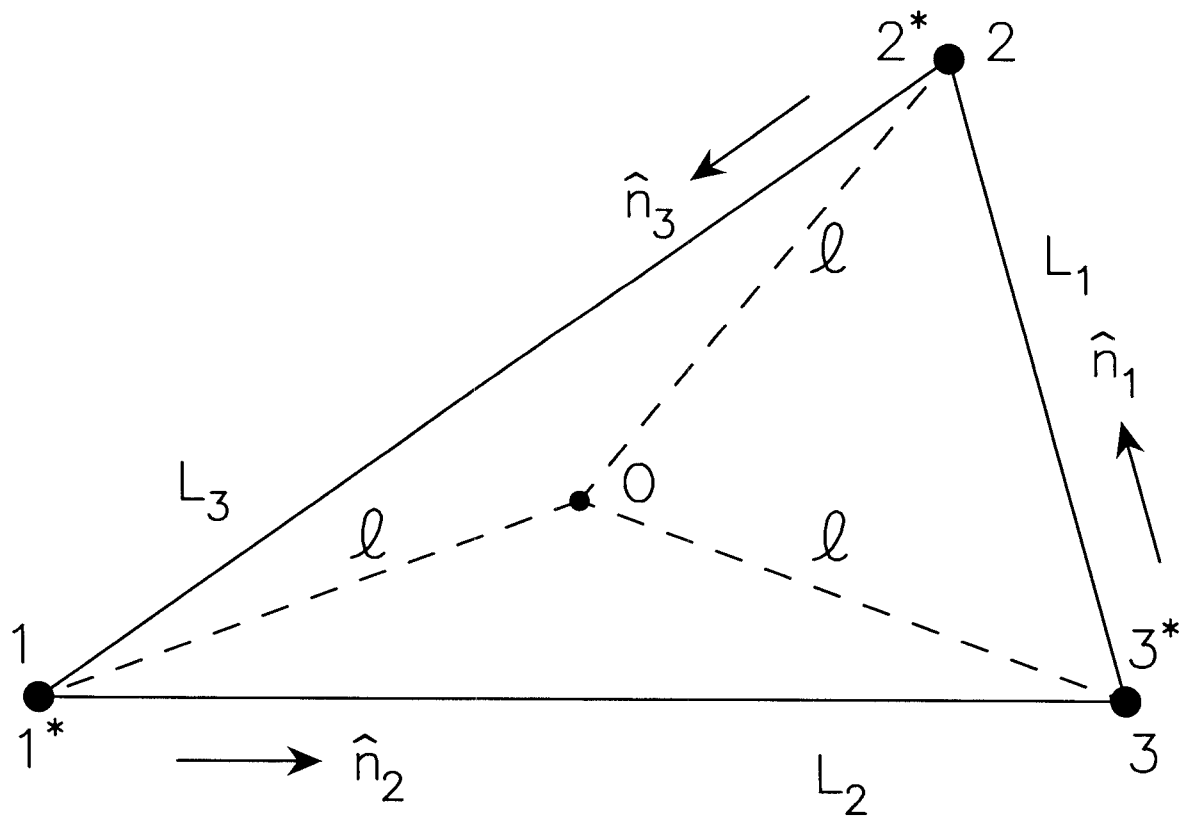
- LISA's can be analyzed symmetrically in terms of Doppler shifts on 6 one-way laser links connecting 6 optical benches on the three spacecraft
- Time-Delay interferometry = methods to cancel principal noises by time-shifting/adding data from single laser links, while retaining the signal
- Use linear systems ideas to build up aggregate GW signal and aggregate instrumental noise for each data combination
- Multiple noise-canceling data streams can be formed simultaneously; robustness against some classes of subsystem failure
- Applications
    - On-orbit noise calibration/isolation of confusion-limited GW background
    - Sensitivity computations
    - Simulations of what the LISA time series will look like (e.g. bursts)

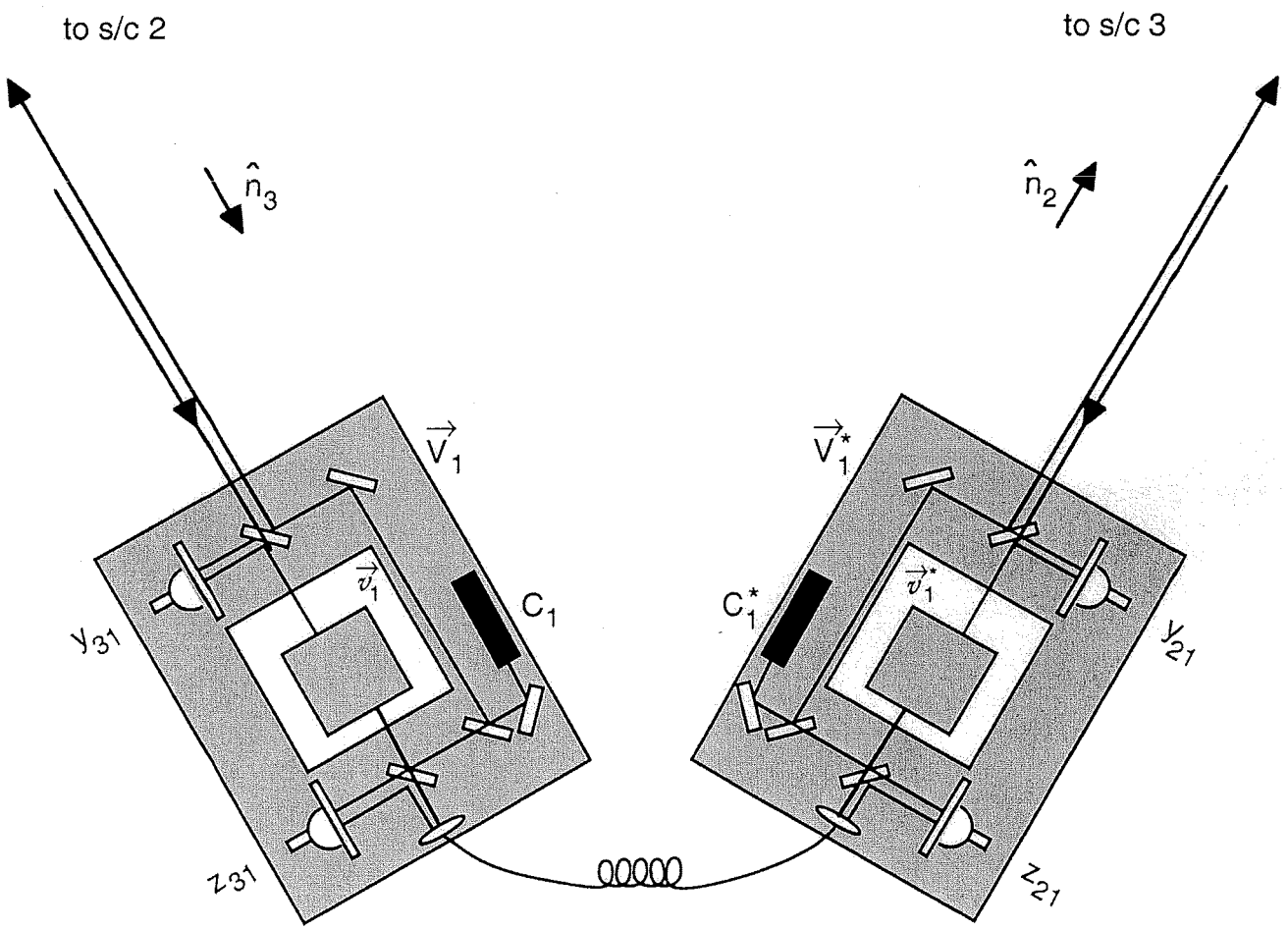
One-Way Links: GW Response

- **Notation and conventions:**
  - **Three LISA spacecraft, equidistant from point "O"**
  - **Unit vectors p_i locate the spacecraft in the plane**
  - **Unit vectors n_i connect spacecraft pairs with the indicated orientation**
  - **$y_{21} = \Delta\nu/\nu_o$ is fractional Doppler shift on link originating at s/c 3, measured at s/c 1; y_{31} is fractional Doppler shift on link originating at s/c 2, measured at s/c 1**
  - **Cyclic permutation of the indices (1 --> 2 --> 3 --> 1) for the other y_{ij}**

- **GW response is "two-pulse" (Wahlquist, GRG, 19, 1101, 1987), e.g.:**

$$y_{21}(t) = [\,1 - (\imath/L_2)(\mu_3 - \mu_1)]\;\;[\Psi_2(t - L_3 - \mu_3\,\imath) - \Psi_2(t - \mu_1\,\imath)]$$


2*
2
$\hat{n}_3$
ℓ
L_1
$\hat{n}_1$
L_3
O
ℓ
ℓ
1
1*
3*
3
$\hat{n}_2$
L_2


to s/c 2
to s/c 3
$\hat{n}_3$
$\hat{n}_2$
$\vec{V}_1$
$\vec{V}_1^*$
$\vec{v}_1$
$\vec{v}_1^*$
C_1
C_1^*
y_{31}
z_{31}
y_{21}
z_{21}

One-Way Links: Noise Response

- **Doppler link noises, in simplest case:**

  laser phase noise: $y_{31} = C^*_2(t - L_3) - C_1(t) = C^*_{2,3} - C_1$

  shot noise: $y_{31} = shot_{31}(t)$

  optical bench and proof mass noise:

$$y_{31} = n_3 \bullet V^*_2(t - L_3) - 2\, n_3 \bullet v^*_1(t) + n_3 \bullet V_1(t)$$

  metrology data: $z_{31} = C^*_1(t) - 2\, n_2 \bullet (v^*_1 - V^*_1) + \eta - C_1(t)$

- **Introduced comma-notation, e.g.** $C_2(t - L_3) = C_{2,3}$

- **C_i's, V_i's, v_i's, $shot_i$, etc. are random variables; ensemble average of squared Fourier transforms of time-domain transfer functions gives spectral modulation**

Noise-Canceling Data Combinations

- **Suitable time-delayed/added linear combinations of the y_{ij} + internal z_{ij} calibration signals can cancel laser phase noise and optical bench noise**

  - **Tinto & Armstrong (1999) *Phys. Rev. D* 59 102003**
  - **Armstrong, Estabrook, & Tinto (1999) *ApJ* 527, 814**
  - **Peterseim, Robertson, Danzmann, Welling, & Bender (2000) *Adv. Space Sci.* 25, 1143.**
  - **Estabrook, Tinto, & Armstrong (2000) *Phys. Rev. D* in press**

- **"Time-delay interferometry"**

Noise-Canceling Data Combinations (cont)

Assume unequal, but at this point not time-varying, armlengths and that the lasers all have the same center frequency. Forming, for example,

$$\alpha = y_{21} - y_{31} + y_{13,2} - y_{12,3} + y_{32,12} - y_{23,13}$$

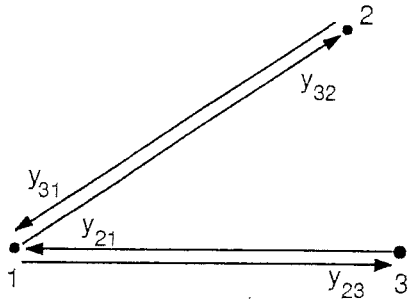
exactly cancels all laser phase noises (C_i and C^*_i)

$$\alpha_{gen} = y_{21} - y_{31} + y_{13,2} - y_{12,3} + y_{32,12} - y_{23,13} + \\ 1/2\,[z_{13,2} + z_{13,13} + z_{21} + z_{21,123} + z_{32,3} + z_{32,12}] + \\ 1/2\,[z_{23,2} + z_{23,13} + z_{31} + z_{31,123} + z_{12,3} + z_{12,12}]$$

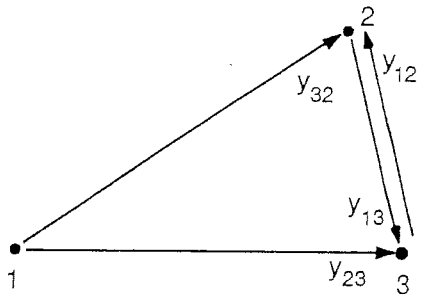
exactly cancels all laser phase (C_i and C^*_i) and optical bench (V_i and V^*_i) noise. Of course non-inertial motion of the proof masses do <u>not</u> cancel:

$$\text{proof mass noise for } \alpha_{gen} = \\ n_1 \bullet (v_{2,3} - v_{2,12} + v^*_{3,2} - v^*_{3,13}) + \\ n_2 \bullet (v_{3,13} - v_{3,2} + v^*_{1} - v^*_{1,123}) + \\ n_3 \bullet (v_{1} - v_{1,123} + v^*_{2,12} - v^*_{2,3})$$

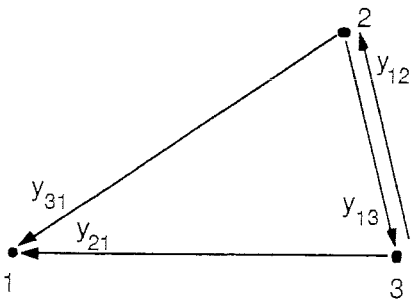
4 LINK / 8 PULSE



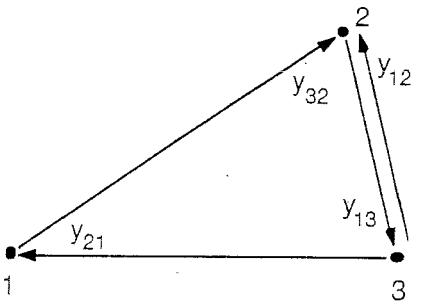
INTERFEROMETER (X,Y,Z)



BEACON (P,Q,R)

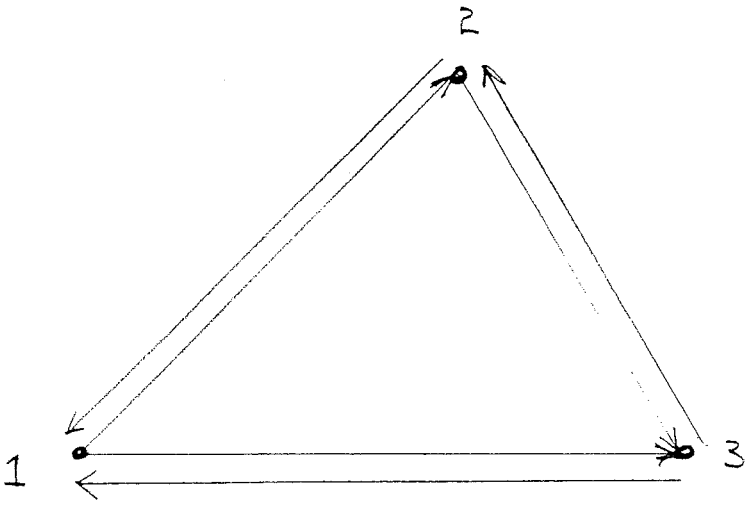


MONITOR (E,F,G)



RELAY (U,V,W)

6 LINK / 6 PULSE



$\alpha, \beta, \gamma, \xi$

$$- \left[1 + \frac{l}{L_1}(\mu_2 - \mu_3)\right][\Psi_1(t - \mu_3 l - L_1 - L_2) - \Psi_1(t - \mu_2 l - L_2)] \,. \tag{18}$$

A δ-function GW signal would produce six pulses in ζ, located at: $\mu_1 l + L_3 + L_2$, $\mu_2 l + L_1 + L_3$, $\mu_3 l + L_1 + L_2$, $\mu_3 l + L_3$, $\mu_2 l + L_2$, and $\mu_1 l + L_1$.

3.3. *Unequal-Arm-Length Interferometric Combinations*

Of course α, β, and γ are not unique; they span a three-space of combinations which produce data free of laser noise. For example, there are combinations which cancel all laser noises while giving eight-pulse responses to gravitational waves:

$$X = y_{32,322} - y_{23,233} + y_{31,22} - y_{21,33} + y_{23,2} - y_{32,3} + y_{21} - y_{31} \,, \tag{19}$$

and Y, Z given by cyclic permutation of the indices. These can be expressed directly in terms α, β, and γ as

$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \zeta \,, \tag{20}$$

$$Y_{,2} = \beta_{,13} - \gamma_{,3} - \alpha_{,1} + \zeta \,, \tag{21}$$

$$Z_{,3} = \gamma_{,21} - \alpha_{,1} - \beta_{,2} + \zeta \,. \tag{22}$$

Note that X does not involve y_{13} and y_{12}, so it would still be available if the laser links between spacecraft 2 and 3 were lost. A similar argument applies for Y and Z. If transponders were used in spacecraft 2 and 3 to slave the frequencies generated there to the uplink frequency received from spacecraft 1, they impose the conditions $y_{32} = y_{23} = 0$ (perhaps then also adding transponder noises which we do not consider here). Then X becomes a combination of data taken only at spacecraft 1:

$$y_{32} = y_{23} = 0 \,, \qquad X \to \Sigma \equiv y_{31,22} - y_{31} - y_{21,33} + y_{21} \,. \tag{23}$$

This is the laser-noise–free unequal-arm interferometer time series first given in Tinto & Armstrong (1999), with its eight-pulse response to a gravitational wave signal. Evidently, X in equation (19) is a *synthesized* interferometer response using one-way Doppler data from all three spacecraft.

Explicitly, for X, the gravitational wave response is

$$\begin{aligned}
X^{\mathrm{gw}} = {} & \left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right][\Psi_3(t - \mu_1 l - 2L_3 - 2L_2) - \Psi_3(t - \mu_2 l - L_3 - 2L_2)] \\
& - \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right][\Psi_2(t - \mu_1 l - 2L_2 - 2L_3) - \Psi_2(t - \mu_3 l - L_2 - 2L_3)] \\
& + \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right][\Psi_3(t - \mu_2 l - L_3 - 2L_2) - \Psi_3(t - \mu_1 l - 2L_2)] \\
& - \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right][\Psi_2(t - \mu_3 l - L_2 - 2L_3) - \Psi_2(t - \mu_1 l - 2L_3)] \\
& + \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right][\Psi_2(t - \mu_1 l - 2L_2) - \Psi_2(t - \mu_3 l - L_2)] \\
& - \left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right][\Psi_3(t - \mu_1 l - 2L_3) - \Psi_3(t - \mu_2 l - L_3)] \\
& + \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right][\Psi_2(t - \mu_3 l - L_2) - \Psi_2(t - \mu_1 l)] \\
& - \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right][\Psi_3(t - \mu_2 l - L_3) - \Psi_3(t - \mu_1 l)] \,,
\end{aligned} \tag{24}$$

Noise-Canceling Data Combinations: Spectral Modulation of Noise

- **If all proof-masses are independent and have same spectrum, and for the LISA equilateral case, ensemble average of FT-squared gives spectral modulation for α's proof mass noise:**

$$[\ 8 \sin^2(3\ \pi\ f\ L) + 16 \sin^2(\pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

- **Similar argument gives proof mass noise for other combinations:**
  - **for Michelson interferometer (X):**

$$[\ 8 \sin^2(4\ \pi\ f\ L) + 32 \sin^2(\ 2\ \pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

  - **for Sagnac combination (ζ):**

$$[\ 24 \sin^2(\pi\ f\ L)\ ]\ S^{\text{proof mass}}$$

  - **and so forth for other combinations and for optical path noise...**

Application of TDI: Sensitivity Calculations

- **Conventional figure-of-merit is RMS sensitivity required for a sinusoidal wave as a function of Fourier frequency**
    - **SNR = 5 in bandwidth B = 1/(one year)**
    - **Averaged over source position and over wave polarization state**

$$h = \frac{noise(f)}{signal(f)} = \frac{5\sqrt{S_{noise}(f)\ B}}{rms\ GW\ response(f)}$$

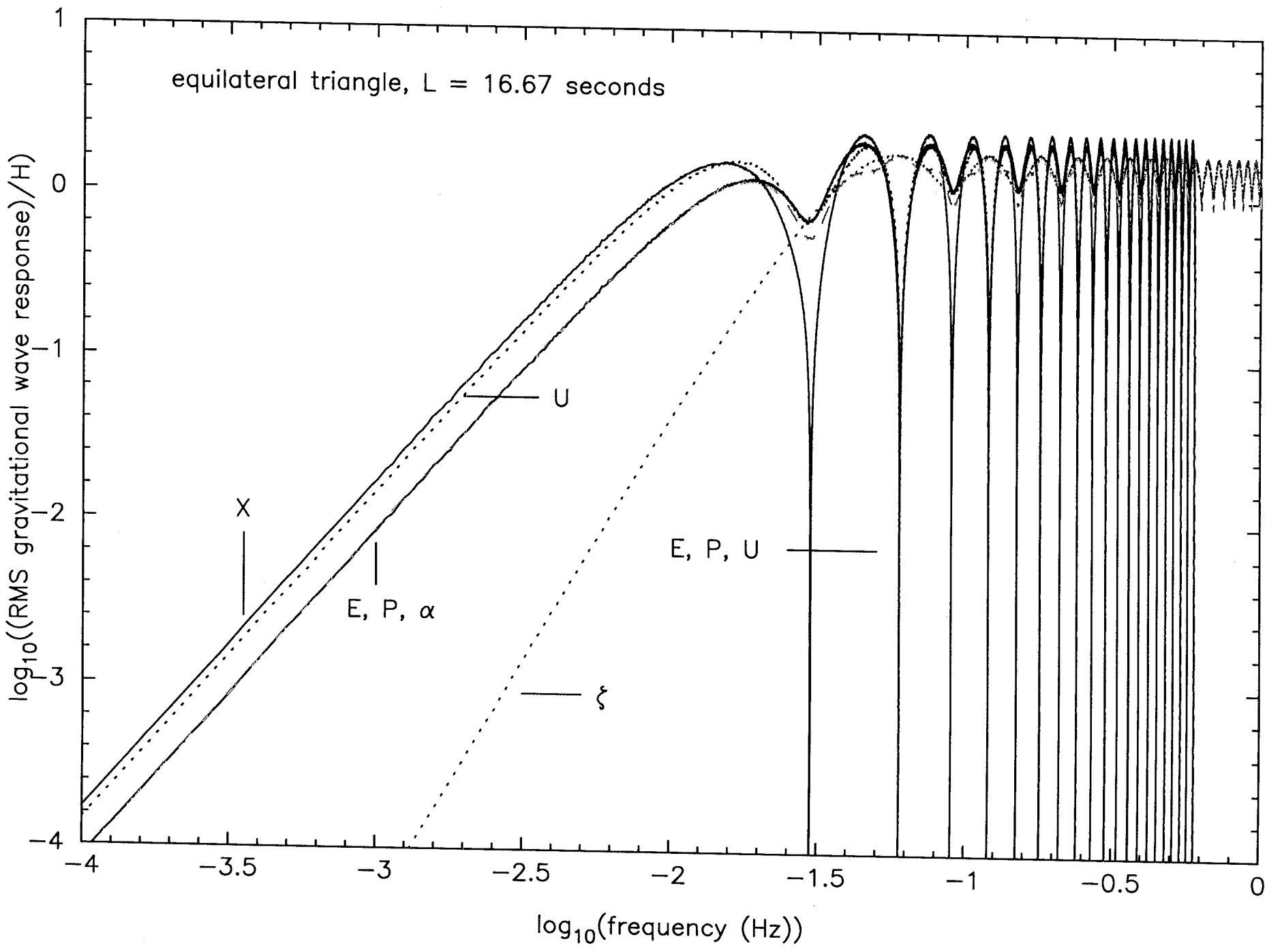
- **Thus need GW signal response for a given noise-canceling data combination and the noise spectra (including transfer functions for both signal and noise) for that data combination**

Procedure: GW Signal

For a given noise-canceling TDI combination (e.g. α, X, P, etc) and for each Fourier frequency in the band:

- **Generate N "sources", uniform on the celestial sphere, radiating GWs with random, general elliptical polarization state (uniform on the Poincare sphere)—4 numbers: "right ascension", "declination", wave ellipticity, and wave tilt**
- **Generate Ψ_i (which depend on the s/c positions, gravitational wave vector and the polarization state)**
- **Compute the y_{ij} (which depend on the Ψ_i, wavevector, arm lengths, and array orientation w.r.t. the source $[p_i]$)**
- **Form desired TDI combination of y_{ij} canceling laser/optical bench noises (if in long-wavelength limit, check calculation against LWL analytical expansion)**
- **Iterate over the N Monte Carlo sources/polarizations to compute the mean-square GW signal response for this data combination at this frequency**
- **Step to next frequency**

- **Examples follow**


equilateral triangle, L = 16.67 seconds
U
X
E, P, α
E, P, U
ζ
log10((RMS gravitational wave response)/H)
log10(frequency (Hz))
1
0
−1
−2
−3
−4
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

Long-Wavelength Limits

Although LISA will not operate exclusively in the long-wavelength limit, LWL analytical results are obviously still very useful. In the LWL, the gravitational wave can be expanded in terms of spatial derivatives, e.g.

$$\mathbf{h}(t-\mu_2 l - L_3) = \mathbf{h}(t) - (\mu_2 l + L_3)\mathbf{h}^{'}(t) + (1/2)(\mu_2 l + L_3)^2 \mathbf{h}^{''}(t) + ...$$

The y_{ij}^{gw} are of order $\mathbf{h}^{'}$, while (α, β, γ), ζ, (E, F, G), (P, Q, R), (U, V, W), and (X, Y, Z) are, for a general triangle LISA configuration, of order $\mathbf{h}^{''}$. The long wavelength expansions are:

$$\begin{aligned} \alpha^{gw} \rightarrow \quad & (1/2)L_1(L_2 - L_3)\ \hat{n}_1 \cdot \mathbf{h}^{''} \cdot \hat{n}_1 \\ & -(1/2)L_2(L_1 + L_3)\ \hat{n}_2 \cdot \mathbf{h}^{''} \cdot \hat{n}_2 \\ & +(1/2)L_3(L_2 + L_1)\ \hat{n}_3 \cdot \mathbf{h}^{''} \cdot \hat{n}_3\ , \end{aligned} \tag{79}$$

$$\begin{aligned} \zeta^{gw} \rightarrow \quad & (1/2)L_1(L_3 - L_2)\ \hat{n}_1 \cdot \mathbf{h}^{''} \cdot \hat{n}_1 \\ & +(1/2)L_2(L_1 - L_3)\ \hat{n}_2 \cdot \mathbf{h}^{''} \cdot \hat{n}_2 \\ & +(1/2)L_3(L_2 - L_1)\ \hat{n}_3 \cdot \mathbf{h}^{''} \cdot \hat{n}_3\ , \end{aligned} \tag{80}$$

$$\begin{aligned} X^{gw} \rightarrow & -2L_2L_3\ \hat{n}_2 \cdot \mathbf{h}^{''} \cdot \hat{n}_2 \\ & +2L_2L_3\ \hat{n}_3 \cdot \mathbf{h}^{''} \cdot \hat{n}_3\ , \end{aligned} \tag{81}$$

$$\begin{aligned} P^{gw} \rightarrow \quad & L_1(L_3 - L_2)\ \hat{n}_1 \cdot \mathbf{h}^{''} \cdot \hat{n}_1 \\ & +L_2L_1\ \hat{n}_2 \cdot \mathbf{h}^{''} \cdot \hat{n}_2 \\ & -L_3L_1\ \hat{n}_3 \cdot \mathbf{h}^{''} \cdot \hat{n}_3\ , \end{aligned} \tag{82}$$

$$\begin{aligned} E^{gw} \rightarrow \quad & L_1(L_2 - L_3)\ \hat{n}_1 \cdot \mathbf{h}^{''} \cdot \hat{n}_1 \\ & -L_1L_2\ \hat{n}_2 \cdot \mathbf{h}^{''} \cdot \hat{n}_2 \\ & +L_1L_3\ \hat{n}_3 \cdot \mathbf{h}^{''} \cdot \hat{n}_3\ , \end{aligned} \tag{83}$$

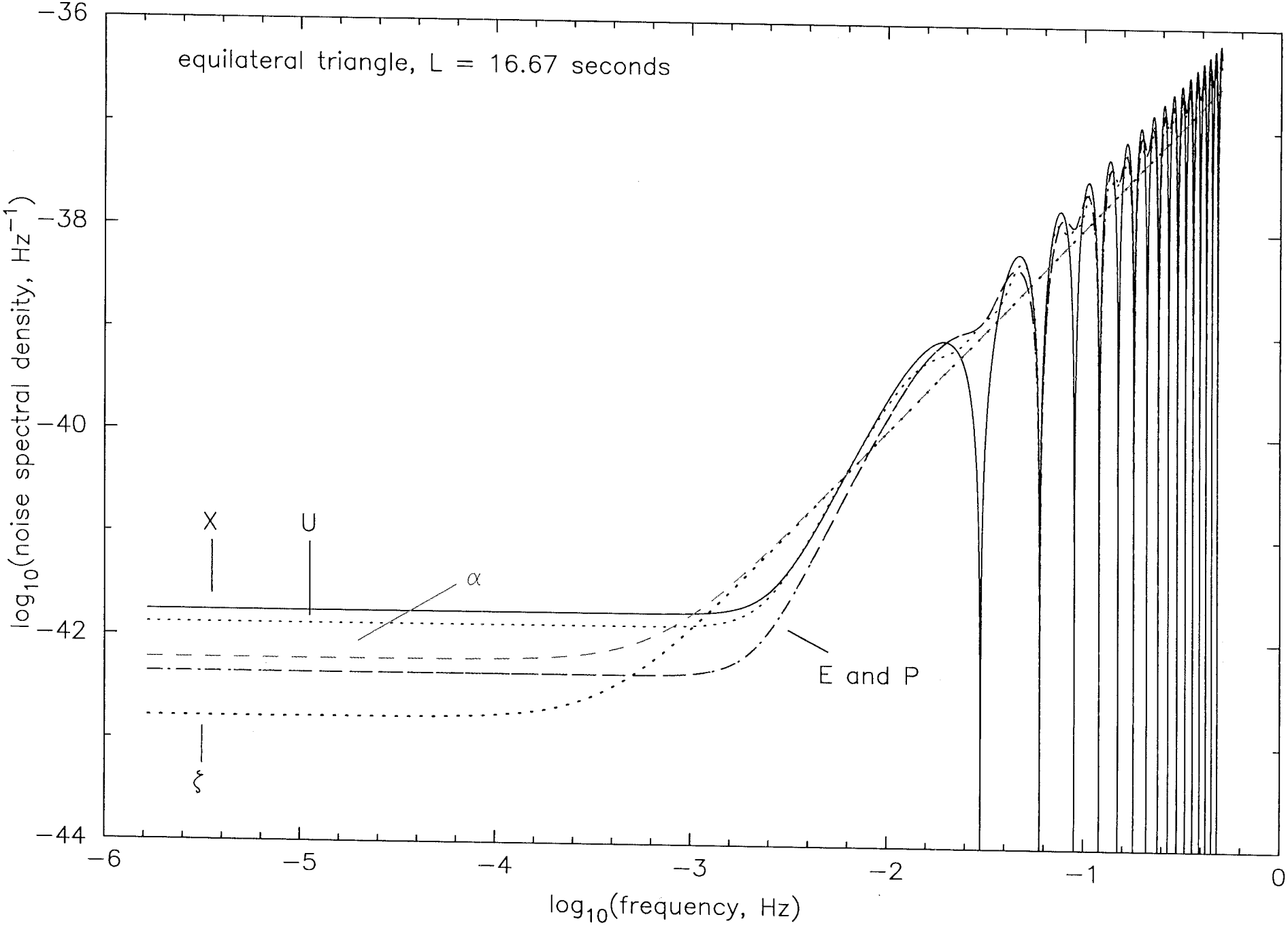
Procedure: Noises

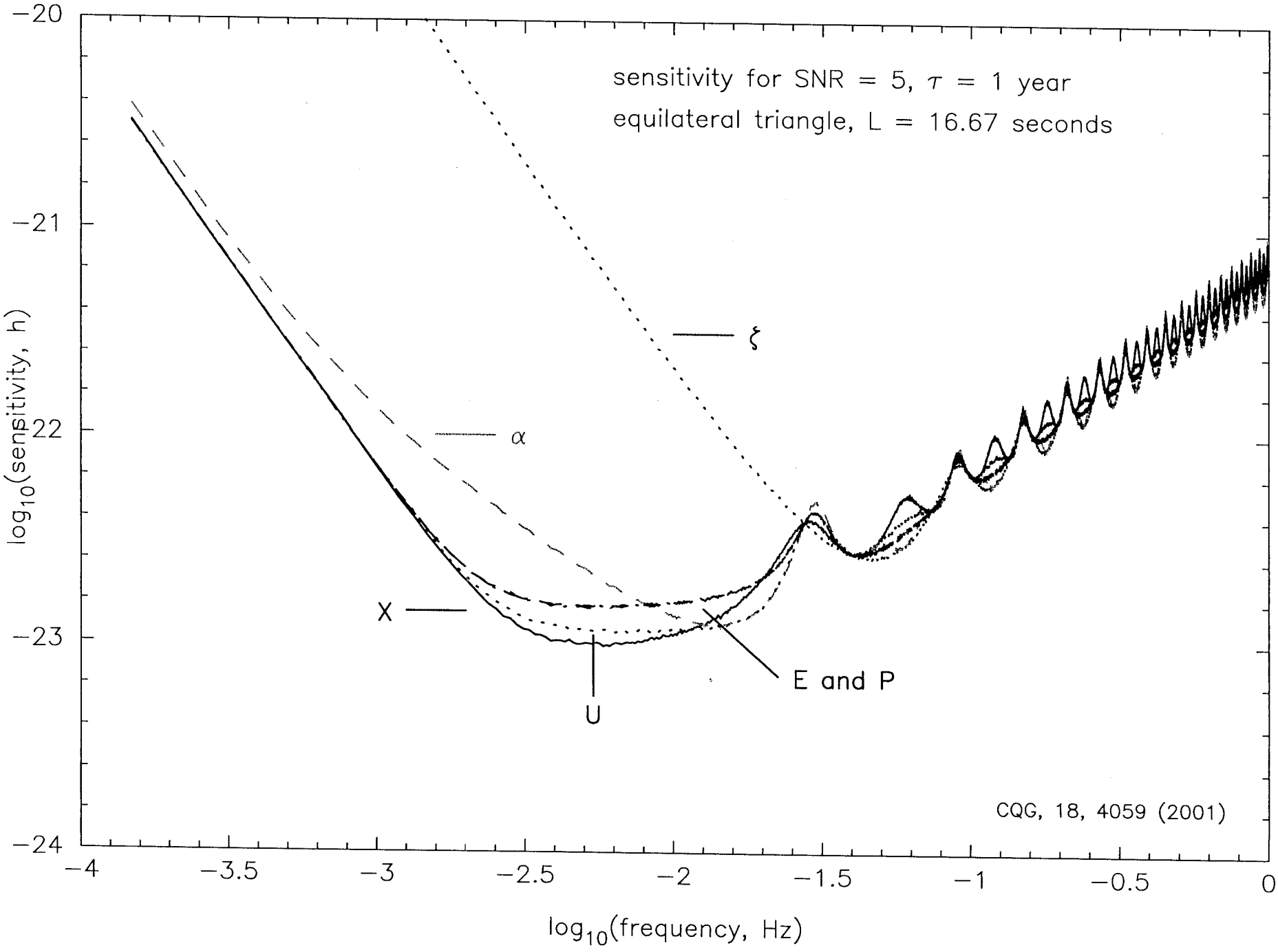
- **Spectra of individual <u>proof-mass</u> noise and single <u>optical path</u> noise (mostly shot + beam pointing) from the LISA Pre-Phase A report—these are expressed in units of length per square-root-Hz**

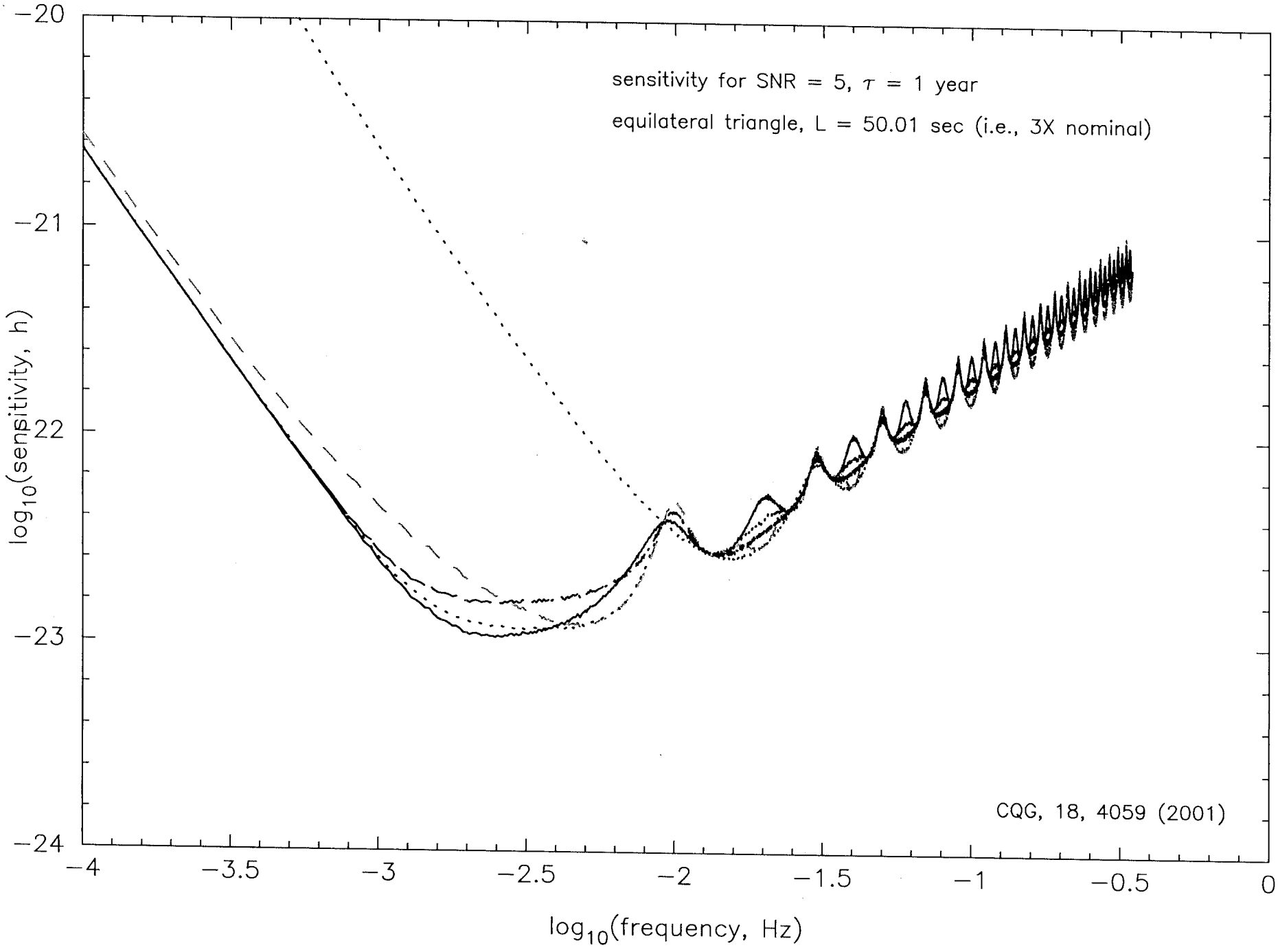
- **Square to get power spectra, convert from units of length^2/Hz to fractional Doppler (derivative theorem for Fourier transforms) to get equivalent velocity spectrum and divide by speed of light squared to convert to spectrum of y: $S^{\text{proof mass}}$ and $S^{\text{optial path}}$**

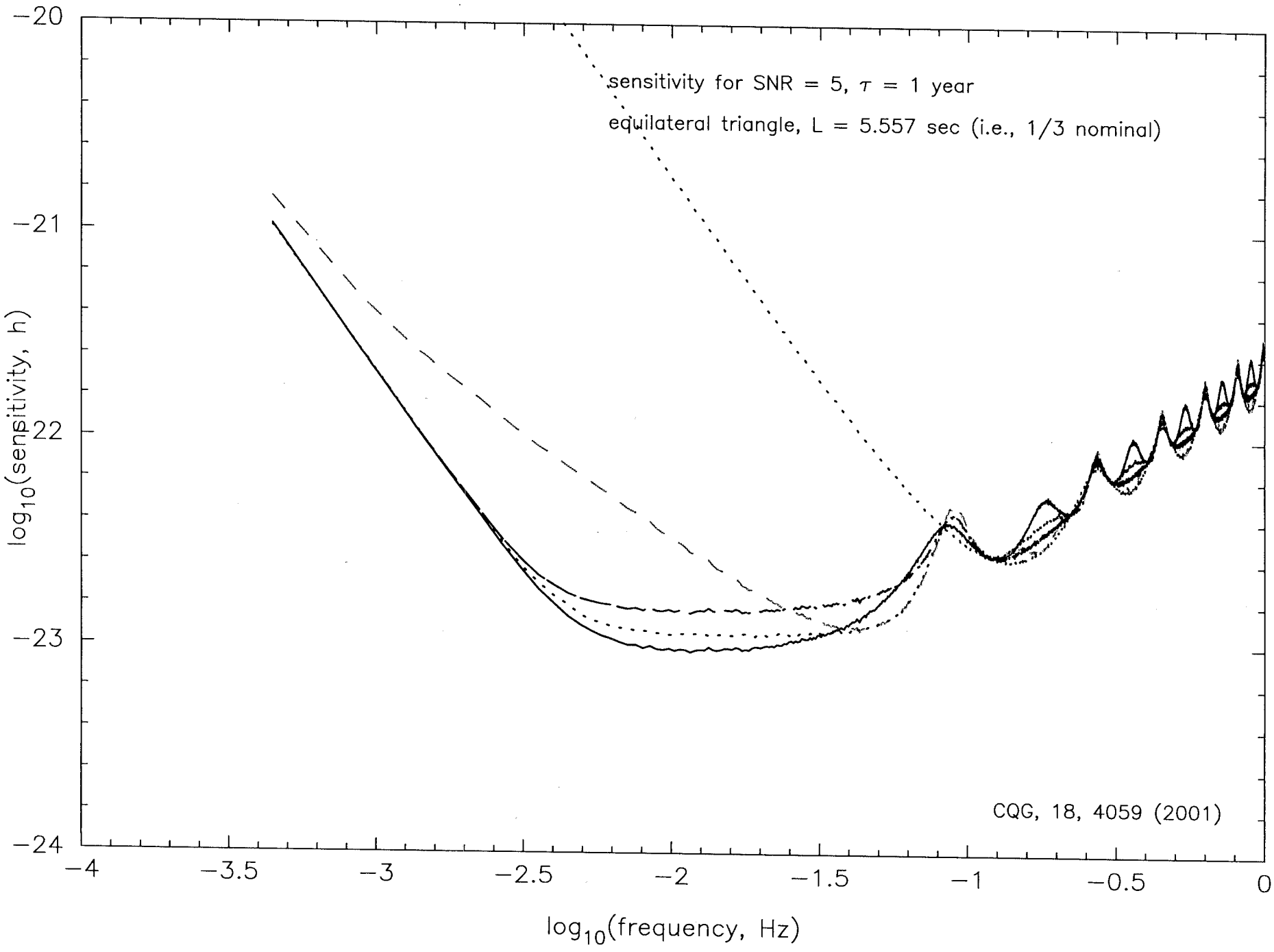
- **Write the noise-canceling data combination under consideration in terms of the defining y_{ij}'s. Square of Fourier-transform gives noise spectrum transfer functions multiplying $S^{\text{proof mass}}$ and $S^{\text{optial path}}$.**

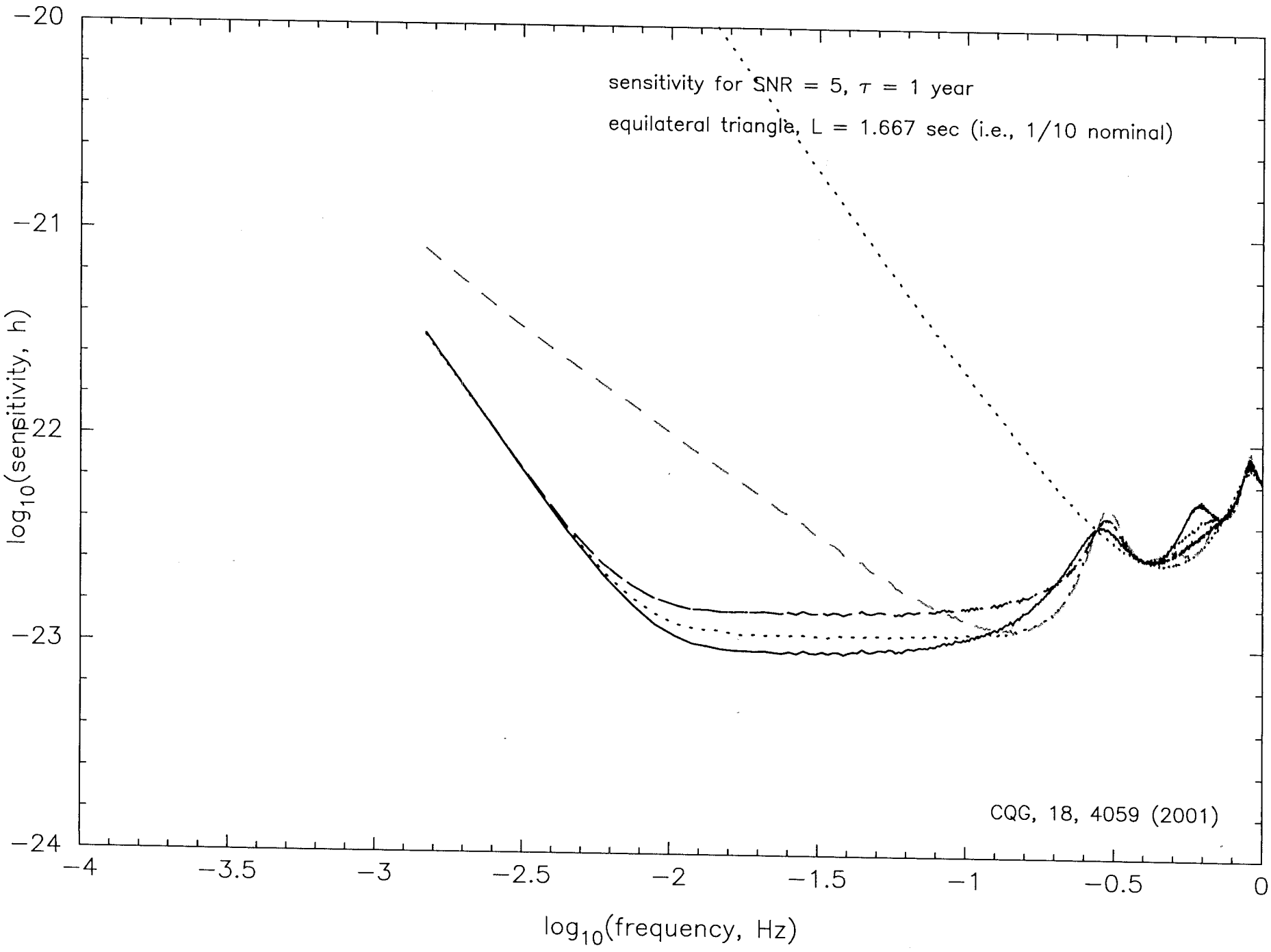
- **Examples: Aggregate noise spectrum for data combinations X, P, etc. in equilateral and non-equilateral triangle cases**


equilateral triangle, L = 16.67 seconds
X
U
α
E and P
ζ
−36
−38
−40
−42
−44
−6
−5
−4
−3
−2
−1
0
log10(noise spectral density, Hz−1)
log10(frequency, Hz)


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 16.67 seconds
ζ
α
X
U
E and P
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 50.01 sec (i.e., 3X nominal)
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 5.557 sec (i.e., 1/3 nominal)
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 1.667 sec (i.e., 1/10 nominal)
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

Application of TDI: On-orbit Noise Calibration and Isolation of GW Background

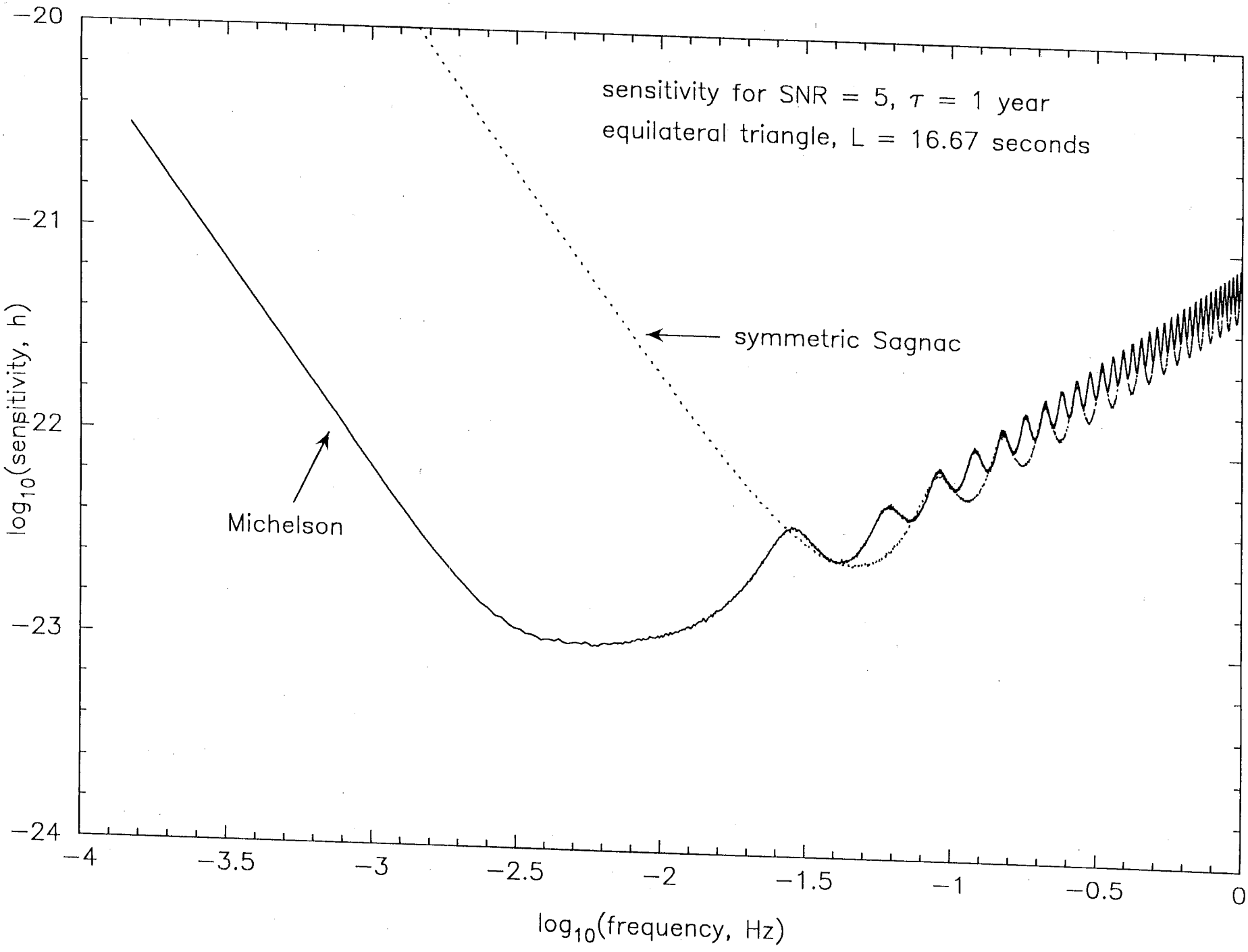
- **Different TDI combinations have different coupling to GWs**

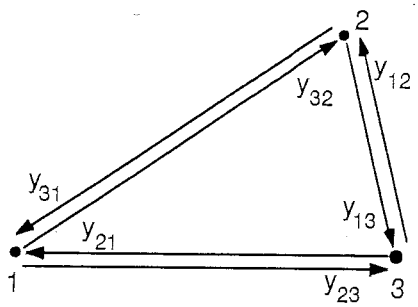
- **In particular, the "symmetrical Sagnac" combination, ζ, is in the equal-armlength case *insensitive* to GWs but contains the noises from all the links**

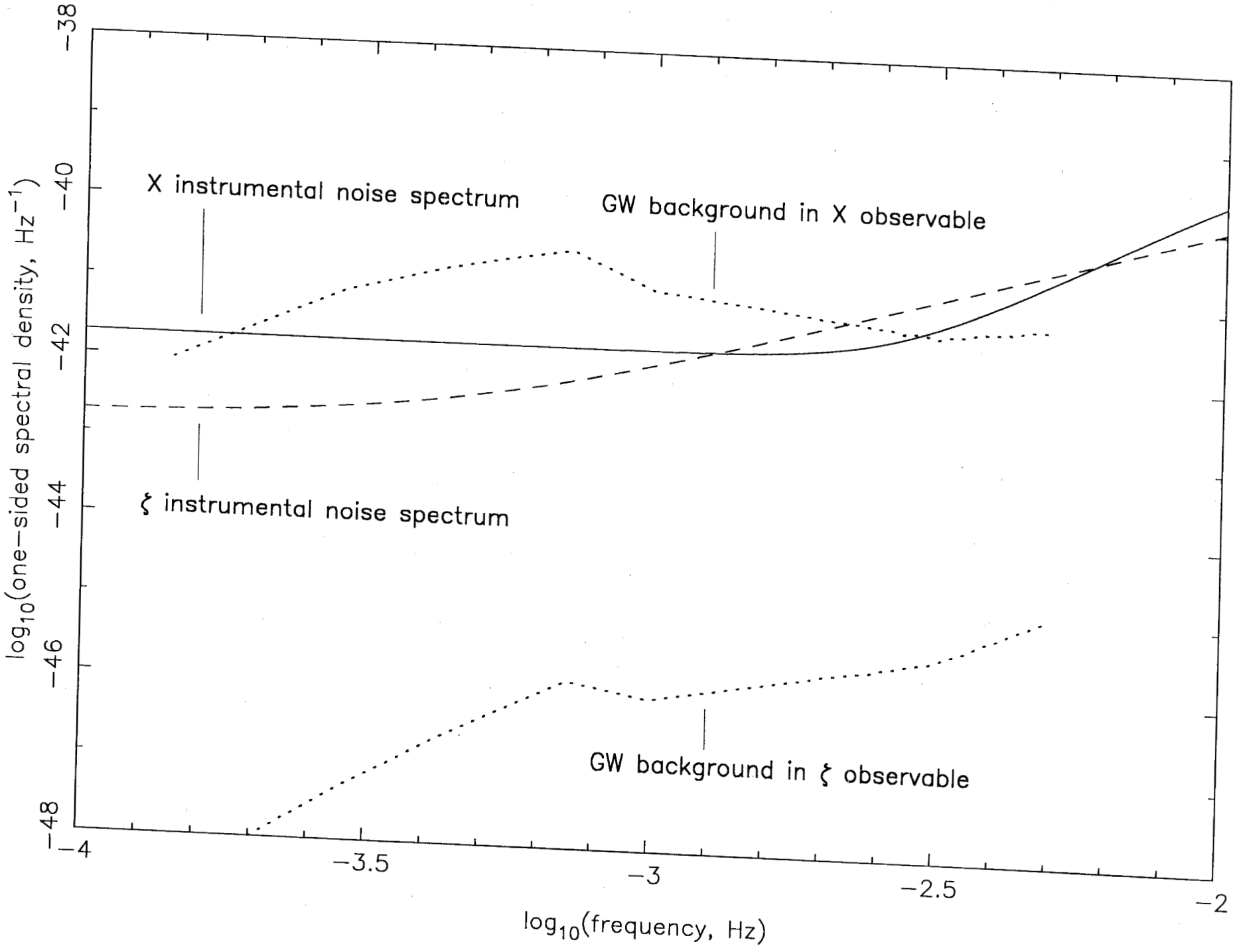
- **Thus ζ allows on-orbit and continuous calibration of the noises**

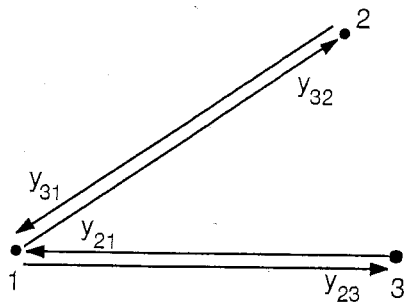
- **Comparison of ζ with a more GW-sensitivity TDI combination (e.g. X) allows unambiguous isolation of a confusion-limited GW background from instrumental noise**

- **Plots from Tinto, et al. "Discriminating a Gravitational Wave Background from Instrumental Noise in the LISA Detector", *Phys Rev. D.* 63, 021101(R) (2000) follow**


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 16.67 seconds
symmetric Sagnac
Michelson
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


2
y_{12}
y_{32}
y_{31}
y_{13}
y_{21}
1
y_{23}
3


X instrumental noise spectrum
GW background in X observable
ζ instrumental noise spectrum
GW background in ζ observable
log10(one−sided spectral density, Hz−1)
log10(frequency, Hz)
−38
−40
−42
−44
−46
−48
−4
−3.5
−3
−2.5
−2



3

Time-Delay-Interferometry (applications)

- Exact method for cancellation of leading noise source—laser phase fluctuations—in interferometer with unequal, time-variable arms
- Explicit used of time-delays across the LISA array to analyze signals, noises, and achievable sensitivity
- Architectural design (including system-level tradeoffs between, e.g., laser stabilization, stability of on-board clock, Doppler shifts due to chosen orbits, stability of optical bench)
- Robustness of mission w.r.t. failures of subsystems
- Alternate LISA configurations offering potential design, implementation or cost advantages

Time-Delay-Interferometry (applications, continued)

- Noise canceling methods and analysis of imperfections in the almost-inertial motions of LISA proof-masses, motions of LISA optical benches, and required internal metrology to achieve LISA's sensitivity goals
- Combinations of data to distinguish GW backgrounds from instrumental noises
- Combinations of data to assess LISA instrumental noise using on-orbit data
- TDI analysis used in our participation in the LISA International Science Team, Working Group #1 to compute instrument response to various signals and noises

Summary

- LISA's can be analyzed symmetrically in terms of Doppler shifts on 6 one-way laser links connecting 6 optical benches on the three spacecraft

- Many ways—not just Michelson-- to combine these links, with appropriate time delays ("time-delay interferometry"), to cancel principal noises but retain GWs

- Laser and optical bench noise canceling data combinations are explicit time-domain transfer functions for the GW signal and the remaining noises.

- Multiple noise-canceling data streams can be formed simultaneously
    - Differing sensitivity to GWs can be exploited
    - Robustness against some classes of subsystem failure

- Applications
    - Isolating instrumental noise from GW confusion with on-orbit data
    - Sensitivity for each noise-free data combination