Addition of Random Run FM Noise to the KPW Time Scale Algorithm

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Motivation

- Jones-Tryon model & Kalman filter ==> TA(NIST) time scale (early 1980s)
- Poor short-term stability observed
- Aims
  - Reproduce TA(NIST) time scale behavior
  - Understand it
  - Improve it
- Succeeded with 2-state clocks (white FM + RWFM): made KPW (Kalman plus weights) algorithm (PTTI 2001)
- Extend to 3-state clocks with RRFM (random walk of drift)

Jones-Tryon Clock Model

$X(t) = [x(t), y(t), z(t)]^T$ = departure from ideal clock at date t

$X(t) = \Phi(\tau)X(t-\tau) + W(t,\tau)$ where

$$\Phi(\tau) = \begin{bmatrix} 1 & \tau & \tau^2/2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix}, \quad W = [w_x, w_y, w_z]^T$$

$$\operatorname{cov} W = q_x \begin{bmatrix} \tau & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + q_y \begin{bmatrix} \tau^3/3 & \tau^2/2 & 0 \\ \tau^2/2 & \tau & 0 \\ 0 & 0 & 0 \end{bmatrix} + q_z \begin{bmatrix} \tau^5/20 & \tau^4/8 & \tau^3/6 \\ \tau^4/8 & \tau^3/3 & \tau^2/2 \\ \tau^3/6 & \tau^2/2 & \tau \end{bmatrix}$$

Hadamard variance $H\sigma_y^2(\tau) = \frac{q_x}{\tau} + \frac{q_y \tau}{6} + \frac{11}{120} q_z \tau^3$

Kalman Filter

Clock difference measurements on n clocks at a sequence of dates:

$$x_{i1}(t) = x_i(t) - x_1(t), \quad i = 2, \ldots, n$$

Kalman filter ==> estimates $\hat{X}_i(t) = \left[\hat{x}_i(t), \hat{y}_i(t), \hat{z}_i(t)\right]^T, \quad i = 1, \ldots, n$

Because measurements are noiseless, estimates satisfy the measurement equations:

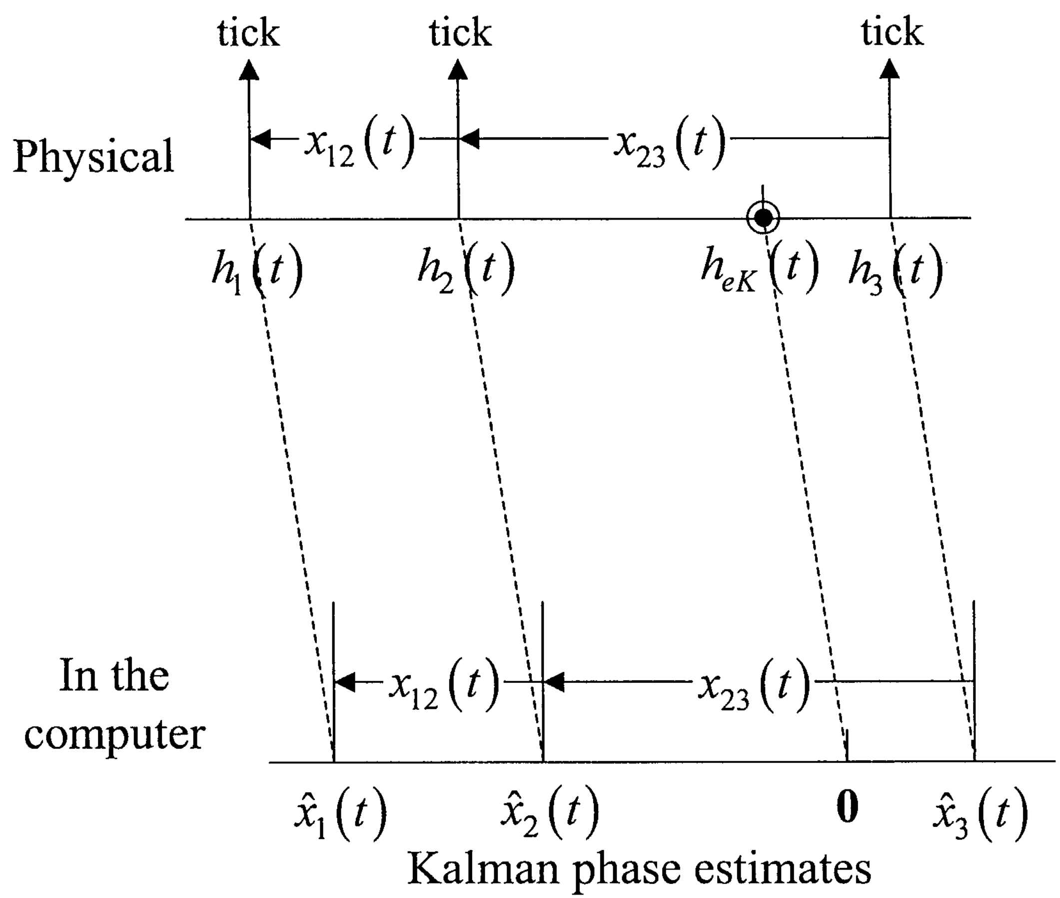
$$x_{i1}(t) = \hat{x}_i(t) - \hat{x}_1(t), \quad i = 1, \ldots, n$$

This leads to the **natural Kalman time scale** (TA(NIST)),

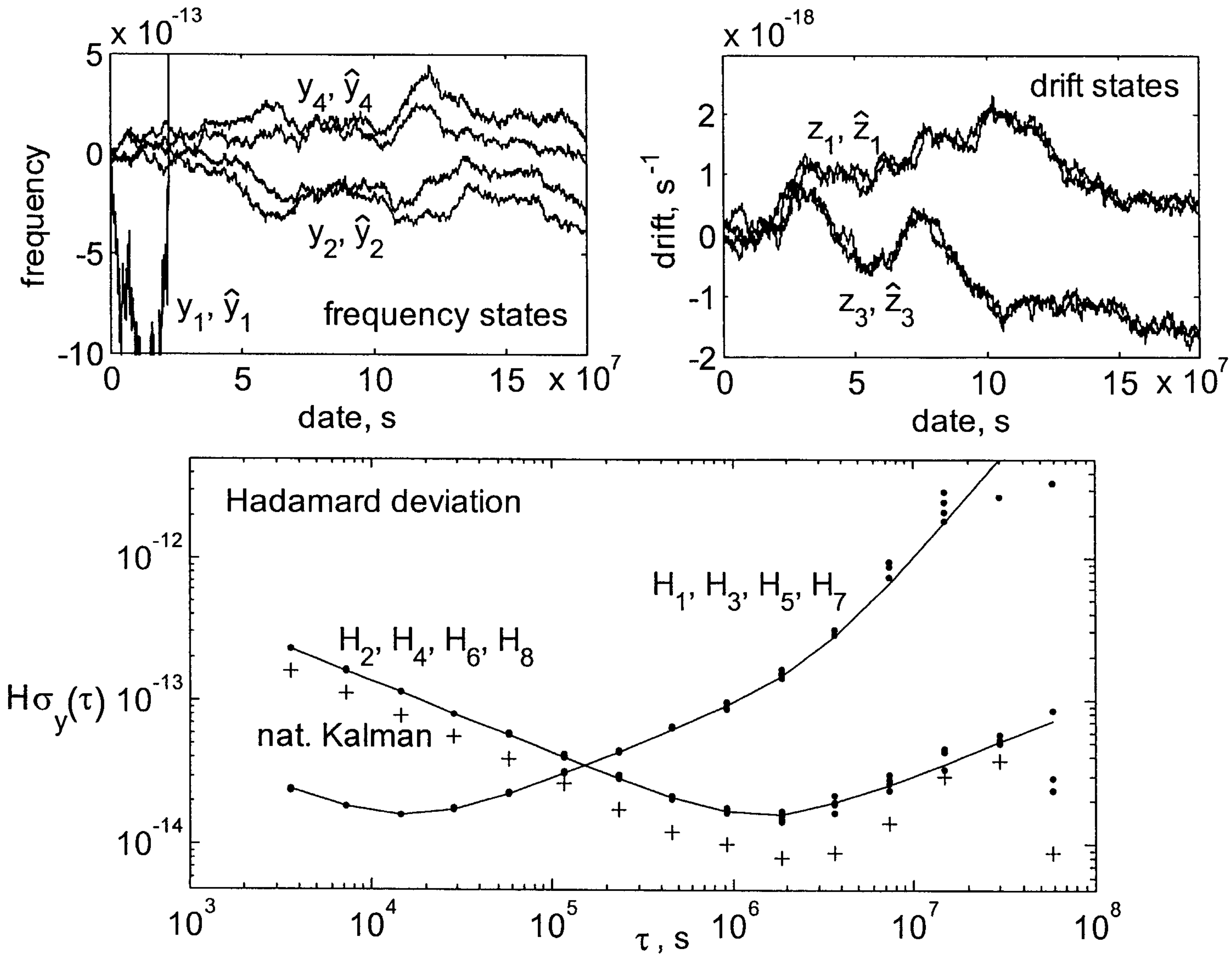
$$x_{eK}(t) = x_i(t) - \hat{x}_i(t)$$

(the same for all i)

Natural Kalman Time Scale, Illustrated



Eight Imaginary Clocks



Basic Time Scale Equation

Form modified by Breakiron (1991) to include drift

Recursive definition of average time scale

$$x_e(t) = x_e(t-\tau) + \sum_{i=1}^{n} w_i(t)\left[x_i(t) - x_i(t-\tau) - \tau\hat{y}_i(t-\tau) - \tfrac{1}{2}\tau^2\hat{z}_i(t-\tau)\right]$$

Calculated offsets from physical clocks

$$x_{ej}(t) = x_{ej}(t-\tau) + \sum_{i=1}^{n} w_i(t)\left[x_{ij}(t) - x_{ij}(t-\tau) - \tau\hat{y}_i(t-\tau) - \tfrac{1}{2}\tau^2\hat{z}_i(t-\tau)\right]$$

Departure from usual practice: $\hat{y}_i$ and $\hat{z}_i$ (the Kalman estimates) are relative to a noiseless ideal clock, not the time scale.

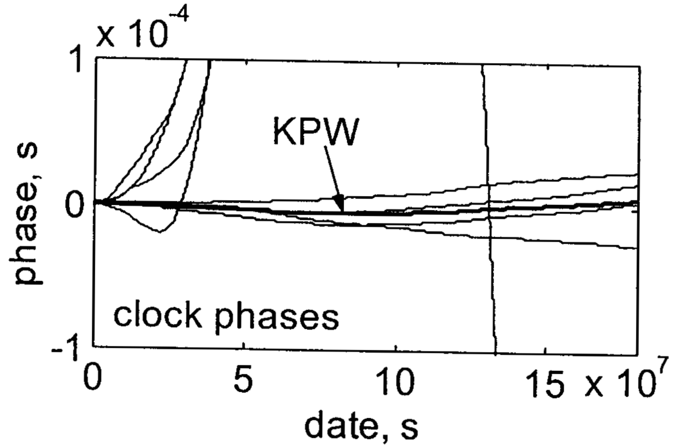
KPW Algorithm

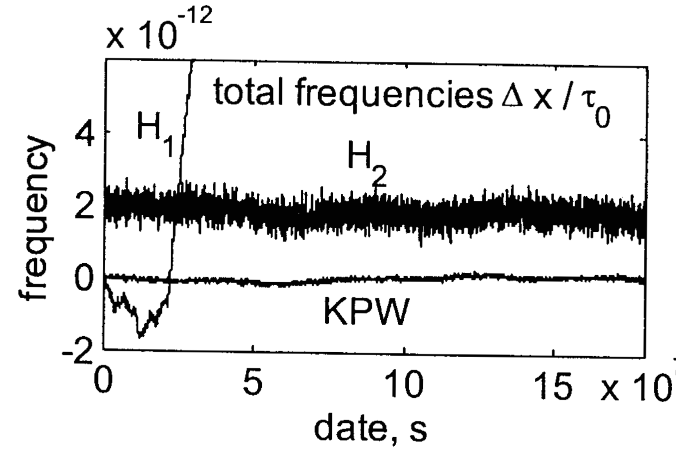
Clock q's assumed to be known

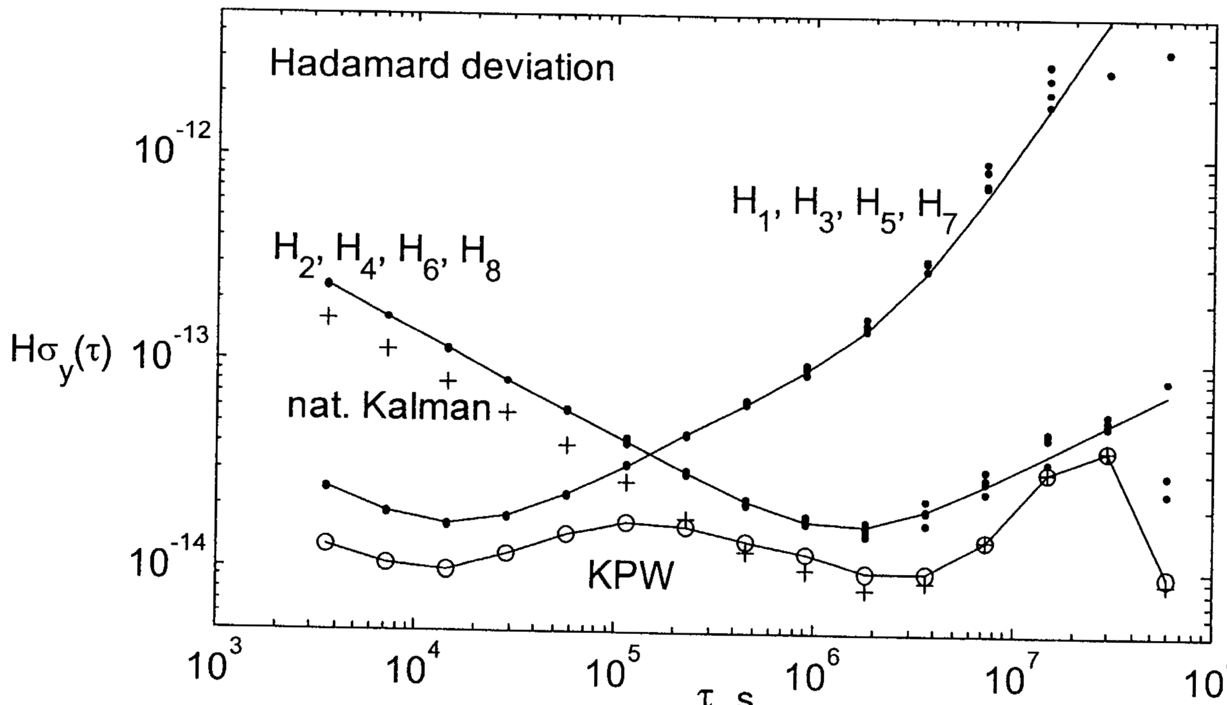
1. Initialize the Kalman filter with state estimates and error covariances.
2. Run the Kalman filter on the clock difference measurements.
3. Throw out the Kalman phase estimates.
4. Use the Kalman frequency and drift estimates in the BTSE, with weights inversely proportional to the white FM noise variances.

The Kalman filter runs independently of the time scale.

Eight-Clock Example with KPW Scale







Conclusions

- Jones-Tryon Kalman filter estimates phase states poorly ==> TA(NIST) is noisy in short term. But the filter estimates frequency and drift states well.
- KPW scale uses only the frequency and drift state estimates. It works well with 3-state clocks in simulation playpen. Observed to be quieter than any component clock at all averaging times, except for extreme ensembles.
- Possible foundation of practical real-time time scale; needs practical add-ons:
  - Ensemble changes
  - Outliers, jumps in phase and frequency
  - Adaptive estimation of q's
  - Steering
- Add white PM and measurement noise