Quantum Information, Computation, and Technology at JPL

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Overview

- Quantum High Precision Measurement
- Linear Optics Quantum Computation
- Quantum Circuit Design
- Quantum Information
The 2\textsuperscript{nd} Quantum Revolution

- Shor’s discovery in 1994 that factoring is polynomial on a quantum computer started the 2\textsuperscript{nd} QR.
- Computer Science and Information Theory obtained new sub-fields: QCS and QIT.
- Reverberates beyond computing, into \textit{physics} of measurement and information.
Background

- Information is physical
  - IT is the theory of relative states of "detectors"
- Computation is dynamics of physical information-carriers
  - An algorithm is equivalent to a physical device
Background

- Until 1994, almost all of CS and IT was based on classical bits and classical laws of interaction.
- Physics went beyond classical in 1900-1925:
  - Quantum Mechanics / QFT
  - Special Relativity
  - General Relativity
Bits to Qubits

\{0,1\} \rightarrow
Relativistic Qubits
Accelerated Qubits
Physical Implementations

- Photon spin
- Electron spin
- Charge (quantum dot)
- Magnetic flux (squid)
- Atomic magnetic moment
- Nuclear spin
- Others?
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High-Precision Measurement

- Any quantum computation is equivalent to an optical interferometer


Quantum circuit diagram
High-Precision Measurement

Equivalent Interferometer:

“Classical” Light

Beam Splitter S₁

Mirror

Beam Splitter S₂

Mirror

Detector C

Detector D
Any computation can be implemented in linear optics, but with exponential cost of resources

From N.J. Cerf, P. Kwiat & C. Adami
High Precision Measurement

• Reverse argument:
  – If quantum communication allows super-classical computation, same techniques should allow for high-precision instruments beyond the classical limit!
High Precision Measurement

- Possible application to LIGO
  - currently limited by shot noise
  - Quantum interferometry can push sensitivity to the quantum limit
Mach-Zehnder Interferometer with non-classical N-photon input

\[ |N\rangle_A |0\rangle_B + e^{iN\phi} |0\rangle_A |N\rangle_B \]

A

| N \rangle_A |0\rangle_B

B

magic BS

N-XOR Gates

N-XOR Gates

Oscillates N times as fast!

1 + \cos \varphi

\frac{1}{2}

uncorrelated

\varphi = kx

\Delta \varphi : \frac{1}{\sqrt{N}} \rightarrow \frac{1}{N}

1 + \cos N\varphi

\frac{1}{2}

correlated
High-Precision Measurements

- Similar arguments lead to following devices with improved sensitivity:
  - Quantum Gradiometer  I. Kulikov, U. Yurtsever, J.P. Dowling & L. Maleki
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Linear Optics Quantum Computation

• Applications:
  – A Quantum Computer!
  – Quantum Key Distribution
  – Quantum Internet
Linear Optical Quantum Computation (LOQC)

- In 2001, Knill, Laflamme, & Milburn [Nature 409, 46 (2001)] showed that the standard LOQC of Cerf, Kwiat & Adami could be implemented efficiently, i.e., without exponential cost in resources, if logic operations are implemented stochastically.
Designing a Linear Optical Quantum Computer

Single and Entangled Photon → Linear-Optical Single and Entangled Photon → Single-photon and homodyne
Realizing quantum computers based on LOQC relies on the development of a number of concepts and devices:

- Generation of single-photons on demand [P. Kok, H. Lee, G. Hockney, J.P. Dowling]
- Optical quantum memory [R.M. Gingrich, P. Kok, H. Lee, F. Vatan & J.P. Dowling]
- Practical quantum repeaters [P. Kok, C.P. Williams, J.P. Dowling]
- Quantum Non-demolition measurement devices [P. Kok, H. Lee, J.P. Dowling]
Example: Quantum Transponder

R.M. Gingrich, P. Kok, H. Lee, F. Vatan & J.P. Dowling
Fiber of length $L$ has a loss of $e^{-\gamma L}$, where $\gamma$ is a property of the fiber. Similarly, the entanglement exhibits decoherence that scales exponentially with the length of the fiber. We therefore need some purification protocol that revives the entanglement.

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Quantum Circuit Design

- Every quantum computation is equivalent to a unitary operation over the input qubit space
Quantum Circuit Design

- This is not practical. But $U$ can be written in terms of one-qubit rotations (phase gates) and two-qubit operations (quantum CNOT) only.
Quantum Circuit Design

- The decomposition of $U$ is not unique. What is the most efficient implementation of $U$?
Quantum Circuit Design

- The *Quantum Circuit Design (QCD) Tool* is a Mathematica-based optimization program with graphical front-end that constructs its circuit decomposition from the Generalized Singular Value Decomposition (GSVD) of the given unitary matrix [C. P. Williams]

- GSVD exploits fact that blocks of a partitioned unitary matrix have highly related singular value decompositions
Quantum Circuit Design

- GSVD decomposition of a $2^n \times 2^n$ unitary matrix

$$U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} = \begin{pmatrix} L_0 & 0 \\ 0 & L_1 \end{pmatrix} \cdot \begin{pmatrix} D_{00} & D_{01} \\ D_{10} & D_{11} \end{pmatrix} \cdot \begin{pmatrix} R_0 & 0 \\ 0 & R_1 \end{pmatrix}$$

- $L_0, L_1, R_0, R_1$, are $2^{n-1} \times 2^{n-1}$ unitary matrices
- Apply decomposition recursively
QCD can detect special structure if it exists
  - E.g. QCD finds a compact circuit for QFT
Example: Compact Circuit for Quantum Wavelet Transform

QCD: Quantum Circuit Designer

```
QuantumWaveletD4[4, Method -> Pyramid]

QuantumWaveletD4[4, Method -> Pyramid]

(circle = RandomRepetitiveCompactifyQuantumCircuit[MatrixToQuantumCircuit[m], 100])

direct[phase(7.85398), i2], direct[i1], rotate(-5.75959), direct[i2], rotate(-90), cos[i2, 1, 2], direct[i2], rotate(-4.18879)]

QuantumCircuitToDiagram[circle]

VerifyQuantumCircuit[circle, m]

True
```
Algorithms

- QCD attempts to find the smallest circuit for a given $U$ by applying circuit compactification rules
  - Eliminate gates from the circuit while preserving correctness, e.g.,
State Generation by QCD

- Goal: to make $W$ state starting from

Complete the matrix with orthonormal vectors.

From this matrix, QCD computes a circuit to synthesize a W state.
QCD

• Versatile Tool that can also be used for:
  – Synthesizing arbitrary mixed states
  – Quantum Signal, Image and Data Processing
  – Non-Unitary (Probabilistic) Quantum Algorithms

• Or in Quantum Hardware Design
  – Charge-based, spin-based and optical qubits are suited to using different elementary quantum gates
  – QCD outputs circuits that use different gate primitives
  – QCD outputs circuits that use encoded gates for decoherence-free operation

C.P. Williams
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Quantum Information

- What happens to the mutual entropy of two correlated detectors moving at relativistic speeds with respect to each other?
- Construct relativistic Quantum Information Theory
- Understand the physics of
  - accelerated qubits
  - qubits in strong gravitational fields
Entanglement of Moving Observers

- But if two satellites (or observers) move with respect to each other, what happens to their entanglement?
- This question has not been previously addressed. It connects two of Einsteins’ most profound contributions: relativity and “Einstein-Podolsky-Rosen” entanglement
So we need to answer the question:

- How does spin-spin entanglement transform under Lorentz transformations?
  - Massive case\(^1\)
  - Massless case\(^2\)

Spin Entanglement

- Many models for quantum computation use spin degrees of freedom for qubit
- When we use spin we are implicitly tracing out the momentum part of the wavefunction, e.g.:

  \[ \rho_{\text{spin}} \]

- We answer how \( \rho_{\text{spin}} \) transforms
Massive Spin-$\frac{1}{2}$ Entanglement

• Lorentz-transform a state, fully entangled in spin, in a Gaussian momentum distribution (centered about zero):

Above is concurrence (C) as a function of rapidity ($\xi$) for two different spreads.
Summary

- Spin behaves differently in relativistic QM
- Spin and momentum degrees of freedom can exchange entanglement when boosted
Information in Curved Space Time

- What happens to entropy in accelerated frames, or in the vicinity of strong gravitational fields?

- Quantum-field theoretic vacuum is non-trivial: particles are surrounded by *Unruh radiation*: they appear to be immersed in a heat bath at temperature
Information in Curved Space Time

- Entropy and Information need to be redefined: only those degrees of freedom *that can be measured locally* can contribute to a local observer’s entropy.