Multi-reference Evaluation of Measurement Uncertainty in Earth Orientation Parameters

T.M. Chin, R.S. Gross, J.O. Dickey

Jet Propulsion Laboratory
California Institute of Technology

The research described in this poster was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (NASA).

Introduction

The earth orientation parameters including the length-of-day variation (LOD) and polar motion (PM) are routinely estimated by measurements from various techniques such as the very long baseline interferometer (VLBI), satellite laser ranging (SLR), and global positioning system (GPS). Objective combination of multiple data sets, such as weighted least-squares or Bayesian statistical (Kalman filter) estimation, requires some quantification of the relative accuracy of these measurements. Statistical sampling of measurement error variances and covariances is usually dependent on some assumed values for the ground-truth earth orientation parameters. Measurement error is difficult to evaluate because the true signal ("ground-truth") values to which the error is referenced are usually not known exactly. A technique that does not require the ground-truth values is thus desirable. When multiple independent measurements are available for the same signal, the error statistics can be determined algebraically under certain assumptions. The *Three-Corner Hat* (TCH) method is such an algebraic technique, applicable when there are three or more independent sets of measurements.

Measurement sets

data set	*label*	*sampling*
Jet Propulsion Lab, Quick-look GPS	GPS.JQ	noon (UTC)
International GPS Service, Rapid	GPS.IR	noon
International GPS Service, Final	GPS.IF	noon
Joint Center Earth System Technology, SLR	SLR	noon
International VLBI Service, combined multibaseline	VLBI.IM	every 2.9 days*†
Goddard Space Flight Center, multibaseline VLBI	VLBI.GM	every 2.7 days*†
Goddard Space Flight Center, intensive VLBI¶	VLBI.GI	every 1.8 days*†
Goddard Space Flight Center, multibaseline VLBI LOD¶§	VLBI.GL	every 2.7 days*
International Earth Rotation Service, C 04	C04	midnight (UTC)
Jet Propulsion Lab, SPACE 2002	SPACE	noon & midnight

* — average time interval.

† — LOD obtained by differentiation of UT1 values.

¶ — PM values not available/used.

§ — This data set is a differentiated version of VLBI.GM.

Method

Let the measured values be y_i, $1 \leq i \leq N$, where the index i specifies the measurement type (a particular data set). We consider the standard, additive-noise model for measurement process as

$$y_i \;=\; x + w_i \tag{1}$$

where x is the true signal and w_i is a zero-mean white noise process representing the measurement error. This model is routinely used in high-level processing (e.g., Kalman filtering and least-squares) of the measurements. The difference among the measurements y_i eliminates the common signal x; we define

$$z_{ij} \;\equiv\; y_i - y_j \;=\; w_i - w_j \tag{2}$$

for $i < j$ and refer to z_{ij} as the *difference process*. The correlation among the difference processes can constrain the statistics of the white noise processes as

$$R_{ik} + R_{j\ell} - R_{i\ell} - R_{jk} \;=\; \langle\, z_{ij} z_{k\ell} \,\rangle \tag{3}$$

where $R_{ij} \equiv \langle\, w_i w_j \,\rangle$, $i \leq j$, are the noise covariances and the angular bracket denotes time average.

We wish to compute the covariances R_{ij} from the empirical correlations $\langle\, z_{ij} z_{k\ell} \,\rangle$ evaluated from the measurements. Given N measurements, the number of distinct covariances R_{ij} is $(N+1)N/2$. Since only $N-1$ linearly independent differences z_{ij} are available, the number of effective constraint equations (3) is only $N(N-1)/2$, which equals to the number of distinct auto- and cross-correlations among the $N-1$ differences. The unknowns out-number the constraints by N. A key feature of the TCH technique is the assumption that N (or more) of the cross-correlations are zero:

$$R_{ij} = 0 \tag{4}$$

for some i and j where $i \neq j$. For example, in a classic application of the Three-Corner Hat technique, there are exactly three *statistically-independent* measurement sets, i.e., $N = 3$ and $R_{12} = R_{13} = R_{23} = 0$. The three variances R_{ii}, $1 \leq i \leq 3$, are then uniquely constrained by (3).

Assumption of independence

There can be some apparent correlation among the noise processes even if the measurements are physically unrelated. The TCH technique can fail when the assumption of independence among the measurements is not satisfied empirically. For example, in the classic case of $N = 3$ the variances are computed as

$$R_{ii} = \langle\, y_i^2 \,\rangle + \langle\, y_j y_k \,\rangle - \langle\, y_i y_j \,\rangle - \langle\, y_i y_k \,\rangle \tag{5}$$

where the signal indices i, j, k are distinct (not equal to each other). The right hand side of (5) can conceivably become negative depending on empirical correlation among the measurements, demonstrating that the TCH method has no inherent algebraic guarantee for positivity of the variance values. On the other hand, if the independence constraint (4) is correct, the last three terms of (5) would be equal to $-\langle\, x^2 \,\rangle$, and correct evaluation can be expected. Fidelity of the independence assumption (4) is thus important to success of the TCH method. Figure 1 displays a plot of the errors in the TCH variance evaluation against the actual correlation present in the noise processes, which have been numerically generated using zero-mean Gaussian distributions. The error increases almost linearly with the correlation coefficient. When the correlation is zero as assumed in (4), the average error is found to be approximately 1%.

Choice of independent pairs

For $N > 3$, at least N cross-variances R_{ij} must be chosen among the $N(N-1)/2$ candidates for application of the independence constraint (4). These choices are usually made based on the physical scenarios. For example, the errors in GPS and SLR can be assumed uncorrelated on the ground that the instrumentation and principle of the two measurements are unrelated. On the other hand, the errors in GPS-based data from different analysis centers should be assumed correlated as the same instrument is shared by the data sets. In addition, there are some algebraic considerations, one of which is as follows:

Lemma 1 *A necessary condition for uniqueness of the TCH solution is that every noise process must be assumed uncorrelated to at least one other noise process.*

This condition is satisfied automatically when N is 3 or 4. Algebraic necessities such as this need to be matched with physical reasoning as an erroneous assumption of independence can lead to failure of the TCH technique as demonstrated previously. For the main eight measurement sets, we assume that the noise processes are independent across the three instrumentation groups of GPS, SLR, and VLBI, while non-zero correlations are allowed within each group (Figure 2).

Result and Summary

- The noise values for LOD are on the order of 10 to 100 micro-seconds, while the signal variability of LOD is approximately half to third of a milli-second. The noise values for the x and y components of PM (PM-X and PM-Y, respectively) have a range of 0.050 to 0.500 milli-arc-second (mas), while the signal variability of both components are on the order of 100 mas.

- The columns labeled *ID* and *SA* in Table 1 represent the evaluation errors due to "interpolation and differentiation" and "stationarity assumption", respectively.

  - The *SA* values are computed as the variability among the TCH evaluations when different time-segments of the data sets are used. The full data segment is divided uniformly into four subsegments, the TCH evaluations are performed on each subsegment, and then variability among the variance values is computed. The range of such time-variability is approximately 10 to 25% for both LOD and PM.

  - To evaluate *ID*, the SPACE LOD series, which is available at both noon and midnight, is appropriately subsampled, interpolated, and compared with the missed data. Numerical integration is also performed to simulate the UT1 data for VLBI. The average daily interpolation (noon to midnight or vise versa) is found negligible at 0.76 μsec.

The interpolation error is found much larger for the VLBI data sets, which have nominal data intervals of 2 to 3 days. Interpolation is performed with a smoothing B-spline scheme which allows analytic differentiation and yields more accurate results than linear interpolation. Interpolation of the UT1 values is found to introduce five to six times error (in magnitude) than the subsequent differentiation procedure. The TCH evaluations for the VLBI-based UT1 series have low signal-to-noise ratios in terms of *ID* error.

- The TCH computation has yielded a set of mostly-intuitive values for the LOD & PM measurement error variances (square-root variances). Still, our results are not a fair assessment of the uncertainty in the VLBI-based measurements, due to strong possibility of interpolation errors in the data. Availability of daily-sampled VLBI-based data sets could remedy this.

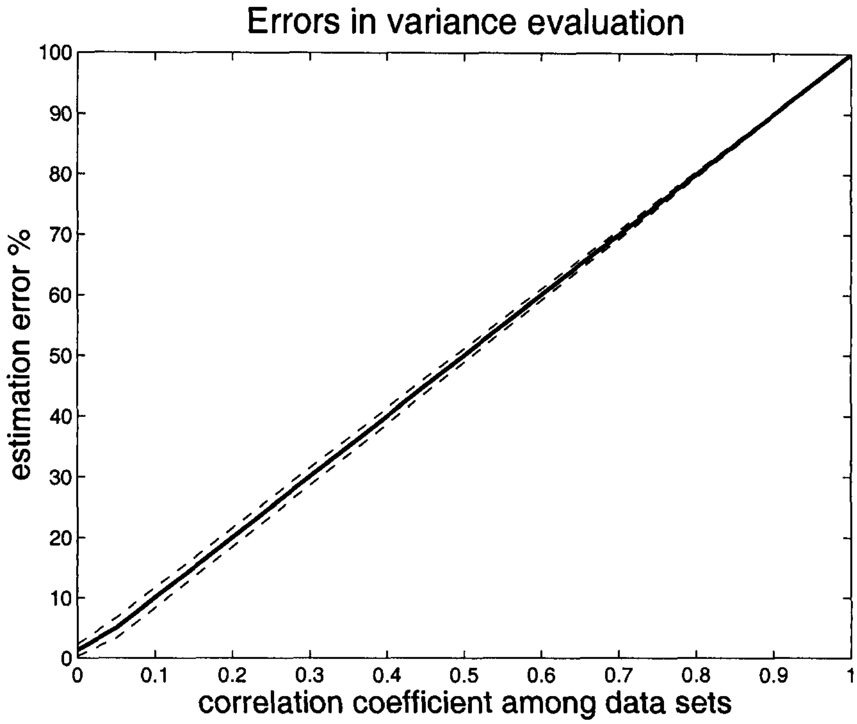

Figure 1. Percent errors in variances computed with the three-corner hat method, as a function of the correlation coefficient among the data sets. The solid line is the mean of 400 simulations, while the dashed lines are the corresponding single-standard-deviation envelops.

	LOD (μsec)				*PM* (mas)			
data set	noon	midnight	*ID*	*SA*	PM-X	*SA*	PM-Y	*SA*
GPS.JQ	38.3	38.1		±4.7	0.301	±0.041	0.240	±0.039
GPS.IR	19.1	18.8		±5.7	0.134	±0.034	0.071	±0.021
GPS.IF	11.8	11.4		±3.9	0.080	±0.025	0.049	±0.028
SLR	132.9	132.3		±18.5	0.171	±0.018	0.150	±0.026
VLBI.IM	27.2	26.9	±24.5	±2.7	0.127	±0.009	0.122	±0.027
VLBI.GM	26.2	26.1	±22.5	±3.5	0.522	±0.217	0.295	±0.145
VLBI.GI	47.6	50.7	±14.4	±13.7				
VLBI.GL	53.3	52.7	±30.9	±13.6				

Table 1. Square-root variances of the measurement errors, evaluated using the Three-Corner Hat method. Columns labeled *ID* and *SA* display estimates of the uncertainty in the evaluation. The former (*ID*) is associated with numerical errors introduced by interpolation and differentiation, while the latter (*SA*) is a measure of deviation from stationarity assumption.

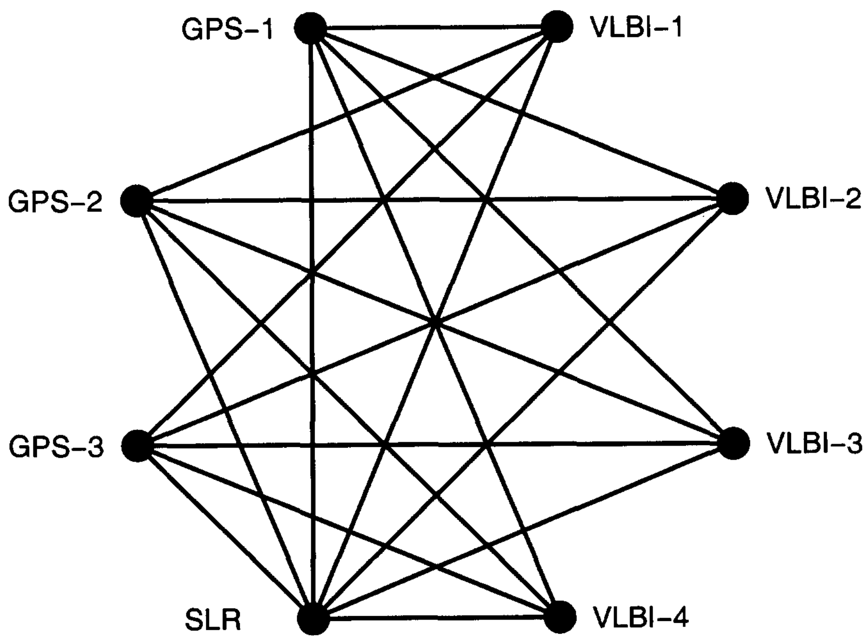

Figure 2. Application of independence constraints among the eight measurement data sets. A pair of data sets connected by a straight line is assumed to be uncorrelated (with respect to the noise processes) during TCH evaluation.

LOD data set	GPS.IR	GPS.IF	SLR	VLBI.IM	VLBI.GM	VLBI.GI	VLBI.GL
GPS.JQ	+.24	+.07	+.02	−.01	−.02	−.03	−.02
GPS.IR		+.24	−.01	+.11	−.07	−.05	−.02
GPS.IF			−.07	+.17	−.03	.00	+.04
SLR				−.03	+.02	+.01	.00
VLBI.IM					+.88	+.13	+.35
VLBI.GM						+.12	+.34
VLBI.GI							+.07

Table 2. Correlation coefficients among the LOD data sets, evaluated with the Three-Corner Hat method.

PM-X data set	GPS.IR	GPS.IF	SLR	VLBI.IM	VLBI.GM
GPS.JQ	+.50	+.39	−.09	+.02	+.02
GPS.IR		+.67	+.12	+.05	−.05
GPS.IF			+.12	−.06	−.02
SLR				−.05	+.01
VLBI.IM					+.27

Table 3. Correlation coefficients among the PM-X data sets, evaluated with the Three-Corner Hat method.

PM-Y data set	GPS.IR	GPS.IF	SLR	VLBI.IM	VLBI.GM
GPS.JQ	+.08	−.42	−.09	−.09	+.09
GPS.IR		−.09	+.13	−.07	−.04
GPS.IF			+.27	+.09	−.17
SLR				+.15	−.16
VLBI.IM					+.25

Table 4. Correlation coefficients among the PM-Y data sets, evaluated with the Three-Corner Hat method.

	LOD (μsec)			*PM* (mas)			
data set	noon	midnight	*SA*	PM-X	*SA*	PM-Y	*SA*
C04	22.8	22.6	±4.5	0.086	±0.013	0.093	±0.015
SPACE	17.8	17.3	±5.2	0.047	±0.017	0.066	±0.026

Table 5. Same as Table 1, except square-root variances of the errors in the multi-measurement combination series are displayed.