Application of a Large Space-Based Diffractive Optical Element Telescope to the Terrestrial Planet-Finder Mission: Design and Tolerance Issues

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Abstract
An analytical description of the scattered light from a 10 meter diameter Diffractive Optical Element lens-based telescope operating at 1 micron wavelength has been formulated. The specifics of the grating and blaze as well as physical manufacturing constraints were made a part of the problem to be solved. A major simplifying approximation made is that a 1 dimensional lens was assumed for the calculations. This simplified model still serves to illustrate the important effects and limitations of a high performance lens used as a telescope. Focal plane light scattering has been rigorously determined for simplified cases.

Keywords- diffractive optical element telescope, diffraction calculation, DOE, telescope, TPF, tolerance, scattered light

Introduction

The purpose of this document is to discuss some design and tolerance issues related to the application of a large Space-Based Diffractive Optical Element Telescope, such as that described in [1], to the Terrestrial Planet-Finder Mission, [2]. The imaging problem for the Terrestrial Planet-Finder is depicted in Figure 1. Light from a distant planet is focussed onto the focal plane of a large lens. Light from the nearby star is also focussed onto the focal plane, at a slightly different location, determined by the angular separation between the planet and the star. For a 10-meter class lens the planet-star angular separations of interest correspond to a separation of 5-10 times the diffraction limit of the lens operating at 1 micron. In general the light from the star will exceed that of the planet by 9 orders of magnitude. Due to imperfections in the lens such as gaps, surface imperfections, and segment alignment some light from the star will find its way into the image of the planet, possibly overwhelming the planet light entirely. Even for a perfect lens diffraction effects will ultimately determine the amount of starlight contaminating the planet's image. The design goals for TPF are to minimize the acceptable angular separation between star and planet and to provide the maximum sensitivity (minimum star light contamination) at that separation.

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In order to allow an exact diffraction calculation for the very large lenses contemplated for TPF, 10^7 wavelengths at 1 micron, a simple one-dimensional lens is considered in the analysis. This simplified model still serves to illustrate the important effects and limitations for the TPF lens. Future work should consider the two-dimensional lens, as discussed in the final section of this document. The study begins with an analysis of the problem of gaps in the lens, and the associated blockage. Next the effect of surface imperfections and the associated stray light is given. The remainder of the document discusses the possibility of minimizing the effects of diffraction by adding some perturbations to the ideal lens surface. The perturbations are chosen to minimize (null) the Point Spread Function (PSF) of the lens over some particular range in the focal plane. If the angular separation between the planet and the star falls within this range the effect of the diffracted light from the star on the planet's image will be minimized. Some example nulled Point Spread Functions and the associated lens perturbations are described. Some fundamental limitations on this approach are also discussed. Conclusions are drawn and directions for future work are suggested.

Gaps and Blockage

Many practical implementations of a 10-meter class space based lens require joints or hinges of some type in order to deploy the lens from a small-envelope, stowed configuration to the full 10-meter diameter. An exception would be a thin flexible structure that could be unrolled in orbit. It is assumed that any required joints will block some of the incoming light, modifying the PSF from that of the unblocked lens. These effects were studied assuming an otherwise ideal lens with perfect phase-front generation and zero surface distortion. The exact diffraction integral was computed numerically using an adaptive integrator with adjustable, guaranteed numerical accuracy. A simple F/D=1, D=10m lens was used in order to amplify any effect of small F/D values. A wavelength of 1μm is assumed here and for all other calculations described in this document.

Figure 2 shows results for a 1 mm gap centered in a 10m wide lens. It is assumed that the gap totally blocks any incoming light from reaching the focal plane. The nominal PSF for the case of no gap, the PSF with gap, and the difference between the two Point Spread Functions is shown. To the accuracy depicted in the plot the two PSFs look identical, however a difference pattern exists at a level 80 dB below the PSF peak. This effect, while small, is not negligible compared to an assumed TPF scattered light (nulling) requirement of 90 dB (9 orders of magnitude). The difference is flat for the portion of the focal plane of interest, as depicted in the plot. This is the result expected using superposition. The difference field in the focal plane is simply that due to the negative of the incident field over the 1 mm gap. The PSF for this field configuration is identical to that of the lens but scaled along the x-axis by a factor of $(10\text{m}/1\text{mm})=10^4$. Therefore it is essentially flat over the plotted range. The field magnitude is also scaled by a ratio of the blocked area to the lens area, a factor of 10^{-4}. This corresponds to a power value of 10^{-8}, or –80 dB as shown on the figure. This result is significant and indicates that even if elaborate lens profiling can be accomplished to reduce the PSF in some region to less than –90 dB the result will be spoiled by the blockage caused by a single 1 mm gap in the lens. Thus the gaps must either be eliminated, reduced in size, or included in the procedure used to optimize the lens profile.

The effect of multiple gaps is depicted in Figure 3. Here nine 1 mm gaps are assumed to be located symmetrically on the lens. The nine scattered fields now interfere with each other in the focal plane. The maximum effect appears on axis where all scattered components add. The blockage in this case is increased by roughly 1 order of magnitude, causing the difference PSF to rise by 20 dB to a value of –60 dB below the PSF peak. Throughout most of the plotted range the effect is much smaller however, in the range of –75dB or less. This indicates that depending on their placement on the lens the effect of multiple gaps on the PSF, in the region of interest, may not be significantly worse than that of a single one. In order to guarantee a difference PSF of –90 dB or less everywhere the total blockage must be reduced to a value of $10^{-4.5}$ or less. For a 10 m linear lens this corresponds to a value of approximately 0.3 mm.

Surface Imperfections

A second important effect on the performance of a Fresnel lens for the TPF application is that due to finite surface imperfections. Random imperfections in the surface will scatter starlight into the focal plane in the region of the incident planet light. Even a small amount of stray starlight is significant in the TPF application so these effects must be considered carefully. Since these effects are presumed to be random they cannot be incorporated into the lens profiling process described in the next section. The significant spectral components of the imperfections must be either eliminated in the manufacturing process or corrected with secondary optics, possibly including a deformable mirror.

Figure 4 shows the assumed form for the Power Spectral Density (PSD) of the surface imperfections for the lens. The spectrum is assumed to be flat for ripple periods greater than about 1mm (small wave numbers) and falling as k^n for wave numbers larger than that corresponding to a period of 1 mm. Curves for the cases of n=-1,-2,-3, and $-\infty$ (rectangular spectrum), are shown. In all cases a total surface RMS of 2 nm is maintained. This RMS value is obtained by integrating the PSD over all values of wave number. As can be seen by the figure the overall height of the spectrum is relatively insensitive to the choice of the power law assumed for the roll-off of the spectrum.

Assuming the surface imperfections are small compared to the wavelength, the spectrum of the scattered light and the PSD are of identical shape. The magnitude of the scattered light is obtained by scaling the PSD by k_0^2, where k_0 is the wave number corresponding to the wavelength of operation, [3]. The scattered light spectra corresponding to the PSD plots of Figure 4 are shown in Figure 5. The overall Point Spread Function of the lens is then composed of two parts (1) the ideal PSF, and (2) a convolution of the ideal PSF with the power density plot depicted in Figure 5. The wave number axis of Figure 5 may be converted to position in the focal plane of a lens with focal length 'F' through the mapping:

$$X_{\text{focal plane}}=(\text{wave number})*F*\lambda/(2\pi).$$

For the TPF application small wave numbers are of utmost importance. These fall well within the flat region of Figure 5, which is at a level of approximately -80 dB for the worst case, rectangular, spectrum. The –80 dB value implies that the PSD of the surface imperfections must be reduced by approximately 10 dB for small wave numbers (large spatial wavelengths)

in order to meet the assumed TPF –90 dB scattered light requirement. If this is not feasible then other corrective measures, such as a deformable mirror located appropriately in the optical train, must be employed after the lens to correct for the large-scale random surface errors of the lens.

As a further check of the surface imperfection effects a Monte Carlo Simulation was conducted. Once again a 10 m diameter (F/D)=1 lens was used to amplify any effects due to small (F/D). A rectangular spectrum (n=∞) for the PSD was assumed. The random surface errors were modeled using a discrete set of 500 Fourier components of equal amplitude but randomly phased (positioned on the lens). The ripple amplitudes were adjusted to give a total RMS surface error of 2 nm. The results of the simulation are shown in Figure 6. Results for five computer runs, each using a different seed value for the random number generator, are included along with the theoretical results described above. The difference between the ideal PSF and the computed PSF are shown in Figure 5. As before, the numerical results are computed using the exact scalar diffraction integral. As expected the numerical results confirm the theoretical prediction of a scattered light level in the –80 dB range for the assumed shape of the PSD and overall RMS surface error. As discussed above, this scattering level is approximately 10 dB higher than that required by our assumptions.

Diffraction Effects and PSF Nulling

If we assume that the effects of blockage and surface imperfections are reduced to a level acceptable for the TPF mission diffraction effects will be the limiting factor in the performance. For example, consider the PSF for a 10 m, (F/D)=10 lens at 1μm, depicted in Figure 7. The search for planet light is expected to be conducted in the focal plane region occupied by the 5-10th sidelobes of the star's PSF. Figure 7 indicates that the star light will only be reduced by a factor of 25 dB from its maximum in this range due to diffraction effects. This is far from the assumed 90 dB for TPF.

One approach for reducing the PSF in this range is to profile the lens in order to scatter additional light into this region, out of phase with the original light, thus canceling it out. This approach will cancel the light in the region of interest at the cost of increasing it elsewhere in the focal plane.

The process of nulling the PSF can be explained quite simply using an approximate analysis assuming a single, small amplitude, sinusoidal ripple. The resulting PSF in wave number space for this situation can be found analytically as a convolution of the undistorted PSF with three delta functions.

$$PSF(k) = PSF_0(k) \otimes (\delta(k) + \left(\frac{\pi \Delta z}{\lambda}\right)(\delta(k-k_k) - \delta(k-k_k))$$

Here Δz is the ripple amplitude, k_x is the ripple wave number, PSF_0 is the undistorted PSF, and δ is the delta function. Clearly the ripple amplitude and wave number may be adjusted to give an overall PSF value of zero at any location, at the expense of doubling it at the negative value of k. When multiple spectral components are present, or the amplitudes are not small,

additional terms which are analogous to inter-modulation products, must be included in the analysis as well.

Figure 7 depicts the nominal PSF and the scattered components for a typical situation. In this case a theoretical PSF of the form sin(x)/x is used in the simulation. The sign and magnitude of the ripple may be chosen to null the overall PSF at X=55 microns or 0.55 μrad. The result is depicted in Figure 8, and the resulting lens profile is given in Figure 9. It should be noted that the main lobe of the PSF is wider than any of the sidelobes. It will be shown that this is a fundamental, limiting factor in our ability to create a broad, deep null in the PSF. Examination of Figure 8 shows that while the null is very deep it is of extremely narrow width in the focal plane. A close examination of Figure 8 reveals that the null is achieved at X=55 microns at the expense of increasing the value of the PSF at X=-55 microns.

In order to create a broader null in the focal plane we need to add more spectral components. A simple MATLAB script was written to solve the theoretical problem in closed form. In this case 'N' spectral components with fixed wave number but variable amplitude are assumed. 'N' points in the focal plane are chosen for exact nulling, giving 'N' equations and 'N' unknowns. Cases of equally distributed spectrum and null points as well as randomly spaced null points within the band were simulated and produced similar results.

Examples of the results obtained using the above approach to null the PSF between 66 and 86 microns in the focal plane (0.66-0.86 μrad), are given in Figures 10-13. In Figure 10 five spectral components are used, resulting in a 60 dB null. Figure 11 shows the resulting lens profile. In this case each of the five spectral components making up the profile depicted in Figure 11 is found to have a magnitude less than two tenths of a wavelength. Examination of the condition number of the matrix used in the solution reveals it is small, indicating a non-singular matrix and a well-posed problem. Figures 12 and 13 show the results for the case of seven spectral components. In this case our 90 dB requirement is met over nearly the whole range. The lens profile in Figure 13 also appears reasonable. Further investigation of the solution reveals difficulties however. The magnitudes of the spectral components reach over 100 wavelengths and have alternating signs, causing a high degree of cancellation that results in the lens profile of Figure 13. The solution matrix is also highly ill-conditioned, indicating a poorly posed problem. Similar results were observed for several other nulling attempts. In all cases the solution matrix became highly ill-conditioned before 90 dB nulling was achieved.

It is believed that the difficulties observed here are closely related to those observed in an analogous problem in the field of antennas involving super directive arrays. In the antenna problem an array of elements may, in principle, be excited to produce an extremely narrow beam, much narrower than one would initially expect from the size of the overall array. While a solution appears possible, close examination of the excitation coefficients in the array indicates very large amplitudes with alternating signs. In practice such arrays cannot be produced due to real-world effects such as finite tolerances on the excitation and resistive loss in the structures. Closer examination of the relationship of this analogous problem to the problem at hand appears warranted.

The 5 term lens profile was used as a starting point for an optimization program employing the exact diffraction integral. The amplitudes of the spectral components were perturbed and an

optimum solution resulting in a 60 dB null from 66-86 microns, (0.66-0.86 μrad), was sought. The results of this optimization are shown in Figure 14. Although the goal was not achieved over the full band a significant null was produced. The results produced here required several thousand iterations of the program and it is believed that the 60 dB goal could be achieved with more iterations. The reduction in error function produced by the optimizer over the first 250 iterations is depicted in Figure 15. A similar calculation was attempted using the 7 term, -90 dB starting point. After many thousand iterations no satisfactory results were obtained. It is believed that this is due to the extreme sensitivity of the starting point, and the finite accuracy of the overall numerical computation. Similar results were found for other starting points and null widths. In general 40 dB nulls were easily achievable, 60 dB nulls were possible with a great deal of effort, and 90 dB nulls were never achieved using the full diffraction calculation. More study is required to determine if a fundamental difficulty exists, as indicated by the high condition number encountered in the matrix solution, or if numerical issues are a factor.

A final consideration is the tolerance required on the amplitudes of the spectral components. The magnitude of this tolerance is easily determined from the equation that appears earlier in this section. The field due to a single ripple of magnitude Δz is of magnitude ($\Delta z * \pi/\lambda$). In order to produce an error at the –90 dB level the field must have a value of $10^{-4.5}$, corresponding to $\Delta z = 10^{-5} * \lambda$, or 0.1 angstroms. This is an extremely tight tolerance on the ripple profile.

Conclusions and Future Work

A simple analysis of a one-dimensional Fresnel lens subject to the TPF requirements has been carried out. The effects of blockage due to gaps in the lens and surface imperfections have been investigated. In order to guarantee adherence to the TPF requirement of –90 dB sensitivity the total blockage must be reduced to a level of 1 part in $10^{4.5}$ or less. If the blockage is distributed this requirement may be reduced, depending on the relative location of the various blockages. The effect of surface imperfections was modeled both theoretically and numerically. Both results indicate that the assumed PSD for the imperfections and overall RMS surface error of 2 nm result in scattered light at a level of approximately –80 dB in the focal plane regions of interest for TPF. This falls slightly short of the –90 dB requirement. Profiling the lens in order to null the PSF in a particular region of interest was also investigated both theoretically and numerically. Nulling to a level of approximately –60 dB was achieved over several lobes of the ideal PSF. The lens profile required to achieve the modified PSF is of modest amplitude and slowly varying. Unfortunately, numerical, theoretical, and tolerance difficulties arise when trying to achieve nulling in the –90 dB range. Further study is required to resolve the numerical difficulties and quadruple precision calculations may be required. Even if the numerical issues are resolved the tolerances required in the production of the profile to produce a 90 dB null in the PSF may make the lens impractical.

Future work should investigate the theoretical limits regarding PSF nulling and the relationship of this problem to the analogous problem of achieving super-gain using antenna arrays. If the required nulls can indeed be achieved theoretically it is possible/likely that the tolerances required on the lens profile will exceed those of the manufacturing process. The possibility of using secondary optics containing a deformable mirror to provide further correction, and the theoretical limitations on this approach, should be investigated. Lastly, the more realistic scenario of a two dimensional, 10 meter diameter lens must be considered, including the effects

of gaps in the structure. This will involve a very large-scale numerical computation, particularly when 10^{-9} accuracy or better is required for the overall calculation.

Acknowledgement
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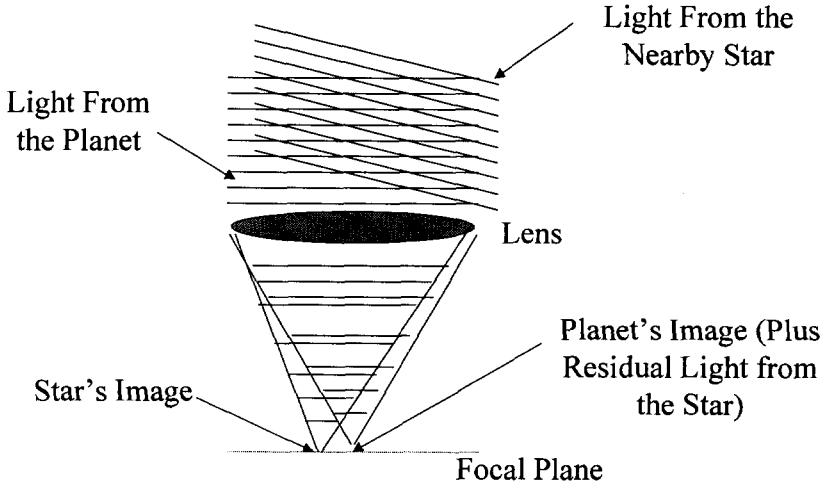

Figure 1. TPF imaging requirements.

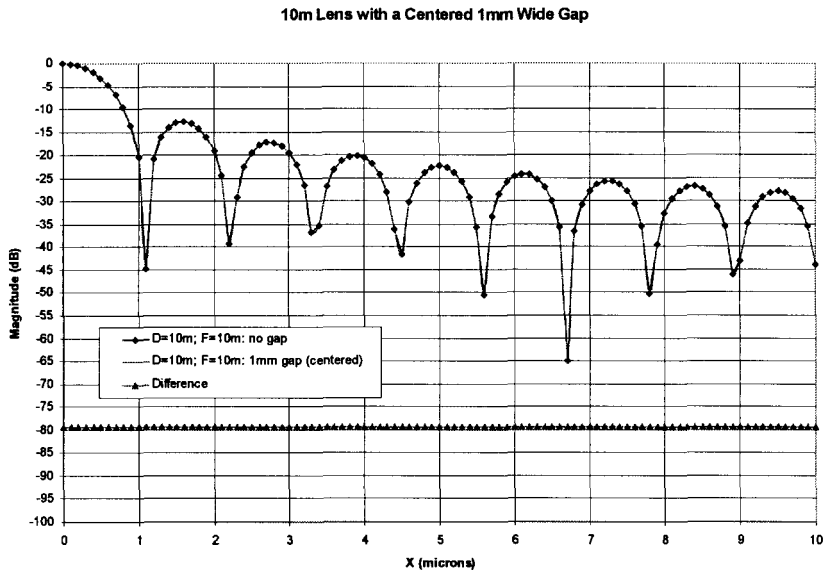

Figure 2. Effect of a single gap.

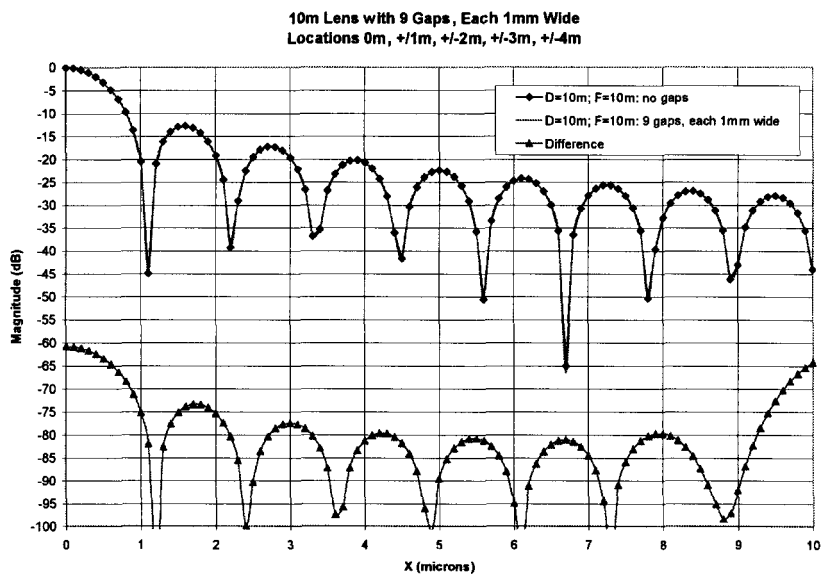

Figure 3. Effect of multiple gaps.

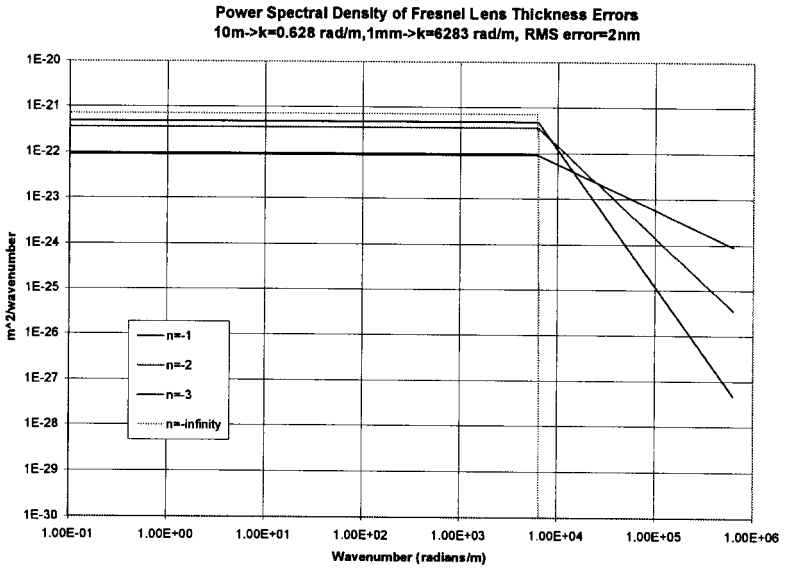

Figure 4. Assumed power spectral density for the surface roughness.

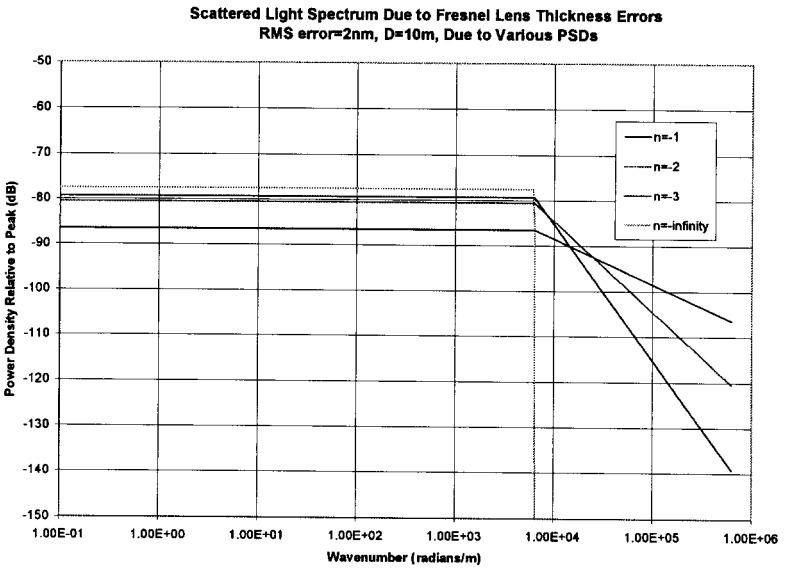

Figure 5. Scattered light spectrum due to the PSD of Figure 4.

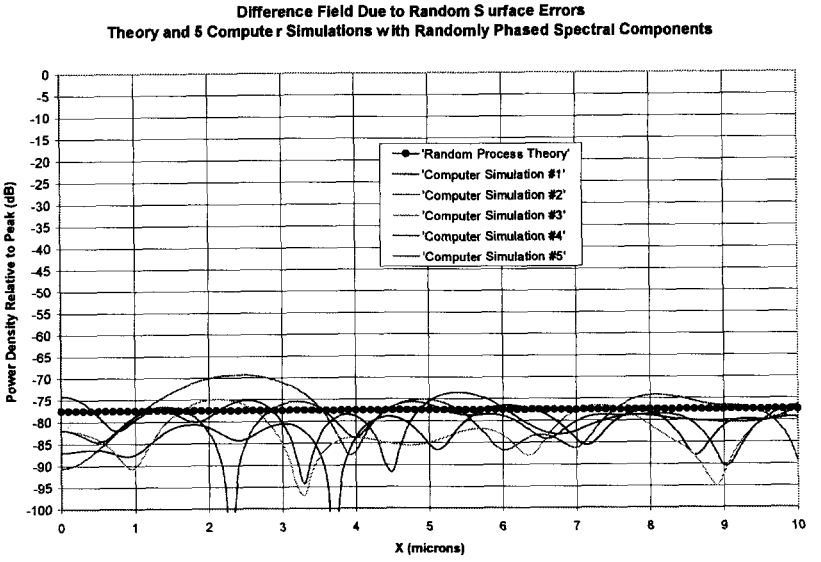

Figure 6. Monte Carlo simulation of surface roughness effects.

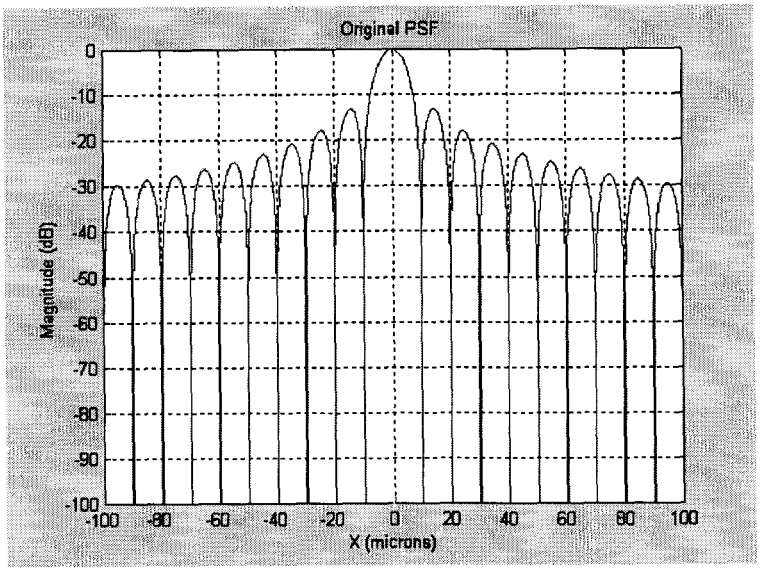

Figure 7. Ideal PSF for a 10m, (F/D)=10 lens when λ=1 micron.

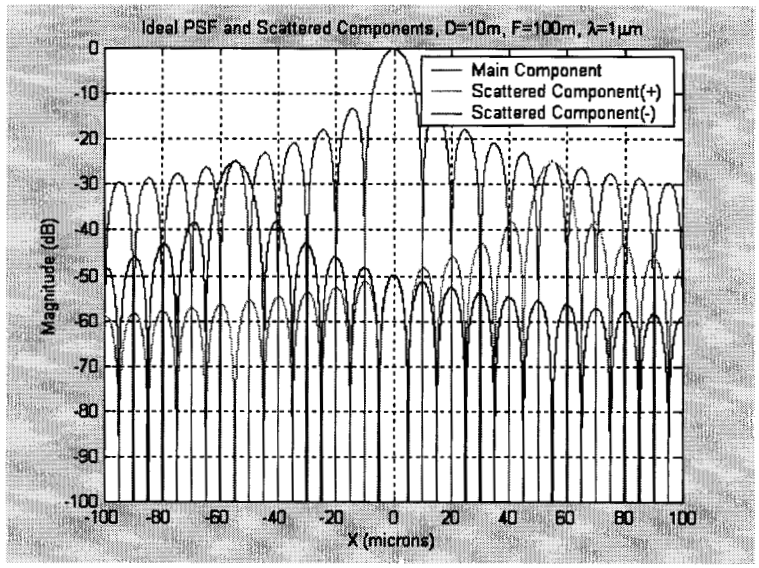

Figure 7. Ideal PSF for a 10m, (F/D)=10 lens when λ=1 micron.

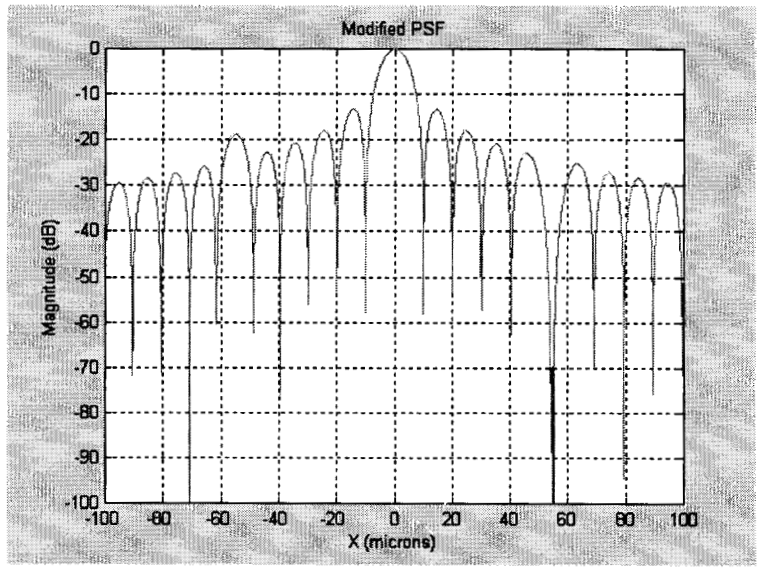

Figure 8.Nulled PSF at X=55 microns.

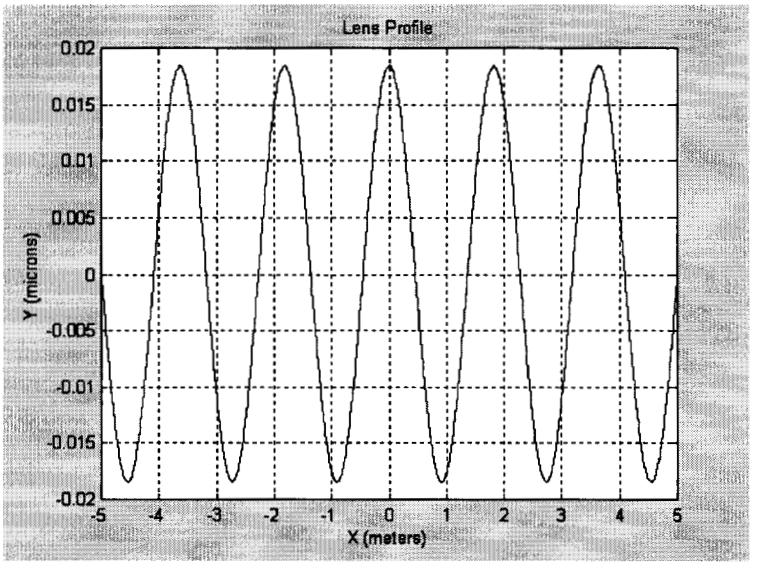

Figure 9.Lens profile for nulled PSF at X=55 microns.

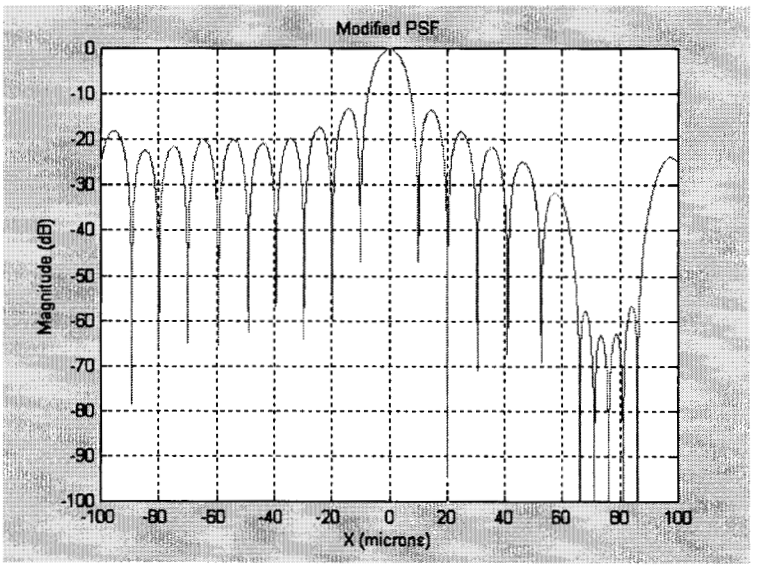

Figure 10. PSF nulling using 5 spectral components.

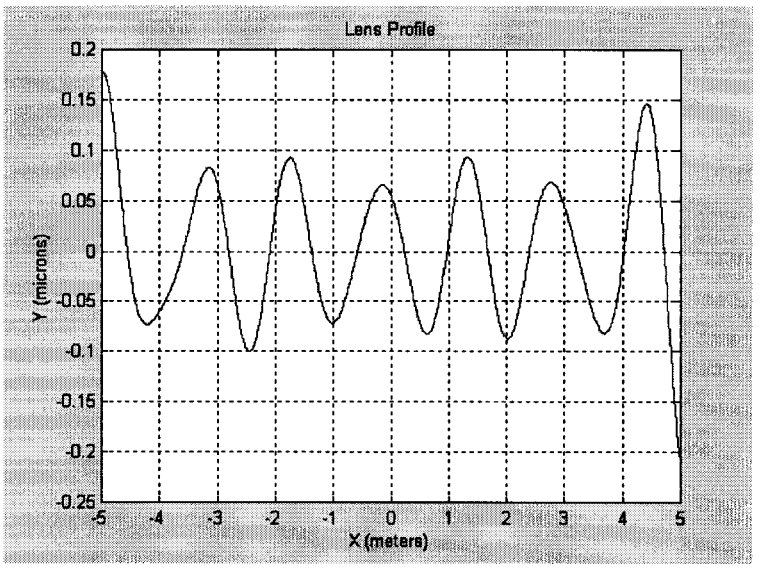

Figure 11. Lens profile using 5 spectral components.

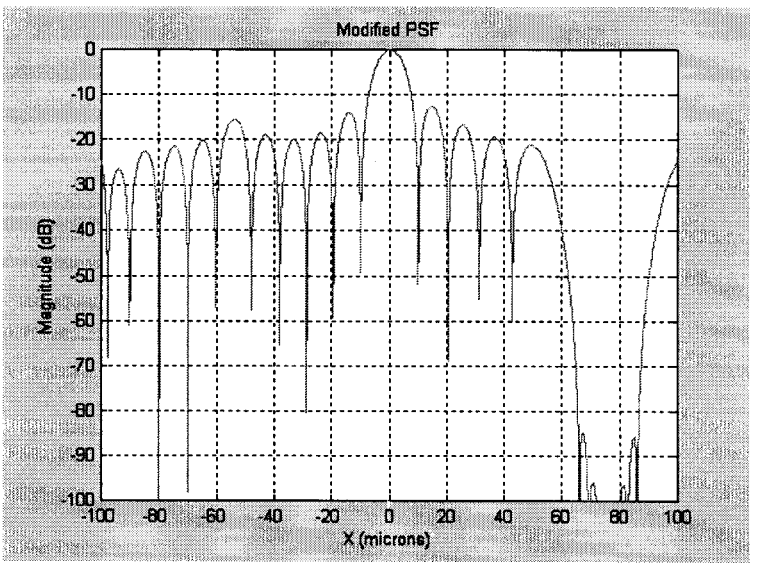

Figure 12. PSF nulling using 7 spectral components.

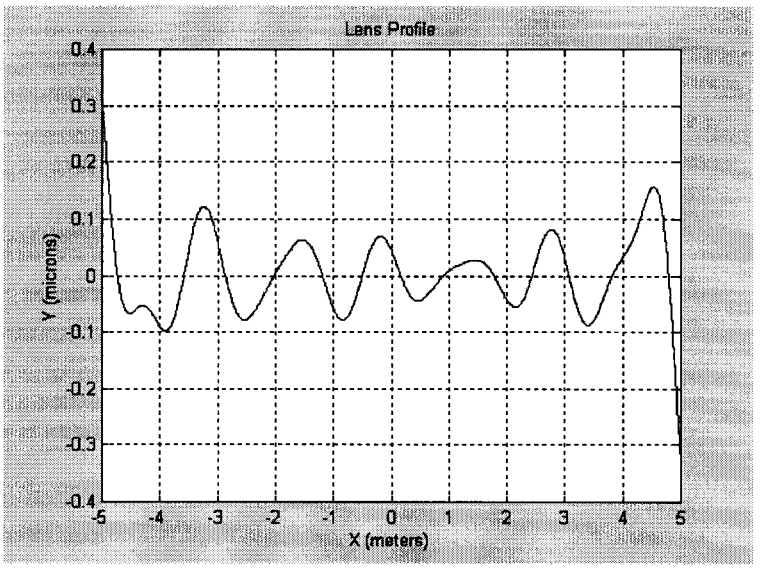

Figure 13. Lens profile using 7 spectral components.

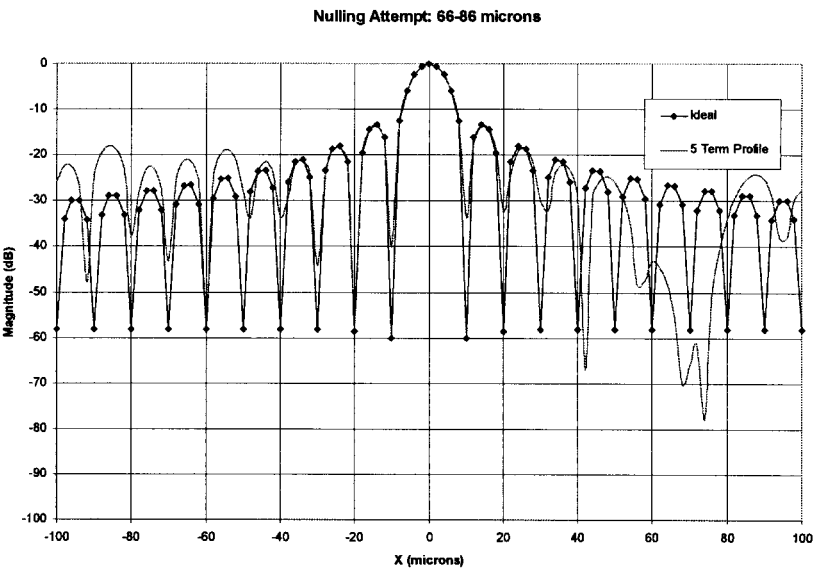

Figure 14. Optimized PSF using a full diffraction calculation.

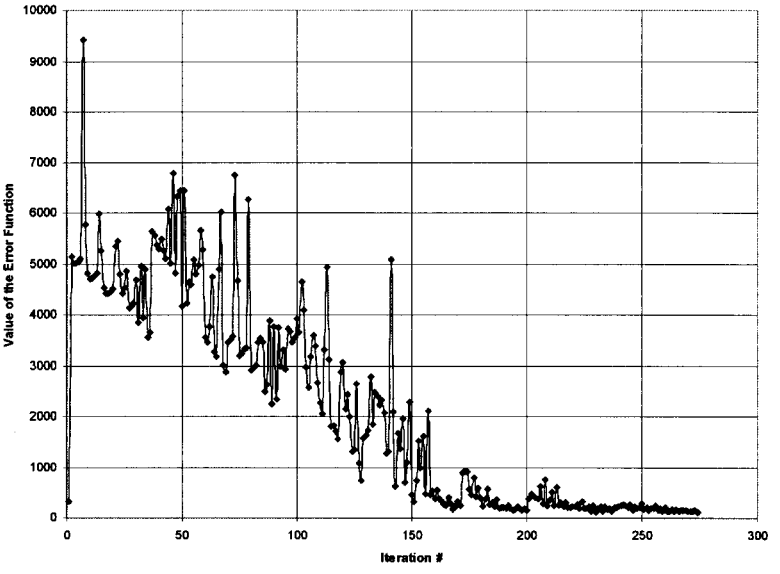

Figure 15. Decrease in the error function over the first 250 iterations.